McGRAW HILL EDUCATION WINNING EDGE (WE) SERIES

Understanding

MECHANICS *for*

JEE Main and Advanced

SECOND EDITION

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Understanding MECHANICS *for* JEE Main and Advanced

SECOND EDITION

by

M.K. Sinha

(Educational Consultant)

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Preface to the Second Edition

Having received an encouraging response from my readers for the first edition, I have great pleasure in presenting to them the revised second edition of *Understanding Mechanics, Volume – I*. Like the earlier edition, this book will also provide students relevant and compact information on the subject of **Mechanics**. The aim has been to make the topic easy and interesting **for aspirants of the IITs and other engineering entrance exams.**

My twelve years of experience in teaching and interaction with students in Kota (Rajasthan) have enabled me to clearly present all concepts and explain the important areas of the subject where students some times need assistance.

All important concepts, including the minute ones, have been discussed in this book.

The theoretical part is followed by a large numbers of solved numerical examples which give the aspirants thorough grounding of the topic.

The text has been divided into eight chapters covering the topics included in engineering entrance examinations. 'Notes', 'Concepts', 'Important Points', 'Summary' and so on have been added to enhance the theory.

Keeping in mind the latest trend of the IIT-JEE question paper, all types of questions asked in engineering entrance examinations have been covered – Subjective Problems, Objective Problems, Single Correct Option Questions, More than One Correct Options Questions, Mark the Correct Statements, Comprehension Questions, Match the Following and Assertion-Reason Questions.

JEE/REE and AIEEE questions from previous years have been included. Questions are followed by Answer Keys, with Solutions and detailed analyses.

I have included tips and tricks of solving problems to help students save valuable time in the examination hall.

To increase its value for aspirants, the second edition has been enhanced with **revision of the theory** portions – Chapters 1 and 8 have been totally revamped – and **addition of more JEE questions. AIEEE questions have also been included.** All these questions are followed by answers and detailed solutions.

Some Useful Tips while Preparing for IIT Entrance

- Be conceptually strong in the topics.
- Finish reading all the material — text, notes etc., at least three days prior to the exam and prepare short notes that will help you during revision.

- Three days prior to the exam, set aside time each day for self-assessment by solving practice problems and review of notes.

Useful Tools for Last Minute Revision

- End of chapter summaries
- Solved numericals
- Practice problems
- Short notes prepared by you

Analyse your weaknesses and create a list of topics of problems that need your attention. Focus on these topics a little more than the others.

Have a healthy routine and ensure you get sufficient sleep before the examination day.

Wish you all success in your endeavours.

M.K. Sinha

Preface to the First Edition

I have great pleasure in presenting this text book *Understanding Mechanics, Volume-I* before you. I was encouraged and inspired by the response to my earlier work *Understanding Optics*. As with my earlier book, this book will also provide students relevant and compact information on Mechanics. I am hopeful that my efforts will make the topic easy and interesting for IIT aspirants and other aspirants of various engmening entrance aspirants. The title is a reflection of the aim and purpose of my book.

My twelve years of experience in teaching and interaction with students in Kota (Rajasthan) have enabled me to clearly present all concepts and explain the important areas of the subject where students sometimes need assistance.

All important concepts, including the minute ones, have been discussed in this book. The theoretical part is followed by solved numerical examples which give the aspirant a thorough grounding of the topic.

Keeping in mind the latest trend of the IIT-JEE question paper, a large number of objective as well as subjective questions have been included at the end of the chapters. All these questions have **solutions with detailed analysis** so that no ambiguity remains in the mind of students. I have included tips and tricks of solving problems to help students save valuable time in the examination hall.

Some Useful Tips while Preparing for IIT entrance

- Be conceptually strong in the topics.
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Wish you all success in your endeavours.

—M. K. Sinha

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I am deeply grateful to my wife Aparna Sinha as without her love, support, encouragement and patience, this book could never been written, and my little kids Sona and Kanha who were patient enough to tolerate me while I spent hours and hours writing this book.

My regards are due to my parents, Mr. Sidhi Prasad Sinha and Mrs. Sarvila Sinha, and my in-laws Mr. Haribansh Prasad and Mrs. Vidya Sinha. I am grateful for the encouragement provided by Mr. Rahul Ranjan (working in Qual Com., U.S.A.), Archana Sinha (working in American Bank, U.S.A.), Dr. Rajesh Kumar, Ragini Sinha and Mini.

I thank my student, Aviral Jain (IIT-Madras), my brother Ramesh Kumar Sinha and Akash Pandey for checking the calculations and solutions. I am grateful indeed to Sanjeev Kumar for taking so keen an interest in bringing the text to this nice form.

Lastly, I would like to thank Mr. V K Bansal (Founder, Bansal Classes, Kota) with whom I have worked and gained a lot of knowledge of the ways in which to teach.

Last but not least, I would like to thank my mentor Mr. Rajan Khare (Ex-HoD Chemistry at Bansal Classes, Kota) for guiding me all the way.

—M. K. Sinha

CONTENTS

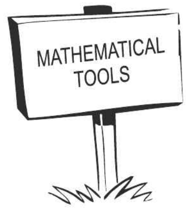
Chapter

1

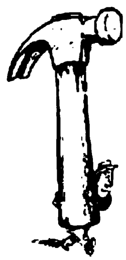
Mathematical Tools

Mathematics is the language of physics. It becomes easier to describe, understand and apply the physical principles, if one has a good knowledge of mathematics.

- Tools are required to do physical work easily and mathematical tools are required to solve numerical problems easily.



Mathematical Tools

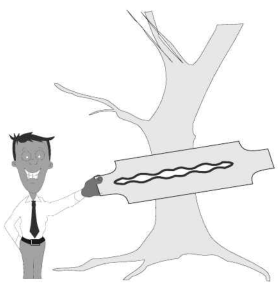


Differentiation

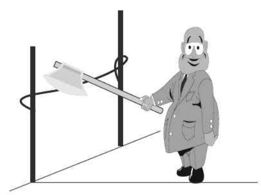
Integration

Vectors

- To solve the problems of physics Newton made significant contributions to Mathematics by inventing differentiation and integration.
- Appropriate choice of tool is very important



Cutting a tree with a blade



Cutting a string with an axe

1. FUNCTION

Function is a rule of relationship between two variables in which one is assumed to be dependent and the other independent variable.

***Example* 1.** The temperatures at which water boils depends on the elevation above sea level (the boiling point drops as you ascend). Here elevation above sea level is the independent and temperature is the dependent variable.

***Example* 2.** The interest paid on a cash investment depends on the length of time for which the investment is held. Here time is the independent and interest is the dependent variable.

In each of the above example, value of one variable quantity (dependent variable), which we might call y, depends on the value of another variable quantity (independent variable), which we might call x. Since the value of y is completely determined by the value of x, we say that y is a function of x and represent it mathematically as $y = f(x)$.

Here f represents the function, x the independent variable and y is the dependent variable.

All possible values of independent variables (x) are called ***domain*** of function.

All possible values of dependent variable (y) are called ***range*** of function.

Think of a function f as a kind of machine that produces an output value $f(x)$ in its range whenever we feed it an input value x from its domain (ref. to figure above).

> When we study circles, we usually call the area A and the radius r. Since area depends on radius, we say that A is a function of r, $A = f(r)$ The equation $A = \pi r^2$ is a rule that tells how to calculate a unique (single) output value of A for each possible input value of the radius r.
>
> $A = f(r) = \pi r^2$ (Here the rule of relationship which describes the function may be described as square & multiply by π).
>
> If $r = 1$ $A = \pi$; if $r = 2$ $A = 4\pi$; if $r = 3$ $A = 9\pi$
>
> The set of all possible input values for the radius is called the domain of the function. The set of all output values of the area is the range of the function.

We usually denote functions in one of the two ways:

1. By giving a formula such as $y = x^2$ that uses a dependent variable y to denote the value of the function.
2. By giving a formula such as $f(x) = x^2$ that defines a function symbol f to name the function.

Strictly speaking, we should call the function f and not $f(x)$,

$y = \sin x$. Here the function is sine, x is the independent variable.

***Example* 1.** The volume V of a ball (solid sphere) of radius r is given by the function $V(r) = \frac{4}{3}\pi\,(r)^3$

The volume of a ball of radius $3m$ is ?

Solution $V(3) = \frac{4}{3}\pi(3)^3 = 36\pi\, m^3$.

***Example* 2.** Suppose that the function F is defined for all real numbers r by the formula $F(r) = 2(r - 1) + 3$. Evaluate F at the input values 0, 2, $x + 2$, and $F(2)$.

Solution In each case we substitute the given input value for r into the formula for F:

$F(0) = 2(0 - 1) + 3 = -2 + 3 = 1$; $F(2) = 2(2 - 1) + 3 = 2 + 3 = 5$

$F(x + 2) = 2(x + 2 - 1) + 3 = 2x + 5$; $F(F(2)) = F(5) = 2(5 - 1) + 3 = 11$.

***Example* 3.** A function $f(x)$ is defined as $f(x) = x^2 + 3$, Find $f(0)$, $F(1)$, $f(x^2)$, $f(x + 1)$ and $f(f(1))$.

Solution $f(0) = 0^2 + 3 = 3$; $f(1) = 1^2 + 3 = 4$;

$f(x^2) = (x^2)^2 + 3 = x^4 + 3$

$f(x + 1) = (x + 1)^2 + 3 = x^2 + 2x + 4$; $f(f(1)) = f(4) = 4^2 + 3 = 19$

Example 4. If function F is defined for all real numbers x by the formula $F(x) = x^2$.
Evaluate F at the input values 0, 2, $x + 2$ and $F(2)$

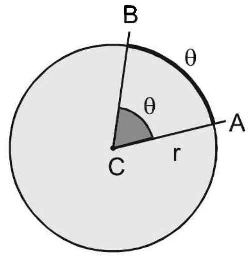
Solution $F(0) = 0$; $F(2) = 2^2 = 4$

$F(x + 2) = (x + 2)^2$; $F(f(2)) = F(4) = 4^2 = 16$

2. TRIGONOMETRY

Measurement of Angle and Relationship Between Degrees and Radian

In navigation and astronomy, angles are measured in degrees, but in calculus it is best to use units called radians because of the way they simplify later calculations.



Let ACB be a central angle in a **circle** of radius r, as in figure.

Then the angle ACB or θ is defined in radius as -

$$\theta = \frac{Arc\ length}{Radius} \Rightarrow \theta = \frac{\overset{\frown}{AB}}{r}$$

If $r = 1$ then $\theta = AB$

The **radian measure** for a circle of unit radius of angle ACB is defined to be the length of the circular arc AB. Since the circumference of the circle is 2π and one complete revolution of a circle is 360°, the relation between radians and degrees is given by: π radians = 180°

Angle Conversion Formulas

$$1 \text{ degree} = \frac{\pi}{180} \quad (\approx 0.02) \text{ radian}$$

1 radian ≈ 57 degrees

Degrees to radians: multiply by $\frac{\pi}{180}$

Radians to degrees; multiply by $\frac{180}{\pi}$

Example 5. (i) Convert 45° to radians.

(ii) Convert $\frac{\pi}{6}$ rad to degrees.

Solution (i) $45 \times \frac{\pi}{180} = \frac{\pi}{4}$ rad

(ii) $\frac{\pi}{6} \times \frac{180}{\pi} = 30°$

Example 6. Convert 30 to radians.

Solution $30 \times \frac{\pi}{180} = \frac{\pi}{6}$ rad

Example 7. Convert $\frac{\pi}{3}$ rad to degrees.

Solution $\frac{\pi}{3} \times \frac{180}{\pi} = 60$

Standard Values

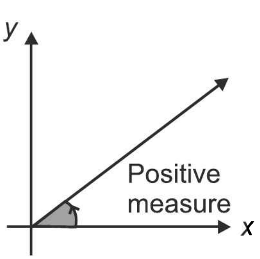
(1) $30^0 = \dfrac{\pi}{6}$ rad (2) $45^\circ = \dfrac{\pi}{4}$ rad (3) $60^\circ = \dfrac{\pi}{3}$ rad

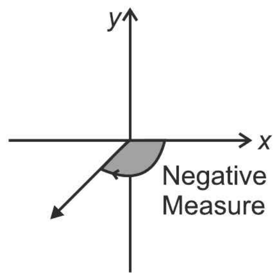
(4) $90^\circ = \dfrac{\pi}{2}$ rad (5) $120^\circ = \dfrac{2\pi}{3}$ rad (6) $135^\circ = \dfrac{3\pi}{4}$ rad

(7) $150^\circ = \dfrac{5\pi}{6}$ rad (8) $180^\circ = \pi$ rad (9) $360^\circ = 2\pi$ rad

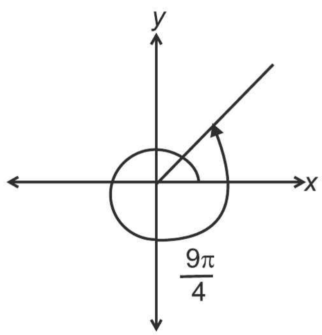
(Check these values yourself to see that they satisfy the conversion formulaes)

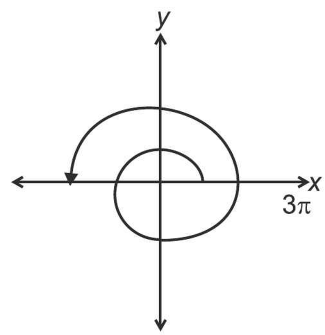
Measurement of Positive and Negative Angles

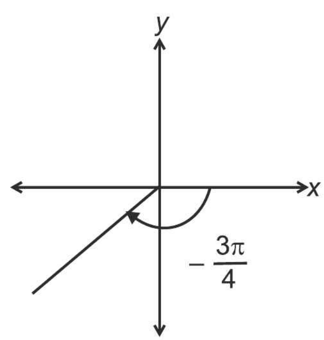


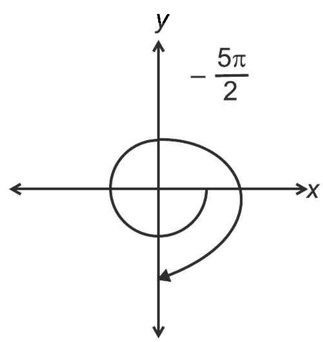


An angle in the xy-plane is said to be in standard position if its vertex lies at the origin and its initial ray lies along the positive x-axis (Fig.). Angles measured counterclockwise from the positive x-axis are assigned positive measures ; angles measured clockwise are assigned negative measures.



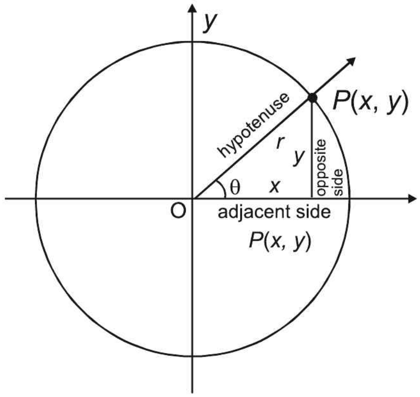






Six Basic Trigonometric Functions

The trigonometric function of a general angle θ are defined in terms of x, y, and r.



Sine: $\sin\theta = \dfrac{opp}{hyp} = \dfrac{y}{r}$

Cosecant: $\text{cosec}\,\theta = \dfrac{hyp}{opp} = \dfrac{r}{y}$

Cosine: $\cos\theta = \dfrac{adj}{hyp} = \dfrac{x}{r}$

Secant: $\sec\theta = \dfrac{hyp}{adj} = \dfrac{r}{x}$

Tangent: $\tan\theta = \dfrac{opp}{adj} = \dfrac{y}{x}$

Cotangent: $\cot\theta = \dfrac{adj}{opp} = \dfrac{x}{y}$

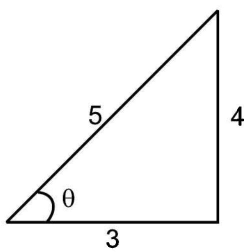
Values of Trigonometric Functions

If the circle in (Fig. above) has radius $r = 1$, the equations defining sin θ and cos θ become

$\cos\theta = x$, $\sin\theta = y$

We can then calculate the values of the cosine and sine directly from the coordinates of P.

Example 8. Find the six trigonometric ratios from given figure



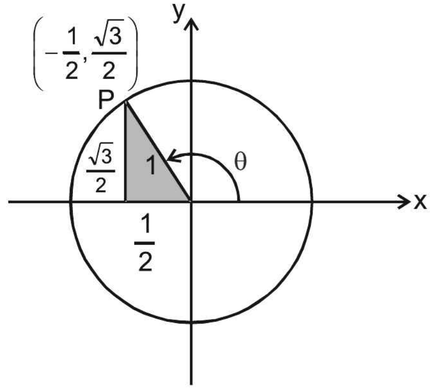
Solution

$\sin\theta = \dfrac{opp}{hyp} = \dfrac{4}{5}$; $\cos\theta = \dfrac{adj}{hyp} = \dfrac{3}{5}$;

$\tan\theta = \dfrac{opp}{adj} = \dfrac{4}{3}$; $\text{cosec}\,\theta = \dfrac{hyp}{opp} = \dfrac{5}{4}$;

$\sec\theta = \dfrac{hyp}{opp} = \dfrac{5}{3}$; $\cot\theta = \dfrac{adj}{opp} = \dfrac{3}{4}$

Example 9. Find the sine and cosine of angle θ shown in the unit circle if coordinate of point p are as shown.



Solution $\cos\theta = x\text{-coordinate of } P = -\dfrac{1}{2}$

$\sin\theta = y\text{-coordinate of } P = \dfrac{\sqrt{3}}{2}.$

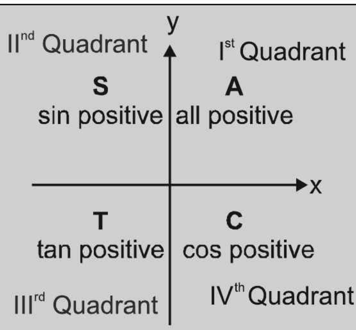
Rules for Finding Trigonometric Ratio of Angles Greater than 90°

Step 1 → Identify the quadrant in which angle lies.

Step 2 → **(a)** If angle = $(n\pi \pm \theta)$ where n is an integer. Then trigonometric function of $(n\pi \pm \theta)$ = same trigonometric function of θ and sign will be decided by CAST Rule.

The Cast Rule

A useful rule for remembering when the basic trigonometric functions are positive and negative is the CAST rule. If you are not very enthusiastic abut CAST. You can remember it as ASTC (After school to college)



(b) If angle = $\left[(2n+1)\dfrac{\pi}{2} \pm \theta\right]$ where n is an integer. Then trigonometric function of $\left[(2n+1)\dfrac{\pi}{2} \pm \theta\right]$ = complimetry trignometric function of θ and sign will be decided by CAST Rule.

Values of sin θ, cos θ and tan θ for some standard angles.

Degree	0	30	37	45	53	60	90	120	135	180
Radians	0	$\pi/6$	$37\pi/180$	$\pi/4$	$53\pi/180$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	π
$\sin\theta$	0	$1/2$	$3/5$	$1/\sqrt{2}$	$4/5$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	0
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$1/\sqrt{2}$	$3/5$	$1/2$	0	$-1/2$	$-1/\sqrt{2}$	-1
$\tan\theta$	0	$1/\sqrt{3}$	$3/4$	1	$4/3$	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	0

Example 10. Evaluate sin 120°

Solution $\sin 120° = \sin (90° + 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

Aliter $\sin 120° = \sin (180° - 60°) = \sin 60° = \frac{\sqrt{3}}{2}$

Example 11. Evaluate cos 135°

Solution $\cos 135° = \cos (90° + 45°) = -\sin 45° = -\frac{1}{\sqrt{2}}$

Example 12. Evaluate cos 210°

Solution $\cos 210° = \cos (180° + 30°) = -\cos 30° = -\frac{\sqrt{3}}{2}$

Example 13. Evaluate tan 210°

Solution $\tan 210° = \tan (180° + 30°) = \tan 30° = \frac{1}{\sqrt{3}}$

General Trigonometric Formulas

1. $\cos^2\theta + \sin^2\theta = 1$

   $1 + \tan^2\theta = \sec^2\theta.$

   $1 + \cot^2\theta = \text{cosec}^2\theta.$

2. $\cos (A + B) = \cos A \cos B - \sin A \sin B$

   $\sin (A + B) = \sin A \cos B + \cos A \sin B$

   $\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

3. $\sin 2\theta = 2 \sin \theta \cos \theta$

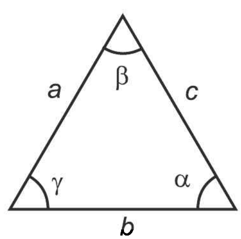
   $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

   $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$;

   $\sin^2\theta = \frac{1 - \cos 2\theta}{2}$

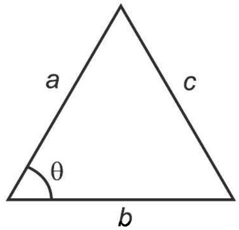
4. **sine rule for triangles**

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$



5. cosine rule for triangles

$c^2 = a^2 + b^2 - 2ab\cos\theta$



3. DIFFERENTIATION

Finite Difference

The finite difference between two values of a physical quantity is represented by Δ notation.

For example:

y_2	100	100	100
y_1	50	99	99.5
$\Delta y = y_2 - y_1$	50	1	0.5

Difference in two values of y is written as Δy as given in the table above.

Infinitely Small Difference

The infinitely small difference means very-very small difference. And this difference is represented by '*d*' notation insted of 'Δ'.

For example infinitely small difference in the values of y is written as 'dy'

if $y_2 = 100$ and $y_1 = 99.99999999$........

then dy = 0.000000...................00001

Definition of Differentiation

Another name for differentiation is derivative. Suppose y is a function of x or $y = f(x)$

Differentiation of y with respect to x is denoted by symbol $f'(x)$

where $f'(x) = \dfrac{dy}{dx}$ dx is very small change in x and dy is corresponding very small change in y.

NOTATION: There are many ways to denote the derivative of a function $y = f(x)$. Besides $f'(x)$, the most common notations are these:

y'	"y prime"	Nice and brief but does not name the independent variable
$\dfrac{dy}{dx}$	"dy by dx"	Names the variables and uses d for derivative

$\frac{df}{dx}$	"df by dx"	Emphasizes the function's name
$\frac{d}{dx}f(x)$	"d by dx of f"	Emphasizes the idea that differentiation is an operation performed on f.
$D_x f$	"dx of f"	A common operator notation
$\dot{y}$	"y dot"	One of Newton's notations, now common for time derivatives i.e. $\frac{dy}{dt}$.

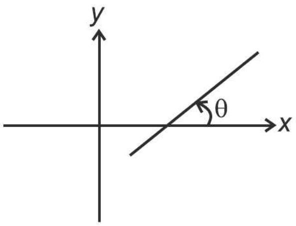
Slope of a Line

It is the tan of angle made by a line with the positive direction of x-axis, measured in anticlockwise direction.

Slope = tan θ (In 1st quadrant tan θ is + *ve* & 2nd quadrant tan θ is – *ve*)

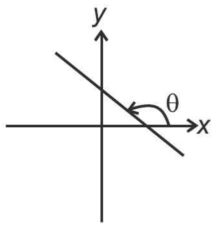
In Figure - 1 slope is positive

$\theta < 90°$ (1st quadrant)



In Figure - 2 slope is negative

$\theta > 90°$ (2nd quadrant)



Average Rate of Change

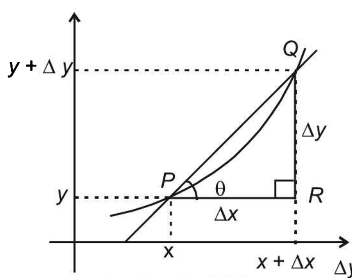
Given an arbitrary function $y = f(x)$ we calculate the average rate of change of y with respect to x over the interval $(x, x + \Delta x)$ by dividing the change in value of y, i.e. $\Delta y = f(x + \Delta x) - f(x)$, by length of interval Δx over which the change occurred.

The average rate of change of y with respect to x over the interval

$$[x, x + \Delta x] = \frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Geometricaly, $\frac{\Delta y}{\Delta x} = \frac{QR}{PR}$ = tan θ = Slope of the line PQ

therefore we can say that average rate of change of y with respect to x is equal to slope of the line joining P & Q.



In triangle QPR $\tan\theta = \frac{\Delta y}{\Delta x}$

The Derivative of a Function

We know that, average rate of change of y w.r.t. x is $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$.

If the limit of this ratio exists as $\Delta x \to 0$, then it is called the derivative of given function $f(x)$ and is denoted as

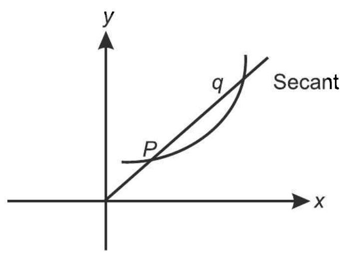
$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Geometrical Meaning of Differentiation

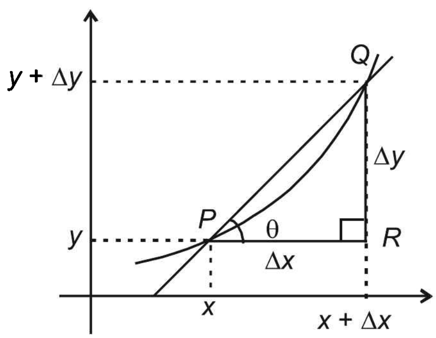
The geometrical meaning of differentiation is very much useful in the analysis of graphs in physics. To understand the geometrical meaning of derivatives we should have knowledge of secant and tangent to a curve.

Secant and Tangent to a Curve

Secant: A secant to a curve is a straight line, which intersects the curve at any two points.



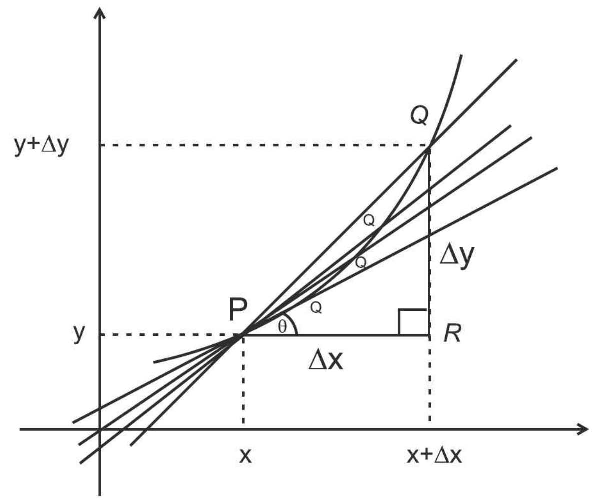
Tangent: A tangent is a straight line, which touches the curve at a particular point. Tangent is a limiting case of secant which intersects the curve at two overlapping points.



In the figure above, if value of Δx is gradually reduced then the point Q will move nearer to the point P. If the process is continuously repeated then the value of Δx will be infinitely small and secant PQ to the given curve will become a tangent at point P.

Therefore $\displaystyle \lim_{\Delta x \to 0}\left(\frac{\Delta y}{\Delta x}\right) = \frac{dy}{dx} = \tan\theta$

we can say that differentiation of y with respect to x,i.e. $\left(\frac{dy}{dx}\right)$ is equal to slope of the tangent at point P (x, y) or $\tan\theta = \frac{dy}{dx}$



(From the figure the average rate of change of y from x to $x + \Delta x$ is identical with the slope of secant PQ.)

Rules for Differentiation

Rule No. 1: Derivative of a Constant

The first rule of differentiation is that the derivative of every constant function is zero.

If c is constant, then $\frac{d}{dx}c = 0$.

Example 14. $\frac{d}{dx}(8) = 0$, $\frac{d}{dx}\left(-\frac{1}{2}\right) = 0$, $\frac{d}{dx}\left(\sqrt{3}\right) = 0$

Rule No. 2: Power Rule

If n is a real number, then $\frac{d}{dx}x^n = nx^{n-1}$.

To apply the power Rule, we subtract 1 from the original exponent (n) and multiply the result by n.

Example 15.

f	x	x^2	x^3	x^4	....
f'	1	$2x$	$3x^2$	$4x^3$	....

Example 16. (i) $\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{d}{dx}(x^{-1}) = (-1)x^{-2} = -\frac{1}{x^2}$

(ii) $\frac{d}{dx}\left(\frac{4}{x^3}\right) = 4\frac{d}{dx}(x^{-3}) = 4(-3)x^{-4} = -\frac{12}{x^4}$.

Example 17. (a) $\frac{d}{dx}(x^{1/2}) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

Function defined for $x \geq 0$ | derivative defined only for $x > 0$

(b) $\frac{d}{dx}(x^{1/5}) = \frac{1}{5}x^{-4/5}$

Function defined for $x \geq 0$ | derivative not defined at $x = 0$

Rule No. 3: The Constant Multiple Rule

> If u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$
>
> In particular, if n is a positive integer, then $\frac{d}{dx}(cx^n) = cn\,x^{n-1}$

Example 18. The derivative formula $\frac{d}{dx}(3x^2) = 3(2x) = 6x$ says that if we rescale the graph of $y = x^2$ by multiplying each y – coordinate by 3, then we multiply the slope at each point by 3.

Example 19. A useful special case

The derivative of the negative of a differentiable function is the negative of the function's derivative. Rule 3 with $c = -1$ gives.

$$\frac{d}{dx}(-u) = \frac{d}{dx}(-1\cdot u) = -1.\ \frac{d}{dx}(u) = -\frac{d}{dx}(u)$$

Rule No. 4: The Sum Rule

> The derivative of the sum of two differentiable functions is the sum of their derivatives.
>
> If u and v are differentiable functions of x, then their sum $u + v$ is differentiable at every point where u and v are both differentiable functions is their derivatives.
>
> $$\frac{d}{dx}(u-v) = \frac{d}{dx}[u+(-1)v] = \frac{du}{dx}+(-1)\frac{dv}{dx} = \frac{du}{dx}-\frac{dv}{dx}$$
>
> The sum Rule also extends to sums of more than two functions, as long as there are only finitely many functions in the sum. If $u_1, u_2,.........u_n$ are differentiable at x, then so is $u_1 + u_2 + .\ .... + u_n$, and
>
> $$\frac{d}{dx}(u_1+u_2+.....+u_n) = \frac{du_1}{dx}+\frac{du_2}{dx}+.......+\frac{du_n}{dx}.$$

Example 20. (a) $y = x^4 + 12x$

$$\frac{dy}{dx} = \frac{d}{dx}(x^4) + \frac{d}{dx}(12x)$$

$$= 4x^3 + 12$$

(b) $y = x^3 + \frac{4}{3}x^2 - 5x + 1$

$$\frac{dy}{dx} = \frac{d}{dx}x^3 + \frac{d}{dx}\left(\frac{4}{3}x^2\right) - \frac{d}{dx}(5x) + \frac{d}{dx}(1)$$

$$= 3x^2 + \frac{4}{3}.2x - 5 + 0$$

$$= 3x^2 + \frac{8}{3}x - 5.$$

Notice that we can differentiate any polynomial term by term, the way we differentiated the polynomials in above example.

Rule No. 5: The Product Rule

If u and v are differentiable at x, then so is their product uv, and $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

The derivative of the product uv is u times the derivative of v plus v times the derivative of u. In prime notation $(uv)' = uv' + vu'$.

While the derivative of the sum of two functions is the sum of their derivatives, the derivative of the product of two functions is not the product of their derivatives. For instance,

$$\frac{d}{dx}(x.x) = \frac{d}{dx}(x^2) = 2x, \quad \text{while } \frac{d}{dx}(x).\ \frac{d}{dx}(x) = 1.1 = 1.$$

Example 21. Find the derivatives of $y = (x^2 + 1)(x^3 + 3)$.

Solution From the product Rule with $u = x^2 + 1$ and $v = x^3 + 3$, we find

we find,
$$\frac{d}{dx}[(x^2+1)(x^3+3)] = (x^2 + 1)(3x^2) + (x^3 + 3)(2x)$$

$$= 3x^4 + 3x^2 + 2x^4 + 6x$$

$$= 5x^4 + 3x^2 + 6x.$$

Example can be done as well (perhaps better) by multiplying out the original expression for y and differentiating the resulting polynomial. We now check: $y = (x^2 + 1)(x^3 + 3) = x^5 + x^3 + 3x^2 + 3$

$$\frac{dy}{dx} = 5x^4 + 3x^2 + 6x.$$

This is in agreement with our first calculation.

There are times, however, when the product Rule must be used. In the following examples. We have only numerical values to work with.

Example 22. Let $y = uv$ be the product of the functions u and v. Find $y'(2)$ if $u'(2) = 3$, $u'(2) = -4$, $v(2) = 1$, and $v'(2) = 2$.

Solution From the Product Rule, in the form

$$y' = (uv)' = uv' + vu',$$

we have $y'(2) = u(2)\,v'(2) + v(2)\,u'(2)$

$= (3)\,(2) + (1)\,(-4) = 6 - 4 = 2.$

Rule No. 6: The Quotient Rule

If u and v are differentiable at x, and $v(x) \neq 0$, then the quotient u/v is differentiable at x, and

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$$

Just as the derivative of the product of two differentiable functions is not the product of their derivatives, the derivative of the quotient of two functions is not the quotient of their derivatives.

Example 23. Find the derivative of $y = \dfrac{t^2 - 1}{t^2 + 1}$

Solution We apply the Quotient Rule with $u = t^2 - 1$ and $v = t^2 + 1$:

$$\frac{dy}{dt} = \frac{(t^2+1).2t - (t^2-1).2t}{(t^2+1)^2}$$

$$\frac{d}{dt}\left(\frac{u}{v}\right) = \frac{v(du/dt) - v(du/dt)}{v^2}$$

$$= \frac{2t^3 + 2t - 2t^3 + 2t}{(t^2+1)^2} = \frac{4t}{(t^2+1)^2}.$$

Rule No. 7: Derivative of Sine Function

$$\frac{d}{dx}(\sin x) = \cos x$$

Example 24. (a) $y = x^2 - \sin x$: $\dfrac{dy}{dx} = 2x - \dfrac{d}{dx}(\sin x)$ Difference Rule

$= 2x - \cos x$

(b) $y = x^2 \sin x$: $\dfrac{dy}{dx} = x^2\dfrac{d}{dx}(\sin x) + 2x\sin x$ Product Rule

$= x^2\cos x + 2x\sin x$

(c) $y = \dfrac{\sin x}{x}$: $\dfrac{dy}{dx} = \dfrac{x.\dfrac{d}{dx}(\sin x) - \sin x.1}{x^2}$ Quotient Rule

$= \dfrac{x\cos x - \sin x}{x^2}.$

Rule No. 8: Derivative of Cosine Function

$$\frac{d}{dx}(\cos x) = -\sin x$$

Example 25. (a) $y = 5x + \cos x$

$$\frac{dy}{dx} = \frac{d}{dx}(5x) + \frac{d}{dx}(\cos x) \qquad \text{Sum Rule}$$

$$= 5 - \sin x$$

(b) $y = \sin x \cos x$

$$\frac{dy}{dx} = \sin x \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(\sin x) \qquad \text{Product Rule}$$

$$= \sin x(-\sin x) + \cos x(\cos x)$$

$$= \cos^2 x - \sin^2 x$$

Rule No. 9: Derivatives of other Trigonometric Functions

Because sin x and cos x are differentiable functions of x, the related functions

$$\tan x = \frac{\sin x}{\cos x}; \qquad \sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}; \qquad \operatorname{cosec} x = \frac{1}{\sin x}$$

are differentiable at every value of x at which they are defined. There derivatives calculated from the Quotient Rule, are given by the following formulas.

$\frac{d}{dx}(\tan x) = \sec^2 x$;	$\frac{d}{dx}(\sec x) = \sec x \tan x$
$\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$;	$\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

Example 26. Find dy / dx if $y = \tan x$.

Solution

$$\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$$

$$= \frac{\cos x \frac{d}{dx}(\sin x) - \sin x \frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$= \frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Example 27. (a) $\frac{d}{dx}(3x + \cot x) = 3 + \frac{d}{dx}(\cot x) = 3 - \text{cosec}^2 x$

(b) $\frac{d}{dx}\left(\frac{2}{\sin x}\right) = \frac{d}{dx}(2\text{cosec } x) = 2\frac{d}{dx}(\text{cosec } x)$

$= 2(-\text{cosec } x \cot x) = -2 \text{ cosec } x \cot x$

Rule No. 10: Derivative of Logarithm and Exponential Functions

$$\frac{d}{dx}(\log_e x) = \frac{1}{x}$$

$$\frac{d}{dx}(e^x) = e^x$$

Example 28. $y = e^x . \log_e (x)$

$$\frac{dy}{dx} = \frac{d}{dx}(e^x). \log(x) +. \frac{d}{dx}[\log_e(x)]\, e^x$$

$$\Rightarrow \frac{dy}{dx} = e^x . \log_e(x) + \frac{e^x}{x}$$

Rule No. 11: Chain Rule or "Outside Inside" Rule

$$\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$$

It sometimes helps to think about the Chain Rule the following way. If $y = f(g(x))$,

$$\frac{dy}{dx} = f'[g(x)].g'(x) .$$

In words: To find dy/dx, differentiate the "outside" function f and leave the "inside" $g(x)$ alone ; then multiply by the derivative of the inside.

We now know how to differentiate $\sin x$ and $x^2 - 4$, but how do we differentiate a composite like $\sin(x^2 - 4)$? The answer is, with the Chain Rule, which says that the derivative of the composite of two differentiable functions is the product of their derivatives evaluated at appropriate points. The Chain Rule is probably the most widely used differentiation rule in mathematics. This section describes the rule and how to use it. We begin with examples.

Example 29. The function $y = 6x - 10 = 2(3x - 5)$ is the composite of the functions $y = 2u$ and $u = 3x - 5$. How are the derivatives of these three functions related ?

Solution We have $\frac{dy}{dx} = 6, \frac{dy}{du} = 2, \frac{du}{dx} = 3.$

Since $6 = 2. 3$, $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$

Is it an accident that $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$?

If we think of the derivative as a rate of change, our intuition allows us to see that this relationship is reasonable. For $y = f(u)$ and $u = g(x)$, if y changes twice as fast as u and u changes three times as fast as x, then we expect y to change six times as fast as x.

Example 30. Let us try this again on another function.

$$y = 9x^4 + 6x^2 + 1 = (3x^2 + 1)^2$$

is the composite of $y = u^2$ and $3x^2 + 1$. Calculating derivatives. We see that

$$\frac{dy}{du}.\frac{du}{dx} = 2\,u.\,6x = 2(3x^2+1).\,6x = 36x^3 + 12\,x$$

and $$\frac{dy}{dx} = \frac{d}{dx}(9x^4 + 6x^2 + 1) = 36x^3 + 12\,x$$

Once again, $$\frac{dy}{du}.\frac{du}{dx} = \frac{dy}{dx}$$

The derivative of the composite function $f(g(x))$ at x is the derivative of f at $g(x)$ times the derivative of g at x.

Example 31. Find the derivative of $y = \sqrt{x^2+1}$

Solution Here $y = f(g(x))$, where $f(u) = \sqrt{u}$ and $g(x) = x^2 + 1$. Since the derivatives of f and g are

$$f'(u) = \frac{1}{2\sqrt{u}} \text{ and } g'(x) = 2x,$$

the Chain Rule gives

$$\frac{dy}{dx} = \frac{d}{dx}f(g(x)) = f'(g(x)).\,g'(x) = \frac{1}{2\sqrt{g(x)}}.\,g'(x) = \frac{1}{2\sqrt{x^2+1}}.\,(2x) = \frac{x}{\sqrt{x^2+1}}.$$

Example 32. $$\frac{d}{dx}\underbrace{\sin}_{\text{outside}}\underbrace{(x^2+x)}_{\text{Inside}} = \underbrace{\cos}_{\text{derivative of the outside}}\underbrace{(x^2+x)}_{\text{Inside Left alone}}.\underbrace{(2x+1)}_{\text{derivative of the inside}}$$

Example 33. We sometimes have to use the Chain Rule two or more times to find a derivative. Here is an example. Find the derivative of $g(t) = \tan(5 - \sin 2t)$

Solution

$$g'(t) = \frac{d}{dt}(\tan(5 - \sin 2t)$$

$$= \sec^2(5 - \sin 2t).\,\frac{d}{dt}(5 - \sin 2t)$$

Derivative of tan u with $u = 5 - \sin 2t$

$$= \sec^2(5 - \sin 2t).\,(0 - (\cos 2t).\,\frac{d}{dt}(2t)$$

Derivative of $5 - \sin u$ with $u = 2t$

$$= \sec^2(5 - \sin 2t).\,(-\cos 2t).\,2$$

$$= -2(\cos 2t)\sec^2(5 - \sin 2t)$$

Example 34. (a) $\frac{d}{dx}(1-x^2)^{1/4} = \frac{1}{4}(1-x^2)^{-3/4}(-2x)$ $\quad u = 1 - x^2$ and $n = 1/4$

↑ Function defined on [–1, 1]

$$= \frac{-x}{2(1-x^2)^{3/4}}$$

↑ derivative defined only on (–1, 1)

(b) $$\frac{d}{dx}(\cos\ x)^{-1/5} = -\frac{1}{5}(\cos x)^{-6/5}\frac{d}{dx}(\cos x)$$

$$= -\frac{1}{5}(\cos x)^{-6/5}(-\sin x) = \frac{1}{5}\sin x\,(\cos x)^{-6/5}$$

(c) $$\frac{d}{dx}\left(e^{\sin\sqrt{x}}\right) = e^{\sin\sqrt{x}}\cdot\frac{d}{dx}\sin\sqrt{x}$$

$$= e^{\sin\sqrt{x}}\cos\sqrt{x}\cdot\frac{d}{dx}\sqrt{x}$$

$$= e^{\sin\sqrt{x}}\cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{x}}e^{\sin\sqrt{x}}\cdot\cos\sqrt{x}$$

(d) $$\frac{d}{dx}\left(\sin\sqrt{x^2+5}\right) = \cos\sqrt{x^2+5}\ \frac{d}{dx}\sqrt{x^2+5}$$

$$= \cos\sqrt{x^2+5}\ \frac{1}{2\sqrt{x^2+5}}\cdot\frac{d}{dx}(x^2+5)$$

$$= \cos\sqrt{x^2+5}\cdot\frac{1}{2\sqrt{x^2+5}}\cdot 2x = \frac{x}{\sqrt{x^2+5}}\cos\sqrt{x^2+5}$$

(e) $$\frac{d}{dx}\sin 2x = \cos 2x\ \frac{d}{dx}\ 2x$$

$$= \cos 2x.\ 2 = 2\cos 2x$$

(f) $$\frac{d}{dt}(A\sin(\omega t+\phi)) = A\cos(\omega t+\phi)\ \frac{d}{dt}(\omega t+\phi)$$

$$= A\cos(\omega t+\phi).\ \omega. = A\,\omega\cos(\omega t+\phi)$$

Rule No. 12: Power Chain Rule

If $u(x)$ is a differentiable function and n is an integer, then u^n is differentiable and

$$\frac{d}{dx}u^n = nu^{n-1}\frac{du}{dx}$$

Example 35. (a) $\frac{d}{dx}\sin^5 x = 5\sin^4 x \frac{d}{dx}(\sin x)$

$= 5\sin^4 x \cos x$

(b) $\frac{d}{dx}(2x+1)^{-3} = -3(2x+1)^{-4}\frac{d}{dx}(2x+1)$

$= -3(2x+1)^{-4}(2) = -6(2x+1)^{-4}$

(c) $\frac{d}{dx}(5x^3 - x^4)^7 = 7(5x^3 - x^4)^6 \frac{d}{dx}(5x^3 - x^4)$

$= 7(5x^3 - x^4)^6 (5.\ 3x^2 - 4x^3)$

$= 7(5x^3 - x^4)^6 (15x^2 - 4x^3)$

(d) $\frac{d}{dx}\left(\frac{1}{3x-2}\right) = \frac{d}{dx}(3x-2)^{-1}$

$= -1(3x-2)^{-2}\frac{d}{dx}(3x-2)$

$= -1(3x-2)^{-2}(3) = -\frac{3}{(3x-2)^2}$

In part (d) we could also have found the derivative with the Quotient Rule.

Example 36. (a) $\frac{d}{dx}(Ax+B)^n$

Solution

Here $u = Ax + B$, $\frac{du}{dx} = A$

$\therefore \quad \frac{d}{dx}(Ax+B)^n = n(Ax+B)^{n-1}.\ A$

(b) $\frac{d}{dx}\sin(Ax+B) = \cos(Ax+B).\ A$

(c) $\frac{d}{dx}\log(Ax+B) = \frac{1}{Ax+B}.\ A$

(d) $\frac{d}{dx}\tan(Ax+B) = \sec^2(Ax+B).\ A$

(e) $\frac{d}{dx}e^{(Ax+B)} = e^{(Ax+B)}.\ A$

Rule No. 13: Radian vs. Degrees

$$\frac{d}{dx}\sin(x^\circ) = \frac{d}{dx}\sin\left(\frac{\pi x}{180}\right) = \frac{\pi}{180}\cos\left(\frac{\pi x}{180}\right) = \frac{\pi}{180}\cos(x^\circ).$$

Double Differentiation

If f is a differentiable function, then its derivative f' is also a function, so f' may have a derivative of its own, denoted by $(f')' = f''$. This new function f'' is called the second derivative of f because it is the derivative of the derivative of f. Using Leibniz notation, we write the second derivative of $y = f(x)$ as

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

Another notation is $f''(x) = D_2 f(x)$.

Interpretation of Double Derivative

We can interpret $f''(x)$ as the slope of the curve $y = f'(x)$ at the point $(x, f'(x))$. In other words, it is the rate of change of the slope of the original curve $y = f(x)$.

In general, we can interpret a second derivative as a rate of change of a rate of change. The most familiar example of this is acceleration, which we define as follows:

If $s = s(t)$ is the position function of an object that moves in a straight line, we known that its first derivative represents the velocity $v(t)$ of the object as a function of time:

$$v(t) = s'(t) = \frac{ds}{dt}$$

The instantaneous rate of change of velocity with respect to time is called the acceleration $a(t)$ of the object. Thus, the acceleration function is the derivative of the velocity function and is therefore the second derivative of the position function:

$a(t) = v'(t) = s''(t)$ or in Leibniz notation, $a = \dfrac{dv}{dt} = \dfrac{d^2s}{dt^2}$

Example 37. If $f(x) = x \cos x$, find $f''(x)$.

Solution Using the Product Rule, we have

$$f'(x) = x\frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(x)$$

$$= -x \sin x + \cos x$$

To find $f''(x)$ we differentiate $f'(x)$:

$$f''(x) = \frac{d}{dx}(-x \sin x + \cos x)$$

$$= -x\frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(-x) + \frac{d}{dx}(\cos x)$$

$$= -x \cos x - \sin x - \sin x = -x \cos x - 2 \sin x$$

Example 39. The position of a particle is given by the equation

$$s = f(t) = t^3 - 6t^2 + 9t$$

where t is measured in seconds and s in meters.

(a) Find the acceleration at time t. What is the acceleration after 4s?

Solution (a) The velocity function is the derivative of the position function:

$$s = f(t) = t^3 - 6t^2 + 9t$$

$$\Rightarrow \quad v(t) = \frac{ds}{dt} = 3t^2 - 12t + 9$$

The acceleration is the derivative of the velocity function:

$$a(t) = \frac{d^2s}{dt^2} = \frac{dv}{dt} = 6t - 12$$

$$\Rightarrow \quad a(4) = 6(4) - 12 = 12 \text{ m/s}^2$$

Application of Derivatives

Differentiation as a Rate of Change

$\frac{dy}{dx}$ is rate of change of 'y' with respect to 'x':

For examples:

(i) $v = \frac{dx}{dt}$ this means velocity 'v' is rate of change of displacement 'x' with respect to time 't'

(ii) $a = \frac{dv}{dt}$ this means acceleration 'a' is rate of change of velocity 'v' with respect to time 't'.

(iii) $F = \frac{dp}{dt}$ this means force 'F' is rate of change of momentum 'p' with respect to time 't'.

(iv) $\tau = \frac{dL}{dt}$ this means torque 'τ' is rate of change of angular momentum 'L' with respect to time 't'

(v) Power $= \frac{dW}{dt}$ this means power 'P' is rate of change of work 'W' with respect to time 't'

(vi) $E = \frac{-d\varphi}{dt}$ this means magnitude of e.m.f. 'E' is rate of change of electric flux 'ϕ' with respect to time 't'

(vii) $I = \frac{dq}{dt}$ this means current 'I' is rate of flow of charge 'q' with respect to time 't'

Example 39. The area A of a circle is related to its diameter by the equation $A = \frac{\pi}{4}D^2$.

How fast is the area changing with respect to the diameter when the diameter is 10 m?

Solution The (instantaneous) rate of change of the area with respect to the diameter is

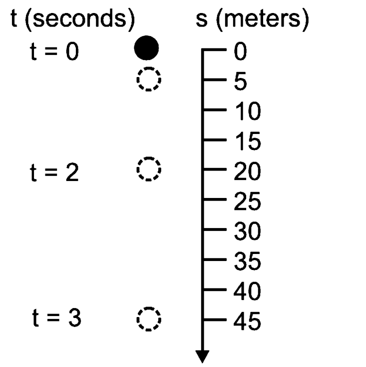
$$\frac{dA}{dD} = \frac{\pi}{4}2D = \frac{\pi D}{2}$$

When $D = 10$ m, the area is changing at rate $(\pi/2)\,10 = 5\pi\ m^2/m$. This means that a small change ΔD m in the diameter would result in a change of about $5\pi\ \Delta D$ m^2 in the area of the circle.

Example 40. Experimental and theoretical investigations revealed that the distance a body released from rest falls in time t is proportional to the square of the amount of time it has fallen. We express this by saying that

$$s = \frac{1}{2} g t^2,$$

where s is distance and g is the acceleration due to Earth's gravity. This equation holds in a vacuum, where there is no air resistance, but it closely models the fall of dense, heavy objects in air. Figure shows the free fall of a heavy ball bearing released from rest at time $t = 0$ sec.



A ball bearing falling from rest

(a) How many meters does the ball fall in the first 2 sec?

(b) What is its velocity, speed, and acceleration then?

Solution

(a) The free – fall equation is $s = 4.9\,t^2$.

During the first 2 sec. the ball falls

$s(2) = 4.9(2)^2 = 19.6\ m$,

(b) At any time t, velocity is derivative of displacement:

$$v(t) = s'(t) = \frac{d}{dt}(4.9t^2) = 9.8\,t.$$

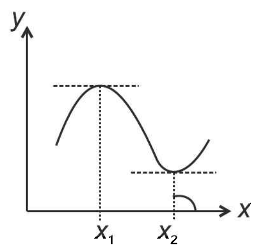
At $t = 2$, the velocity is $v(2) = 19.6$ m/sec

in the downward (increasing s) direction. The speed at $t = 2$ is

speed $= |v(2)| = 19.6$ m/sec. $\quad a = \frac{d^2 s}{dt^2} = 9.8$ m/s^2

Maxima and Minima

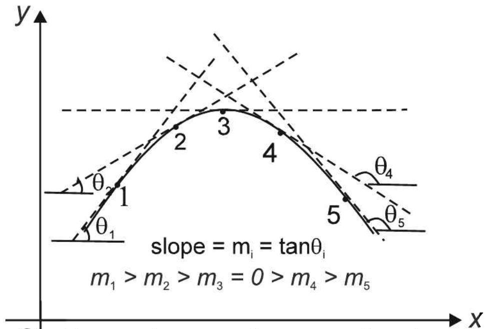
Suppose a quantity y depends on another quantity x in a manner shown in the figure. It becomes maximum at x_1 and minimum at x_2. At these points the tangent to the curve is parallel to the x–axis and hence its slope is $\tan\theta = 0$. Thus, at a maximum or a minimum,



$$\text{slope} = \frac{dy}{dx} = 0.$$

Maxima

Just before the maximum the slope is positive, at the maximum it is zero and just after the maximum it is negative. Thus, $\frac{dy}{dx}$ decreases at a maximum and hence the rate of change of $\frac{dy}{dx}$ is negative at a maximum i.e. $\frac{d}{dx}\left(\frac{dy}{dx}\right) < 0$ at maximum.



The quantity $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ is the rate of change of the slope. It is written as $\frac{d^2y}{dx^2}$.

Conditions for maxima are:– (a) $\frac{dy}{dx} = 0$ (b) $\frac{d^2y}{dx^2} < 0$

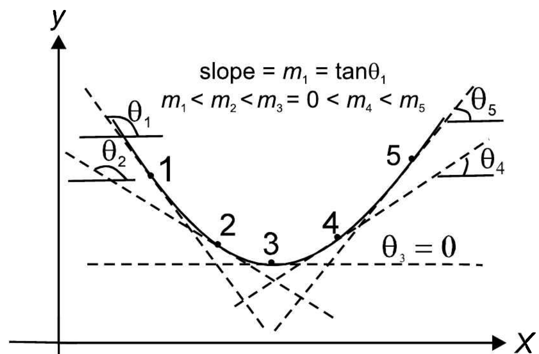
Minima

Similarly, at a minimum the slope changes from negative to positive.

Hence with the increases of x the slope is increasing that means the rate of change of slope with respect to x is positive hence $\frac{d}{dx}\left(\frac{dy}{dx}\right) > 0$.

Conditions for minima are:– (a) $\frac{dy}{dx} = 0$ (b) $\frac{d^2y}{dx^2} > 0$

Quite often it is known from the physical situation whether the quantity is a maximum or a minimum. The test on $\frac{d^2y}{dx^2}$ may then be omitted.



Example 41. Particle's position as a function of time is given as $x = 5t^2 - 9t + 3$. Find out the maximum value of position co-ordinate? Also, plot the graph.

Solution

$$x = 5t^2 - 9t + 3$$

$$\frac{dx}{dt} = 10t - 9 = 0$$

$\therefore$ $t = 9/10 = 0.9$

Check, whether maxima or minima exists. $\frac{d^2x}{dt^2} = 10 > 0$

$\therefore$ there exists a minima at $t = 0.9$

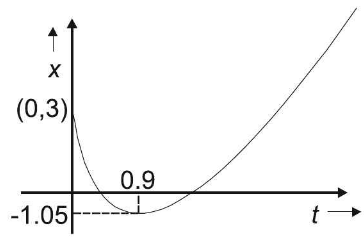
Now, Check for the limiting values.

When $t = 0; \quad x = 3$

$t = \infty; \quad x = \infty$

So, the maximum position co-ordinate does not exist.

Graph



Putting $t = 0.9$ in the equation

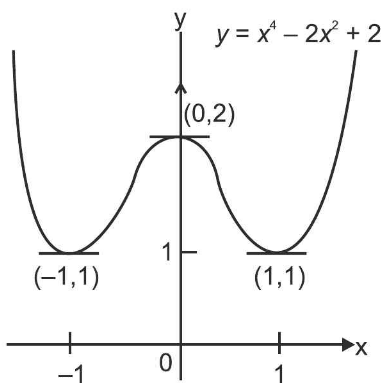
$$x = 5(0.9)^2 - 9(0.9) + 3 = -1.05$$

NOTE

If the coefficient of t^2 is positive, the curve will open upside.

Solved Examples on Application of Derivative

Example 42. Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangents ? If so, where ?



Solution The horizontal tangents, if any, occur where the slope dy/dx is zero. To find these points. We

1. Calculate dy/dx: $\dfrac{dy}{dx} = \dfrac{d}{dx}(x^4 - 2x^2 + 2) = 4x^3 - 4x$

2. Solve the equation: $\dfrac{dy}{dx} = 0$ for x: $4x^3 - 4x = 0$

$$4x(x^2 - 1) = 0$$

$$x = 0, 1, -1$$

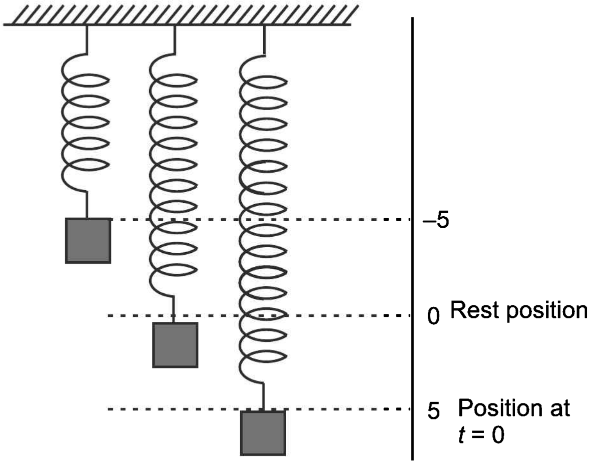
The curve $y = x^4 - 2x^2 + 2$ has horizontal tangents at $x = 0, 1,$ and -1.

The corresponding points on the curve are (0,2) (1,1) and (– 1,1). See figure.

Example 43. A body hanging from a spring (fig.) is stretched 5 units beyond its rest position and released at time $t = 0$ to oscillate up and down. Its position at any later time t is

$$s = 5 \cos t$$

What are its velocity and acceleration at time t ?



Solution We have

Position: $s = 5 \cos t$

Velocity: $v = \dfrac{ds}{dt} = \dfrac{d}{dt}(5 \cos t) = 5\dfrac{d}{dt}(\cos t) = -5 \sin t$

Acceleration: $a = \dfrac{dv}{dt} = \dfrac{d}{dt}(-5 \sin t) = 5\dfrac{d}{dt}(\sin t) = -5 \cos t$

Example 44. A sudden change in acceleration is called a "jerk". When a ride in a car or a bus is jerky. It is not that the accelerations involved are necessarily large but that the changes in acceleration are abrupt. Jerk is what spills your soft drink. The derivative responsible for jerk is d^3s/dt^3.

Jerk is the derivative of acceleration. If a body's position at time t is $s = f(t)$, the body's jerk at time t is

$$J = \frac{da}{dt} = \frac{d^3s}{dt^3}$$

Recent tests have shown that motion sickness comes from accelerations whose changes in magnitude or direction take us by surprise. Keeping an eye on the road helps us to see the changes coming. A driver is less likely to become sick than a passenger reading in the backseat.

(a) The jerk of the constant acceleration of gravity ($g = 32\ ft/\text{sec}^2$) is zero: $j = \dfrac{d}{dt}(g) = 0$

(b) The jerk of the simple harmonic motion in Example 2 is: $j = \dfrac{da}{dt} = \dfrac{d}{dt}(-5 \cos t)$

It has its greatest magnitude when $\sin t = \pm 1$, not at the extremes of the displacement but at the origin, where the acceleration changes direction and sign.

Example 45. A hot air balloon rising straight up from a level field is tracked by a range finder 500 ft from the lift-off point. At the moment the range finder's elevation angle is $\pi/4$, the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at the moment ?

Solution We answer the question in six steps.

Step 1: Draw a picture and name the variables and constants (Figure). The variables in the picture are θ = the angle the range finder makes with the ground (radians)

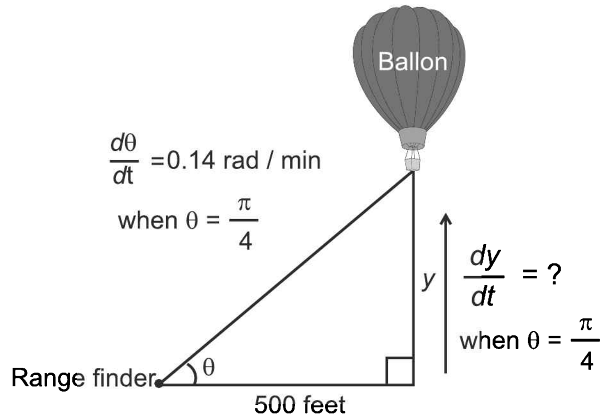
y = the height of the balloon (feet).

We let t represent time and assume θ and y to be differentiable functions of t.

The one constant in the picture is the distance from the range finder to the lift-off point (500 ft.) There is no need to give it a special symbol s.

Step 2: Write down the additional numerical information.

$$\frac{d\theta}{dt} = 0.14 \text{ rad/min} \text{ when } \theta = \pi/4$$



Step 3: Write down what we are asked to find. We want dy/dt when $\theta = \pi/4$.

Step 4: Write an equation that relates the variables y and θ. $\frac{y}{500} = \tan\theta$, or

$y = 500 \tan\theta$

Step 5: Differentiate with respect to t using the Chain Rule. The result tells how dy/dt (which we want) is related to $d\theta/dt$ (which we know).

$$\frac{dy}{dt} = 500 \sec^2\theta \, \frac{d\theta}{dt}$$

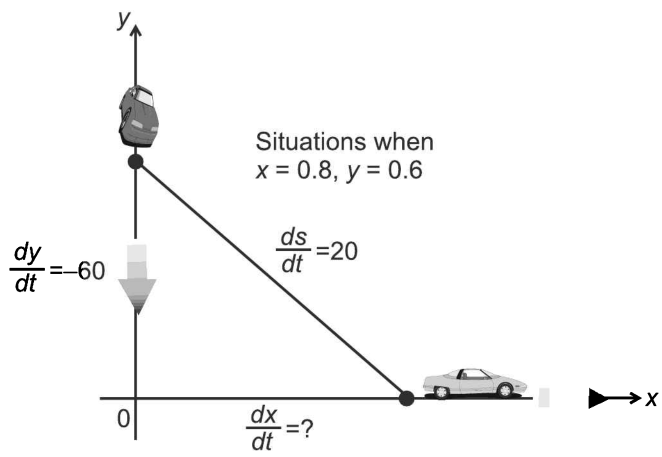
Step 6: Evaluate with $\theta = \pi/4$ and $d\theta/dt = 0.14$ to find dy/dt.

$$\frac{dy}{dt} = 500\left(\sqrt{2}\right)^2 (0.14) = (1000)\,(0.14) = 140 \quad \left(\sec\frac{\pi}{4} = \sqrt{2}\right)$$

At the moment in question, the balloon is rising at the rate of 140 ft./min.

***Example* 46.** A police cruiser, approaching a right-angled intersection from the north, is chasing a speeding car that has turned the corner and is now moving straight east. When the Cruiser is 0.6 mi north of the intersection and the car is 0.8 mi to the east, the police determine with radar that the distance between them and the car is increasing at 20 mph. If the cruiser is moving at 60 mph at the instant of measurement, what is the speed of the car?

Solution We carry out the steps of the basic strategy.



Step 1: Picture and variables. We picture the car and cruiser in the coordinate plane, using the positive x-axis as the eastbound highway and the positive y-axis as the northbound highway (Figure). We let t represent time and set

x = position of car at time t.

y = position of cruiser at time t,

s = distance between car and cruiser at time t.

We assume x, y and s to be differentiable functions of t.

$x = 0.8$ mi, $\quad y = 0.6$ mi, $\quad \dfrac{dy}{dt} = -60$ mph ,

$\dfrac{ds}{dt} = 20$ mph

(dy/dt is negative because y is decreasing.)

Step 2: To find: $\dfrac{dx}{dt}$

Step 3: How the variables are related: $s^2 = x^2 + y^2$ (Pythagorean Theorem)

(The equation $s = \sqrt{x^2 + y^2}$ would also work.)

Step 4: Differentiate with respect to t. $2s\dfrac{ds}{dt} = 2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt}$ (Chain Rule)

$$\frac{ds}{dt} = \frac{1}{s}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right) = \frac{1}{\sqrt{x^2+y^2}}\left(x\frac{dx}{dt} + y\frac{dy}{dt}\right)$$

Step 5: Evaluate, with $x = 0.8$, $y = 0.6$, $dy/dt = -60$, $ds/dt = 20$, and solve for dx/dt.

$$20 = \frac{1}{\underbrace{\sqrt{(0.8)^2 + (0.6)^2}}_{1}}\left(0.8\frac{dx}{dt} + (0.6)(-60)\right) \Rightarrow 20 = 0.8\frac{dx}{dt} - 36 \Rightarrow \frac{dx}{dt} = \frac{20+36}{0.8} = 70$$

At the moment in question, the car's speed is 70 mph.

4. INTEGRATION

In mathematics, for each mathematical operation, there has been defined an inverse operation.

For example- Inverse operation of addition is subtraction, inverse operation of multiplication is division and inverse operation of square is square root. Similarly there is a inverse operation for differentiation which is known as integration

Anti-derivatives or Indefinite Integrals

Definitions

A function $F(x)$ is an anti-derivative of a function $f(x)$ if $F'(x) = f(x)$ for all x in the domain of f. The set of all anti-derivatives of f is the indefinite integral of f with respect to x, denoted by $\int f(x)dx$.

The symbol $\int$ is an integral sign. The function f is the integrand of the integral and x is the variable of integration.

For example $f(x) = x^3$ then $f'(x) = 3x^2$

So the integral of $3x^2$ is x^3

Similarly if $f(x) = x^3 + 4$ then $f'(x) = 3x^2$

So the integral of $3x^2$ is $x^3 + 4$

there for general integral of $3x^2$ is x^3 + c where c is a constant

One anti derivative F of a function f, the other anti derivatives of f differ from F by a constant. We indicate this in integral notation in the following way:

$$\int f(x)dx = F(x) + C. \qquad \text{...(i)}$$

The constant C is the constant of integration or arbitrary constant, Equation (1) is read, "The indefinite integral of f with respect to x is $F(x) + C$." When we find $F(x) + C$, we say that we have integrated f and evaluated the integral.

Example 47. Evaluate $\int 2x\, dx$.

Solution

$$\int 2x dx = x^2 + C$$

(an anti derivative of $2x$ → x^2; the arbitrary constant → C)

The formula $x^2 + C$ generates all the anti derivatives of the function $2x$. The function $x^2 + 1$, $x^2 - \pi$, and $x^2 + \sqrt{2}$ are all anti derivatives of the function $2x$, as you can check by differentiation.

Many of the indefinite integrals needed in scientific work are found by reversing derivative formulas.

Integral Formulas

	Indefinite Integral	Reversed Derivative Formula
1.	$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$, n rational	$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^2$
	$\int dx = \int 1 dx = x + C$ (special case)	$\frac{d}{dx}(x) = 1$
2.	$\int \sin kx\, dx = -\frac{\cos kx}{k} + C$	$\frac{d}{dx}\left(-\frac{\cos kx}{k}\right) = \sin kx$
3.	$\int \cos kx\, dx = \frac{\sin kx}{k} + C$	$\frac{d}{dx}\left(\frac{\sin kx}{k}\right) = \cos kx$
4.	$\int \sec^2 x\, dx = \tan x + C$	$\frac{d}{dx} \tan x = \sec^2 x$
5.	$\int \csc^2 x\, dx = -\cot x + C$	$\frac{d}{dx}(-\cot x) = \csc^2 x$
6.	$\int \sec x \tan x\, dx = \sec x + C$	$\frac{d}{dx} \sec x = \sec x \tan x$
7.	$\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$	$\frac{d}{dx}(-\csc x) = \csc x \cot x$

Example 48. Examples based on above formulas:

(a) $\int x^5 dx = \frac{x^6}{6} + C$ Formula 1 with $n = 5$

(b) $\int \frac{1}{\sqrt{x}} dx = \int x^{-1/2} dx = 2x^{1/2} + C = 2\sqrt{x} + C$ Formula 1 with $n = -1/2$

(c) $\int \sin 2x\, dx = \frac{-\cos 2x}{2} + C$ Formula 2 with $k = 2$

(d) $\int \cos \frac{x}{2} dx = \int \cos \frac{1}{2} x dx = \frac{\sin (1/2)x}{1/2} + C = \int 2 \sin \frac{x}{2} + C$ Formula 3 with $k = 1/2$

***Example* 49.** **Right** : $\int x \cos x\, dx = x \sin x + \cos x + C$

Reason : The derivative of the right-hand side is the integrand:

Check : $\frac{d}{dx}(x \sin x + \cos x + C) = x \cos x + \sin x - \sin x + 0 = x \cos x.$

Wrong : $\int x \cos x\, dx = x \sin x + C$

Reason : The derivative of the right-hand side is not the integrand:

Check : $\frac{d}{dx}(x \sin x + C) = x \cos x + \sin x + 0 \neq x \cos x.$

Rules for Integration

Rule No. 1: Constant Multiple Rule

A function is an anti derivative of a constant multiple kf of a function f if and only if it is k times an antiderivative of f.

$$\int kf(x)dx = k \int f(x)dx$$

***Example* 50.** Rewriting the constant of integration

$\int 5 \sec x \tan x\, dx = 5 \int \sec x \tan x\, dx$ Rule 1

$= 5 (\sec x + C)$ Formula 6

$= 5 \sec x + 5C$ First form

$= 5 \sec x + C'$ Shorter form, where C' is 5

$= 5 \sec x + C$ Usual form – no prime. Since 5 times an arbitrary constant is an arbitrary constant, we rename C'.

What about all the different forms in Example? Each one gives all the antiderivatives of $f(x) = 5 \sec x \tan x$. so each answer is correct. But the least complicated of the three, and the usual choice, is

$$\int 5 \sec x \tan x\, dx = 5 \sec x + C.$$

Just as the Sum and Difference Rule for differentiation enables us to differentiate expressions term by term, the Sum and Difference Rule for integration enables us to integrate expressions term by term. When we do so, we combine the individual constants of integration into a single arbitrary constant at the end.

Rule No. 2: Sum and Difference Rule

> A function is an antiderivative of a sum or difference $f \pm g$ if and only if it is the sum or difference of an antiderivative of f an antiderivative of g.
>
> $$\int [f(x) \pm g(x)dx] = \int f(x)dx \pm \int g(x)dx$$

Example 51. Term – by – term integration

Evaluate: $\int (x^2 - 2x + 5)\,dx$.

Solution If we recognize that $(x^3/3) - x^2 + 5x$ is an antiderivative of $x^2 - 2x + 5$, we can evaluate the integral as

$$\int (x^2 - 2x + 5)dx = \overbrace{\frac{x^3}{3} - x^3 + 5x}^{\text{anti derivative}} + \underset{\text{arbitrary constant}}{c}$$

If we do not recognize the antiderivative right away, we can generate it term by term with the sum and difference Rule:

$$\int (x^2 - 2x + 5)dx = \int x^2 dx - \int 2x dx + \int 5dx$$

$$= \frac{x^3}{3} + C_1 - x^2 + C_2 + 5x + C_3.$$

This formula is more complicated than it needs to be. If we combine C_1, C_2 and C_3 into a single constant $C = C_1 + C_2 + C_3$, the formula simplifies to

$$\frac{x^3}{3} - x^2 + 5x + C$$

and still gives all the antiderivatives there are. For this reason we recommend that you go right to the final form even if you elect to integrate term by term. Write

$$\int (x^2 - 2x + 5)dx = \int x^2 dx - \int 2x dx + \int 5dx = \frac{x^3}{3} - x^2 + 5x + C.$$

Find the simplest antiderivative, you can for each part add the constant at the end.

Example 52. We can sometimes use trigonometric identities to transform integrals, we do not know how to evaluate into integrals we do know how to evaluate. The integral formulas for $\sin^2 x$ and $\cos^2 x$ arise frequently in applications.

(a) $$\int \sin^2 x\,dx = \int \frac{1 - \cos 2x}{2} dx \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$= \frac{1}{2}\int (1 - \cos 2x)\,dx = \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x\,dx$$

$$= \frac{1}{2} \text{x} \left(-\frac{1}{2}\right) \frac{\sin 2x}{2} + C = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

(b) $\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}dx$ $\qquad \cos^2 x = \frac{1+\cos 2x}{2}$

$= \frac{x}{2} + \frac{\sin 2x}{4} + C$ As in part (a), but with a sign change

Example 53. Find a body velocity from its acceleration and initial velocity. The acceleration of gravity near the surface of the earth is 9.8 m/sec^2. This means that the velocity v of a body falling freely in a vacuum changes at the rate of $\frac{dv}{dt}$ = 9.8 m/sec^2. If the body is dropped from rest, what will its velocity be t seconds after it is released?

Solution In mathematical terms, we want to solve the initial value problem that consists of

The differential condition: $\frac{dv}{dt} = 9.8$

The initial condition: $v = 0$ when $t = 0$ (abbreviated as $v(0) = 0$)

We first solve the differential equation by integrating both sides with respect to t:

$$\frac{dv}{dt} = 9.8 \quad \text{The differential equation}$$

$$\int \frac{dv}{dt}dt = \int 9.8dt \quad \text{Integrate with respect to } t.$$

$$v + C_1 = 9.8t + C_2 \quad \text{Integrals evaluated}$$

$$v = 9.8t + C. \quad \text{Constants combined as one}$$

This last equation tells us that the body's velocity t seconds into the fall is $9.8t + C$ m/sec. For value of C: What value? We find out from the initial condition:

$$v = 9.8t + C$$

$$0 = 9.8(0) + C \qquad v(0) = 0$$

$$C = 0.$$

Conclusion: The body's velocity t seconds into the fall is

$$v = 9.8t + 0 = 9.8t \text{ m/sec.}$$

The indefinite integral $F(x) + C$ of the function $f(x)$ gives the general solution $y = F(x) + C$ of the differential equation $dy/dx = f(x)$. The general solution gives all the solutions of the equation (there are infinitely many, one for each value of C). We solve the differential equation by finding its general solution. We then solve the initial value problem by finding the particular solution that satisfies the initial condition $y(x_o) = y_o$ (y has the value y_o when $x = x_o$.).

Rule No. 3: Rule of Substitution

$\int f(g(x)) \cdot g'(x)\,dx = \int f(u)du$	1. Substitute $u = g(x)$, $du = g'(x)\,dx$.
$= F(u) + C$	2. Evaluate by finding an antiderivative $F(u)$ of $f(u)$. (any one will do.)
$= F(g(x)) + C$	3. Replace u by $g(x)$.

***Example* 54.** Evaluate $\int (x+2)^5\,dx$

We can put the integral in the form

$$\int u^n du$$

by substituting

$$u = x + 2, \qquad du = d(x+2) = \frac{d}{dx}(x+2).\,dx$$

$$= 1.dx = dx.$$

Then $\int (x+2)^5 dx = \int u^5 du$ u $= x + 2$, $du = dx$

$$= \frac{u^6}{6} + C$$ Integrate, using rule no. 3 with $n = 5$.

$$= \frac{(x+2)^6}{6} + C.$$ Replace u by $x + 2$.

***Example* 55.** Evaluate $\int \sqrt{1+y^2} \bullet 2y\,dy = \int u^{1/2} du$ Let $u = 1 + y^2$, $du = 2y\,dy$.

$$= \frac{u^{(1/2)+1}}{(1/2)+1}$$ Integrate, using rule no. 3 with $n = 1/2$.

$$= \frac{2}{3}u^{3/2} + C$$ Simpler form

$$= \frac{2}{3}(1+y^2)^{3/2} + C$$ Replace u by $1 + y^2$.

***Example* 56.** Evaluate $\int \sqrt{4t-1}dt = \int u^{1/2} \bullet \frac{1}{4} du$ Let u = $4t - 1$, du = 4 dt, $(1/4)du = dt$.

$$= \frac{1}{4}\int u^{1/2} du$$ With the 1/4 out front, the integral is now in standard form.

$$= \frac{1}{4} \bullet \frac{u^{3/2}}{3/2} + C$$ Replace u by $1 + y^2$.

$$= \frac{1}{6}u^{3/2} + C$$ Simpler form

$$= \frac{1}{6}(4t-1)^{3/2} + C$$ Replace u by $4t - 1$.

***Example* 57.** Evaluate $\int \cos\,(7\theta + 5)\,d\theta = \int \cos u \cdot \frac{1}{7} du$ Let $u = 7\theta + 5$, du = $7d\theta$, $(1/7)\,du = d\theta$.

$$= \frac{1}{7}\int \cos u\,du$$ With (1/7) out front, the integral is now in standard form.

$= \frac{1}{7}\sin u + C$ Integrate with respect to u.

$= \frac{1}{7}\sin(7\theta + 5) + C$ Replace u by $7\theta + 5$.

Example 58. Evaluate $\int x^2 \sin(x)^3 dx = \int \sin(x)^3 \bullet x^2\, dx$

$= \int \sin u \bullet \frac{1}{3} du$ Let $u = x^3$, $du = 3x^2 dx$, $(1/3)\, du = x^2 dx$.

$= \frac{1}{3}\int \sin u\, du$

$= \frac{1}{3}(-\cos u) + C$ Integrate with respect to u.

$= -\frac{1}{3}\cos(x^3) + C$ Replace u by x^3.

Example 59. $\int \frac{1}{\cos^2 2\theta} d\theta = \int \sec^2 2\theta\, d\theta$ $\sec 2\theta = \frac{1}{\cos 2\theta}$

$= \int \sec^2 u \bullet \frac{1}{2} du$ Let $u = 2\theta$, $du = 2d\theta$, $d\theta = (1/2)du$.

$= \frac{1}{2}\int \sec^2 u\, du$

$= \frac{1}{2}\tan u + C$ Integrate, using eq. (4).

$= \frac{1}{2}\tan 2\theta + C$ Replace u by 2θ.

Check: $\frac{d}{d\theta}\left(\frac{1}{2}\tan 2\theta + C\right) = \frac{1}{2} \bullet \frac{d}{d\theta}(\tan 2\theta) + 0$

$= \frac{1}{2} \bullet \left(\sec^2 2\theta \cdot \frac{d}{d\theta} 2\theta\right)$ Chain Rule

$= \frac{1}{2} \bullet \sec^2 2\theta \bullet 2 = \frac{1}{\cos^2 2\theta}$.

Example 60. $\int (x^2 + 2x - 3)^2 (x + 1)\, dx = \int u^2 \cdot \frac{1}{2} du$

Let $u = x^2 + 2x - 3$, $du = 2xdx + 2dx = 2(x + 1)dx$, $(1/2)du = (x + 1)\, dx$.

$= \frac{1}{2}\int u^2 du$

$= \frac{1}{2} \bullet \frac{u^3}{3} + C = \frac{1}{6}u^3 + C$ Integrate with respect to u.

$= \frac{1}{6}(x^2 + 2x - 3)^3 + C$ Replace u.

Example 61. $\int \sin^4 t \cos t\, dt = \int u^4 du$ Let $u = \sin t$, $du = \cos t\, dt$.

$= \frac{u^5}{5} + C$ Integrate with respect to u.

$= \frac{\sin^5 t}{5} + C$ Replace u.

The success of the substitution method depends on finding a substitution that will change an integral we cannot evaluate directly into one that we can. If the first substitution fails, we can try to simplify the integrand further with an additional substitution or two.

Example 62. Evaluate: $\int \frac{2zdz}{\sqrt[3]{z^2+1}}$

We can use the substitution method of integration as exploratory tool: substitute for the most troublesome part of the integrand and see how things work out. For the integral here, we might try $u = z^2 + 1$ or we might even press our luck and take u to be the entire cube root. Here is what happens in each case.

Solution 1. Substitute $u = z^2 + 1$. $\int \frac{2zdz}{\sqrt[3]{z^2+1}} = \int \frac{du}{u^{1/3}}$ Let $u = z^2 + 1$, $du = 2zdz$.

$= \int u^{-1/3} du$ In the form $\int u^n du$

$= \frac{u^{2/3}}{2/3} + C$ Integrate with respect to u.

$= \frac{3}{2} u^{2/3} + C$

$= \frac{3}{2}(z^2 + 1)^{2/3} + C$ Replace u by $z^2 + 1$.

Solution 2. Substitute $u = \sqrt[3]{z^2+1}$ instead.

$\int \frac{2zdz}{\sqrt[3]{z^2+1}} = \int \frac{3u^2 du}{u}$

Let $u = \sqrt[3]{z^2+1}$, $u^3 = z^2 + 1$, $3u^2\, du = 2zdz$

$= 3 \cdot \int u\, du$

$= 3 \cdot \frac{u^2}{2} + C$ Integrate with respect to u.

$= \frac{3}{2}(z^2 + 1)^{2/3} + C$ Replace u by $(z^2 + 1)^{1/3}$

Definite Integration or Integration with Limits

The function is the integrand.
Upper limit of integration → b
Integral sign → $\int_a^b f(x)dx$ ← x is the variable of integration
Lower Limit of integration → a
Integral of f from a to b

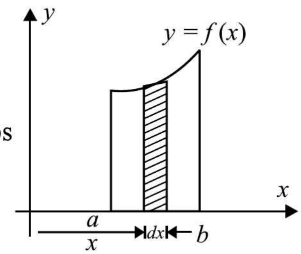
$$\int_a^b f(x)\,dx = [g(x)]_a^b = g(b) - g(a)$$

where $g(x)$ is the antiderivative of $f(x)$ i.e. $g'(x) = f(x)$

Example 63. $\int_{-1}^{4} 3dx = 3\int_{-1}^{4} dx = 3[x]_{-1}^{4} = 3[4-(-1)] = (3)(5) = 15$

$$\int_0^{\pi/2} \sin x dx = [-\cos x]_0^{\pi/2} = -\cos\left(\frac{\pi}{2}\right) + \cos(0) = -0 + 1 = 1$$

Application of Definite Integral: Calculation of Area of a Curve



From graph shown in figure if we divide whole area in infinitely small strips of dx width.

We take a strip at x positon of dx width.

Small area of this strip $dA = f(x)\,dx$

So, the total area between the curve and x-axis = sum of area of all strips = $\int_a^b f(x)dx$

Let $f(x) \geq 0$ be continuous on $[a,b]$. The area of the region between the graph of f and the x-axis is

$$A = \int_a^b f(x)\,dx$$

Example 64. Using an area to evaluate a definite integral

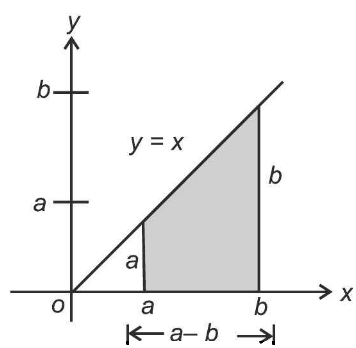
Evaluate $\int_a^b x\,dx \quad 0 < a < b.$

Solution We sketch the region under the curve $y = x$, $a \leq x \leq b$ (figure) and see that it is a trapezoid with height $(b - a)$ and bases a and b. The value of the integral is the area of this trapezoid:

$$\int_a^b x\,dx = (b-a)\bullet\frac{a+b}{2} = \frac{b^2}{2} - \frac{a^2}{2}.$$

Thus $\int_1^{\sqrt{5}} x\,dx = \frac{(\sqrt{5})^2}{2} - \frac{(1)^2}{2} = 2$ and so on.

Notice that $x^2/2$ is an antiderivative of x, further evidence of a connection between antiderivatives and summation.



The region in Example

EXERCISES

Q.1 Find $\frac{dy}{dx}$ for (Assume a, b, p, q as constants)

1. $y = \frac{1}{x}$
2. $y = x^{1/2}$
3. $y = x^{-3}$
4. $y = x^{-1/3}$
5. $y = 4\sqrt[3]{x}$
6. $y = px^{2q}$
7. $y = \frac{ax^6}{b}$
8. $y = ax^p$
9. $y = \frac{6}{\sqrt[5]{x^3}}$
10. $y = \left(x + \frac{1}{x}\right)^2$
11. $y = \sqrt{x} + \frac{1}{\sqrt{x}}$
12. $y = (x^2 + 1)\left(\frac{x}{2} + 1\right)$

Q.2 Find the following integrals. (Assume a, b as constants)

1. $\int 3x^7 dx$
2. $\int \frac{dx}{\sqrt{x}}$
3. $\int (x^3 - 5x^2 + 7x - 11)dx$
4. $\int \frac{d\theta}{\theta}$
5. $\int (x^{1/2} + x^{-1/2})dx$
6. $\int \frac{1}{2\sqrt{2x^3}} dx$
7. $\int \frac{dx}{ax + b}$
8. $\int \sqrt{ax + b}\, dx$
9. $\int (ax + b)^2 dx$
10. $\int e^{3x-1} dx$
11. $\int (e^{ax} - e^{-ax})dx$

ANSWER KEY

Q.1

1. $-\frac{1}{x^2}$
2. $\frac{1}{2\sqrt{x}}$
3. $-3x^{-4}$
4. $-\frac{1}{3}x^{-4/3}$
5. $\frac{4}{3\sqrt[3]{x^2}}$
6. $2pqx^{2q-1}$
7. $\frac{6a}{b}x^5$
8. pax^{p-1}
9. $-\frac{18}{5}\frac{1}{x^{8/5}}$
10. $2x - \frac{2}{x^3}$
11. $\frac{1}{2\sqrt{x}} - \frac{1}{2x^{3/2}}$
12. $\frac{1}{2}(3x^2 + 4x + 1)$

Q.2

1. $\frac{3}{8}x^8 + C$
2. $2\sqrt{x} + C$
3. $\frac{x^4}{4} - \frac{5}{3}x^3 + \frac{7}{2}x^2 - 11x + C$
4. $\ln\theta + C$
5. $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$
6. $\frac{-1}{\sqrt{2}}x^{-1/2} + C$
7. $\frac{1}{a}\ln(ax + b) + C$
8. $\frac{2}{3a}(ax + b)^{3/2} + C$
9. $\frac{1}{3a}(ax + b)^3 + C$
10. $\frac{e^{3x-1}}{3} + C$
11. $\frac{e^{ax} + e^{-ax}}{a} + C$

5. VECTOR

In physics we deal with two type of physical quantity one is scalar and other is vector. Each scalar quantities has magnitude and unit.

For example mass = 4*kg*

Magnitude of mass = 4

and unit of mass = *kg*

Example of scalar quantities: mass, speed, distance etc.

Scalar quantities can be added, subtracted and multiplied by simple laws of algebra.

Definition of Vector

If a physical quantity in addition to magnitude -

(a) has a specified direction.

(b) obeys the law of parallelogram of addition, then and then only it is said to be a vector. If any of the above conditions is not satisfied the physical quantity cannot be a vector.

If a physical quantity is a vector it has a direction, but the converse may or may not be true, i.e. if a physical quantity has a direction, it may or may not a be vector. e.g. time, pressure, surface tension or current etc. have directions but are not vectors because they do not obey parallelogram law of addition.

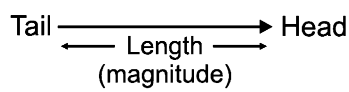
The magnitude of a vector ($\vec{A}$) is the absolute value of a vector and is indicated by

$|\vec{A}|$ or A.

Example of vector quantity: Displacement, velocity, acceleration, force etc.

Representation of Vector

Geometrically, the vector is represented by a line with an arrow indicating the direction of vector as

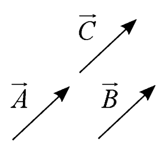


Mathematically, vector is represented by $\vec{A}$

Sometimes it is represented by bold letter $\boldsymbol{A}$.

Important Points

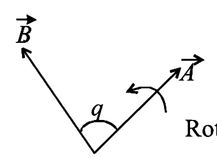
- If a vector is displaced parallel to itself it does not change (see Figure)



$\vec{A} = \vec{B} = \vec{C}$

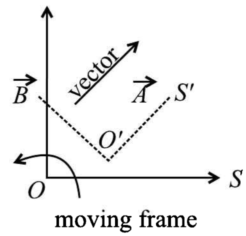
Transition of a vector parallel to itself

- If a vector is rotated through an angle other than multiple of 2π (or 360°) it changes (see Figure).



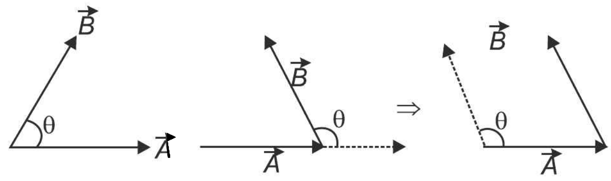
$\vec{A} \neq \vec{B}$

Rotation of a vector



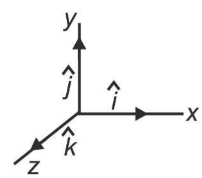
- If the frame of reference is translated or rotated the vector does not change (though its components may change). (see Figure).

- Two vectors are called equal if their magnitudes and directions are same, and they represent values of same physical quantity.



- Angle between two vectors means smaller of the two angles between the vectors when they are placed tail to tail by displacing either of the vectors parallel to itself (i.e. $0 \le \theta \le \pi$).

Unit Vector

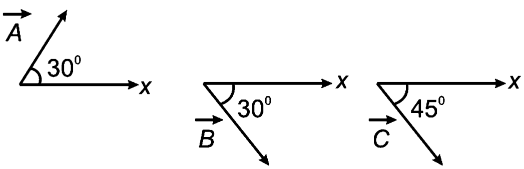


Unit vector is a vector which has a unit magnitude and points in a particular direction. Any vector ($\vec{A}$) can be written as the product of unit vector ($\hat{A}$) in that direction and magnitude of the given vector.

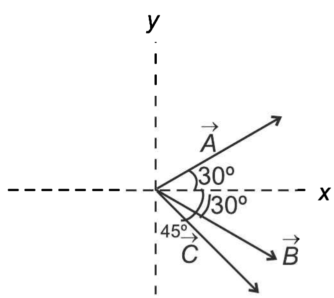
$$\vec{A} = |A|\hat{A} \text{ or } \hat{A} = \frac{\vec{A}}{|A|}$$

A unit vector has no dimensions and unit. Unit vectors along the positive *x*-, *y*- and z-axes of a rectangular coordinate system are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively such that $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.

Example 65. Three vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ are shown in the figure. Find angle between (i) $\vec{A}$ and $\vec{B}$, (ii) $\vec{B}$ and $\vec{C}$, (iii) $\vec{A}$ and $\vec{C}$.



Solution To find the angle between two vectors we connect the tails of the two vectors. We can shift $\vec{B}$ such that tails of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are connected as shown in figure.



Now we can easily observe that angle between $\vec{A}$ and $\vec{B}$ is 60º, $\vec{B}$ and $\vec{C}$ is 15º and between $\vec{A}$ and $\vec{C}$ is 75º.

Example 66. A unit vector along East is defined as $\hat{i}$. A force of 10^5 dynes acts west wards. Represent the force in terms of $\hat{i}$.

Solution $\vec{F} = -10^5\ \hat{i}$ dynes

Multiplication of a Vector by a Scalar

Multiplying a vector $\vec{A}$ with a positive number λ gives a vector $\vec{B}$ $(= \lambda\vec{A})$ whose magnitude is changed by the factor λ but the direction is the same as that of $\vec{A}$. Multiplying a vector $\vec{A}$ by a negative number λ gives a vector $\vec{B}$ whose direction is opposite to the direction of $\vec{A}$ and whose magnitude is – λ times $|\vec{A}|$.

Example 67. A physical quantity (m = 3kg) is multiplied by a vector $\vec{a}$ such that $\vec{F} = m\vec{a}$. Find the magnitude and direction of $\vec{F}$ if

(i) $\vec{a}$ = 3m/s^2 East wards

(ii) $\vec{a}$ = – 4m/s^2 North wards

Solution (i) $\vec{F} = m\vec{a}$ = 3 × 3 ms^{-2} East wards = 9 N East wards

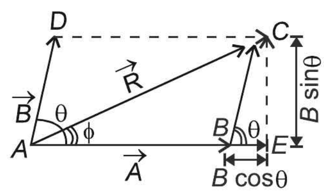
(ii) $\vec{F} = m\vec{a}$ = 3 × (– 4) N North wards

= – 12N North wards = 12 N South wards

Addition of Vectors

Addition of vectors is done by parallelogram law or its corallary, the triangle law:

(a) Parallelogram law of addition of vectors: If two vectors $\vec{A}$ and $\vec{B}$ are represented by two adjacent sides of a parallelogram both pointing outwards (and their tails coinciding) as shown. Then the diagonal drawn through the intersection of the two vectors represents the resultant (i.e., vector sum of $\vec{A}$ and $\vec{B}$).



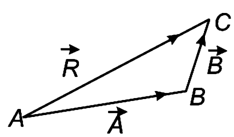
$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

The direction of resultant vector $\vec{R}$ from $\vec{A}$ is given by

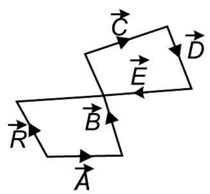
$$\tan\phi = \frac{CE}{AE} = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\varphi = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

(b) Triangle law of addition of vectors: To add two vectors $\vec{A}$ and $\vec{B}$ shift any of the two vectors parallel to itself until the tail of $\vec{B}$ is at the head of $\vec{A}$. The sum $\vec{A}$ + $\vec{B}$ is a vector $\vec{R}$ drawn from the tail of $\vec{A}$ to the head of $\vec{B}$, i.e., $\vec{A} + \vec{B} = \vec{R}$. As the figure formed is a triangle, this method is called ' triangle method' of addition of vectors.



$\vec{A} + \vec{B} = \vec{R}$



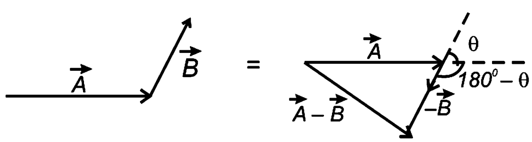
$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{E}$

If the 'triangle method' is extended to add any number of vectors in one operation as shown. Then the figure formed is a polygon and hence the name Polygon Law of addition of vectors is given to such type of addition.

IMPORTANT POINTS

- To a vector only a vector of same type can be added that represents the same physical quantity and the resultant is a vector of the same type.
- As $R = [A^2 + B^2 + 2AB \cos\theta]^{1/2}$ so R will be maximum when, $\cos\theta$ = max = 1, i.e., $\theta = 0°$, i.e. vectors are like or parallel and $R_{max} = A + B$.
- The resultant will be minimum if, $\cos\theta$ = min = -1, i.e., $\theta = 180°$, i.e. vectors are antiparallel and
$$R_{min} = A \sim B.$$
- If the vectors A and B are orthogonal, i.e., $\theta = 90°$, $R = \sqrt{A^2 + B^2}$
- As previously mentioned that the resultant of two vectors can have any value from $(A \sim B)$ to $(A + B)$ depending on the angle between them and the magnitude of resultant decreases as θ increases 0° to 180°
- Minimum number of unequal coplanar vectors whose sum can be zero is three.
- The resultant of three non-coplanar vectors can never be zero, or minimum number of non coplanar vectors whose sum can be zero is four.
- Subtraction of a vector from a vector is the addition of negative vector, i.e.,
$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

(a) From figure it is clear that $\vec{A} - \vec{B}$ is equal to addition of $\vec{A}$ with reverse of $\vec{B}$



$$|\vec{A} - \vec{B}| = [(A)^2 + (B^2) + 2AB \cos(180° - \theta)]^{1/2}$$
$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

(b) Change in a vector physical quantity means subtraction of initial vector from the final vector.

Example 68. Find the resultant of two forces each having magnitude F_0, and angle between them is θ.

Solution
$$F^2_{Resultant} = F_0^2 + F_0^2 + 2F_0^2 \cos\theta$$
$$= 2F_0^2 (1 + \cos\theta)$$
$$= 2F_0^2 (1 + 2\cos^2\frac{\theta}{2} - 1)$$
$$= 2F_0^2 \times 2\cos^2\frac{\theta}{2}$$

$$F_{\text{resultant}} = 2F_0 \cos\frac{\theta}{2}$$

Example 69. Two non zero vectors $\vec{A}$ and $\vec{B}$ are such that $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$. Find angle betwen $\vec{A}$ and $\vec{B}$?

Solution $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}| \Rightarrow A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$$\Rightarrow 4AB\cos\theta = 0 \quad \Rightarrow \quad \cos\theta = 0 \quad \Rightarrow \quad \theta = \frac{\pi}{2}$$

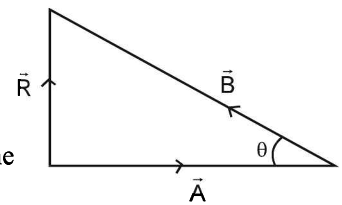
Example 70. The resultant of two velocity vectors $\vec{A}$ and $\vec{B}$ is perpendicular to $\vec{A}$. Magnitude of Resultant $\vec{R}$ is equal to half magnitude of $\vec{B}$. Find the angle between $\vec{A}$ and $\vec{B}$?

Solution Since $\vec{R}$ is perpendicular to $\vec{A}$. Figure shows the three vectors $\vec{A}$, $\vec{B}$ and $\vec{R}$.

angle between $\vec{A}$ and $\vec{B}$ is $\pi - \theta$

$$\sin\theta = \frac{R}{B} = \frac{B}{2B} = \frac{1}{2}$$

$\Rightarrow \theta = 30° \Rightarrow$ angle between A and B is 150°.



Example 71. If the sum of two unit vectors is also a unit vector. Find the magnitude of their difference?

Solution Let $\hat{A}$ and $\hat{B}$ are the given unit vectors and $\hat{R}$ is their resultant then

$$|\hat{R}| = |\hat{A} + \hat{B}|$$

$$1 = \sqrt{(\hat{A})^2 + (\hat{B})^2 + 2|\hat{A}||\hat{B}|\cos\theta}$$

$$1 = 1 + 1 + 2\cos\theta \quad \Rightarrow \quad \cos\theta = -\frac{1}{2}$$

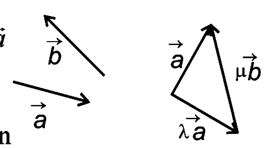
$$|\vec{A} - \vec{B}| = \sqrt{(\hat{A})^2 + (\hat{B})^2 - 2|\hat{A}||\hat{B}|\cos\theta} = \sqrt{1 + 1 - 2\times 1\times 1(-\frac{1}{2})} = \sqrt{3}$$

Resolution of Vectors

If $\vec{a}$ and $\vec{b}$ be any two nonzero vectors in a plane with different directions and $\vec{A}$ be another vector in the same plane. $\vec{A}$ can be expressed as a sum of two vectors - one obtained by multiplying $\vec{a}$ by a real number and the other obtained by multiplying $\vec{b}$ by another real number.

$$\vec{A} = \lambda\,\vec{a} + \mu\,\vec{b} \quad \text{(where } \lambda \text{ and } \mu \text{ are real numbers)}$$

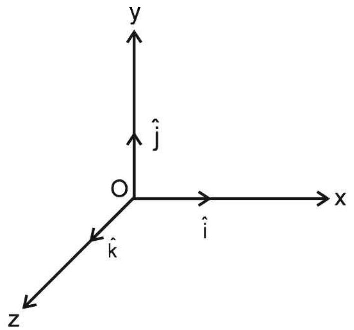
We say that $\vec{A}$ has been resolved into two component vectors namely $\lambda\vec{a}$ and $\mu\vec{b}$



$\lambda\,\vec{a}$ and $\mu\,\vec{b}$ along $\vec{a}$ and $\vec{b}$ respectively. Hence one can resolve a given vector into two component vectors along a set of two vectors – all the three lie in the same plane.

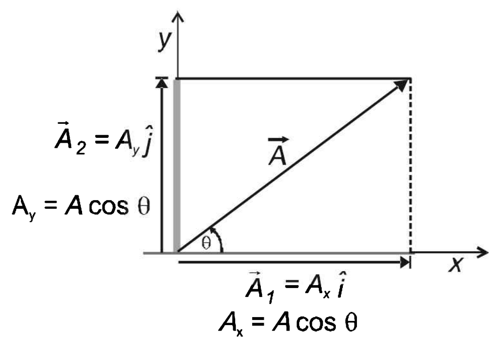
Resolution along Rectangular Component

It is convenient to resolve a general vector along axes of a rectangular coordinate system using vectors of unit magnitude, which we call as unit vectors. $\hat{i}, \hat{j}, \hat{k}$ are unit vector along *x*,*y* and z-axis as shown in figure below:



Resolution in Two Dimension

Consider a vector $\vec{A}$ that lies in *xy* plane as shown in figure,



$$\vec{A} = \vec{A}_1 + \vec{A}_2$$

$$\vec{A}_1 = A_x\hat{i},\ \vec{A}_2 = A_y\hat{j} \quad \Rightarrow \quad \vec{A} = A_x\hat{i} + A_y\hat{j}$$

The quantities A_x and A_y are called x- and y-components of the vector $\vec{A}$. A_x is itself not a vector but $A_x\hat{i}$ is a vector and so is $A_y\hat{j}$.

$$A_x = A\cos\theta \text{ and } A_y = A\sin\theta$$

Its clear from above equation that a component of a vector can be positive, negative or zero depending on the value of θ. A vector $\vec{A}$ can be specified in a plane by two ways:

(a) its magnitude *A* and the direction θ it makes with the *x*-axis; or

(b) its components A_x and A_y.$A = \sqrt{A_x^2 + A_y^2}$, $\theta = \tan^{-1}\dfrac{A_y}{A_x}$

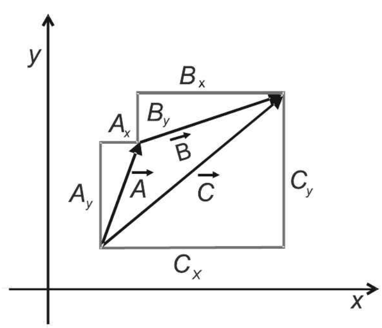
NOTE

If $A = A_x \Rightarrow A_y = 0$ and if $A = A_y \Rightarrow A_x = 0$ i.e.

components of a vector perpendicular to itself is always zero.

The rectangular components of each vector and those of the sum $\vec{C} = \vec{A} + \vec{B}$ are shown in figure. We saw that

$\vec{C} = \vec{A} + \vec{B}$ is equivalent to both

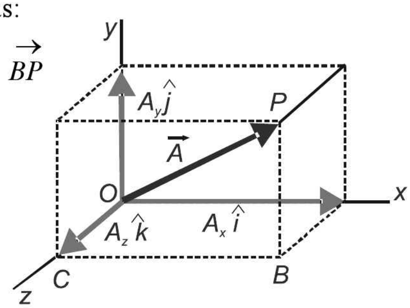


$C_x = A_x + B_x$

and $C_y = A_y + B_y$

Resolution in Three Dimensions

A vector $\vec{A}$ in components along *x*-, *y*- and z-axis can be written as:



$$\overrightarrow{OP} = \overrightarrow{OB} + \overrightarrow{BP} = \overrightarrow{OC} + \overrightarrow{CB} + \overrightarrow{BP}$$

$$\Rightarrow \quad \vec{A} = \vec{A}_Z + \vec{A}_x + \vec{A}_y = \vec{A}_x + \vec{A}_y + \vec{A}_Z$$

$$= A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

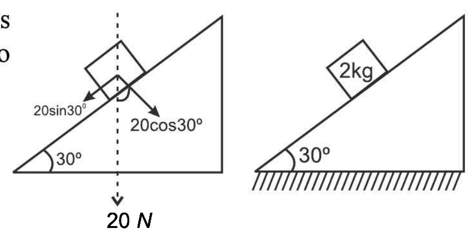
$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$A_x = A\cos\alpha,\ A_y = A\cos\beta,\ A_z = A\cos\gamma$

where cos α, cos β and cos γ are termed as **Direction Cosines** of a given vector $\vec{A}$.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Example **72.** A mass of 2 kg lies on an inclined plane as shown in figure. Resolve its weight along and perpendicular to the plane. (Assume $g = 10\ \text{m/s}^2$)



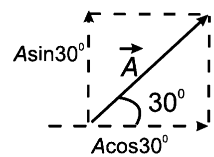
Solution Component along the plane

$= 20\sin 30 = 10\,N$

component perpendicular to the plane

$= 20\cos 30 = 10\sqrt{3}\,N$

Example **73.** A vector makes an angle of 30° with the horizontal. If horizontal component of the vector is 250. Find magnitude of vector and its vertical component?



Solution Let vector is $\vec{A}$

$$A_x = A\cos 30^0 = 250 = \frac{A\sqrt{3}}{2} \quad \Rightarrow \quad A = \frac{500}{\sqrt{3}}$$

$$A_y = A\sin 30^0 = \frac{500}{\sqrt{3}} \times \frac{1}{2} = \frac{250}{\sqrt{3}}$$

Example **74.** $\vec{A} = \hat{i} + 2\hat{j} - 3\hat{k}$, when a vector $\vec{B}$ is added to $\vec{A}$, we get a unit vector along *x*-axis. Find the value of $\vec{B}$? Also find its magnitude

Solution $\vec{A} + \vec{B} = \hat{i}$

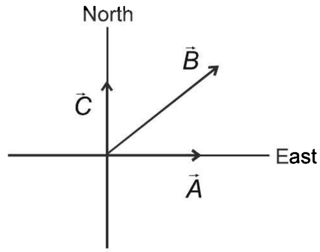
$$\vec{B} = \hat{i} - \vec{A} = \hat{i} - (\hat{i} + 2\hat{j} - 3\hat{k}) = -2\hat{j} + 3\hat{k}$$

$$\Rightarrow \quad |\vec{B}| = \sqrt{(2)^3 + (3)^2} = \sqrt{13}$$

Example **75.** In the above question find a unit vector along $\vec{B}$?

Solution $$\hat{B} = \frac{\vec{B}}{B} = \frac{-2\hat{j} + 3\hat{k}}{\sqrt{13}}$$

Example 76. Vector $\vec{A}$, $\vec{B}$ and $\vec{C}$ have magnitude 5, $5\sqrt{2}$ and 5 respectively, direction of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are towards east, North-East and North respectively. If $\hat{i}$ and $\hat{j}$ are unit vectors along East and North respectively. Express the sum $\vec{A} + \vec{B} + \vec{C}$ in terms of $\hat{i}$, $\hat{j}$. Also Find magnitude and direction of the resultant.



Solution $\vec{A} = 5\hat{i}$ $\vec{C} = 5\hat{j}$

$$\vec{B} = 5\sqrt{2}\cos 45\hat{i} + 5\sqrt{2}\sin 45\hat{j} = 5\hat{i} + 5\hat{j}$$

$$\vec{A} + \vec{B} + \vec{C} = 5\hat{i} + 5\hat{i} + 5\hat{j} + 5\hat{j} = 10\hat{i} + 10\hat{j}$$

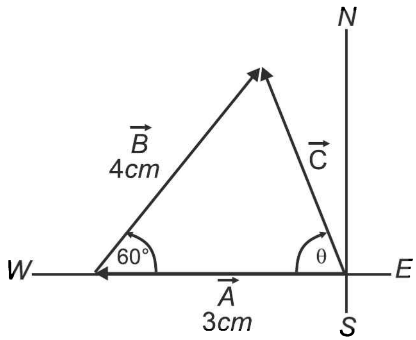
$$|\vec{A} + \vec{B} + \vec{C}| = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2}$$

$$\tan\theta = \frac{10}{10} = 1 \Rightarrow \theta = 45^\circ \text{ from East}$$

Example 77. You walk 3 *Km* west and then 4 *Km* headed 60° north of east. Find your resultant displacement

(a) graphically and

(b) using vector components.



Solution Picture the Problem: The triangle formed by the three vectors is not a right triangle, so the magnitudes of the vectors are not related by the Pythagorean theorem. We find the resultant graphically by drawing each of the displacements to scale and measuring the resultant displacement.

(a) If we draw the first displacement vector 3 cm long and the second one 4 cm long, we find the resultant vector to be about 3.5 cm long. Thus the magnitude of the resultant displacement is 3.5 Km. The angle θ made between the resultant displacement and the west direction can then be measured with a protractor. It is about 75°.

(b)

1. Let $\vec{A}$ be the first displacement and choose the *x*-axis to be in the easterly direction. Compute A_x and A_y, $A_x = -3$, $A_y = 0$
2. Similarly, compute the componets of the second displacement $\vec{B}$, $B_x = 4\cos 60^\circ = 2$, $B_y = 4\sin 60^\circ = 2\sqrt{3}$
3. The components of the resultant displacement $\vec{C} = \vec{A} + \vec{B}$ are found by addition,

$$\vec{C} = (-3+2)\hat{i} + (2\sqrt{3})\hat{j} = -\hat{i} + 2\sqrt{3}\hat{j}$$

4. The Pythagorean theorem gives the magnitude of $\vec{C}$.

$$C = \sqrt{1^2 + (2\sqrt{3})^2} = \sqrt{13} = 3.6$$

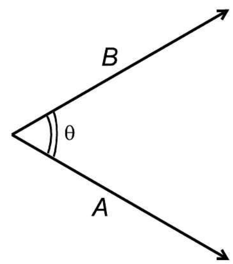
5. The ratio of C_y to C_x gives the tangent of the angle θ between $\vec{C}$ and the *x* axis.

$$\tan\theta = \frac{2\sqrt{3}}{-1} \Rightarrow \theta = -74°$$

Remark: Since the displacement (which is a vector) was asked for, the answer must include either the magnitude and direction, or both components. in (b) we could have stopped at step 3 because the *x* and *y* components completely define the displacement vector. We converted to the magnitude and direction to compare with the answer to part (a). Note that in step 5 of (b), a calculator gives the angle as – 74°. But the calculator can't distinguish whether the *x* or *y* components is negative. We noted on the figure that the resultant displacement makes an angle of about 75° with the negative *x* axis and an angle of about 105° with the positive *x* axis. This agrees with the results in (a) within the accuracy of our measurement.

5.6 MULTIPLICATION OF VECTORS

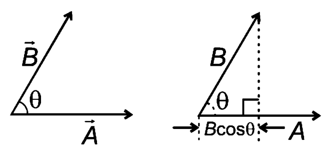
The Scalar Product

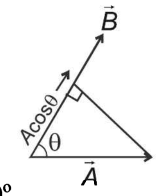


The scalar product or dot product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A}.\vec{B}$ (read $\vec{A}$ dot $\vec{B}$) is defined as the product of their magnitude with cosine of angle between them. Thus, $\vec{A}.\vec{B} = AB\cos\theta$ {here θ is the angle between the vectors}

PROPERTIES

- It is always a scalar which is positive if angle between the vectors is acute (i.e. < 90º) and negative if angle between them is obtuse (i.e. 90º < θ ≤ 180º)
- It is commutative, i.e., $\vec{A}.\vec{B} = \vec{B}.\vec{A}$
- It is distributive, i.e. $\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$
- As by definition $\vec{A}.\vec{B} = AB\cos\theta$. The angle between the vectors $\theta = \cos^{-1}\left[\frac{\vec{A}.\vec{B}}{AB}\right]$
- $\vec{A}.\vec{B} = A(B\cos\theta) = B(A\cos\theta)$
- Geometrically, $B\cos\theta$ is the projection of $\vec{B}$ onto $\vec{A}$ and $A\cos\theta$ is the projection of $\vec{A}$ onto $\vec{B}$ as shown. So $\vec{A}.\vec{B}$ is the product of the magnitude of $\vec{A}$ and the component of $\vec{B}$ along $\vec{A}$ and vice versa.





Component of $\vec{B}$ along $\vec{A} = B\cos\theta = \dfrac{\vec{A}.\vec{B}}{A} = \hat{A}.\vec{B}$

Component of $\vec{A}$ along $\vec{B} = A\cos\theta = \dfrac{\vec{A}.\vec{B}}{B} = \vec{A}.\hat{B}$

- Scalar product of two vectors will be maximum when cos θ = max = 1, i.e., θ = 0°, i.e., vectors are parallel $\Rightarrow (\vec{A}.\vec{B})_{max} = AB$
- If the scalar product of two nonzero vectors vanishes then the vectors are perpendicular.
- The scalar product of a vector by itself is termed as self dot product and is given by

  $(\vec{A})^2 = \vec{A}.\vec{A} = AA\cos\theta = A^2 \quad \Rightarrow \quad A = \sqrt{\vec{A}.\vec{A}}$
- In case of unit vector $\hat{n}$,

  $\hat{n}.\hat{n} = 1 \times 1 \times \cos 0° = 1 \quad \Rightarrow \quad \hat{n}.\hat{n} = \hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$
- In case of orthogonal unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$; $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$
- $\vec{A}.\vec{B} = (\hat{i}A_x + \hat{j}A_y + \hat{k}A_z).(\hat{i}B_x + \hat{j}B_y + \hat{k}B_z) = [A_xB_x + A_yB_y + A_zB_z]$

Example 78. If the Vectors $\vec{P} = a\hat{i} + a\hat{j} + 3\hat{k}$ and $\vec{Q} = a\hat{i} - 2\hat{j} - \hat{k}$ are perpendicular to each other. Find the value of a?

Solution If vectors $\vec{P}$ and $\vec{Q}$ are perpendicular

$\Rightarrow \quad \vec{P}\cdot\vec{Q} = 0 \quad \Rightarrow \quad (a\hat{i} + a\hat{j} + 3\hat{k}).(a\hat{i} - 2\hat{j} - \hat{k}) = 0$

$\Rightarrow \quad a^2 - 2a - 3 = 0 \quad \Rightarrow \quad a^2 - 3a + a - 3 = 0$

$\Rightarrow \quad a(a-3) + 1(a-3) = 0 \quad \Rightarrow \quad a = -1, 3$

Example 79. Find the component of $3\hat{i} + 4\hat{j}$ along $\hat{i} + \hat{j}$?

Solution Componant of $\vec{A}$ along $\vec{B}$ is given by $\dfrac{\vec{A}\cdot\vec{B}}{B}$ hence required component

$$= \frac{(3\hat{i} + 4\hat{j})\cdot(\hat{i} + \hat{j})}{\sqrt{2}} = \frac{7}{\sqrt{2}}$$

Example 80. Find angle between $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 12\hat{i} + 5\hat{j}$?

Solution We have $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{(3\hat{i} + 4\hat{j})\cdot(12\hat{i} + 5\hat{j})}{\sqrt{3^2 + 4^2}\sqrt{12^2 + 5^2}}$

$$\cos\theta = \frac{36 + 20}{5 \times 13} = \frac{56}{65}$$

$$\theta = \cos^{-1}\frac{56}{65}$$

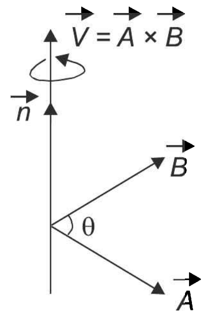
Vector Product

The vector product or cross product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A} \times \vec{B}$ (read $\vec{A}$ cross $\vec{B}$) is defined as:

$$\vec{A} \times \vec{B} = AB \sin\theta \, \hat{n}$$

Here θ is the angle between the vectors and the direction $\hat{n}$ is given by the right-hand-thumb rule.

Right-Hand-Thumb Rule



To find the direction of $\hat{n}$, draw the two vectors $\vec{A}$ *and* $\vec{B}$ with both the tails coinciding. Now place your stretched right palm perpendicular to the plane of $\vec{A}$ *and* $\vec{B}$ in such a way that the fingers are along the vector $\vec{A}$ and when the fingers are closed they go towards $\vec{B}$. The direction of the thumb gives the direction of $\hat{n}$.

PROPERTIES

- Vector product of two vectors is always a vector perpendicular to the plane containing the two vectors i.e. orthogonal to both the vectors $\vec{A}$ *and* $\vec{B}$, though the vectors $\vec{A}$ *and* $\vec{B}$ may or may not be orthogonal.
- Vector product of two vectors is not commutative i.e. $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$.

  But $\left|\vec{A}\times\vec{B}\right| = \left|\vec{B}\times\vec{A}\right| = AB\sin\theta$

The vector product is distributive when the order of the vectors is strictly maintained i.e.

$$A\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}.$$

The magnitude of vector product of two vectors will be maximum when sinθ = max = 1, i.e,, θ = 90°

$$|\vec{A}\times\vec{B}|_{max} = AB$$

i.e., magnitude of vector product is maximum if the vectors are orthogonal.

The magnitude of vector product of two non – zero vectors will be minimum when |sinθ| = minimum = 0,i.e.,

θ = 0° or 180° and $|\vec{A}\times\vec{B}|_{min} = 0$ i.e., if the vector product of two non – zero vectors vanishes, the vectors are collinear.

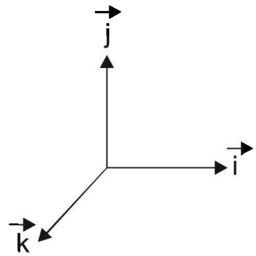
The self cross product i.e. product of a vector by itself vanishes i.e. is a null vector.

$$\vec{A}\times\vec{A} = AA\sin 0^\circ \, \hat{n} = \vec{0}.$$

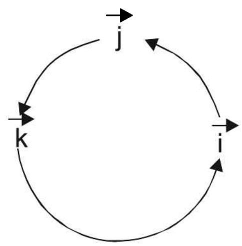
In case of unit vector $\hat{n}$, $\hat{n} \times \hat{n} = \vec{0} \Rightarrow \hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$

In case of orthogonal unit vectors $\hat{i}, \hat{j}$ *and* $\hat{k}$ in accordance with right-hand-thumb-rule,

$$\hat{j}\times\hat{k} = \hat{i} \quad \hat{k}\times\hat{i} = \hat{j}$$



(A)



(B)

In terms of components, $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$\vec{A} \times \vec{B} = \hat{i}\,(A_y B_z - A_z B_y) + \hat{j}\,(A_z B_x - A_x B_z) + \hat{k}\,(A_x B_y - A_y B_x)$$

Solved Examples

Example 81. $\vec{A}$ is Eastwards and $\vec{B}$ is downwards. Find the direction of $\vec{A} \times \vec{B}$?

Solution Applying right hand thumb rule we find that $\vec{A} \times \vec{B}$ is along North.

Example 82. If $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|$, find angle between $\vec{A}$ and $\vec{B}$

Solution $\vec{A} \cdot \vec{B} = |\vec{A} \times \vec{B}|\, AB \cos\theta = AB \sin\theta \tan\theta = 1 \quad \Rightarrow \quad \theta = 45^\circ$

Example 83. Two vectors $\vec{A}$ and $\vec{B}$ are inclined to each other at an anlge θ. Find a unit vector which is perpendicular to both $\vec{A}$ and $\vec{B}$

Solution $\vec{A} \times \vec{B} = AB \sin\theta\, \hat{n} \quad \Rightarrow \quad \hat{n} = \dfrac{\vec{A} \times \vec{B}}{AB \sin\theta}$ here $\hat{n}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

Example 83. Find $\vec{A} \times \vec{B}$ if $\vec{A} = \hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{B} = 2\hat{i} - \hat{j} + 2\hat{k}$.

Solution $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 4 \\ 3 & -1 & 2 \end{vmatrix} = \hat{i}\,(-4-(-4)) - \hat{j}\,(2-12) + \hat{k}\,(-1-(-6)) \quad = 10\hat{j} + 5\hat{k}$

Solved Miscellaneous Problems

1 Find the value of

(a) $\sin(-\theta)$

(b) $\cos(-\theta)$

(c) $\tan(-\theta)$

(d) $\cos\left(\dfrac{\pi}{2} - \theta\right)$

(e) $\sin\left(\dfrac{\pi}{2} + \theta\right)$

(f) $\cos\left(\dfrac{\pi}{2} + \theta\right)$

(g) $\sin(\pi - \theta)$

(h) $\cos(\pi - \theta)$

(i) $\sin(\frac{3\pi}{2} - \theta)$ **(j)** $\cos(\frac{3\pi}{2} - \theta)$

(k) $\sin(\frac{3\pi}{2} + \theta)$ **(l)** $\cos(\frac{3\pi}{2} + \theta)$

(m) $\tan(\frac{\pi}{2} - \theta)$ **(n)** $\cot(\frac{\pi}{2} - \theta)$

Sol.

(a) $-\sin\theta$ **(b)** $\cos\theta$
(c) $-\tan\theta$ **(d)** $\sin\theta$
(e) $\cos\theta$ **(f)** $-\sin\theta$
(g) $\sin\theta$ **(h)** $-\cos\theta$
(i) **$-\cos\theta$** **(j)** **$-\sin\theta$**
(k) **$-\cos\theta$** **(l)** **$\sin\theta$**
(m) $\cot\theta$ **(n)** **$\tan\theta$**

2 (i) For what value of m the vector $\vec{A} = 2\hat{i} + 3\hat{j} - 6\hat{k}$ is perpendicular to $\vec{B} = 3\hat{i} - m\hat{j} + 6\hat{k}$

(ii) Find the components of vector $\vec{A} = 2\hat{i} + 3\hat{j}$ along the direction of $\hat{i} + \hat{j}$?

Sol. (i) $m = -10$

(ii) $\frac{5}{\sqrt{2}}$.

3 (i) $\vec{A}$ is North – East and $\vec{B}$ is downwards, find the direction of $\vec{A} \times \vec{B}$.

(ii) Find $\vec{B} \times \vec{A}$ if $\vec{A} = 3\hat{i} - 2\hat{j} + 6\hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + \hat{k}$.

Sol. (i) North - West.

(ii) $-4\hat{i} - 3\hat{j} + \hat{k}$

Comprehension Questions

$\hat{i}$ and $\hat{j}$ are unit vectors along x – and y – axis respectively.

1. What is the magnitude and direction of the vectors $\hat{i} + \hat{j}$?
2. What is the magnitude and direction of the vectors $\hat{i} - \hat{j}$?
3. What are the components of a vector $A = 2\hat{i} + 3\hat{j}$ along the directions of $\hat{i} + \hat{j}$ and $\hat{i} - \hat{j}$?

Ans. (1) $\sqrt{2}$,450 with the x – axis ;

(2) $\sqrt{2}$, – 450 with the x – axis,

(3) $(5/\sqrt{2}, -1/\sqrt{2})$

Comprehension Questions

If $S = ut + \frac{1}{2}at^2$

Where ; S is displacement, u - initial velocity (constant), v - final velocity, a - acceleration(constant) & t - time taken then -

$$S = ut + \frac{1}{2}at^2$$

1. Differentiation of 'S' w.r.t. 't' will be -

(A) $u + \frac{at}{2}$ (B) $u + at$

(C) $u + 2at$ (D) $\frac{ut^2}{2} + \frac{at^3}{6}$

Sol. $S = ut + \frac{1}{2}at^2$

$u = \frac{ds}{dt} = u(1) + \frac{1}{2}a\,(2t) = u + at.$ **[Ans.** (B)**]**

2. Differentiation of above result w.r.t. 't' will be -

(A) a (B) $u + a$

(C) u (D) none

Sol. $\frac{du}{dt} = 0 + a(1) = a.$ **[Ans.** (A)**]**

Comprehension Questions

If a function is written as:

$y_1 = \sin(4x^2)$ & another function is $y_2 = \ln(x^3)$ then:

1. $\frac{dy_1}{dx}$, will be:

(A) $8x \cos(4x^2)$ (B) $\cos(4x^2)$ (C) $-\cos(4x^2)$ (D) $-8x\cos(4x^2)$

Sol. $\frac{dy_1}{dx} = \cos(4x^2).8x = 8x\cos(4x^2).$ **[Ans.** (A)**]**

2. $\frac{dy_2}{dx}$ will be $\frac{dy_2}{dx}$

(A) $\frac{1}{x^3}$ (B) $\frac{3}{x}$ (C) $-\frac{1}{x^3}$ (D) $\frac{3}{x^2}$

Sol. $y_2 = \ell n x^3$ $\frac{dy_2}{dx} = \frac{1}{x^3}.(3x^2) = \frac{3}{x}$ **[Ans.** (B)**]**

3. $\frac{d}{dx}\left(\frac{y_1}{y_2}\right)$ will be

(A) $\frac{8x\cos(4x^2)-\frac{3}{x}}{[\ell n(x^3)]^2}$ (B) $\frac{\ell n(x^3)-\sin 4x^2}{[\ell n(x^3)]^2}$

(C) $\frac{8x\ell n(x^3)\cos(4x^2)-\frac{3}{x}\sin(4x^2)}{[\ell n(x^3)]^2}$ (D) none

Sol. $\frac{y_1}{y_2} = \frac{\sin(4x^2)}{\ell n(x^3)}$

$$\frac{d}{dx}\left(\frac{y_1}{y_2}\right) = \frac{\ell n(x^3)\left[\cos(4x^2).8x\right]-\left[\frac{1}{x^3}.3x^2\right].\sin 4x^2}{[\ell nx^3]^2}$$

$$= \frac{8\times \ell nx^3\cos(4x^2)-\frac{3}{x}\sin(4x^2)}{[\ell n(x^3)]^2}.$$ **[Ans.** (C)**]**

Comprehension Questions

If $a = (3t^2 + 2t + 1)$m/s^2 is the expression according to which the acceleration of a particle varies. Then -

1. The expression for instantaneous velocity at any time 't' will be (if the particle was initially at rest) -

(A) $t^3 + 2t + 1$ (B) $t^3 + t + 1$ (C) $t^3 + t^2 + t$ (D) $t^3 + t^2 + t + C$

Sol. $a = 3t^2 + 2t + 1$

$$\int_0^t dv = \int_0^t 3t^2 + 2t + 1)\,dt$$

$v = t^3 + t^2 + t$ **[Ans.** (C)**]**

2. The change in velocity after 3 seconds of its start is:

(A) 30 m/s (B) 39 m/s (C) 3 m/s (D) 20 m/s

Sol. $V(t\text{-}0) = 0$

$$V_{t=3} = (3)^3 + (3)^2 + 3$$
$$= 27 + 9 + 3$$
$$= 39$$
$$\Delta V = 39 - 0 = 39 \text{ m/s}^2$$ **[Ans.** (B)**]**

3. Find displacement of the particle after 2 seconds of start -

(A) 26 m (B) 26/3 m (C) 30/7 m (D) 26/7 m

Sol. $\int_0^s dS = \int_0^2 (t^3 + t^2 + t)dt$

$$S = \left[\frac{t^4}{4}+\frac{t^3}{3}+\frac{t^2}{2}\right]_0^2$$

$$S = 4 + \frac{8}{3} + 2$$

$$S = \frac{12+8+6}{3} = \frac{26}{3}$$

[Ans. (B)**]**

Comprehension Questions

If charge flown through a wire is given by $q = 3\sin(3t)$ then

1. Find out the amount of charge flown through the wire at $t = \left(\frac{\pi}{6}\right)$ seconds.

$$t = \left(\frac{\pi}{6}\right)$$

(A) 3 coulombs (B) 6 coulombs (C) 1 coulomb (D) Zero coulomb

Sol. $q = 3\sin 3t$

$q = 2\sin 3 \times \frac{\pi}{6} = 3$ coluombs **[Ans.** (A)**]**

2. Find out the current flown through the wire at $t = \frac{\pi}{9}$ second.

$$t = \frac{\pi}{9}$$

(A) 4.5 Amp (B) $4.5\sqrt{3}$ Amp (C) $\sqrt{3}/2$ Amp (D) 9 Amp.

Sol. $i = \frac{dq}{dt} = 3 \times 3\cos(3t) \Rightarrow i\big|_{t=\frac{\pi}{9}} = 9\cos\left(3.\frac{\pi}{9}\right) = 9\cos\left(\frac{\pi}{3}\right) = \frac{9}{2}A$ **[Ans.** (A)**]**

3. Find out the area under $i-t$ curve from $t = \frac{\pi}{9}$ to $t = \frac{\pi}{6}$ seconds:

(A) $3\left[\frac{2-\sqrt{3}}{2}\right]$ (B) $3\left[\frac{2+\sqrt{3}}{2}\right]$ (C) $\left[\frac{2-\sqrt{3}}{2}\right]$ (D) $\left[\frac{2+\sqrt{3}}{2}\right]$

[Ans. (A)**]**

Comprehension Questions

Velocity of a particle varies as -

$v = 2t^3 - 3t^2$ in km/hr

If ; $t = 0$ is taken at 12:00 noon

1. Find the expression for the acceleration of the particle.

(A) $3t^2 + 3t$ (B) $6t(t-1)$ (C) $6t^2 + 3t$ (D) none

Sol. $v = 2t^2 - 3t^2$

$a = \frac{dv}{dt} = 6t^2 - 6t \quad \Rightarrow \quad = 6t\,(t-1)$ **[Ans.** (B)**]**

2. Find the time between 12:00 noon and 1:00 pm at which speed is maximum.

(A) 12:00 noon (B) 1:00 pm (C) 11:00 am (D) 2:00 pm

[Ans. (B)**]**

3. The time at which speed of the particle is minimum.

(A) 12:00 noon (B) 1:00 pm (C) 11:00 am (D) 2:00 pm

Sol. $v = 2t^3 - 3t^2$

$\frac{dv}{dt} = a = 6t\,(t-1)$

$\frac{dv}{dt} = 0$

$t = 0$, 1 sec.

$\frac{d^2v}{dt^2} = 12t - 6 \quad \Rightarrow \quad \left(\frac{d^2v}{dt^2}\right)_{t=0} = -6$ **[Ans.** (A)**]**

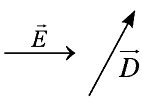
4. What is the velocity of the particle at 12:00 noon?

(A) 0.5 km/hr (B) zero (C) 1 km/hr (D) 2 km/hr

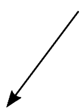
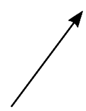
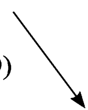
Sol. $v = 2t^3 - 3t^2 \Rightarrow t = 0\ v = 0$ **[Ans.** (B)**]**

Comprehension Questions

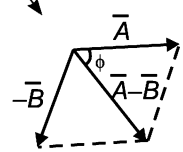
Two vectors $\vec{A}$ *and* $\vec{B}$ of unknown magnitudes along $\vec{E}$ & $\vec{D}$ (as shown below) respectively:

1. 

Then $(\vec{A} - \vec{B})$ could be -

(A)  (B)  (C)  (D) 

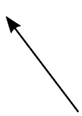
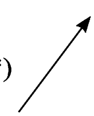
Sol. Angle between $\bar{A}$ and $\bar{B}$ is fixed and is an acute angle. Let it be θ.

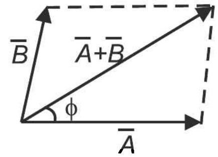


$$\tan\phi = \frac{B\sin\theta}{A - B\cos\theta}$$

Depending on values of *A* & *B*, ϕ can vary between *O* to θ. **[Ans.** (A), (B), (D)**]**

2. $\vec{A} + \vec{B}$ could be:

(A)  (B*)  (C)  (D) 



Sol. $\tan\phi = \dfrac{B\sin\theta}{A + B\cos\theta}$

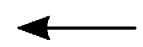
ϕ will vary between 0 to θ.

3. Angle between $\vec{A}$ *and* $\vec{B}$ is

(A) obtuse
(B) Acute
(C) obtuse or acute depending upon there magnitudes
(D) none

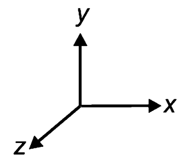
Sol. Angle between $\overline{A} \times \overline{B}$ is acute. **[Ans.** (B)]

4. If $\vec{C}$ is another vector represented as then $(\vec{A} - \vec{B} + \vec{C})$ could be

(A)  (B) Null vector (C)  (D) 

Sol. All the options are possible. **[Ans.** (A), (B), (C), (D)]

Comprehension Questions

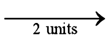


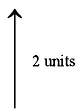
Position vector $\vec{A}$ is $2\hat{i}$

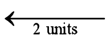
Position vector $\vec{B}$ is $3\hat{j}$

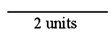
$\hat{i}, \hat{j}, \hat{k}$ are along the shown x,y and z axes:

1. Geometrical representation of $\vec{A}$ is

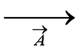
(A) 

(B) 

(C) 

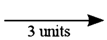
(D) 

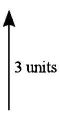
Sol. $\vec{A} = 2\hat{i}$

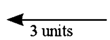


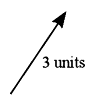
[Ans. (A)]

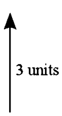
2. Geometrical representation of $\vec{B}$ is:

(A) 

(B) 

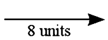
(C) 

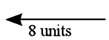
(D) 

Sol. $\vec{B} = 3\,\text{j}$ 

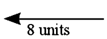
[Ans. (B)]

3. $-4\vec{A}$ can be represented as

(A) 

(B) 

(C) $\vec{A}$ (D) $\vec{A}$

Sol. $-4\vec{A} = 8\text{i}$ 

[Ans. (B)]

Comprehension Questions

There are two vectors $\vec{A}$ & $\vec{B}$, the *x* and *y* components of vector $\vec{A}$ are 4 m and 6 m respectively. The *x*, *y* components of vector $\vec{A}+\vec{B}$ are 10 m and 9 m respectively. Find:

1. The magnitude of $\vec{B}$. $\vec{B}$

Ans. $\sqrt{45}$

Sol. $\bar{A} = 4\hat{i} + 6\hat{j}$

$\bar{A} + \bar{B} = 10\hat{i} + 9\hat{j}$

$\bar{B} = (\bar{A} + \bar{B}) - \bar{B}$

$= 10\hat{i} + 9\hat{j} - 4\hat{i} - 6\hat{j} \;=\; 6\hat{i} + 3\hat{j}$

$|\bar{B}| = \sqrt{6^2 + 3^2} \;=\; \sqrt{45}$.

2. Angle between $\vec{B}$ and X-axis.

Ans. $\tan^{-1}\frac{1}{2}$

Sol. Angle between $\bar{B}$ & x – axis = $\tan^{-1}\left(\frac{3}{6}\right) = \tan^{-1}\left(\frac{1}{2}\right)$.

3. Angle between ($\vec{A}+\vec{B}$) & $\vec{B}$

Ans. $\cos^{-1}\frac{29}{\sqrt{905}}$

Sol. $\cos\alpha = \frac{(\bar{A}+\bar{B}).\bar{B}}{|\bar{A}+\bar{B}|} \;=\; \frac{29}{\sqrt{905}}$.

4. Unit vector along $\vec{A}$

Ans. $\frac{2\hat{i}+3\hat{j}}{\sqrt{13}}$

Sol. Unit vector along $\bar{A} \;=\; \frac{\bar{A}}{|\bar{A}|} \;=\; \frac{4\hat{i}+6\hat{j}}{\sqrt{52}} \;=\; \frac{2\hat{i}+3\hat{j}}{\sqrt{13}}$

Match the following:

1 Match the following columns:

(a)	sin 37°	(P)	$-\frac{3}{5}$
(b)	cos 127°	(Q)	$\frac{3}{5}$
(c)	tan 307°	(R)	$-\frac{4}{3}$

(d) cos 307° (S) $\frac{4}{3}$

(e) cos (– 53°) (T) $\frac{3}{4}$

Ans.(a) → *Q*, (b) → *P*, (c) → *R*, (d) → *Q*, (e) → *Q*.

Sol. **(a)** $\sin 37^\circ = \frac{3}{5}$

(b) $\cos 127^\circ = \cos(180^\circ - 53^\circ) = -\cos 53^\circ = -\frac{3}{5}$

Since, $\cos(180^\circ - \theta) = -\cos\theta$

(c) $\tan 307^\circ = \tan(360^\circ - 53^\circ) = -\tan 53^\circ = -\frac{4}{3}$

(d) $\cos 307^\circ = \cos(360^\circ - 53^\circ) = \cos 53^\circ = \frac{3}{5}$

(e) $\cos(-53^\circ) = \cos 53^\circ = \frac{3}{5}$.

2 Match the following:

(A) Acceleration (P) sin t

(B) $6x$ (Q) $6x$

(C) zero (R) x^3

(D) – sin t (S) displacement

Ans. $A \to S, B \to R, C \to Q, D \to P$.

Sol. Column I is double derivative of coulmn II.

3 Match the following:

(A) $\sin(\omega t + \phi)$ (P) $C + \frac{1}{\omega}\tan(\omega t + \phi)$

(B) $\cos(\omega t + \phi)$ (Q) $\frac{1}{\omega}\sin(\omega t + \phi) + C$

(C) $\tan(\omega t + \phi)\sec(\omega t + \phi)$ (R) $-\frac{1}{\omega}\cos(\omega t + \phi) + C$

(D) $\sec^2(\omega t + \phi)$ (S) $\frac{1}{\omega}\sec(\omega t + \phi) + C$

Ans. $A \to R, B \to Q, C \to S, D \to P$.

Assertion Reason Questions

1 **Assertion:** Anti derivative of $(3x^2 + 2)$ is equal to $(x^3 + 2x)$.

Reason: Derivative of $(x^3 + 2x)$ is equal to $(3x^2 + 2)$.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(C) if Assertion is true, but the Reason is false.

(D) if Assertion is false, but the Reason is true.

Sol. $\int(3x^2+2)dx = x^3 + 2x + C$

$\frac{d}{dx}(x^3+2x) = 3x^2 + 2$ **[Ans.** (D)**]**

2 Assertion: If slope of a given curve is zero at a given point then there may exist a maxima at that point.

Reason: At a maxima, slope changes its sign.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(C) if Assertion is true, but the Reason is false.

(D) if Assertion is false, but the Reason is true. **[Ans.** (A)**]**

Sol. $\frac{dy}{dx} = 0 \Rightarrow$ there may be a maxima or minima.

So, Assertion is true.

At maxima, sign of slope changes, so if there is change of sign there must be a zero in between.

3 Assertion: Position of a particle is given by $S = \frac{t^2}{2} - t + 3$. Then position will be maximum at $t = 1\ s$.

Reason: Maximum is obtained when slope is zero.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(C) if Assertion is true, but the Reason is false.

(D) if Assertion is false, but the Reason is true.

Sol. $\frac{ds}{dt} = \frac{2t}{2} - 1 = 0 \Rightarrow t = 1$ sec.

$\frac{d^2s}{dt^2}\ t = 1 = 1 \quad = \quad +ve$

$\Rightarrow$ So at $t = 1$, minima exists. **[Ans.** (D)**]**

4 **Assertion:** If $y = x^3 + 3x^2$ then there will be a maxima at $x = -2$.

Reason: Slope of $(y - x)$ curve will be, zero at $x = -2$.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(C) if Assertion is true, but the Reason is false.

(D) if Assertion is false, but the Reason is true.

Sol. $\frac{dy}{dx} = 3x^2 + 6x = 0$

$\Rightarrow \quad 3x(x + 2) = 0$

$\Rightarrow \quad x = 0, x = -2$

$$\frac{d^2y}{dx^2} = 6x + 6$$

At, $x = 0, \frac{d^2y}{dx^2} = 6 > 0$

$x = 0$ is a point of minima

At, $x = 2, \frac{d^2y}{dx^2} = -6 < 0$

$x = 0$ is point of maxima. **[Ans.** (A)**]**

5 **Statement 1:** Two vectors are always coplanar vectors.

Statement 2: Three points are always coplanar.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

Sol. We can draw a plane from two vectros so two vectors one always coplanar.

From three point we can draw two vectors so they are coplanar. **[Ans.** (A)**]**

EXERCISE

1. (a) If $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{b} = 3\hat{i} + 4\hat{j}$, then $\frac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}} =$

(A) 7/5 (B) 5/7 (C) 4/9 (D) None

(b) Two forces $\hat{i} + \hat{j} + \hat{k}\ N$ and $\hat{i} + 2\hat{j} + 3\hat{k}\ N$ act on a particle and displace it from (2, 3, 4) to point (5, 4, 3). Displacement is in m. Work done is

(A) 5 J (B) 4 J (C) 3 J (D) None of these

2. The vector $i + xj + 3k$ is rotated through an angle θ and doubled in magnitude, then it becomes $4i + (4x - 2)j + 2k$. The values of x are

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) 2

3. If the resultant of three forces $F_1 = pi + 3j - k$, $F_2 = -5i + j + 2k$ and $F_3 = 6i - k$ acting on a particle has magnitude equal to 5 units, then the value (s) of p is (are)

(A) – 6 (B) – 4 (C) 2 (D) 4

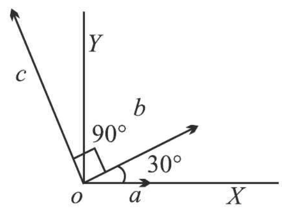
4. A vector $\vec{B}$ which has a magnitude 8.0 is added to a vector $\vec{A}$ which lie along the x-axis. The sum of these two vectors is a third vector which lie along the y-axis and has a magnitude that is twice the magnitude of $\vec{A}$. The magnitude of $\vec{A}$ is ___.

5. Three vectors as shown in the fig have magnitudes

$|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{c}| = 10$.

(i) Find the x and y components of these vectors.

(ii) Find the numbers p and q such that $\vec{c} = p\vec{a} + q\vec{b}$.



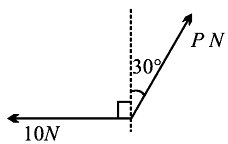
6. A buoy is attached to three tugboats by three ropes. The tugboats are engaged in a tug – of – war. One tugboat pulls west on the buoy with a force $\vec{F_1}$ of magnitude 1000 N. The second tugboat pulls south on the buoy with a force $\vec{F_2}$ of magnitude 2000 N. The third tugboat pulls northeast (that is, half way between north and east), with a force $\vec{F_3}$ of magnitude 2000 N.

(A) Draw a free body diagram of forces acting on the buoy to represent this situation.

(B) Express each force in unit vector form $(\hat{i}, \hat{j})$.

(C) Calculate the magnitude of the resultant force.

7. Two horizontal forces of magnitudes 10 N & P N act on a particle. The force of magnitude 10 N acts due west & the force of magnitude P N acts on a bearing of 30° east of north as shown in figure. The resultant of these two force acts due north. Find the magnitude of this resultant.



8. The position vectors of two balls are given by

$\vec{r}_1 = 2(m)i + 7(m)j$

$\vec{r}_2 = -2(m)i + 4(m)j$

What will be the distance between the two balls?

9. A man is travelling in east direction with a velocity of 6 m/s. Rain is falling down vertically with a speed of 4 m/s. Find the velocity of rain with respect to man and its angle with the vertical using the concept that relative velocity of B w.r.t A = $\vec{v}_B - \vec{v}_A$.

10. The sum of three forces $\vec{F_1} = 100\text{N}$, $\vec{F_2} = 80\text{N}$ & $\vec{F_3} = 60\text{N}$ acting on a particle is zero. The angle between $\vec{F_1}$ & $\vec{F_2}$ is nearly

(A) 53° (B) 143° (C) 37° (D) 127°

(E) 90°

11. A particle is moving westward with a velocity $\vec{v}_1 = 5$ m/s. Its velocity changed to $\vec{v}_2 = 5$ *m/s* northward. The change in velocity vector $(\Delta\vec{V} = \vec{v}_2 - \vec{v}_1)$ is:

(A) $5\sqrt{2}$ *m/s* towards north east
(B) 5 *m/s* towards north west
(C) zero
(D) $5\sqrt{2}$ *m/s* towards north west

12. A force $\vec{F} = 3\hat{i} + c\hat{j} + 2\hat{k}$ acting on a particle causes a displacement $\vec{d} = -4\hat{i} + 2\hat{j} + 3\hat{k}$. If the work done is 6J then the value of '*c*' is

(A) 12
(B) 0
(C) 6
(D) 1

13. A force $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$ newton produces acceleration 1 m/s^2 in a body. The mass of the body is (in kg)

(A) $6\hat{i} - 8\hat{j} + 10\hat{k}$
(B) 100
(C) $10\sqrt{2}$
(D) 10

14. If *a*, *b*, *c* are three unit vectors such that $a + b + c = 0$, then $a.b + b.c + c.a$ is equal to

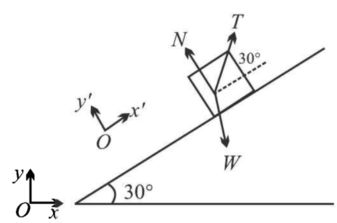
(A) – 1
(B) 3
(C) 0
(D) $-\frac{3}{2}$

15. A particle is displaced from $A \equiv (2, 2, 4)$ to $B \equiv (5, -3, -1)$. A constant force of 34N acts in the direction of $\vec{AP}$. where $P \equiv (10, 2, -11)$. (Coordinates are in m).

(i) Find the ($\vec{F}$).
(ii) Find the work done by the force to cause the displacement.

16. A 300 *gm* mass has a velocity of $\vec{v} = 3\hat{i} + 4\hat{j}$ *m/s* at certain instant. Its kinetic energy is ______.

17. A body is supported on a smooth plane inclined at 30^0 to the horizontal by a string attached to the body and held at an angle of 30^0 to the plane. Draw a diagram showing the forces acting on the body and resolve each of these forces(forces are strings Tension T, weight W & normal reaction by plane)



(A) horizontally and vertically.
(B) parallel and perpendicular to the plane.

18. A particle whose speed is 50 *m/s* moves along the line from *A* (2, 1) to *B* (9, 25). Find its velocity vector in the form of $a\hat{i} + b\hat{j}$.

19. $\vec{A} + \vec{B} = 2\hat{i}$ and $\vec{A} - \vec{B} = 4\hat{j}$ then angle between $\vec{A}$ *and* $\vec{B}$ is

(A) 127°
(B) 143°
(C) 53°
(D) 37°

20. Two forces *P* and *Q* are in ratio *P*: *Q* = 1: 2. If their resultant is at an angle $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$ to vector *P*, then angle between *P* and *Q* is:

(A) $\tan^{-1}\left(\frac{1}{2}\right)$
(B) 45°
(C) 30°
(D) 60°

21. Given the vectors

$$\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k};\ \vec{B} = 3\hat{i} - 2\hat{j} - 2\hat{k}\ \&\ \vec{C} = p\hat{i} + p\hat{j} + 2p\hat{k}$$

Find the angle between $(\vec{A} - \vec{B})$ & $\vec{C}$

(A) $\theta = \cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (B) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (C) $\theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (D) none of these

22. A force of 35 N acts in the direction parallel to $2\hat{i} + 3\hat{j} + 6\hat{k}$ and it displaces a body from (1*m*, 0*m*, 3*m*) to (3*m*, 4*m*, 1*m*)
 (A) Express the force vector (in unit vector form)
 (B) Find the work done.

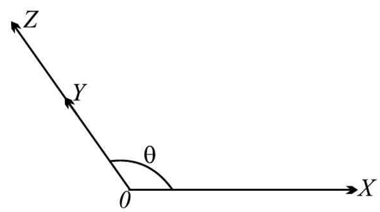
23. A particle travels with speed 50*m/s* from the point (3, – 7) in a direction $7\hat{i} - 24\hat{j}$. Find its position vector after 3 seconds.

24. A force $\vec{F} = 5\hat{i} + 2\hat{j} + \hat{k}$ displaces a body from a point of coordinate (1, 1, 1) to another point of coordinates (2, 0, 3). Calculate the work done by the force.

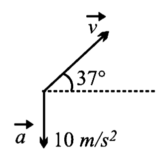
25. An object of weight W is fastened to one end of a string whose other end is fixed and is pulled sideways by a horizontal force *P* until the string is inclined at 37^0 to the vertical. Draw a diagram showing the forces acting on the object and resolve each force parallel and perpendicular to the string.

26. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Gind speed after 10s.

27. Forces *X*, *Y* and *Z* have magnitudes 10 *N*, $5\left(\sqrt{3} - 1\right)$ *N* and $5\left(\sqrt{3} + 1\right)$ *N*. The forces *Y* and *Z* act in the same direction as shown in the diagram. The resultant of *X* and *Y* and the resultant of *X* and *Z* have the same magnitude. Find θ°, the angle between *X* and *Y*.



28. After firing, a bullet is found to move at an angle of 37° to horizontal. Its acceleration is 10 m/s^2 downwards. Find the component of acceleration in the direction of the velocity.

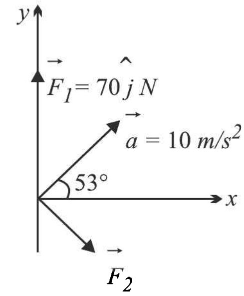


(A) $-6\ m/s^2$ (B) $-4\ m/s^2$ (C) $-8\ m/s^2$
(D) $-5\ m/s^2$

29. A man moves in an open field such that after moving 10 *m* in a straight line, he makes a sharp turn of 60° to his left. Find the total displacement of the man just after 7 such turns.
(A) 10 *m* (B) 20 *m* (C) 70 *m* (D) 30 *m*

30. $\vec{A} = \hat{i} + \hat{j} - \hat{k};\ \vec{B} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ angle between $\vec{A}$ and $\vec{B}$ is
(A) 120° (B) 90° (C) 60° (D) 30°

31. A particle of $m = 5$ kg is momentarily at rest at $x = 0$ at $t = 0$. It is acted upon by two forces $\vec{F}_1$ and $\vec{F}_2$. $\vec{F}_1 = 70\hat{j}\,N$. The direction and magnitude of $\vec{F}_2$ are unknown. The particle experiences a constant acceleration, $\vec{a}$, in the direction as shown. Neglect gravity.



(A) Find the missing force $\vec{F}_2$.

(B) What is the velocity vector of the particle at $t = 10$ sec?

(C) What third force, $\vec{F}_3$ is required to make the acceleration of the particle zero? Either give magnitude and direction of $\vec{F}_3$ or its components.

32. At $t = 0$, a particle at (1, 0, 0) moves with velocity vector = $(15i + 20j + 60k)$ *m/s*. Find its position vector at time $t = 2$ sec.
33. A spy plane is being tracked by a radar. At $t = 0$, its position is reported as (100*m*, 200*m*, 1000*m*). 130 second later its position is reported to be (2500*m*, 1200*m*, 1000*m*). Find a unit vector in the direction of plane velocity and the magnitude of its average velocity.
34. (A) Calculate $\vec{r} = \vec{a} - \vec{b} + \vec{c}$ where $\vec{a} = 5\hat{i} + 4\hat{j} - 6\hat{k}$, $\vec{b} = -2\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{c} = 4\hat{i} + 3\hat{j} + 2\hat{k}$.

    (B) Calculate the angle between $\vec{r}$ and the *z*-axis.

    (C) Find the angle between $\vec{a}$ *and* $\vec{b}$
35. The area '*A*' of a blot of ink is growing such that after *t* sec. its area is given by $A = t^2 \text{cm}^2$. Calculate the rate of increase of area at $t = 5$ sec.
36. A particle moves along a straight line such that at time *t* its displacement from a fixed point O on the line is $3t^2 - 2$. The velocity of the particle when $t = 2$ is:

    (A) 8 ms^{-1} (B) 4 ms^{-1} (C) 12 ms^{-1} (D) 0
37. The momentum of a particle moving in straight line is given by $p = \ln t + \frac{1}{t}$ (in *m/s*) find the (time $t > 0$) at which the net force acting on particle is 0 and it's momentum at that time.
38. The velocity of the particle is given as $v = 3t^3 + t - \frac{1}{t^2}$. Calculate the net force acting on the body at time $t = 2$ sec, if the mass of the body is 5 kg _________.
39. The charge flowing through a conductor begining with time $t = 0$ is given by the formula $q = 2t^2 + 3t + 1$ (coulombs). Find the current $i = \frac{dq}{dt}$ at the end of the 5th second.
40. The angle θ through which a pulley turns with time t is specified by the function $\theta = t^2 + 3t - 5$. Find the angular velocity $\omega = \frac{d\theta}{dt}$ at $t = 5$ sec.
41. The motion of a particle in a straight line is defined by the relation $x = t^4 - 12t^2 - 40$ where *x* is in metres and *t* is in sec. Determine the position *x*, velocity *v* and accleration *a* of the particle at $t = 2$sec.

42. A point moves in a straight line so that its distance from the start in time t is equal to $s = \frac{1}{4}t^4 - 4t^3 + 16t^2$.
    (A) At what times was the point at its starting position?
    (B) At what times is its velocity equal to zero?
43. A body whose mass is 3 kg performs rectilinear motion according to the formula $s = 1 + t + t^2$, where s is measured in centimeters & t in seconds. Determine the kinetic energy $\frac{1}{2}mv^2$ of the body in 5 sec after its start
44. A force of 40N is responsible for the motion of a body governed by the equation $s = 2t + 2t^2$ where s is in metres and t in sec. What is the momentum of the body at $t = 2$ sec?
    [**Hint:** Find acc. then $m = F/a$ & $p = mv$]
45. The angle rotated by a disc is given by $\theta = \frac{2}{3}t^3 - \frac{25}{2}t^2 + 77t + 5$, where θ is in rad. and t in seconds.
    (A) Find the times at which the angular velocity of the disc is zero.
    (B) Its angular acceleration at these times.
46. Temperature of a body varies with time as $T = (T_0 + \alpha t^2 + \beta \sin t)K$, where T_0 is the temperature in Kelvin at $t = 0$ sec. & $\alpha = 2/\pi$. K/s^2 & $\beta = -4$ K, then rate of change of temperature at $t = \pi$sec. is
    (A) $8\,K$ (B) 8^0K (C) $8K$/sec (D) 8^0K/sec
47. A particle moves in a straight line, according to the law $x = 4a\left[t + a\sin\left(\frac{t}{a}\right)\right]$, where x is its position in meters, t in sec. & a is some constants, then the velocity is zero at
    (A) $x = 4a^2\pi$ meters (B) $t = \pi$ sec. (C) $t = 0$ sec (D) none
48. A point moves in a straight line so that its displacement is x m at time t sec, given by $x^2 = t^2 + 1$. Its acceleration in m/s^2 at time t sec is:
    (A) $\frac{1}{x}$ (B) $\frac{1}{x} - \frac{1}{x^2}$ (C) $-\frac{t}{x^2}$ (D) $-\frac{t^2}{x^3}$
    (E) $\frac{1}{x^3}$
49. A particle moves in space such that
    $x = 2t^3 + 3t + 4$; $y = t^2 + 4t - 1$; $z = 2\sin \pi t$
    where x, y, z are measured in meter and t in second. The acceleration of the particle at $t = 3s$ is
    (A) $36\hat{i} + 2\hat{j} + \hat{k}\ ms^{-2}$ (B) $36\hat{i} + 2\hat{j} + \pi\hat{k}\ ms^{-2}$
    (C) $36\hat{i} + 2\hat{j}\ ms^{-2}$ (D) $12\hat{i} + 2\hat{j}\ ms^{-2}$
50. The velocity of a particle moving in straight line depends on it's position as $v = 3\sin\left(x + \frac{\pi}{2}\right)$ m/s. Find the acceleration of the particle when he is at $x = \frac{\pi}{4}$.

51. In a certain interval of time, the position of a particle is represented by x at time t. Find the velocity of the particle at time t.

(A) $x = \dfrac{t \ln t}{e^t}$ (B) $x = \dfrac{t^2 - 1}{\tan t}$

52. The length, breadth & height of a cuboid depends on time t as
$L = 1 + \sin t; \quad b = t^2 - 1; \quad h = (t + 1)$

find the rate of change of volume with time at $t = \dfrac{\pi}{2}$ sec.

53. A body of mass 1 kg moves in x-y plane such that its position vector is given by $\vec{r} = \sin(t)\hat{i} + \cos(t)\hat{j}$

(i) Find its equation of trajectory.

(ii) Find the velocity of the particle at the initial time.

(iii) Find the component of its acceleration in the direction of velocity at time t.

(iv) Find angle between velocity vector and acceleration vector at time t.

54. The position vector of a body of mass $m = 6kg$ is given as $r = i(3t^2\text{-}6t) + j(\text{-}4t^3)$m. Find:

(A) the force ($F = ma$) acting on the particle

(B) the power ($P = F \cdot v$) generated by the force.

(C) the momentum($p = mv$)

55. A wheel rotates so that the angle of rotation is proportional to the square of time. The first revolution was performed by the wheel for 8 sec. Find the angular velocity ω, 32 sec after the wheel started.

[**Hint:** Consider $\theta = kt^2$, find k]

Answer Key

1. (a) A, (b) A

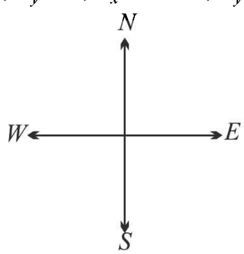
2. AD

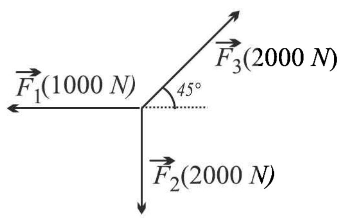
3. BC

4. $\dfrac{8}{\sqrt{5}}$

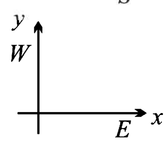
5. $a_x = 3$, $a_y = 0$; $b_x = 2\sqrt{3}$, $b_y = 2$; $c_x = \text{-}5$, $c_y = 5\sqrt{3}$; $p = \text{-}20/3$, $q = 5\sqrt{3}/2$

6. (A)





(B)



$\vec{F}_1 = -1000\hat{i}$ (N),

$\vec{F}_2 = -2000\hat{j}$ (N),

$\vec{F}_3 = 2000(\cos 45°\ \hat{i} + \sin 45°\ \hat{j})$ (N) $= 1000\sqrt{2}\ \hat{i} + 1000\sqrt{2}\ \hat{j}$

(C) $\vec{F}_{\text{resultant}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 1000(\sqrt{2}-1)\hat{i} - 1000\ (\sqrt{2}-1)\hat{j}N$

$F_x = 1000\ (\sqrt{2} - 1)\ N$

$F_y = -1000(2 - \sqrt{2})\ N$

$|\vec{F}| = \sqrt{F_x^2 + F_y^2} = 1000(\sqrt{2} - 1) \times 3 = 3000\ (\sqrt{2} - 1)\ N$

7. $10\sqrt{3}\ N$

8. $= |-4\vec{i} - 3\vec{j}| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{25} = 5\ m$

9. $|\vec{v}| = \sqrt{52}\ m/s$, angle with vertical = $\tan^{-1}(3/2)$ towards west

10. *B*

11. *A*

12. *C*

14. *D*

15. $16\hat{i} - 30\hat{k}$, $198\ J$

16. $3.75\ J$

17. $T_x = \frac{T}{2}; T_y = \frac{T\sqrt{3}}{2}$ $T_y' = \frac{T\sqrt{3}}{2}$; $T_y' = \frac{T}{2}$

$W_x = 0$; $W_y = -W$; $W_X' = -\frac{W}{2}$; $W_y' = -\frac{W\sqrt{3}}{2}$; $N_x = -\frac{N}{2}$; $N_y = \frac{N\sqrt{3}}{2}$; $N_X' = 0$; $N_y' =$ N

18. $2(7\hat{i} + 24\hat{j})$

19. *A*

20. *D*

21. *C*

22. $\vec{F} = 10\hat{i} + 21\hat{j} + 30\hat{k}$, $\vec{d} = 2\hat{i} + 4\hat{j} - 2\hat{k}$, $w = 44J$

23. $45\ i - 151\ j$

24. 5 units.

25. $P_{\parallel} = \frac{3P}{5}, W_{\parallel} = \frac{4W}{5}, T_{\parallel} = T, P_{\perp} = \frac{4P}{5}, W_{\perp} = \frac{3W}{5}, T_{\perp} = 0$

26. $7\sqrt{2}$ units

27. $\theta = 150°$

28. *A*

29. *A*

30. B

31. (A) $\vec{F}_2 = 30\hat{i} - 30\hat{j}$ or $30\sqrt{2}$ at 45°

(B) $\vec{v} = (60\hat{i} + 80\hat{j})$ or 100 at 53° from horizontal

(C) $\vec{F}_3 = -30\hat{i} - 40\hat{j}$ or 50 *N* at $\theta = 233°$ *ACW*

$\theta = 127°$ *CW*

18. $2(7\hat{i} + 24\hat{j})$

19. *A*

20. *D*

21. *C*

32. $(31i + 40j + 120k)\ m$

33. $\frac{12\hat{i} + 5\hat{j}}{13}$, $20\ m/s$

34. (A) $11\hat{i}+5\hat{j}-7\hat{k}$, (B) $\cos^{-1}\left(\frac{-7}{\sqrt{195}}\right)$, (C) $\cos^{-1}\left(\frac{-20}{\sqrt{1309}}\right)$

35.	10 cm^2/sec	36.	*C*
37.	1 *kg m*/sec.	38.	186.25 *N*
40.	13 *rad/s*	41.	-72, -16, 24
42.	(A) 0, 8 sec (B) 0, 4, 8 sec	43.	1.815×10^5 ergs
44.	100 *kgm/s*	45.	(A) 7, $\frac{11}{2}$ (B) 3, – 3
46.	*C*	47.	*A*
48.	*E*	49.	*C*
50.	$\frac{-9}{2} m/s^2$	52.	$\frac{3}{2}\pi^2 + 2\pi - 2$

53. $x^2+y^2 = 1,\ \hat{i}, 0, \frac{\pi}{2}$

54. (A) $36\hat{i} - 144t\hat{j}$, (B) $216t + 1728t^3 - 216$, (C) $(36t-36)\hat{i} - 72t^2\hat{j}$

55. 2π rad/sec.

UNITS AND DIMENSIONS

Physical Quantities

The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities. Till class *x* we have studied many physical quantities eg. length, velocity, acceleration, force, time, pressure, mass, density etc.

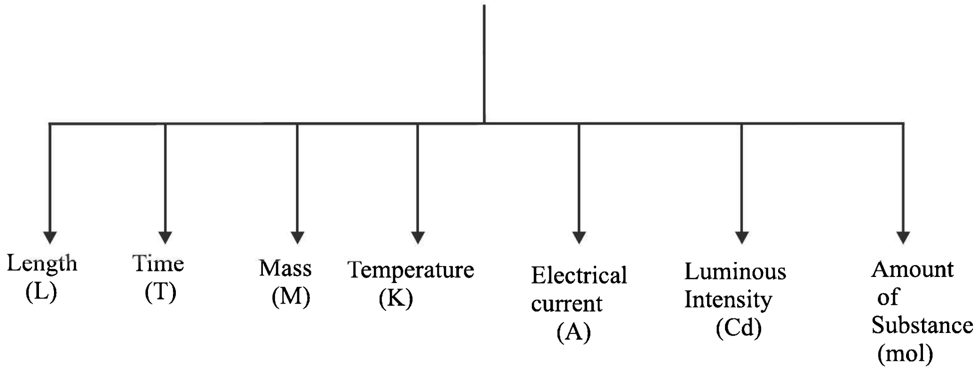
Physical quantities are of three types

- Fundamental or Basic Quantities
- Derived Quantities
- Supplementary Quantities

1. Fundamental (Basic) Quantities:

- These are the elementary quantities which covers the entire span of physics.
- Any other quantities can be derived from these.
- All the basic quantities are chosen such that they should be different, that means independent of each other. (i.e., distance, time and velocity cannot be chosen as basic quantities as $V = \frac{d}{t}$).

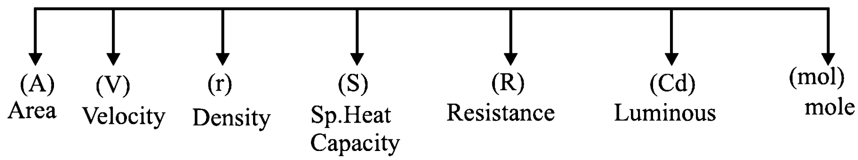
An International Organization named CGPM: General Conference on weight and Measures, chose seven physical quantities as basic or fundamental.



These are the elementary quantities (in our planet) that's why chosen as basic quantities.

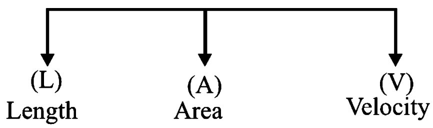
In fact any set of independent quantities can be chosen as basic quantities by which all other physical quantities can be derived.

i.e.,



Can be chosen as basic quantities (on some other planet, these might also be used as basic quantities)

But



cannot be used as basic quantities as

Area = (Length)^2 so they are not independent.

2. Derived Quantities

Physical quantities which can be expressed in terms of basic quantities (M,L,T....) are called derived quantities.

i.e., Momentum

$$P = mV$$

$$= (m)\frac{displacement}{time} = \frac{ML}{T} \; M^1 L^1 T^{-1}$$

Here $[M^1 L^1 T^{-1}]$ is called dimensional formula of momentum, and we can say that momentum has

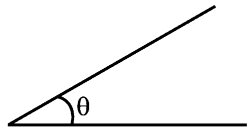
1 Dimension in M (mass)

1 Dimension in L (meter)

and – 1 Dimension in T (time)

The representation of any quantity in terms of basic quantities (M,L,T....) is called dimensional formula and in the representation, the powers of the basic quantities are called dimensions.

3. Supplementary Quantities

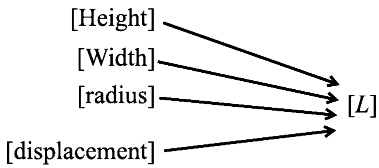


Besides seven fundamental quantities two supplementary quantities are also defined. They are

- Plane angle (The angle between two lines)
- Solid angle

Dimensions

- Height, width, radius, displacement etc. are a kind of length. So we can say that their dimension is $[L]$



here [Height] can be read as "Dimension of Height"

- Area = Length × Width

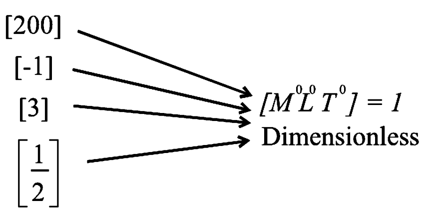
$$[\text{Area}] = [\text{Length}] \times [\text{Width}] = [L] \times [L] = [L^2]$$

For circle

$$\text{Area} = \pi r^2$$

$$[\text{Area}] = [\pi]\,[r^2] = [1]\,[L^2] = [L^2]$$

Here π is not a kind of length or mass or time so π shouldn't effect the dimension of Area. Hence its dimension should be 1 ($M^0L^0T^0$) and we can say that it is dimensionless. From similar logic we can say that all the numbers are dimensionless.



- [Volume] = [Length] × [Height] × [Width]

$$= L \times L \times L = [L^3]$$

For sphere

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$[\text{Volume}] = \left[\frac{4}{3}\pi\right][r^3]$$

$$= (1)\ [L^3] = [L^3]$$

So dimension of volume will be always $[L^3]$ whether it is volume of a cuboid or volume of sphere.

Dimension of a physical quantity will be same, it doesn't depend on which formula we are using for that quantity.

- Density = $\dfrac{mass}{volume}$

  $$[\text{Density}] = \frac{[mass]}{[volume]} = \frac{M}{L^3} = [M^1L^{-3}]$$

- Velocity (V) = $\dfrac{displacement}{time}$

  $$[V] = \frac{[Displacement]}{[time]} = \frac{L}{T} = [M^0L^1T^{-1}]$$

- Acceleration (a) = $\dfrac{dV}{dt}$

  $$[a] = = \frac{LT^{-1}}{T} = LT^{-2}$$

- Momentum $(P) = mV$

  $$[P] = [M]\ [V]$$
  $$= [M]\ [LT^{-1}]$$
  $$= [M^1L^1T^{-1}]$$

- Force $(F) = ma$

  $$[F] = [m]\ [a]$$
  $$= [M]\ [LT^{-2}]$$
  $$= [M^1L^1T^{-2}]$$

- Work or Energy = force × displacement

  [Work] = [force] [displacement]

  $$= [M^1L^1T^{-2}]\ [L]$$
  $$= [M^1L^2T^{-2}]$$

- Power = $\dfrac{work}{time}$

  $$[\text{Power}] = \frac{[work]}{[time]} = \frac{M^1L^2T^{-2}}{T} = [M^1L^2T^{-3}]$$

- Pressure = $\dfrac{Force}{Area}$

$$[\text{Pressure}] = \frac{[Force]}{[Area]} = \frac{M^1L^1T^{-2}}{L^2} = M^1L^{-1}T^{-2}$$

1. Dimensions of Angular Quantities:

- Angle (θ)

(Angular displacement) $\theta = \frac{Arc}{radius}$

$$[\theta] = \frac{[Arc]}{[radius]} = \frac{L}{L} = [M^0L^0T^0] \text{ (Dimensionless)}$$

- Angular velocity $(\omega) = \frac{\theta}{t}$

$$[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = [M^0L^0T^{-1}]$$

- Angular acceleration $(\alpha) = \frac{d\omega}{dt}$

$$[\alpha] = \frac{[d\omega]}{[dt]} = \frac{M^0L^0T^{-1}}{T} = [M^0L^0T^{-2}]$$

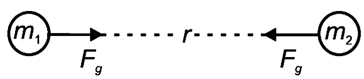
- Torque = Force × Arm length

[Torque] = [force] × [arm length]

$= [M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$

2. Dimensions of Physical Constants

- **Gravitational Constant:**



If two bodies of mass m_1 and m_2 are placed at r distance, both feel gravitational attraction force, whose value is,

Gravitational force $F_g = \frac{Gm_1m_2}{r^2}$

where G is a constant called Gravitational constant

$$[F_g] = \frac{[G][m_1][m_2]}{[r^2]}$$

$$[M^1L^1T^{-2}] = \frac{[G][M][M]}{[L^2]}$$

$$[G] = M^{-1}L^3T^{-2}$$

- **Specific heat capacity:** To increase the temperature of a body by ΔT, Heat required is $Q = ms\,\Delta T$

Here s is called specific heat capacity.

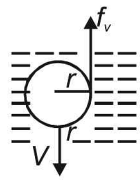
$$[Q] = [m]\,[s]\,[\Delta T]$$

Here Q is heat: A kind of energy so $[Q] = M^1L^2T^{-2}$

$$[M^1L^2T^{-2}] = [M]\,[S]\,[K]$$

$$[S] = [M^0L^2T^{-2}K^{-1}]$$

- **Coefficient of viscosity:** If any spherical ball of radius r moves with velocity v in a viscous Liquid, then viscous force acting on it is given by



$$F_v = 6\pi\eta rv$$

Here η is coefficient of viscosity

$$[F_v] = [6\pi]\,[\eta]\,[r]\,[v]$$

$$M^1L^1T^{-2} = (1)\,[\eta]\,[L]\,[LT^{-1}]$$

$$[\eta] = M^1L^{-1}T^{-1}$$

- **Planck's constant:** If light of frequency υ is falling, energy of a photon is given by

$$E = h\upsilon$$

Here h = Planck's constant

$$[E] = [h]\,[\upsilon]$$

$$\upsilon = \text{frequency} = \frac{1}{Time\ Period}$$

$$\Rightarrow \quad [\upsilon] = \frac{1}{[Time\ Period]} = \left[\frac{1}{T}\right]$$

so

$$M^1L^2T^{-2} = [h]\,[T^{-1}]$$

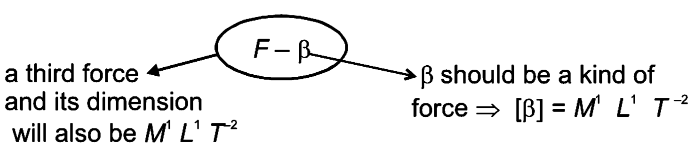
$$[h] = M^1L^2T^{-1}$$

3. Some Special Features of Dimensions

- Suppose in any formula, $(L + a)$ term is coming (where L is length). As length can be added only with a length, so α should also be a kind of length.

  So $[a] = [L]$

- Similarly consider a term $(F - \beta)$ where F is force. A force can be added/substracted with a force only and give rise to a third force. So β should be a kind of force and its result $(F - \beta)$ should also be a kind of force.



Rule No. 1

One quantity can be added / substracted with a similar quantity only and give rise to the similar quantity.

Example 1. $\frac{\alpha}{t^2} = \text{Fv} + \frac{\beta}{x^2}$

Find dimension formula for [α] and [β]

(here t = time, F = force, V = velocity, x = distance)

Solution Since $[Fv] = M^1L^2T^{-3}$,

so $\left[\frac{\beta}{x^2}\right]$ should also be $M^1L^2T^{-3}$

$$\frac{[\beta]}{[x^2]} = M^1\,L^2T^{-3}$$

$$[\beta] = M^1L^4T^{-3}$$

and $\left[Fv + \frac{\beta}{x^2}\right]$ will also have dimension $M^1L^2T^{-3}$

so $$\frac{[\alpha]}{[t^2]} = M^1L^2T^{-3}$$

$$[\alpha] = M^1L^2T^{-1}$$

Example 2. For n moles of gas, Vander waal's equation is $\left(P - \frac{a}{V^2}\right)(V - b) = nRT$

Find the dimensions of a and b, where P is gas pressure, V = volume of gas T = temperature of gas

Solution $$\left(P - \frac{a}{V^2}\right) \qquad (v - b) \qquad = \qquad nRT$$

↑ should be a kind of pressure ↑ should be a kind of volume

So $$\frac{[a]}{[V^2]} = M^1L^{-1}T^{-2}$$

So $$[b] = L^3$$

$$\frac{[a]}{[L^3]^2} = M^{-1}\,L^{-1}\,T^{-2}$$

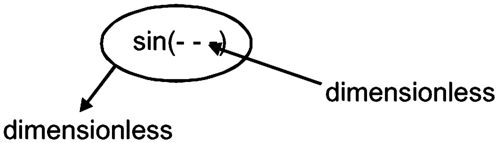
$\Rightarrow$ $$[a] = M^1\,L^5\,T^{-2}$$

Rule No. 2

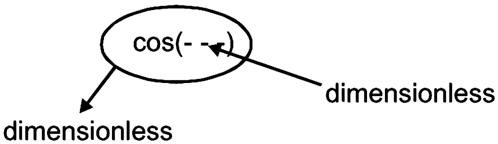
Consider a term sin(θ)

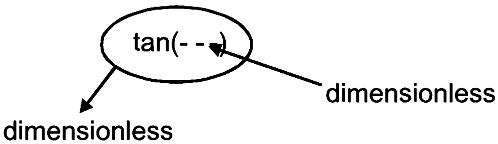
Here θ is dimensionless and sinθ $\left(\dfrac{Perpendicular}{Hypoteneous}\right)$ is also dimensionless.

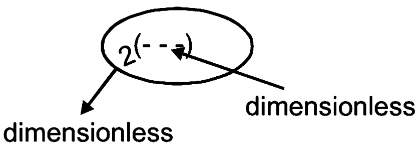
⇒ Whatever comes in sin(......) is dimensionless and entire [sin (.......)] is also dimensionless.

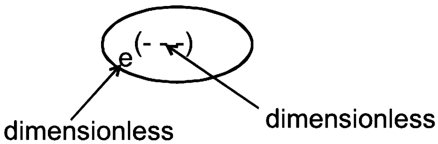
⇒ 

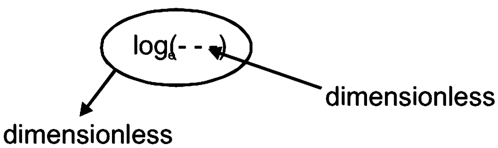
Similarly:

⇒ 

⇒ 

⇒ 

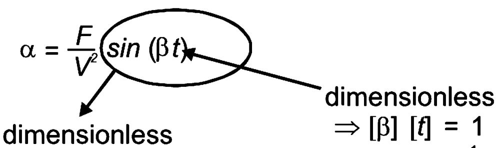
⇒ 

⇒ 

Example **3.** $\alpha = \dfrac{F}{V^2}\sin(\beta t)$ (here V = velocity, F = force, t = time)

Find the dimension of α and β

Solution



$\Rightarrow [\beta]\,[t] = 1$

$[\beta] = [T^{-1}]$

So $[\alpha] = \dfrac{[F]}{[V^2]} = \dfrac{[M^1L^1T^{-2}]}{[L^1T^{-1}]^2} = M^1L^{-1}T^0$

Example 4. $\alpha = \dfrac{FV^2}{\beta^2}\log_e\left(\dfrac{2\pi\beta}{V^2}\right)$ where F = force, V = velocity

Find the dimensions of α and β.

Solution

$$\alpha = \frac{FV^2}{\beta^2}\left(\log_e \frac{2\pi\beta}{V^2}\right)$$

dimensionless   dimensionless

$$\Rightarrow \quad [\alpha] = \frac{[F][V^2]}{[\beta^2]}$$

$$\Rightarrow \quad \frac{[2\pi][\beta]}{[V^2]} = 1$$

$$\Rightarrow \quad \frac{[1][\beta]}{L^2T^{-2}} = 1$$

$$\Rightarrow \quad [\beta] = L^2T^{-2}$$

$$\Rightarrow \quad [\alpha] = \frac{[M^1L^1T^{-2}][L^2T^{-2}]}{[L^2T^{-2}]^2}$$

$$\Rightarrow \quad [\alpha] = M^1L^{-1}T^0$$

4. Uses of Dimensions

- **To check the correctness of the formula:** If the dimensions of the L.H.S and R.H.S are same, then we can say that this eqn. is at least dimensionally correct. So this equation may be correct.

  But if dimensions of L.H.S and R.H.S is not same then the equation is not even dimensionally correct.

  So it cannot be correct.

  ei → A formula is given centrifugal force

  $$F_e = \frac{mv^2}{r} \qquad \text{(where } m = \text{mass, } v = \text{velocity, } r = \text{radius)}$$

  we have to check whether it is correct or not.

  Dimension of L.H.S is

  $$[F] = [M^1L^1T^2]$$

  Dimension of R.H.S is

  $$\frac{[m]\,[v^2]}{[r]} = \frac{[M]\,[LT^{-1}]^2}{[L]} = [M^1L^1T^{-2}]$$

  So this eqn. is at least dimensional correct.

  ⇒ we can say that this equation may be correct.

Example 5. Check whether this equation may be correct or not

$$\text{Pressure} = P_r \frac{3FV^2}{\pi^2 t^2 x}$$

Solution $P_r = \dfrac{3\,FV^2}{\pi^2 t^2 x}$ (where F = force, V = velocity, t = time, x = distance)

Dimension of L.H.S = $[P_r] = M^1L^{-1}T^{-2}$

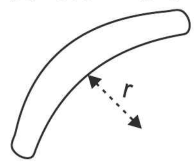
Dimension of R.H.S = $\dfrac{[3]\,[F]\,[v^2]}{[\pi]\,[t^2]\,[x]}$

$$= \frac{[M^1L^1T^{-2}]\,[L^2T^{-2}]}{[T^2]\,[L]} = M^1L^2T^{-6}$$

Dimension of L.H.S and R.H.S are not same. So the relation cannot be correct.

Sometimes a question is asked which is beyond our syllabus, then certainly it must be the question of dimensional analysis.

Example 6. A Boomerang has mass m surface Area A, radius of curvature of lower surface = r and it is moving with velocity V in air of density ρ. The resistive force on it should be –



(A) $\dfrac{2\rho VA}{r^2}\log\left(\dfrac{\rho m}{\pi Ar}\right)$ (B) $\dfrac{2\rho V^2A}{r}\log\left(\dfrac{\rho A}{\pi m}\right)$

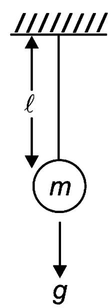
(C) $\dfrac{2\rho V^2A}{r}\log\left(\dfrac{\rho Ar}{\pi m}\right)$ (D) $\dfrac{2\rho V^2A}{r^2}\log\left(\dfrac{\rho Ar}{\pi m}\right)$

Solution Only C is dimensionally correct.

- **We can derive a new formula roughly:**

  If a quantity depends on many parameters, we can estimate, to what extent, the quantity depends on the given parameters !

Example 7. So we can say that expression of T should be in this form



$$T = (\text{Some Number})\,(m)^a\,(\ell)^b (g)^c$$

$$M^0L^0T^1 = (1)\,[M^1]^a\,[L^1]^b\,[L^1T^{-2}]^c$$

$$M^0L^0T^1 = M^a\,L^{b+c}\,T^{-2c}$$

Comparing the powers of M, L and T,

get $a = 0,\ b + c = 0,\ -2c = 1$

so $a = 0,\ b = \dfrac{1}{2},\ c = \dfrac{1}{2}$

so $T = (\text{some Number})\,M^0\,L^{1/2}\,g^{-1/2}$

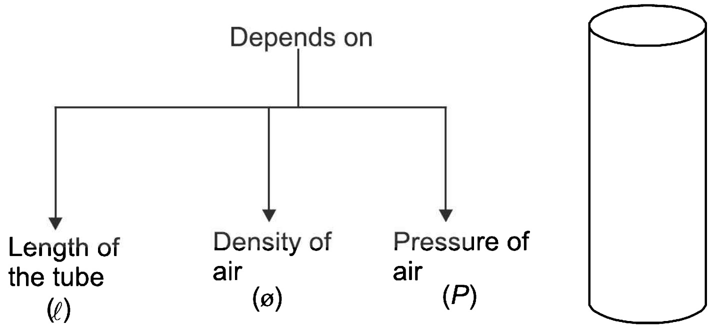
$$T = (\text{Some Number})\sqrt{\frac{\ell}{g}}$$

The quantity "Some number" can be found experimentally. Measure the length of a pendulum and oscillate it, find its time period by stopwatch.

Suppose for $\ell = 1m$, we get $T = 2$ sec. so

$2 = (\text{Some Number})\sqrt{\dfrac{1}{9.8}}$ $\Rightarrow$ "Some number" $= 6.28 \approx 2\pi$.

Example 8. Natural frequency (f) of a closed pipe



So we can say that $f = (\text{some Number})\ (\ell)^a\ (\rho)^b\ (P)^c$

Equating dimensions of both the sides get

$a = -1,\ b = -1/2,\ c = 1/2$

So $f = (\text{some number})\ \dfrac{1}{\ell}\sqrt{\dfrac{P}{\rho}}$

- **We can express any quantity in terms of the given basic quantities.**

Example 9. If velocity (V), force (F) and time (T) are chosen as fundamental quantities, express (i) mass and (ii) energy in terms of V, F and T

Solution Let $M = (\text{some Number})\ (V)^a\ (F)^b\ (T)^c$

Equating dimensions of both the sides

$$M^1L^0T^0 = (1)\ [L^1T^{-1}]^a\ [M^1L^1T^{-2}]\ [T^1]^c$$

$$M^1L^0T^0 = M^b\ L^{a+b}\ T^{-a-2b+c}$$

get $a = -1,\ b = 1,\ c = 1$

$$M = (\text{Some Number})\ (V^{-1}\ F^1\ T^1) \quad \Rightarrow \quad [M] = [V^{-1}\ F^1\ T^1]$$

Similarly we can also express energy in terms of V, F, T

Let $[E] = [\text{some Number}]\ [V]^a\ [F]^b\ [T]^c$

$\Rightarrow$ $[MLT^{-2}] = [M^0L^0T^0]\ [LT^{-1}]^a\ [MLT^{-2}]^b\ [T]^c$

$\Rightarrow$ $[M^1L^1T^{-2}] = [M^b\ L^{a-2b+c}\ T^{-a-2b+c}]$

$\Rightarrow$ $1 = b;\ 1 = a - 2b + c;\ -2 = -a - 2b + c$

get $a = 1\ ;\ b = 1\ ;\ c = 1$

$\therefore$ $E = (\text{some Number})\ V1F1T1$ or $[E] = [V^1][F^1][T^1]$.

- **To find out unit of a physical quantity:**

***Example* 10.** [Force] = $[M^1L^1T^{-2}]$

As unit of M is kilogram (kg), unit of L is meter (m) and unit of T is second (s) so unit force can be written as = $(kg)^1$ $(m)^1$ $(s)^{-2}$ = $kg\ m/s$ in *MKS* system.

In CGS system, unit of force can be written as = $(g)^1$ (cm)^1 $(s)^{-1}$ = g cm/s.

Limitations of Dimensional Analysis:

From Dimensional analysis we get T = *(Some Number)* $\sqrt{\dfrac{\ell}{g}}$

so the expression of T can be

$$T = 2\sqrt{\frac{\ell}{g}} \qquad\qquad T = \sqrt{\frac{\ell}{g}}\ \sin(.....)$$

or   or

$$T = 50\sqrt{\frac{\ell}{g}} \qquad\qquad T = \sqrt{\frac{\ell}{g}}\ \log(......)$$

or   or

$$T = 2\pi\sqrt{\frac{\ell}{g}} \qquad\qquad T = \sqrt{\frac{\ell}{g}} + (t_0)$$

- Dimensional analysis doesn't give information about the "some Number":The dimensional constant.
- This method is useful only when a physical quantity depends on other quantities by multiplication and power relations.

  (i.e., $f = x^a y^b z^c$)

  It fails if a physical quantity depends on sum or difference of two quantities

  (i.e.$f = x + y - z$)

  i.e., we cannot get the relation

  $$S = ut + \frac{1}{2}at^2 \quad \text{from dimensional analysis.}$$

- This method will not work if a quantity depends on another quantity as sine or cosine, logarithmic or exponential relation. The method works only if the dependence is by power functions.
- We equate the powers of M, L and T hence we get only three equations. So we can have only three variable (only three dependent quantities)

  So dimensional analysis will work only if the quantity depends only on three parameters, not move than that.

Example 11. Can Pressure (P), density (ρ) and velocity (v) be taken as fundamental quantities ?

Solution P, ρ and v are not independent, they can be related as $P = \rho v^2$, so they cannot be taken as fundamental variables.

To check whether the 'P', 'ρ', and 'V' are dependent or not, we can also use the following mathematical method:

$$[P] = [M^1 L^{-1} T^{-2}]$$
$$[\rho] = [M^1 L^{-3} T^0]$$
$$[V] = [M^0 L^1 T^{-1}]$$

Check the determinates of their powers:

$$= 1\,(3) - (-1)(-1) - 2\,(1) = 0,$$

So these three terms are dependent.

Unit

Measurement of any physical quantity is expressed in terms of an internationally accepted certain basic standard called unit.

SI Units: In 1971, an international Organization "CGPM": (General Conference on weight and Measure) decided the standard units, which are internationally accepted. These units are called SI units (International system of units)

1. SI units of basic quantities

Base Quantity	*SI Units*		
	Name	*Symbol*	*Definition*
Length	meter	m	The meter is the length of the path traveled by light in vacuum during a time interval of 1/299, 792, 458 of a second (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)
Time	second	s	The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967)
Electric Current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2 x 10-7 Newton per metre of length. (1948)

Base Quantity	*SI Units*		
	Name	*Symbol*	*Definition*
Thermodynamic Temperature	kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of Substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous Intensity	candela	Cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 x 1012 hertz and that has a radiant intensity in that direction of 1/683 watt per steradian (1979).

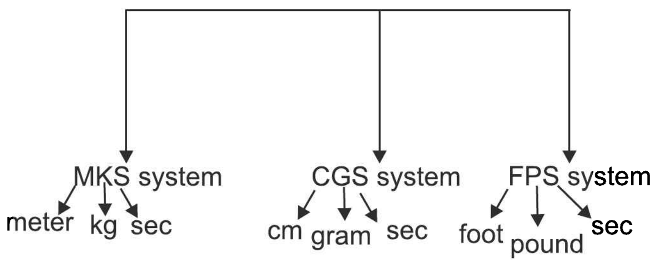
2. Two supplementary units were also defined

- Plane angle – Unit = radian (rad)
- Solid angle – Unit = Steradian (sr)

3. Other classification

If a quality involves only length, mass and time (quantities in mechanics), then its unit can be written in MKS, CGS or FPS system.

- **For MKS system:** In this system Length, mass and time are expressed in meter, *kg* and sec. respectively. It comes under SI system.
- **For CGS system:** In this system, Length, mass and time are expressed in cm, gram and sec. respectively.



- **For FPS system:** In this system, length, mass and time are measured in foot, pound and sec. respectively.

4. SI units of derived quantities

- Velocity = $\frac{\text{displacement}}{\text{time}}$ $\begin{matrix}\rightarrow \text{meter} \\ \rightarrow \text{second}\end{matrix}$

  So unit of velocity will be m/s

- Acceleration = $\frac{\textit{change} \text{ in velocity}}{\textit{time}} = \frac{m/s}{s} = \frac{m}{s^2}$

- Momentum = mV

  so unit of momentum will be = $(kg)\ (m/s) = kg\ m/s$

- Force = ma

  Unit will be = $(kg) \times (m/s^2) = kg\ m/s^2$ called newton (N)

- Work = FS

  unit = $(N) \times (m) = N\,m$ called joule (J)

- Power = $\dfrac{work}{time}$

  Unit = J / s called watt (w)

5. Units of some physical constants

- Unit of "Universal Gravitational Constant" (G)

$$F = \frac{G(m_1)(m_2)}{r^2} \quad \Rightarrow \quad \frac{kg \times m}{s^2} = \frac{G(kg)(kg)}{m^2}$$

so unit of $G = \dfrac{m^3}{\text{kg } s^2}$

- **Unit of specific heat capacity (S)**

$$Q = ms\,\Delta T$$

$$J = (kg)\,(S)\,(K)$$

$$\text{Units of } S = J / kg\,K$$

- **Unit of μ_0**

force per unit length between two long parallel wires is: $\dfrac{F}{\ell} = \dfrac{\mu_0}{4\pi}\dfrac{i_1 i_2}{r^2}$

$$\frac{N}{m} = \frac{\mu_0}{(1)}\frac{(A)\,(A)}{(m^2)} \qquad \text{Unit of } \mu_0 = \frac{N.m}{A^2}$$

6. SI prefix

- Suppose distance between kota to Jaipur is 3000 m. so

$$d = 3000 \text{ m} = 3 \times \underset{\text{kilo (k)}}{\underset{\downarrow}{(1000)}} \text{ m}$$

= 3 km (here 'k' is the prefix used for 1000 (10^3))

- Suppose thickness of a wire is 0.05 m

$$d = 0.05\,m = 5 \times \underset{\text{centi (c)}}{\underset{\downarrow}{(10^{-2})}} \text{ m}$$

= 5 cm (here 'c' is the prefix used for (10^{-2}))

Similarly, the magnitude of physical quantities very over a wide range. So in order to express the very large magnitude as well as very small magnitude more compactly, "CGPM" recommended some standard prefixes for certain power of 10.

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

Example 12. Convert all in meters (m):

(i) $5\ \mu m$. (ii) $3\ km$ (iii) 20 mm (iv) $73\ pm$

(v) $7.5\ nm$

Solution

(i) $5\ \mu m = 5 \times 10^{-6} m$

(ii) $3\ km = 3 \times 10^{3}\ m$

(iii) $20\ \text{mm} = 20 \times 10^{-3} m$

(iv) $73\ pm = 73 \times 10^{-12} m$

(v) $7.5\ nm = 7.5 \times 10^{-9}\ m$

Example 13. $F = 5\ N$ convert it into CGS system

Solution

$$F = 5\ \frac{kg \times m}{s^2}$$

$$= (5)\ \frac{(10^3 g)(100\ \text{cm})}{s^2}$$

$$= 5 \times 10^{5}\ \frac{g\ \text{cm}}{s^3}\ \text{(in } CGS \text{ system).}$$

This unit $\left(\frac{g\ \text{cm}}{s^3}\right)$ is also called dyne

Example 14. $G = 6.67 \times 10^{-11}\ \frac{kg\ \text{m}^3}{s^2}$ convert it into CGS system.

Solution

$$G = 6.67 \times 10^{-11}\ \frac{kg\ \text{m}^3}{s^2}$$

$$= (6.67 \times 10^{-11})\ (1000\ g)\ \frac{(100\ \text{cm})^3}{s^2}$$

$$= 6.67 \times 10^{-2}\ \frac{g\ \text{cm}^3}{s^2}$$

Example 15. $\rho = 2\ g/\text{cm}^3$

convert it into MKS system

Solution $\rho = 2\ g/cm^3$

$$= (2)\ \frac{10^{-3}\ \text{kg}}{(10^{-2}\text{m})^3}$$

$$= 2 \times 10^3\ \frac{kg}{m^3}$$

Example 16. $V = 90\ km$ / hour

convert it into m/s

Solution $V = 90\ km$ / hour

$$= (90)\ \frac{(1000\ \text{m})}{(60 \times 60\ \text{second})}$$

$$V = (90)\left(\frac{1000}{3600}\right)\frac{m}{s}$$

$$V = 90 \times \frac{5}{18}\ \frac{m}{s}$$

$$V = 25\ m/s$$

Point to Remember

To convert $\frac{km}{hour}$ into $\frac{m}{\text{sec}}$, multiply by $\frac{5}{18}$.

Example 17. Convert 7 pm into μm

Solution

Let $7\ pm = (x)\ \mu m$

$$7 \times (10^{-12})\ m = (x)\ x\ 10^{-6} m$$

get $x = 7 \times 10^{-6}$

So $7\ pm = (7 \times 10^{-6})\ \mu m$

Some SI units of derived quantities are named after the scientist, who has contributed in that field a lot

8. SI derived units, named after the scientist

S.N	*Physical Quantity*	*SI Units*			
		Unit name	*Symbol of the unit*	*Expression in terms of other units*	*Expression in terms of base units*
1.	Frequency ($f = \frac{1}{T}$)	hertz	Hz	$\frac{\text{Oscilliation}}{S}$	s^{-1}
2.	Force ($F = ma$)	Newton	N	- - - - -	Kg m / s^2

S.N	Physical Quantity	SI Units			
		Unit name	Symbol of the unit	Expression in terms of other units	Expression in terms of base units
3.	Energy, Work, Heat ($W = Fs$)	Joule	J	Nm	Kg m^2 / s^2
4.	Pressure, stress ($P = \frac{F}{A}$)	Pascal	Pa	N / m^2	Kg / $m\ s^2$
5.	Power, ($Power = \frac{W}{t}$)	watt	W	J / s	Kg m^2 / s^3
6.	Electric charge ($q = it$)	coulomb	C	- - - - - -	$A\ s$
7.	Electric Potential Emf. ($V = \frac{U}{q}$)	volt	V	J / C	Kg m^3 / s3 A
8.	Capacitance ($C = \frac{q}{v}$)	Farad	F	C / V	A s4 kg -1 m^{-2}
9.	Electrical Resistance ($V = i R$)	ohm	Ω	V / A	kg m2 s-3 A^{-2}
10.	Electrical Conductance ($C = \frac{1}{R} = \frac{i}{V}$)	Siemens (mho)	S, ℧	A / V	kg^{-1} m-2 s^3 A^2
11.	Magnetic field	Tesla	T	Wb / m^2	kg s^{-2} A^{-1}
12.	Magnetic flux	Weber	Wb	$V\ s$ or J/A	kg m^2 s^{-2} A^{-1}
13.	Inductance	Henry	H	Wb / A	kg m^2 s-2 A_{-2}

S.N	Physical Quantity	SI Units			
		Unit name	Symbol of the unit	Expression in terms of other units	Expression in terms of base units
14.	Activity of radioactive material	Becquerel	*Bq*	$\frac{\text{D sintegration}}{\text{second}}$	s_{-1}

9. Some SI units expressed in terms of the special names and also in terms of base units:

Physical Quantity	SI Units	
	In terms of special names	In terms of base units
Torque (τ = Fr)	$N\,m$	Kg m^2 / s^2
Dynamic Viscosity ($F_v = q\,A\,\frac{dv}{dr}$)	Poiseiulles (Pℓ) or $Pa\,s$	$Kg / m\,s$
Impulse (J = F Δt)	$N\,s$	$Kg\,m / s$
Modulus of elasticity (Y = $\frac{\text{stress}}{\text{strain}}$)	N / m^2	$Kg / m\,s^2$
Surface Tension Constant (T = $\frac{F}{l}$)	N/m or J/m^2	Kg / s^2
Specific Heat capacity (Q = msΔT)	$J/kg\,K$ (old unit s $\frac{cAL}{g°C}$)	$m^2 s^{-2} K^{-1}$
Thermal conductivity ($\frac{dQ}{dt}$ = KA $\frac{dT}{dr}$)	$W / m\,K$	$m\,kg\,s^{-3} K^{-1}$
Electric field Intensity E = $\frac{F}{q}$	V/m or N/C	$m\,kg\,s^{-3} A^{-1}$
Gas constant (R) (PV = nRT) or molar Heat Capacity (C = $\frac{Q}{M\Delta T}$)	$J / K\,mol$	$m^2\,kg\,s^{-2}\,k^{-1}\,mol^{-1}$

Change of numerical value with the change of unit:

Suppose we have

$$\ell = 7\text{ cm} \xrightarrow[\textit{it}\text{ into meters, we get}]{\textit{If}\text{ we convert}} = \frac{7}{100}m$$

we can say that if the unit is increased to 100 times (cm $\rightarrow$ *m*), the numerical value became

$$\frac{1}{100} \text{ times } \left(7 \rightarrow \frac{7}{100}\right)$$

So we can say

Numerical value $\propto \dfrac{1}{unit}$

we can also tell if in a formal way like the following: –

Magnitude of a physical quantity = (Its Numerical value) (unit) = (*n*) (*u*)

Magnitude of a physical quantity always remains constant, it wont change if we express it in some other unit.

So

$$(n)\,(u) = \text{constant}$$

$$n \propto \frac{1}{u} \qquad n_1u_1 = n_2u_2$$

or Numerical value $\propto \dfrac{1}{unit}$

Example 18. If unit of length is doubled, the numerical value of Area will be.. ....

Solution As unit of length is doubled, unit of Area will become four times. So the numerical value of Area will became one fourth. Because numerical value $\propto \dfrac{1}{unit}$,

Example 19. Force acting on a particle is 5*N*. If unit of length and time are doubled and unit of mass is halved than the numerical value of the force in the new unit will be.

Solution Force = $5\ \dfrac{kg \times m}{sec^2}$

If unit of length and time are doubled and the unit of mass is halved.

Then the unit of force will be $\left(\dfrac{\frac{1}{2} \times 2}{(2)^2}\right) = \dfrac{1}{4}$ times

Hence the numerical value of the force will be 4 times.

SIGNIFICANT FIGURES

Significant figures in the measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement greater is the accuracy of the measurement.

"All accurately known digits in a measurement plus the first uncertain digit together form significant figures."

For example, when we measure the length of a straight line using a metre scale and it lies between 7.4 cm and 7.5 cm, we may estimate it as l = 7.43 cm. This expression has three significant figures out of these 7 and 4 are precisely known but the last digit 3 is only approximately known.

Rules of Counting Significant Figures: For counting significant figures, we use the following rules:

1. All non-zero digits are significant. For example $x = 2567$ has four significant figures.
2. The zeros appearing between two non-zero digits are counted in significant figures. For example 6.028 has 4 significant figures.
3. The zeros occurring to the left of last non-zero digit are *NOT* significant.

   For example 0.0042 has two significant figures.
4. In a number without decimal, zeros to the right of non-zero digit are *NOT* significant. However when some value is recorded on the basis of actual measurement the zeros to the right of non-zero digit become significant. For example $L = 20$ m has two significant figures but $x = 200$ has only one significant figure.
5. In a number with decimal, zeros to the right of last non-zero digits are significant. For example $x = 1.400$ has four significant figures.
6. The powers of ten are *NOT* counted as significant digits. For example 1.4×10^{-7} has only two significant figures 1 and 4.
7. Change in the units of measurement of a quantity does not change the number of significant figures. For example, suppose distance between two stations is 4067 m. It has four significant figures. The same distance can be expressed as 4.067 *km* or 4.067×10^5 cm. In all these, number of significant figures continues to be four.

Example

Measured Value	*Number of significant figures*	*Rule*
12376	5	1
6024.7	5	2
0.071	2	3
410 m	3	4
720	2	4
2.40	3	5
1.6×10^{10}	2	6

Rounding off a digit: Following are the rules for rounding off a measurement:

1. If the number lying to the right of cut-off digits is less than 5, then the cut-off digit is retained as such. However, if it is more than 5, then the cut-off digit is increased by 1.

   For example $x = 6.24$ is rounded off to 6.2 to two significant digits and $x = 5.328$ is rounded off to 5.33 to three significant digits.
2. If the digit to be dropped is 5 followed by digits other than zero then the preceding digit is increased by 1.

   For example $x = 14.252$ is rounded off to $x = 14.3$ to three significant digits.
3. If the digit to be dropped is simply 5 or 5 followed by zeros, then the preceding digit is left unchanged if it is even. For example $x = 6.250$ or $x = 6.25$ becomes $x = 6.2$ after rounding off to two significant digits.

4. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one if it is odd.

   For example $x = 6.350$ or $x = 6.35$ becomes $x = 6.4$ after rounding off two significant digits.

Example

Measured Value	*Rounding off to three significant digits*	*Rule*
7.364	7.36	1
7.367	7.37	1
8.3251	8.33	2
9.445	9.44	3
9.4450	9.44	3
15.75	15.8	4
15.7500	15.8	4

Algebraic Operations with significant Figures: In addition, subtraction, multiplication or division inaccuracy in the measurement of any one variable affects the accuracy of the final result. Hence, in general, the final result shall have significant figures corresponding to their number in the least accurate variable involved. To understand this, let us consider a chain of which all links are strong except the one. The chain will obviously break at the weakest link. Thus the strength of the chain cannot be more than the strength of the weakest link in the chain.

(i) **Addition and Subtraction:** Suppose in the measured values to be added or subtracted, the least number of significant digits after the decimal is n. Then in the sum or difference also, the number of significant digits after the decimal should be n.

   Ex. $1.2 + 3.45 + 6.789 = 11.439 \approx 11.4$

   Here, the least number of significant digits after the decimal is one. Hence, the result will be 11.4 (when rounded off to smallest number of decimal places).

   Ex. $12.63 - 10.2 = 2.43 \approx 2.4$

(ii) **Multiplication or Division:** Suppose in the measured values to be multiplied or divided, the least number of significant digits be n, then in the product or quotient, the number of significant digits should also be n.

   Ex. $1.2 \times 36.72 = 44.064 \approx 44$

   The least number of significant digits in the measured values are two. Hence, the result when rounded off to two significant digits become 44. Therefore the answer is 44.

   Ex. $\frac{1100}{10.2} = 107.8431373 \approx 110$

   As 1100 has minimum number of significant figures (i.e., 2), therefore the result should also contain only two significant digits. Hence, the result when rounded off to two significant digits becomes 110.

Ex. $\frac{1100 \text{ m/s}}{10.2 \text{ m/s}} = 107.8431373 \approx 108$

- *In this case answer becomes 108. Think why?*

ERROR ANALYSIS

No measurement is perfect, as the errors involved in a measurement cannot be removed completely. Measurement value is always somewhat different from the true value. The difference is called an error.

Errors can be classified in two ways. First classification is based on the cause of error. Systematic errors and random errors fall in this group.

Second classification is based on the magnitude of error. Absolute error, mean absolute error and relative (or fractional) error lie on this group. Now let us discuss them separately.

(i) **Systematic errors:** These are the errors whose cause are known to us. Such errors can therefore be minimized. Following are few-causes of these errors.

(A) Instrumental errors may be due to erroneous instruments. These errors can be reduced by using more accurate instruments and applying zero correction, when required.

(B) Sometimes errors arise on account of ignoring certain facts. For example in measuring time period of simple pendulum error may creap because no consideration is taken of air resistance. These errors can be reduced by applying proper corrections to the formula used.

(C) Change in temperature, pressure, humidity etc. may also sometimes cause errors in the result. Relevant corrections can be made to minimized their effects.

(ii) **Random errors:** The causes of random errors are not known. Hence, it is not possible to remove them completely. These errors may arise due to a variety of reasons. For example the reading of a sensitive beam balance may change by the vibrations caused in the building due to persons moving in the laboratory or vehicles running nearby. The random errors can be minimized by repeating the observation a large number of times and taking the arithmetic mean of all the observations. The mean value would be very close to the most accurate reading. Thus,

$$\boxed{a_{mean} = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n}}$$

(iii) **Absolute errors:** The difference between the true value and the measured value of a quantity is called an absolute error. Usually the mean value a_m is taken as the true value. So, if

$$\boxed{a_m = \frac{a_1 + a_2 + \ldots\ldots + a_n}{n}}$$

Then by definition, absolute errors in the measured values of the quantity are,

$$\Delta a_1 = a_m - a_1$$

$$\Delta a_2 = a_m - a_2$$

$$\ldots\ldots \quad \ldots\ldots \quad \ldots\ldots$$

$$\Delta a_n = a_m - a_n$$

Absolute error may be positive or negative.

(iv) **Mean absolute error:** It is the arithmetic mean of the magnitudes of absolute errors. Thus,

$$\Delta a_{mean} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots\ldots + |\Delta a_n|}{n}$$

The final result of measurement can be written as

$$a = a_m \pm \Delta a_{mean}$$

This implies that value of a is likely to lie between $a_m + \Delta a_{\text{mean}}$ and $a_m - \Delta a_{\text{mean}}$.

(v) **Relative or Fractional error:** The ratio of mean absolute error to the mean value of the quantity measured is called relative or fractional error. Thus,

$$\text{Relative error} = \frac{\Delta a_{mean}}{a_m}$$

Relative error expressed in percentage is called as the percentage error i.e.,

$$\text{Percentage error} = \frac{\Delta a_{mean}}{a_m} \times 100$$

COMBINATION OF ERRORS

(i) **Errors in sum or difference:** Let $x = a \pm b$.

Further, let Δa is the absolute error in the measurement of a, Δb the absolute error in the measurement of b and Δx is the absolute error in the measurement of x. Then

$$x + \Delta x = (a \pm \Delta a) \pm (b \pm \Delta b) = (a \pm b) \pm (\Delta a \pm \Delta b) = x \pm (\pm \Delta a + \Delta b)$$

or $\quad \Delta x = \pm \Delta a \pm \Delta b$

The four possible values of Δx are $(\Delta a - \Delta b)$ $(-\Delta a - \Delta b)$ and $(-\Delta a + \Delta b)$ Therefore, the maximum absolute error in x is;

$$\Delta x = \pm(\Delta a + \Delta b)$$

i.e., the maximum absolute error in sum and difference of two quantities is equal to sum of the absolute errors in the individual quantities.

Example 7. The volumes of two bodies are measured to be $V_1 = (10.2 \pm 0.02)\,\text{cm}^3$ and $V_2 = (6.4 \pm 0.01)\,\text{cm}^3$. Calculate sum and difference in volumes with error limits.

Solution $\quad V_1 = (10.2 \pm 0.02)\,cm^3$

and $V_2 = (6.4 \pm 0.01)\,cm^3$

$$\Delta V = \pm(\Delta V_1 + \Delta V_2) = \pm(0.02 + 0.01)\,cm^3 = \pm 0.03\,cm^3$$

$$V_1 + V_2 = (10.2 + 6.4)\ \text{cm}^3 = 16.6\ \text{cm}^3$$

and $\quad V_1 - V_2 = (10.2 - 6.4)\ \text{cm}^3 = 3.8\ \text{cm}^3$

Hence, sum of volumes = $(16.6 \pm 0.03)\,cm^3$

and difference of volumes = $(3.8 \pm 0.03)\,cm^3$

(ii) **Error in a product:** Let $x = ab$

Then $(x \pm \Delta x) = (a \pm \Delta a)(b \pm \Delta b)$

or $x\left(1 \pm \frac{\Delta x}{x}\right) = ab\left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)$

or $1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a} \pm \frac{\Delta a}{a}.\frac{\Delta b}{b}$ (as $x = ab$)

or $\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b} \pm \frac{\Delta a}{a}.\frac{\Delta b}{b}$

Here, $\frac{\Delta a}{a}.\frac{\Delta b}{b}$ is a small quantity, so can be neglected.

Hence, $\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \pm \frac{\Delta b}{b}$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right), \left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right), \left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$ *and* $\left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$

Hence, maximum possible value of

$$\boxed{\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)}$$

Therefore, maximum fractional error in product of two (or more) quantities is equal to sum of fractional errors in the individual quantities.

(iii) **Error in division:** Let $x = \frac{a}{b}$

Then, $x \pm \Delta x = \frac{a \pm \Delta a}{b \pm \Delta b}$ or $x\left(1 \pm \frac{\Delta x}{x}\right) = \frac{a\left(1 \pm \frac{\Delta a}{a}\right)}{b\left(1 \pm \frac{\Delta b}{b}\right)}$

or $\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \pm \frac{\Delta b}{b}\right)^{-1}$ $\left(as\ x = \frac{a}{b}\right)$

As $\frac{\Delta b}{b} << 1$, so expanding binomially, we get

$$\left(1 \pm \frac{\Delta x}{x}\right) = \left(1 \pm \frac{\Delta a}{a}\right)\left(1 \mp \frac{\Delta b}{b}\right) \quad \text{or} \quad 1 \pm \frac{\Delta x}{x} = 1 \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b} \pm \frac{\Delta a}{a}.\frac{\Delta b}{b}$$

Here, $\frac{\Delta a}{a}.\frac{\Delta b}{b}$ is a small quantity, so can be neglected.

Hence, $\pm \frac{\Delta x}{x} = \pm \frac{\Delta a}{a} \mp \frac{\Delta b}{b}$

Possible values of $\frac{\Delta x}{x}$ are $\left(\frac{\Delta a}{a} - \frac{\Delta b}{b}\right), \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right), \left(-\frac{\Delta a}{a} - \frac{\Delta b}{b}\right)$ and $\left(-\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$. Therefore, the maximum value of;

$$\boxed{\frac{\Delta x}{x}=\pm\left(\frac{\Delta a}{a}+\frac{\Delta b}{b}\right)}$$

Or, the maximum value of fractional error in division of two quantities is equal to the sum of fractional errors in the individual quantities.

(iv) **Error in quantity raised to some power:**

Let $x=\frac{a^n}{b^m}$ Then, $\ln(x)=n\ln(A)-m\ln(B)$

Differentiating both sides, we get

$$\frac{dx}{x}=n.\frac{da}{a}-m\frac{db}{b}$$

In terms of fractional error we may write,

$$\pm\frac{\Delta x}{x}=\pm n\frac{\Delta a}{a}\mp m\frac{\Delta b}{b}$$

Therefore maximum value of

$$\frac{\Delta x}{x}=\pm\left(n\frac{\Delta a}{a}+m\frac{\Delta b}{b}\right)$$

Example 8. Calculate percentage error in determination of time period of a pendulum.

$T=2\pi\sqrt{\frac{l}{g}}$ where l and g are measured with ±1% and ±2% errors.

Solution

$$\frac{\Delta T}{T}\times100=\pm\left(\frac{1}{2}\times\frac{\Delta l}{l}\times100+\frac{1}{2}\times\frac{\Delta g}{g}\times100\right)=\pm\left(\frac{1}{2}\times1+\frac{1}{2}\times2\right)=\pm1.5\%$$ **[Ans.]**

Example 9. The mass and density of a solid sphere are measured to be (12.4 ± 0.1) kg and (4.6 ± 0.2) kg/m^3. Calculate the volume of the sphere with error limits.

Solution

Here $m\pm\Delta m=(12.4\pm0.1)\,kg$

and $\rho\pm\Delta\rho=(4.6\pm0.2)\,kg/m^3$

$Volume\,V=\frac{m}{\rho}=\frac{12.4}{4.6}=2.69\,m^3=2.7\,m^3$ (rounding off to one decimal place)

Now, $$\frac{\Delta V}{V}=\pm\left(\frac{\Delta m}{m}+\frac{\Delta\rho}{\rho}\right)$$

or $$\Delta V=\pm\left(\frac{\Delta m}{m}+\frac{\Delta\rho}{\rho}\right)\times V=\pm\left(\frac{0.1}{12.4}+\frac{0.2}{4.6}\right)\times2.7=\pm0.14$$

∴ $V\pm\Delta V=(2.7\pm0.14)\,m^3$ **[Ans.]**

Example 10. Calculate focal length of a spherical mirror from the following observations. Object distance u = (50.1 ± 0.5) cm and image distance v = (20.1 ± 0.2) cm.

Solution

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}$$

or $$f=\frac{uv}{u+v}=\frac{(50.1)(20.1)}{(50.1+20.1)}=14.3\,cm$$

Also $$\frac{\Delta f}{f}=\pm\left[\frac{\Delta u}{u}+\frac{\Delta v}{v}+\frac{\Delta u+\Delta v}{u+v}\right]=\pm\left[\frac{0.5}{50.1}+\frac{0.2}{20.1}+\frac{0.5+0.2}{50.1+20.1}\right]$$

$= [0.00998 + 0.00995 + 0.00997] = \pm\ (0.0299)$

∴ $\Delta f = 0.0299 \times 14.3 = 0.428 = 0.4$ cm

∴ $f = (14.3 \pm 0.4)$ cm **[Ans.]**

VERNIER CALLIPERS

Vernier constant, i.e., least count of the Vernier Calliper is the minimum length which can be measured with the help of this instrument. It is the difference between I main scale division (i.e. M.S.D.) and I vernier scale division (i.e. V.S.D.).

$$\text{Least count, LC} = \frac{\text{Value of M.S.D.}}{\text{No. of divisions on vernier scale}}$$

Generally, vernier scale has 10 divisions and these 10 divisions coincide with 9 main scale divisions 1 *M.S.D.* is generally equal to 1 mm i.e.

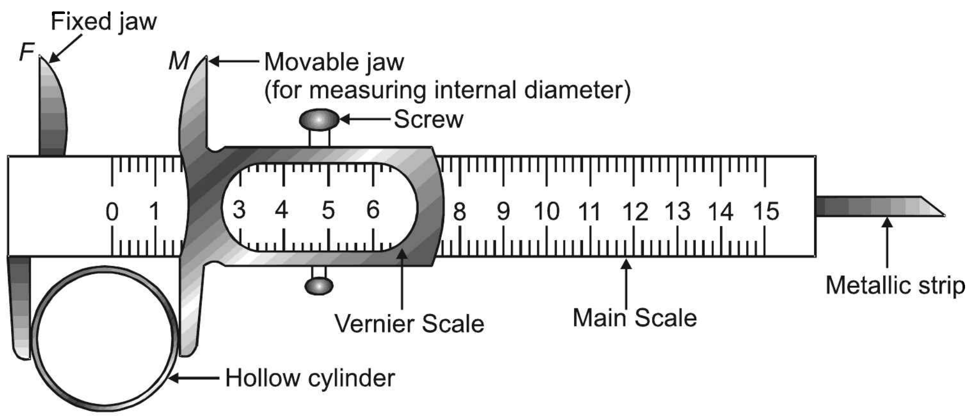
$$=\frac{1}{10}cm$$

∴ Vernier constant = 1 *M.S.D.* – 1 *V.S.D.*

$$1\text{M.S.D.} - \frac{9}{10}\text{M.S.D.} \quad (\because 10\ V.S.D. = 9\,M.S.D.)$$

$$=\frac{1}{10}-\frac{9}{10}\times\frac{1}{10}=\frac{1}{100}cm=0.1mm$$

- If zero mark of the vernier scale is not coinciding with the zero mark of the main scale and is towards its right then the ***zero error*** is called positive ***zero error***. For correction, this error is substracted from the observed value. If zero mark of the vernier scale is towards the left of zero mark of main scale then the zero error is called ***negative zero error***. For correction, this error is added to the observed value.



The vernier constant of the vernier calliper is found as explained above. The jaws are pressed so as to find the zero error. The article is held lengthwise between the jaws without applying undue pressure. The main scale reading is noted just before the zero of the vernier scale and the number of vernier scale divisions coinciding with some of the main scale divisions. To find the diameter of cylinder jaws *F M* are used by inserting them inside the cylinder.

Main scale reading = x

No. of Vernier divisions coinciding = y

Observed length, $l' = x + y\ (y \times V.C.)$

Correct Diameter = $l + l' \pm$ zero error (cm)

% age error = $\dfrac{\Delta l}{l} \times 100 = \dfrac{LC}{l} \times 100$ and so on

Example 11. A vernier callipers has 20 divisions on vernier scale which coincides with 19 on main scale. The least count of the instrument is 0.1 mm. Find the value of main scale divisions.

Solution Least count (LC) = 1 $M.S.D.$ – 1 $V.S.D.$

$$0.1 \text{ mm} = 1\ M.S.D. - 1\ V.S.D.$$

$$= 1 \text{ M.S.D.} - \frac{19}{20} \text{ M.S.D.}$$

$$0.1\ mm = \text{M.S.D.}\left(1 - \frac{19}{20}\right)$$

$$\Rightarrow 1\ M.S.D. = 0.1 \times 20 \text{ mm} = 2 \text{ mm}$$

Example 12. In a vernier callipers each centimeter on main scale is divided in m equal parts and n vernier divisions coincide with **(n – 1)** scale divisions. Find the vernier constant of the calliper.

Solution

$$\text{n V.S.D.} = (\text{n} - 1)\text{M.S.D.}$$

$$\Rightarrow 1 \text{ V.S.D.} = \frac{\text{n}-1}{\text{n}} \text{M.S.D.}$$

Vernier constant = 1 $M.S.D.$ – 1 $V.S.D.$

$$= 1\,M.\text{S.D.} - \frac{\text{n}-1}{\text{n}}\text{M.S.D.} \quad = 1\,M.\text{S.D.}\left(1 - \frac{\text{n}-1}{\text{n}}\right)$$

$$\text{Vernier constant} = \frac{1}{\text{mn}}$$

Example 13. The side of cube is measured by vernier callipers (10 divisions of vernier scale coincide with 9 divisions of main scale where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 *gm*. Find the density of cube in appropriate significant figures.

Solution 10 $V.S.D.$ = 9 $M.S.D.$

$$\Rightarrow 1 \text{V.S.D.} = \frac{9}{10} M.S.D.$$

Vernier constant or Least Count = 1 $M.S.D.$ – 1 $V.S.D.$

$$=1M.\text{S.D.}-\frac{9}{10}M.S.D. \;=M.\text{S.D.}\left(1-\frac{9}{10}\right)=1mm\times\frac{1}{10}$$

Side of the cube $=10mm+\frac{1}{10}mm\times 1$ = 10.1 mm = 1.01 cm

Density of the cube $=\frac{2.736}{(1.01)^3}$ = 2.66 gm/cm^3

SCREW GAUGE (MICROMETER)

Least Count: To find the least count of screw gauge, the pitch is determined first zero of circular scale is brought against a zero of main scale.

About four complete rotations are given to the circular scale and again the reading of the main scale is noted, then;

$$Pitch=\frac{\text{Distance travelled on the main scale}}{Number\text{ of rotations}}$$

$$\text{Hence, Least count, LC}=\frac{\text{Pitch}}{\text{Total }number\text{ of divisions on circular scale}}$$

Zero Error: Sometimes the zero of the circular scale may not coincide with the reference line, when we bring two jaws A and B in contact. This error is called **zero error.**

This happens due to certain manufacturing defect or due to wear and tear of the jaws.

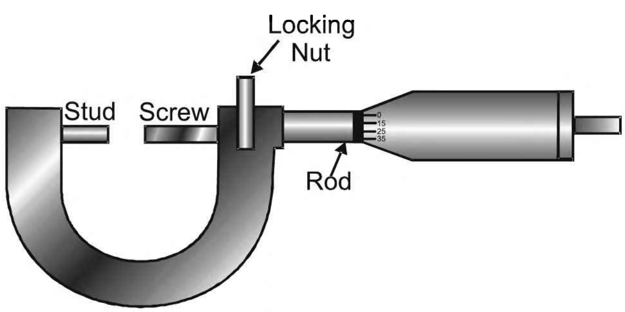
If the zero of the circular scale crosses the reference line, the screw gauge is said to possess negative zero error.

∴ Zero correction = + (number of divisions crossed) × $L.C.$

If the zero of the circular **scale is left** behind the reference line, then screw gauge is said to possess positive zero error.

∴ Zero correction = – (number of divisions left behind) × $L.C.$

Screw gauge, a wire or a sphere or a plate.



Least count of screw gauge found as explained above.

The article is held between the jaws and the screw is turned so that the wire (or sphere or plate) is held without any undue pressure. The reading on the main scale is noted and the number of divisions coincide with the reference line on the main scale. The zero correction is found and applied to find the correct diameter or thickness.

Main Scale Reading = x mm

Circular Scale Division reference line = n

Observed Diameter $D' = x + (n \times L.C.)$

Corrected Diameter $D = D' \pm$ zero error mm

% age error in diameter = $\frac{\Delta d}{d} \times 100 = \frac{LC}{d} \times 100$

and % age error in thickness = $\frac{\Delta t}{t} \times 100 = \frac{LC}{t} \times 100$

Example 14. A screw gauge having 100 equal dimensions and a pitch of length 1mm is used to measure the diameter of a wire of length 5.6 cm. The main scale reading is 1 mm and 47[th] circular division coincides with the main scale. Find the surface area of wire in cm^2 to appropriate significant. figure $(\pi = 22/7)$.

Solution

Least Count = $\frac{\text{Pitch}}{\text{No. of divisions on circular scale}} = \frac{1}{100} mm = 0.01$ mm

Diameter of wire = 1 mm + 47 × LC = 1 mm + 47 × 0.01 = 1.47 mm

Surface area = $2\pi r.\ell = \pi d \ell$

$= \frac{22}{7} \times 1.47 mm \times 5.6 cm = \frac{22}{7} \times \frac{1.47}{10} cm \times 5.6 cm = 2.587$ cm^2

Surface area = 2.6 cm^2. **[Ans.]**

EXERCISE

1.1 Match the following

(a)	Resistivity	(*P*)	$[K^{-1}]$
(b)	Coefficient of volume expansion	(*Q*)	$[M^1 L^2 T^{-3} A^{-1}]$
(c)	Gravitational potential	(R)	$[M^1 L^3 T^{-3} A^{-2}]$
(d)	Electric potential	(*S*)	$[L^2 T^{-2}]$

(a) *R* (b) *P* (c) *S* (d) *Q* **[Ans.]**

1.2 Three vectors $\vec{A}, \vec{B}$ *and* $\vec{C}$ are given such that $\vec{A} = \frac{3\hat{i} + 4\hat{j}}{5}$, $\vec{B} = \frac{-4\hat{i} + 3\hat{j}}{5}$ and $\vec{C} = 4\hat{i} + 3\hat{j}$.

Two or more vectors is given in column I. Certain statements are given in column II. Match the vectors given in column I with the statements in column II.

Column I		Column II	
(A)	$\vec{A}, \vec{B}$ and $\vec{A} \times \vec{B}$	(p)	are all unit vectors
(B)	$\vec{A}, \vec{B}$ and $\vec{C}$	(q)	are mutually perpendicular vectors
(C)	$\vec{A} \times \vec{B}$ and $\vec{A}$	(r)	are collinear vectors
(D)	$\vec{C} \times \vec{A}$ and $\vec{A} + \vec{B}$	(s)	are coplanar vectors

Ans. (A) *p*, *q* (B) *s* (C) *p*, *q*, *s* (D) *q*, *s*

Solution (A) $\vec{A}$ *and* $\vec{B}$ are mutually perpendicular unit vectors

$\therefore$ $\vec{A}\times\vec{B}$ is also a unit vector equal to $\hat{k}$.

$\therefore$ $\vec{A}, \vec{B}$ and $\vec{A}\times\vec{B}$ are unit vectors and mutually perpendicular.

(B) $\vec{A}, \vec{B}$ *and* $\vec{C}$ all lie in x-y plane and are hence coplanar.

(C) $\vec{A}\times\vec{B}$ and $\vec{A}$ are mutually perpendicular and coplanar.

(D) $\vec{C}\times\vec{A}$ and $\vec{A}+\vec{B}$ are mutually perpendicular and coplanar.

1.3 If $\vec{A}=\hat{i}-\hat{j}+\hat{k}$, $\vec{B}=\hat{i}+\hat{k}$, $\vec{C}=\hat{j}-\hat{k}$. Match the algebraic operations in Column I with the corresponding results in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

Column I	**Column II**
(A) $\vec{A}+\vec{B}+\vec{C}$	(p) $-\hat{i}+\hat{j}$
(B) $2\vec{A}-\vec{B}+3\vec{C}$	(q) $\hat{k}$
(C) $(\vec{A}\times\vec{B})+\vec{C}$	(r) $2\hat{i}+\hat{k}$
(D) $\vec{A}+(\vec{B}\times\vec{C})$	(s) $\hat{i}+\hat{j}-2\hat{k}$

Solution (A) *r* (B) *s* (C) *p* (D) *q*

1.4 Consider three vectors given as $\vec{A}=\hat{i}+\hat{j}$, $\vec{B}=-\hat{i}+\hat{j}$, $\vec{C}=\hat{i}+3\hat{j}$. Match the terms in Column I with the corresponding statements in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the *OMR*.
Match the terms in Column I with the corresponding statements in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the *OMR*.

Column I	**Column II**
(A) $\vec{A}$ & $\vec{B}$	(p) perpendicular vectors
(B) $\vec{B}$ & $\vec{C}$	(q) coplanar vectors
(C) $\vec{A}$ & $\vec{C}$	(r) parallel vectors
(D) $\vec{A}\times\vec{B}$ & $\vec{C}$	(s) antiparallel vectors

Solution (A) *p*, *q* (B) *q* (C) *q* (D) *p*, *q*

1.5 In Column – I, some physical quantities are given and same possible SI units are given in column – II. Match the the physical quantities in Column I with the units in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the *OMR*.

Column – I	**Column – II**
(a) $\dfrac{B^2qv}{\Phi}$	(P) $\dfrac{watt\cdot \text{sec}ond}{meter^3}$

v – Magnitude of velocity
B – Magnetic field

q – Charge
Φ – Magnetic flux

(b) hgR (Q) $\dfrac{farad \cdot volt^2}{second}$

h – Planck's constant
g – Gravitational acceleration
R – Rydberg constant

(c) $\dfrac{\sigma b^4}{A}$ (R) $\dfrac{Newton}{metre^2}$

σ – Stefan's constant
b – Wien's constant
A – Area

(d) $\dfrac{\eta}{RC}$ (S) $\dfrac{Newton \cdot metre}{second}$

η – Coefficient of viscosity
R – Resistance
C – Capacitance

(A) *p*, *r* (B) *q*, *s* (C) *q*, *s* (D) *p*, *r* **[Ans.]**

Solution (a) $\dfrac{B^2qv}{\varphi} = \dfrac{(qvB)B}{BA} = \dfrac{F}{A} = \dfrac{Newton}{metre^2}$ = unit of pressure

(b) hgR = (Joule-second) $\left(\dfrac{metre}{second^2}\right)$ $(\text{metre}^{-1}) = \dfrac{joule}{second}$ = unit of power

(c) $\dfrac{\sigma b^4}{A} = \dfrac{\sigma \lambda^4 T^4}{A} = \dfrac{(\sigma A T^4)\lambda^4}{A^2} = \dfrac{(joule/\text{sec})(metre)^4}{(metre^2)^2} = \dfrac{joule}{second}$ = unit of power

(d) $\dfrac{\eta}{RC} = \left(\dfrac{F}{6\pi rv}\right)\dfrac{1}{(RC)} = \dfrac{Newton}{\left(metre \cdot \dfrac{metre}{second}\right)} \cdot \dfrac{1}{(second)} = \dfrac{Newton}{metre^2}$ = unit of pressure

(P) $\dfrac{watt \cdot second}{metre^3} = \dfrac{\dfrac{joule}{second} \cdot second}{metre^3} = \dfrac{Newton \cdot metre}{metre^3} = \dfrac{Newton}{metre^2}$ = unit of pressure

(Q) $\dfrac{(farad\, volt^2)}{second} = \dfrac{joule}{second}$ = unit of power

(R) $\dfrac{Newton}{metre^2}$ = unit of pressure

(S) $\dfrac{Newton \cdot metre}{second} = \dfrac{joule}{second}$ = unit of power

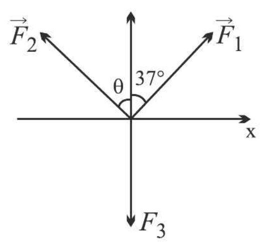
EXERCISE

Exercise–1: Subjective Problems

1. Use the approximation $(1 + x)^n \approx 1 + nx$, $|x| << 1$, to find approximate value for

   (A) $\sqrt{99}$ (B) $\frac{1}{1.01}$ (C) $124^{1/3}$

2. A particle is in a uni-directional potential field where the potential energy (U) of a particle depends on the x-cordinate given by $U_x = k(1 - \cos ax)$ and k and 'a' are constants. Find the physical dimensions of 'a' and k.
3. An enclosed ideal gas A has its pressure P as a function of its volume V as $P = P_0 - \alpha V^2$, where P_0 and α are constants. Find the physical dimensions of α.
4. Use the small angle approximations to find approximate values for (A) sin 8° and (B) tan 5°
5. When two forces of magnitude P and Q are perpendicular to each other, their resultant is of magnitude R. When they are at an angle of 180° to each other their resultant is of magnitude $\frac{R}{\sqrt{2}}$. Find the ratio of P and Q.
6. A particle moves along the space curve $\vec{r} = (t^2 + t)\hat{i} + (3t - 2)\hat{j} + (2t^3 - 4t^2)\hat{k}$.(t in sec, r in m) Find at time $t = 2$ the (A) velocity, (B) acceleration, (C) speed or magnitude of velocity and (D) magnitude of acceleration.
7. The time period (T) of a spring mass system depends upon mass (m) and spring constant (k) & length of the spring (l) [$k = \frac{Force}{length}$]. Find the relation among, (T), (m), (l) & (k) using dimensional method.
8. A body acted upon by 3 given forces is under equilibrium.



   (A) If $|\vec{F}_1| = 10\,Nt.$, $|\vec{F}_2| = 6\,Nt.$

   Find the values of $|\vec{F}_3|$ and angle (θ).

   (B) Express $\vec{F}_2$ in unit vector form.

9. A particle is acted upon by the forces

   $$\vec{F}_1 = 2\hat{i} + a\hat{j} - 3\hat{k},\ \vec{F}_2 = 5\hat{i} + c\hat{j} - b\hat{k}, \vec{F}_3 = b\hat{i} + 5\hat{j} - 7\hat{k},\ \vec{F}_4 = c\hat{i} + 6\hat{j} - a\hat{k},.$$

   Find the values of the constants a, b, c in order that the particle will be in equilibrium.
10. A satellite is orbiting around a planet. Its orbital velocity (v_0) is found to depend upon

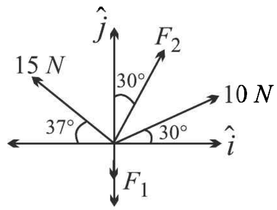
    (A) Radius of orbit (R)

    (B) Mass of planet (M)

    (C) Universal gravitation constant (G)

    Using dimensional analysis find an expression relating orbital velocity (v_0) to the above physical quantities.

11. If the four forces as shown are in equilibrium. Express $\vec{F}_1$ & $\vec{F}_2$ in unit vector form.



12. The equation of state for a real gas at high temperature is given by $P = \dfrac{nRT}{V-b} - \dfrac{a}{T^{1/2}V(V+b)}$

where n, P, V & T are number of moles, pressure, volume & temperature respectively & R is the universal gas constant. Find the dimensions of constant 'a' in the above equation.

13. The distance moved by a particle in time t from centre of a ring under the influence of its gravity is given by $x = a \sin\omega t$ where a & ω are constants. If ω is found to depend on the radius of the ring (r), its mass (m) and universal gravitational constant (G), find using dimensional analysis an expression for ω in terms of r, m and G.

14. If the velocity of light c, Gravitational constant G & Plank's constant h be chosen as fundamental units, find the dimension of mass, length & time in the new system.

15. A plane body has perpendicular axes OX and OY marked on it and is acted on by following forces

    $5P$ in the direction OY

    $4P$ in the direction OX

    10P in the direction OA where A is the point $(3a, 4a)$

    15P in the direction AB where B is the point $(-a, a)$

Express each force in the unit vector form & calculate the magnitude and direction of sum of the vector of these forces.

16. Two vectors have magnitudes 3 unit and 4 unit respectively. What should be the angle between them if the magnitude of the resultant is (A) 1 unit, (B) 5 unit and (C) 7 unit.

17. A vector $\vec{A}$ of length 10 units makes an angle of 60° with a vector $\vec{B}$ of length 6 units. Find the magnitude of the vector difference $\vec{A} - \vec{B}$ and the angle it makes with vector $\vec{A}$.

18. At time t the position vector of a particle of mass $m = 3kg$ is given by $\vec{r} = 6t\hat{i} - t^3\hat{j} + \cos t\hat{k}$.

Find the resultant force $\vec{F}(t)$, magnitude of its acceleration when $t = \dfrac{\pi}{2}$ & speed when $t = \pi$.

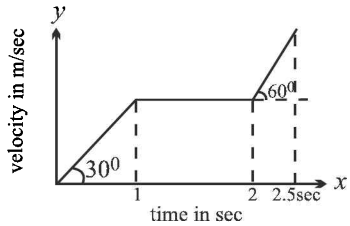
19. Given that the position vector of a particle moving in x-y plane is given by $\vec{r} = (t^2 - 4)\hat{i} + (t - 4)\hat{j}$.

Find

(A) Equation of trajectory of the particle

(B) Time when it crosses x-axis and y-axis

20. The velocity time graph of a body moving in a straight line is shown. Find its

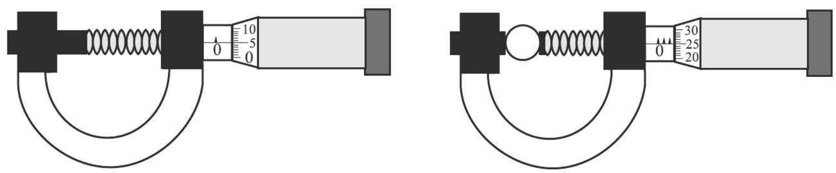


(A) instantaneous velocity at $t = 1.5$ sec.
(B) average acceleration from $t = 1.5$ sec. to $t = 2.5$ sec.
(C) draw its acceleration time graph from $t = 0$ to $t = 2.5$ sec

21. The curvilinear motion of a particle is defined by $v_x = 50\text{-}16t$ and $y = 100\text{-}4t^2$, where v_x is in metres per second, y is in metres and t is in seconds. It is also known that $x = 0$ when $t = 0$. Determine the velocity (v) and acceleration(A) when the position $y = 0$ is reached.

22. The force acting on a body moving in a straight line is given by $F = (3t^2 - 4t + 1)$ Newton where t is in sec. If mass of the body is $1kg$ and initially it was at rest at origin. Find
(A) displacement between time $t = 0$ and $t = 2$ sec.
(B) distance travelled between time $t = 0$ and $t = 2$ sec.

23. The circular divisions of shown screw gauge are 50. It moves 0.5 mm on main scale in one rotation. The diameter of the ball is **[JEE 2006]**



(A) 2.25 mm (B) 2.20 mm (C) 1.20 mm (D) 1.25 mm

Exercise–2: Subjective Problems

1. If force, acceleration and time are taken as fundamental quantities, then the dimensions of length will be:
(A) FT^2 (B) $F^{-1}A^2T^{-1}$ (C) FA^2T (D) AT^2

2. In a certain system of units, 1 unit of time is 5 sec, 1 unit of mass is 20 kg and unit of length is 10 m. In this system, one unit of power will correspond to
(A) 16 watts (B) $\frac{1}{16}$ watts (C) 25 watts (D) none of these

3. Three forces P, Q & R are acting at a point in the plane. The angle between P & Q and Q & R are 150º & 120º respectively, then for equilibrium, forces P, Q & R are in the ratio
(A) 1: 2: 3 (B) 1: 2: $\sqrt{3}$ (C) 3: 2: 1 (D) $\sqrt{3}$: 2: 1

4. The resultant of two forces F_1 and F_2 is P. If F_2 is reversed, then resultant is Q. Then the value of $(P^2 + Q^2)$ in terms of F_1 and F_2 is
(A) $2(F_1^2 + F_2^2)$ (B) $F_1^2 + F_2^2$ (C) $(F_1 + F_2)^2$ (D) none of these

5. A man rows a boat with a speed of 18*km/hr* in northwest direction. The shoreline makes an angle of 15° south of west. Obtain the component of the velocity of the boat along the shoreline.
   (A) 9 km/hr (B) $18\frac{\sqrt{3}}{2}$ km/hr (C) 18 cos15°km/hr (D) 18 cos75° km/hr

6. A bird moves from point (1, −2, 3) to (4, 2, 3). If the speed of the bird is 10 m/sec, then the velocity vector of the bird is:
   (A) $5\left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$ (B) $5\left(4\hat{i} + 2\hat{j} + 3\hat{k}\right)$
   (C) $0.6\hat{i} + 0.8\hat{j}$ (D) $6\hat{i} + 8\hat{j}$

7. The dimensions $ML^{-1}T^{-2}$ can correspond to:
   (A) moment of a force or torque (B) surface tension
   (C) pressure (D) co-efficient of viscosity.
   (useful relation are $\vec{\tau} = \vec{r} \times \vec{F}$, $S = F/l$, $F = 6\pi\eta r v$, where symbols have usual meaning)

8. The pressure of 10^6 dyne/cm^2 is equivalent to
   (A) 10^5 N/m^2 (B) 10^6 N/m^2 (C) 10^7 N/m^2 (D) 10^8 N/m^2

9. If area (A) velocity (v) and density (ρ) are base units, then the dimensional formula of force can be represented as.
   (A) $Av\rho$ (B) $Av^2\rho$ (C) $Av\rho^2$ (D) $A^2v\rho$

10. If the resultant of two forces of magnitudes *P* and *Q* acting at a point at an angle of 60° is $\sqrt{7}$ *Q*, then *P* / *Q* is
    (A) 1 (B) 3 / 2 (C) 2 (D) 4

11. For a particle moving in a straight line, the position of the particle at time (t) is given by
    $$x = t^3 - 6t^2 + 3t + 7$$
    what is the velocity of the particle when it's acceleration is zero ?
    (A) -9 ms^{-1} (B) -12 ms^{-1} (C) 3 ms^{-1} (D) 42 ms^{-1}

12. If the angle between the unit vectors $\hat{a}$ and $\hat{b}$ is 60°, then $|\hat{a} - \hat{b}|$ is
    (A) 0 (B) 1 (C) 2 (D) 4

13. In a book, the answer for a particular question is expressed as
    $$b = \frac{ma}{k}\left[\sqrt{1 + \frac{2kl}{ma}}\right]$$
    here *m* represents mass, a represents accelerations, *l* represents length. The unit of *b* should be
    (A) m/s (B) m/s^2 (C) meter (D) / sec.

14. The resultant of two forces, one double the other in magnitude is perpendicular to the smaller of the two forces. The angle between the two forces is
    (A) 150° (B) 90° (C) 60° (D) 120°

15. Which of the following can be a set of fundamental quantities
    (A) length, velocity, time
    (B) momentum, mass, velocity
    (C) force, mass, velocity
    (D) momentum, time, frequency

16. If 1 unit of mass = 4 *kg*; 1 unit of length = $\frac{1}{4}m$ and 1 unit of time = 5 sec, then 1 Joule = x units of energy in this system where x =
(A) 100 units (B) 0.01 units (C) 200 units (D) 0.02 units

17. A man moves towards 3 m north then 4 m towards east and finally 5m towards south west. His approximate displacement from origin is
(A) $5\sqrt{2}\,m$ (B) $0\,m$ (C) $12\,m$ (D) $5\,m$
(E) $1\,m$

18. Kinetic energy (K) depends upon momentum (p) and mass (m) of a body as $K\,\alpha\,p^a m^b$
(A) $a = 1; b = 1$ (B) $a = 2; b = -1$ (C) $a = 2; b = 1$ (D) $a = 1; b = 2$

Exercise–3: General Physics

Only One Correct Option

1. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. What are the dimensions of Y in *MKSQ* system? **[JEE 1995]**
(A) $[M^{-3}L^{-1}T^3Q^4]$ (B) $[M^{-3}L^{-2}T^4Q^4]$
(C) $[M^{-2}L^{-2}T^4Q^4]$ (D) $[M^{-3}L^{-2}T^4Q]$
Ans. (B)

2. The dimensions of $\frac{1}{2}\varepsilon_0 E^2$ (ε_0 : permittivity of free space; E: electric field) is **[JEE 2000]**
(A) $[MLT^{-1}]$ (B) $[ML^2T^{-2}]$
(C) $[MLT^{-2}]$ (D) $[ML^{-1}T^{-2}]$
Ans. (D)

3. A quantity X is given by $\varepsilon_0 L\frac{\Delta V}{\Delta t}$, where ε_0 is the permittivity of free space, L is a length, ΔV is a potential difference and Δt is a time interval. The dimensional formula for X is the same as that of **[JEE 2001]**
(A) resistance (B) charge
(C) voltage (D) current
Ans. (D)

4. A cube has a side of length 1.2×10^{-2} m. Calculate its volume **[JEE 2003]**
(A) $1.7\times10^{-6}\,m^3$ (B) $1.73\times10^{-6}\,m^3$ (C) $1.70\times10^{-6}\,m^3$ (D) $1.732\times10^{-6}\,m^3$
Ans. (A)

5. In the relation $p = \frac{\alpha}{\beta}e^{-\frac{\alpha Z}{k\theta}}$; p is pressure, z is distance, k is Boltzmann constant and θ is the temperature. The dimensional formula of β will be **[JEE 2004]**
(A) $[M^0L^2T^0]$ (B) $[ML^2T]$ (C) $[ML^0T^{-1}]$ (D) $[M^0L^2T^{-1}]$
Ans. (A)

6. A wire has a mass $(0.3 \pm 0.003)\,g$, radius (0.5 ± 0.005) mm and length (6 ± 0.06) cm. The maximum percentage error in the measurement of its density is **[JEE 2004]**
(A) 1 (B) 2 (C) 3 (D) 4
Ans. (D)

7. Which of the following sets have different dimensions? **[JEE 2005]**
(A) Pressure, Young's modulus, Stress (B) Emf, Potential difference, Electric potential
(C) Heat, Work done, Energy (D) Dipole moment, Electric flux, Electric field
Ans. (D)

8. Let $[\varepsilon_0]$ denote the diensional formula of the permittivity of the vacuum. If M = mass, L = length, T = Time and A = electric current, then **[JEE Main 2013]**
(A) $[\varepsilon_0] = [M^{-1}L^{-3}T^{4}A]$ (B) $[\varepsilon_0] = [M^{-1}L^{-3}T^{4}A^{2}]$
(C) $[\varepsilon_0] = [M^{-2}L^{2}T^{-1}A^{-2}]$ (D) $[\varepsilon_0] = [M^{-1}L^{2}T^{-1}A^{2}]$
Ans. (B)

9. The diameter of a cylinder is measured using a vernier callipers with no zero error. It is found the zero of the vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The vermier scale has 50 division equivalent to 2.45 cm. The 24th division of the vernier scale exactly coinciders with one of the main scale divisions. The diameter of the cyliner is **[JEE Advanced 2013]**
(A) 5.112 cm (B) 5.124 cm (C) 5.136 cm (D) 5.148 cm
Ans. (B)

10. During Searle's experiment, zero of the vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2}m of the main scale. The 20th division of the vernier scale exactly coincides with one of the main divisions. When an additional loas of 2 kg is applied to the wire, the zero of the vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of the Vernier scale coincides with one of the main scale division. The length of the thin metallic wire is 2 m and its cross-sectional area is 8×10^{-7} m^2. The least count of the vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is **[JEE Advanced (Integer Type) 2014]**
Ans. (4)

11. The current voltage relation of diode is given by $I = (e^{1000v/T} - 1)mA$ where the applied voltage V is in volts and the temperature T is in degree Kelvin. If a student makes an error measuring $\pm\, 0.01V$ while measuring the current of $5mA$ at $300K$, what will be the error in the value of current in mA? **[JEE Main 2004]**
(A) 0.02 mA (B) 0.5 mA (C) 0.05 mA (D) 0.2 mA
Ans. (D)

12. A student measured the rod and wrote it as 3.50 cm. Which instrument did he use to measure it? **[JEE Mains 2014]**
(A) A vernier calliper where the 10 divisions in vernier scale matches with 9 division in main scale and main scale has 10 division in 1 cm.
(B) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm.
(C) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm.
(D) A meter scale
Ans. (A)

13. The period of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 securing wrist watch of 1 sec resolution. The accuracy in the determination of g is: **[JEE Main 2015]**

(A) 2% (B) 3% (C) 1% (D) 5%

Ans. (B)

Match the Columns

1. **Column I** gives three physical quantities. Select the appropriate units for the choices given in **Column II**. Some of the physical quantities may have more than one choice. **[JEE 1990]**

Column I	**Column II**
Capacitance	ohm-second
Inductance	coulomb^2-joule^{-1}
Magnetic induction	coulomb (volt)$^{-1}$,
	newton (ampere metre)$^{-1}$,
	volt-second (ampere)$^{-1}$.

Ans. I	**II**
Capacitance	coulomb-volt coulomb^2 joule^{-1}
Inductance	ohm-sec, volt second ampere^{-1}
Magnetic induction	newton (ampere-metre)$^{-1}$

2. Match the physical quantities given in **Column I** with dimensions expressed in terms of mass (M), length (L), time (T), charge (Q) given in **Column II** and write the correct answer against the matched quantity in a tabular form in your answer book. **[JEE 1993]**

Column I	**Column II**
(A) Angular momentum	(p) $[ML^2T^{-2}]$
(B) Latent heat	(q) $[ML^2Q^{-2}]$
(C) Torque	(r) $[ML^2T^{-1}]$
(D) Capacitance	(s) $[ML^3T^{-1}Q^{-2}]$
(E) Inductance	(t) $[M^{-1}L^{-2}T^2Q^2]$
(F) Resistivity	(u) $[L^2T^{-2}]$

Ans. (*A-r*); (*B-u*); (*C-p*); (*D-t*); (*E-q*); (*F-s*)

3. Some physical quantities are given in **Column I** and some possible SI units in which these quantities may be expressed are given in **Column II**. Match the physical quantities in **Column I** with the units in **Column II.** **[JEE 2007]**

Column I	**Column II**
(A) GM_eM_s	(p) (volt) (coulomb) (metre)

G – universal gravitational constant,
M_e – mass of the earth,
M_s – mass of the sun.

(B) $\frac{3RT}{M}$ (q) (kilogram) (metre)^3 (second)$^{-2}$

R – universal gas constant,
T – absolute temperature,
M – molar mass.

(C) $\frac{F^2}{q^2B^2}$ (r) (metre)^2 (second)$^{-2}$

F – force,
q – charge,
B – magnetic field.

Ans. (*A-p, q*); (*B-r, s*); (*C-r, s*); (*D-r, s*)

4. Match Column I with Column II and select the correct answer using the codes given. **[JEE Advanced 2013]**

	Column I		Column II
P.	Boltzmann constant	1.	$[ML^2T^{-1}]$
Q.	Coefficient of viscosity	2.	$[ML^{-1}T^{-1}]$
R.	Plank constant	3.	$[MLT^{-3}K^{-1}]$
S.	Thermal conducitivity	4.	$[ML^2T^{-2}K^{-1}]$

Codes:

	P	Q	R	S
(A)	3	1	2	4
(B)	4	2	1	3
(C)	3	2	1	4
(D)	4	1	2	3

Ans. (B)

One or More than One Correct Option

1. L, C and R represent the physical quantities inductance, capacitance and resistance respectively. The combinations which have the dimensions of frequency are **[JEE 1984]**

(A) $\frac{1}{RC}$ (B) $\frac{R}{L}$ (C) $\frac{1}{\sqrt{LC}}$ (D) $\frac{C}{L}$

Ans. (*A, B, C*)

2. The dimensions of the quantities in one (or more) of the following pairs are the same. Identify the pair(s) **[JEE 1986]**

(A) torque and work (B) angular momentum and work
(C) energy and Young's modulus (D) light year and wavelength

Ans. (*A, D*)

3. The pairs of physical quantities that have the same dimensions is/(are) **[JEE 1995]**

(A) Reynolds number and coefficient of friction
(B) Curie and frequency of a light wave
(C) Latent heat and gravitational potential
(D) Planck's constant and torque

Ans. (*A,B,C*)

4. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of the vacuum and $[\mu_0]$ that of the permeability of the vacuum. If M = mass, L = length, T = time and I = electric current. **[JEE 1998]**

   (A) $[\varepsilon_0] = [M^{-1}L^{-3}T^2I]$ (B) $[\varepsilon_0] = [M^{-1}L^{-3}T^4I^2]$

   (C) $[\mu_0] = [MLT^{-2}I^{-2}]$ (D) $[\mu_0] = [ML^2T^{-1}I]$

   [Ans.] *(B,C)*

5. The *SI* unit of the inductance, the henry can by written as **[JEE 1998]**

   (A) weber/ampere (B) volt-second/ampere

   (C) joule/(ampere)^2 (D) ohm-second

   [Ans.] *(A,B,C,D)*

6. A student uses a simple pendulum of exactly 1m length to determine g, the acceleration due to gravity. He uses a stop watch with the least count of 1 s for this and records 40 s for 20 oscillations. For this observation, which of the following statement(s) is/are true?

   (A) Error ΔT in measuring T, the time period, is 0.05 *s*

   (B) Error ΔT in measuring T, the time period, is 1 *s*

   (C) Percentage error in the determination of g is 5%

   (D) Percentage error in the determination of g is 2.5%

   [Ans.] *(A,C)*

Fill in the Blanks

1. Planck's constant has dimensions.. .... **[JEE 1985]**

   Ans. $[ML^2T^{-1}]$

2. In the formula $X = 3YZ^2$, X and Z have dimensions of capacitance and magnetic induction respectively. The dimensions of y in *MKSQ* system are.. .... **[JEE 1988]**

   Ans. $[M^{-3}L^{-2}T^4Q^4]$

3. The dimensions of electrical conductivity is................ **[JEE 1997]**

   Ans. $[M^{-1}L^{-3}T^3A^2]$

4. The equation of state of a real gas is given by $\left(p + \frac{a}{V^2}\right)(V - b) = RT$; where p, V and T are pressure, volume and temperature respectively and R is the universal gas constant. The dimensions of the constant a in the above equation is.. .... **[JEE 1997]**

   Ans. $[ML^5T^{-2}]$

Analytical & Descriptive Questions

1. Give the *MKS* units for each of the following quantities **[JEE 1980]**

   (a) Young's modulus, (b) Magnetic induction,

   (c) Power of a lens.

   Ans. (a) N/m^2 (b) Tesla (c) m^{-1}

2. A gas bubble, from an explosion under water, oscillates with a period T proportional to $p^a d^b E^c$, where p is the static pressure, d is the density of water and E is the total energy of the explosion. Find the values of a, b and c. **[JEE 1981]**

   [Ans.] $a = \frac{-5}{6}, b = \frac{1}{2}, c = \frac{1}{3}$

3. Write the dimensions of the following in terms of mass, time, length and charge. **[JEE 1982]**
   (a) Magnetic flux
   (b) Rigidity modulus.
   [Ans.] (a) $[ML^2T^{-1}Q^{-1}]$ (b) $[ML^{-1}T^{-2}]$

4. Let $[\varepsilon_0]$ denotes the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = Time period and A = electric current, then

   (A) $[\varepsilon_0] = [M^{-1}L^{-3}T^2A]$
   (B) $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$
   (C) $[\varepsilon_0] = [M^{-2}L^2T^{-1}A^{-2}]$
   (D) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A^2]$

   [Ans.] (B)

Chapter

2

Kinematics

RECTILINEAR MOTION

Mechanics

Mechanics is the branch of physics which deals with the cause and effects of motion of a particle, rigid objects and deformable bodies etc. Mechanics is classified under two streams namely *Kinematics* and *Dynamics*.

Mechanics

Kinematics

The word kinematics means 'science of motion'. It is a branch of mechanics which deals with study of motion without going into the cause of motion, i.e, force, torque etc.

Dynamics (or Kinetics)

It is branch of mechanics which is concerned with the causes (i.e. the force, torque) that cause motion of bodies.

In kinematics we define parameters of motion and we study relations between them. We do not study the cause of motion or cause of change in motion, which is discussed in dynamics. Here is the discription of parameters.

Motion and Rest

Motion is a combined property of the object and the observer. There is no meaning of rest or motion without the observer. Nothing is in absolute rest or in absolute motion.

An object is said to be in motion with respect to observer, its position changes with respect to that of observer. It may happen both ways, either observer moves or object moves.

Rectilinear Motion

Rectilinear motion is motion along a straight line or in one dimension. It deals with the kinematics of a particle in one dimension.

Position

The position of a particle refers to its location in the space at a certain moment of time. It is concerned with the position of *the particle at a particular moment of time.*

Displacement

The change in the position of a moving object is known as displacement. It is the vector joining the initial position of the particle to its final position during an interval of time.

Explanatory Notes on Position and Displacement

Change in position vector is called **displacement**.

Its magnitude is minimum distance between final and initial point, and is directed from initial position to final position.

For a particle moving along x-axis, motion from one position x_1 to another position x_2 is displacement or Δx, where $\Delta x = x_2 - x_1$

If the particle moves from x_1 = 4m to x_2 = 12 m, then Δx = (12m) – (4m) = +8m. The positive result indicates that the motion is in the positive direction. If the particle then returns to x = 4m, the displacement for the full trip is zero. The actual number of meter covered for the full trip is irrelevant; ***displacement involves only the original and final position.***

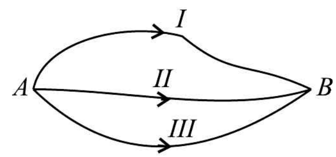
Distance

The length of the actual path travelled by a particle during a given time interval is called distance. The distance travelled is a scalar quantity which is quite different from displacement. In general, the distance travelled between two points may not be equal to the magnitude of the displacement between the same points.

Explanatory Notes on Distance

Length of path traversed by a body is called **distance**.

It is dependent on the path chosen, thus for motion between two fixed points A and B we can have many different values of distance traversed. It is a **scalar** quantity, as length of path has no indication of direction in it.

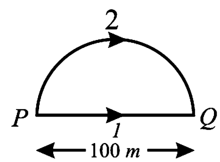


Its SI unit is meter (m) and dimensions is (L).

Example 1. Ram takes path 1 (straight line) to go from P to Q and Shyam takes path 2 (semicircle).

(a) Find the distance travelled by Ram and Shyam?

(b) Find the displacement of Ram and Shyam?



Solution

(a) Distance travelled by Ram = 100 m

Distance travelled by Shyam = π(50 m) = 50π m

(b) Displacement of Ram = 100 m

Displacement of Shyam = 100 m

Average Velocity (in an Interval)

The average velocity of a moving particle over a certain time interval is defined as the displacement divided by the lapsed time.

$$\text{Average Velocity} = \frac{\text{displacement}}{\text{time interval}}$$

for straight line motion, along x–axis, we have

$$v_{av} = \overline{v} = \langle v \rangle = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

The average velocity is a vector in the direction of displacement. For motion in a straight line, directional aspect of a vector can be taken care of by +ve and -ve sign of the quantity.

Explanatory Notes on Average Velocity and Instantneous Velocity

The average velocity V_{avg}, is the ratio of the total displacement Δs, that occurs during a particular time interval Δt, to that interval Δt. It should be noted that V_{avg} is independent of path as displacement is independent of path.

$$V_{avg} = \frac{\Delta \vec{s}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} \quad \text{... (1)}$$

The position is $\boldsymbol{r}_1$ at time t_1 and then r_2 at time t_2.Unit for V_{avg} is the meter per second (m/s). The average velocity V_{avg} always has the same sign as the displacement Δs because Δt in Eq. 1 is always positive.

Instantaneous velocity is the value that $\vec{v}_{avg} = \frac{\vec{r}_2 - \vec{r}_1}{\Delta t}$ approaches in the limit as we shrink the time interval Δt so we are able to find instantaneous velocity about that instant. Using the language of calculus, we may write $\vec{v}$ as the derivative

$$\vec{v} = \frac{d\vec{r}}{dt}$$

It is a vector quantity, directed along tangent of path, in the sense of motion. Its SI unit is m/s.

From now on when we use word velocity it will mean instantaneous velocity.

Average Speed (in an Interval)

Average speed is defined as the total path length travelled divided by the total time interval during which the motion has taken place. It helps in describing the motion along the actual path.

$$\text{Average Speed} = \frac{\text{distance travelled}}{\text{time interval}}$$

Note

(a) Average speed is always positive in contrast to average velocity which being a vector, can be positive or negative.

(b) If the motion of a particle is along a straight line and in same direction then, average velocity = average speed.

(c) Average speed is, in general, greater than the magnitude of average velocity. *The dimension of velocity and speed is* $[LT^{-1}]$ *and their SI unit is meters per second (m/s)*

Explanatory Notes on Average Spped and Instantaneous Speed

Rate of traversing distance is called **speed**.

Average speed, v_{avg} gives overall effect of motion in a given period. The average speed involves the total distance covered which is independent of direction

$$v_{avg} = \frac{\text{total distance}}{\Delta t}$$

Because average speed does not include direction, it is not written with sign.

Instantaneous speed is the magnitude of instantaneous velocity, that is, Instantaneous speed is Instantaneous velocity that does not have any indication of direction, either in words or via an algebraic sign. *(Caution: Speed and average speed can be quite different.)* An instantaneous velocity of +5m/s and one of –5m/s both have an associated instantaneous speed of 5 m/s. The speedometer in a car measures the instantaneous speed, not the instantaneous velocity, because it cannot determine the direction.

Example 2. In the example 1, if Ram takes 4 seconds and Shyam takes 5 seconds to go from P to Q, find

(a) Average speed of Ram and Shyam?

(b) Average velocity of Ram and Shyam?

Solution

(a) Average speed of Ram $= \frac{100}{4}$ m/s = 25 m/s

Average speed of Shyam $= \frac{50\pi}{5}$ m/s = 10π m/s

(b) Average velocity of Ram $= \frac{100}{4}$ m/s = 25 m/s

Average velocity of Shyam $= \frac{100}{5}$ m/s = 20 m/s

Example 3. A particle travels half of total distance with speed v_1 and next half with speed v_2 along a straight line. Find out the average speed of the particle?

Solution Let total distance travelled by the particle be 2s.

Time taken to travel first half $= \frac{s}{v_1}$

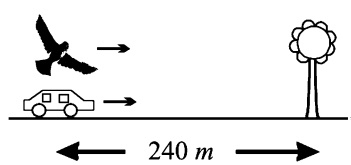
Time taken to travel next half $= \frac{s}{v_2}$

$$\text{Average speed} = \frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$$

Example 4. A particle covers $\frac{3}{4}$ of total distance with speed v_1 and next $\frac{1}{4}$ with v_2. Find the average speed of the particle?

Solution $\frac{4v_1v_2}{v_1+3v_2}$

Example 5. A car is moving with speed 60 Km/h and a bird is moving with speed 90 km/h along the same direction as shown in figure. Find the distance travelled by the bird till the time car reaches the tree?



Solution [**Ans.** 360m]

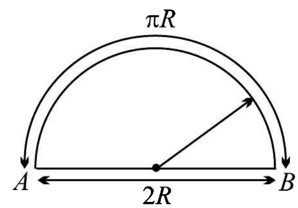
Example 6. You drive an old car along a straight road for 8.4 km at 70 km/h, at this point it runs out of petrol and stops. Over the next 30 min, you walk another 2.0 km further along the road to a petrol pump.

(a) What is your average velocity v_{avg} from the beginning of your drive to your arrival at the petrol pump?

(b) Suppose to pump the petrol, pay for it, and walk back to the Car takes you another 45 min. What is your average speed from the beginning of your drive to your return to the truck with the petrol?

Solution (a) 16.8 ≈ 17 km/hr; (b) 9.1 km/hr.]

Example 7. If a particle traverses on a semicircular path of radius R from A to B in time T then find average speed and average velocity.



Solution [Ans. average speed = $\frac{\pi R}{T}$; average velocity = $\frac{2R}{T}$ (from A to B)]

Note

1. If a particle moves on a straight line path without changing direction then
   (a) distance = | displacement | , hence
   (b) average speed = | average velocity |
2. If a particle traverses curved path or changes direction on a straight line path
   | average velocity | < average speed
3. Always valid for all paths | the instantaneous velocity | = the instantaneous speed

Instantaneous Velocity (At an Instant)

The velocity at a particular instant of time is known as instantaneous velocity. The term *"velocity"* usually means instantaneous velocity.

$$V_{inst.} = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) = \frac{dx}{dt}$$

In other words, the instantaneous velocity at a given moment (say , t) is the limiting value of the average velocity as we let Δt approach zero. The limit as $\Delta t \to 0$ is written in calculus notation as dx/dt and is called the derivative of x with respect to t.

Average Acceleration (in an Interval)

The average acceleration for a finite time interval is defined as:

$$\text{Average acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

Average acceleration is a vector quantity whose direction is same as that of the change in velocity.

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Since for a straight line motion the velocities are along a line, therefore

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

(where one has to substitute v_f and v_i with proper signs in one dimensional motion)

Instantaneous Acceleration (At an Instant)

The instantaneous acceleration of a particle is its acceleration at a particular instant of time. It is defined as the derivative (rate of change) of velocity with respect to time. We usually mean instantaneous acceleration when we say *"acceleration"*. For straight motion we define instantaneous acceleration as:

$$a = \frac{dv}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta x}{\Delta t}\right) \text{ and in general } \vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0}\left(\frac{\Delta \vec{v}}{\Delta t}\right)$$

The dimension of acceleration is $[LT^{-2}]$ and its SI unit is m/s^2.

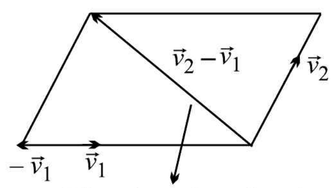
Acceleration

Rate of change of velocity is called acceleration and it is directed along the change in velocity.

If change in velocity is $\Delta\vec{v}$ in Δt time then

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} \qquad \Delta\vec{v} = \vec{v}_2 - \vec{v}_1$$

Direction of $\vec{a}$ is in the direction of change in $\vec{v}$ not in $\vec{v}$:



(Emphasis on vector subtraction)

Dimensions → $M^0L^1T^{-2}$ Units → Its SI unit is ms^{-2}

Important Points

1. It is a vector quantity
2. Direction is not along the velocity but change in velocity.
3. For any change in velocity either in magnitude or direction or both acceleration must be present. Without acceleration neither direction nor magnitude of velocity can be changed.

When a particle's velocity changes, the particle is said to undergo acceleration (or to accelerate).

The **average acceleration a_{avg}** over a time interval Δt is

$$a_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \qquad ...(2)$$

where the particle has velocity v_1 at time t_1 and then velocity v_2 at time t_2.

The **instantaneous acceleration** (or simply acceleration) is the derivative of the velocity with respect to time.

$$a = \frac{d\vec{v}}{dt} \qquad ...(2)$$

In words, the acceleration of a particle at any instant is the rate at which its velocity is changing at that instant.

$$a = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \frac{d^2\vec{r}}{dt^2} \qquad ...(3)$$

In other words, the acceleration of a particle at any instant is the second derivative of its position vector with respect to time.

Acceleration has both magnitude and direction (it is yet another vector quantity). For motion on a straight line its algebraic sign represents its direction on an axis just as for displacement and velocity; that is, acceleration with a positive value is in the positive direction of an axis, and acceleration with a negative value is in the negative direction.

Note

Many times it is misunderstood that positive acceleration means that the speed of an object is increasing, and negative acceleration means that the speed is decreasing (the object is decelerating). ***The sign of an acceleration indicates a direction, not whether an object's speed is increasing or decreasing.***

For example, if a car with an initial velocity $v = -25$ m/s is braked to a stop in 5.0s, then $a_{avg} = +5.0$ m/s^2. The acceleration is positive, but the car's speed has decreased. The reason is the difference in signs: the direction of the acceleration is opposite to that of the velocity.

If the signs of the velocity and acceleration of a particle are the same, the speed of the particle increases. If the signs are opposite, the speed decreases.

Example 8. What is meant by $\frac{d}{dt}|\vec{v}|$ and $\left|\frac{d\vec{v}}{dt}\right|$, can these be equal?

Can (i) $\frac{d}{dt}|\vec{v}| = 0$ while $\left|\frac{d\vec{v}}{dt}\right|$ (ii) $\frac{d}{dt}|\vec{v}| \neq 0$ while $\left|\frac{d\vec{v}}{dt}\right| = 0$

Solution $\frac{d}{dt}|\vec{v}|$ means the time rate of change of speed and $\left|\frac{d\vec{v}}{dt}\right|$ means the magnitude of acceleration.

- When a particle moves with uniform velocity.
- When a particle moves with constant acceleration along straight line path.

(i) $\frac{d}{dt}|\vec{v}| = 0$ i.e. speed constant $\left|\frac{d\vec{v}}{dt}\right| \neq 0 \, |\vec{a}| \neq 0$ may be due to change in direction.

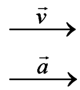
(ii) $\left|\frac{d\vec{v}}{dt}\right| = 0$, $\frac{d}{dt}|\vec{v}| \neq 0$ not possible.

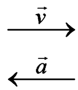
Concept

Difference between magnitude of rate of change of velocity and rate of change of magnitude of velocity

$$|\vec{a}| = \left|\frac{d\vec{v}}{dt}\right| \neq \frac{d|\vec{v}|}{dt};$$

Change in velocity: ***Only magnitude***

(i) acceleration should be parallel to velocity  velocity will increase

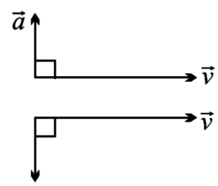
(ii) anti parallel to velocity  velocity will decrease

(iii) corresponds to one dimensional motion

Only direction

(i) acceleration should be perpendicular to velocity

(ii) corresponds to Uniform Circular Motion and any general curved path with constant speed.



Both:

Acceleration should have an angle θ with velocity where θ ≠ 0, 180°, 90° etc. Here acceleration will have two components

(a) along velocity → will change magnitude

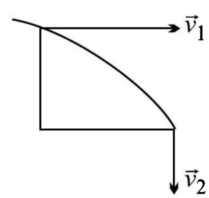
(b) perpendicular velocity → direction

(ii) corresponds to projectile and any general curved path.

Concept

When there is either a change in magnitude or direction of velocity or both there will be non zero acceleration.

Example 9. Find the magnitude of average acceleration of minute hand of clock (length = 20 cm) rotate an angle of π/2.



Solution $|\Delta v| = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos 90°} = \sqrt{2}v$ t = 15 min = 15 × 60 sec

$$|\vec{v}| = \frac{2\pi r}{T} = \frac{2\pi \times 0.20}{60 \times 60}$$

$$|\vec{a}| = |\vec{a}| = \frac{\sqrt{2}(2\pi \times 0.2)}{3600 \times 15 \times 60} \text{ m/s}^2$$

Example 10. Position of a particle as a function of time is given as $x = 5t^2 + 4t + 3$. Find the velocity and acceleration of the particle at $t = 2$ s?

Solution Velocity; $v = \frac{dx}{dt} = 10t + 4$

At $t = 2$ s $\quad v = 10(2) + 4 \quad \Rightarrow \quad v = 24$ m/s

Acceleration – $a = \dfrac{d^2x}{dt^2} = 10$

Acceleration is constant, so at t = 2 s

$a = 10$ m/s²

Example 11. The position of a particle moving on X-axis is given by $x = At^3 + Bt^2 + Ct + D$.

The numerical values of A, B, C, D are 1, 4, -2 and 5 respectively and SI units are used. Find (a) the dimensions of A, B, C and D, (b) the velocity of the particle at t=4 s, (c) the acceleration of the particle at t = 4s, (d), the average velocity during the interval t =0 to t = 4s, (e) the average acceleration during the interval t = 0 to t= 4 s.

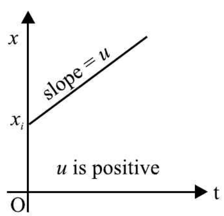
Solution [(a) [A] = [LT^{-3}], [B] = [LT^{-2}], [C] = [LT^{-1}] and [D] = [L] ; (b) 78 m/s ; (c) 32 m/s²; (d) 30 m/s ; (e) 20 m/s²]

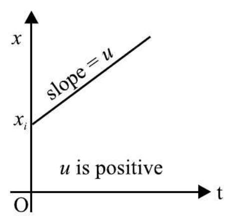
Motion with Uniform Velocity

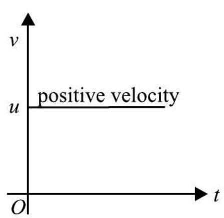
Consider a particle moving along x–axis with uniform velocity u starting from the point $x = x_i$ at $t = 0$.

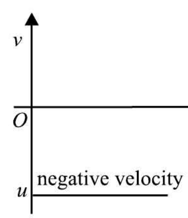
Equations of x, v, a are: $x(t) = x_i + ut$; $v(t) = u$; $a(t) = 0$

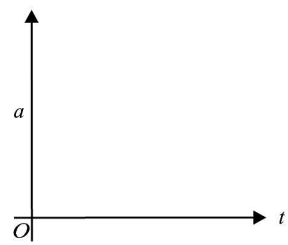
- $x - t$ graph is a straight line of slope u through x_i.
- as velocity is constant, $v - t$ graph is a horizontal line.
- $a–t$ graph coincides with time axis because $a = 0$ at all time instants.







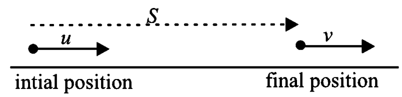




Uniformly Accelerated Motion

If a particle is accelerated with constant acceleration in an interval of time, then the motion is termed as uniformly accelerated motion in that interval of time.

For uniformly accelerated motion along a straight line (x–axis) during a time interval of t seconds, the following important results can be used.



(a) $v = u + at$

(b) $s = ut + 1/2\ at^2$

$s = vt - 1/2\ at^2$

$x_f = x_i + ut + 1/2\ at^2$

(c) $v^2 = u^2 + 2as$

(d) $s = 1/2\ (u + v)\ t$

(e) $s_n = u + a/2\ (2n - 1)$

u = initial velocity (at the beginning of interval)

a = acceleration

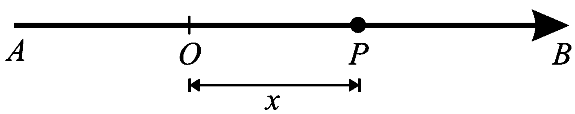
v = final velocity (at the end of interval)

s = displacement $(x_f - x_i)$

x_f = final coordinate (position) x_i = initial coordinate (position)

s_n = displacement during the n[th] sec

Explanatory Notes on Motion in One Dimension



Consider a particle moving on a straight line AB. For the analysis of motion we take origin, O at any point on the line and x–axis along the line. Generally, we take origin at the point from where particle starts its motion and rightward direction as positive x-direction. At any moment if particle is at P then its position is given by OP = x.

Velocity is defined as, $$v = \frac{dx}{dt}$$

Acceleration is defined as, $$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{v\,dv}{dx}$$

Motion in a Straight Line with Uniform Velocity

If motion takes place with a uniform velocity v on a straight line, then displacement in time t, $s = v\,.\,t$, acceleration of particle is zero ... (1)

Motion in a Straight Line with Uniform Acceleration-Equations of Motion

Let a particle move in a straight line with initial velocity u (velocity at time $t = 0$) and with uniform acceleration a. Let its velocity be v at the end of the interval of time t (final velocity at time t). Let S be its displacement at the instant t.

Now, acceleration $$a = \frac{v - u}{t}$$

or, $$v = u + at \qquad ...(2)$$

If u and a are in the same direction, a is positive and hence the velocity increases with time. If a is opposite to the direction of u, a is negative and the velocity decreases with time.

Displacement during the time interval t = average velocity $\times t$

$$S = \frac{u+v}{2} \times t \qquad \dots (3)$$

Eliminating v from equations (2) and (3) we get

$$S = \frac{u+u+at}{2} \times t$$

or $$S = ut + \frac{1}{2} at^2 \qquad \dots (4)$$

Another equation is obtained by eliminating t from equations (2) and (3)

$$v = u + at \quad \text{or} \quad a = \frac{v-u}{t}$$

$$S = \frac{v+u}{2} \times t$$

or $$aS = \frac{v-u}{t} \times \frac{v+u}{2} \times t = \frac{v^2-u^2}{2}$$

or $$v^2 - u^2 = 2aS$$

$$v^2 = u^2 + 2aS \qquad \dots (5)$$

Distance traversed by the particle in the n^{th} second of its motion

The velocity at the beginning of the n^{th} second = $u + a(n-1)$

The velocity at the end of the n^{th} second = $u + an$

Average velocity during the n^{th} second = $\frac{u+a(n-1)+u+an}{2} = u + \frac{1}{2} a(2n-1)$

Distance traversed during this one second

$$S_n = \text{average velocity} \times \text{time} = \left[u + \frac{1}{2}a(2n-1)\right] \times 1$$

i.e. $$Sn = u + \frac{1}{2} a(2n-1) \qquad \dots (6)$$

The five equations derived above are very important and are to be memorized. They are very useful in solving problems in straight–line motion.

Calculus method of deriving equations of motion:

The acceleration of a body is defined as

$$a = \frac{dv}{dt} \quad \text{i.e.} \quad dv = a\,dt$$

Integrating, we get, $v = at + A$

where A is constant of integration. By the initial condition when $t = 0$, $v = u$ (initial velocity), we get $A = u$

$\therefore$ $$v = u + at$$

we know that the instantaneous velocity $v = \frac{dS}{dt}$

Displacement of body for duration dt from time t to $t + dt$ is given by

$$dS = v\,dt\,(u + at)\,dt$$

On integration we get,

$$S = ut + \frac{1}{2}at^2 + B, \text{ where } B \text{ is integration constant}$$

At, $t = 0, S = 0$ yields $B = 0$

$$S = ut + \frac{1}{2}at^2$$

$$\text{Acceleration} = \frac{dv}{dt} = \frac{dv}{dS} \cdot \frac{dS}{dt} = v \cdot \frac{dv}{dS} \qquad \therefore$$

$$\therefore \quad a = v\frac{dv}{dS}$$

$$a \,.\, dS = v \,.\, dv$$

Integrating we get,

$$aS = \frac{v^2}{2} + C, \text{ where } C \text{ is integration constant}$$

Applying initial condition, when $S = 0$, $v = u$ we get

$$0 = \frac{u^2}{2} + C \qquad \text{or} \qquad C = \frac{u^2}{2}$$

$$\therefore \quad aS = \frac{v^2}{2} - \frac{u^2}{2} \qquad \therefore \quad v^2 - u^2 = 2\,aS \quad \Rightarrow \quad v^2 = u^2 + 2aS$$

If S_1 and S_2 are the distances traversed during n seconds and $(n - 1)$ seconds

$$S_1 = un + \frac{1}{2}an^2$$

$$S_2 = u(n-1) + \frac{1}{2}a(n-1)^2$$

Displacement in n[th] second

$$S_n = S_1 - S_2 = u_n + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2$$

$$S_n = u + \frac{1}{2}a(2n-1)$$

Example 12. A certain automobile manufacturer claims that its super–deluxe sport's car will accelerate from rest to a speed of 21.0 ms^{-1} in 4.0 s. Under the important assumption that the acceleration is constant. Find:

(a) the acceleration of car in ms^{-1}.
(b) the distance the car travels in 4.0 s.
(c) the distance the car travels in 4^{th} second.

Solution

(a) We are given that $u = 0$ and velocity after 4 s is 21 m/s, so we can used $v = u + at$ to find acceleration.

$$a = \frac{v-u}{t} = \frac{21.0-u}{4.0} = 5.25 \text{ ms}^{-2}$$

(b) Distance travelled in 4.0 s

We can use, $S = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 5.25 \times 4^2 = 42.00$ m

(c) Distance travelled in 4^{th} second

We have $S_n = u + (2n - 1)\frac{a}{2} = (2 \times 4 - 1) \times \frac{5.25}{2} = 18.375$ m

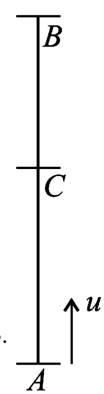
Vertical Motion Under Gravity

When a body is thrown vertically upward or dropped from a height, it moves in a vertical straight line. If the air resistance offered by air to the motion of the body is neglected, all bodies moving freely under gravity will be acted upon by its weight only. This causes a constant vertical acceleration g having value 9.8 m/s^2, so the equation for motion in a straight line with constant acceleration can be used with proper sign convantion.

Projection of a body vertically upwards:

Suppose a body is projected vertically upwards from a point A with velocity u.

If we take upward direction as positive



(i) At time t, its velocity $v + u = gt$

(ii) At time t, its displacement from A is given by

$$S = ut - \frac{1}{2}gt^2$$

(iii) Its velocity when it has a displacement S is given by

$$v^2 = u^2 - 2gS$$

(iv) When it reaches the maximum height from A, its velocity $v = 0$.

This happens when $t = \frac{u}{g}$. The body is instantaneously at rest at the highest point B.

(v) The maximum height reached, $H = \frac{u^2}{2g}$

(vi) Total time to go up and return to the point of projection = $\frac{2u}{g}$

Since, $S = 0$ at the point of projection,

$$S = ut - \frac{1}{2}gt^2$$

$$0 = ut - \frac{1}{2}gt^2 \text{ or } t = \frac{2u}{g}$$

Since the time of ascent $= \frac{u}{g}$, the time of descent $= \frac{2u}{g} - \frac{u}{g} = \frac{u}{g}$

(vii) At any point C between A and B, where AC = S, the velocity v is given by ,

$$v = \pm\sqrt{u^2 - 2gS}$$

The velocity of body while crossing C upwards = + $\sqrt{u^2 - 2gS}$ and while crossing C downwards is $-\sqrt{u^2 - 2gS}$. The magnitudes of the velocities are the same.

Example 13. A body is projected upwards with a velocity 98 m/s. Find:

(a) the maximum height reached.

(b) the time taken to reach the maximum height.

(c) its velocity at a height 196 m from the point of projection.

(d) velocity with which it will cross down the point of projection.

(e) the time taken to reach back the point of projection.

Solution

(a) The maximum height reached

Initial upward velocity $u = 98$ m/s

Acceleration $a = (-g) = -9.8$ m/s^2

Maximum height reached H is given by,

$$v^2 = u^2 + 2aS$$

$$0 = 98^2 + 2(-9.8)H$$

$$H = \frac{98^2}{2 \times 9.8} = 490\ m$$

(b) The time taken to reach the maximum height

$$t = \frac{u}{g} = \frac{98}{9.8} = 10\ s$$

(c) Velocity at a height of 196 m from the point of projection

$$v^2 = u^2 + 2aS$$

$$v^2 = 982 + 2(-9.8)\ 196$$

$$v = \pm\sqrt{5762.4} = \pm 75.91 \text{ m/s}$$

+ 75.91 m/s while crossing the height upward and – 75.91 m/s while crossing it downward

(d) Velocity with which it will cross down the point of projection

$$v^2 = u^2 + 2gS$$

At the point of projection S = 0

$\therefore$ $v = \pm u$

While crossing the point of projection downwards, $v = -u = -98$ m/s

The velocity has the same magnitude as the initial velocity but reversed in direction

(e) The time taken to reach back the point of projection

$$t = \frac{2u}{g} = \frac{2\times 98}{9.8} = 20\ s$$

Derivation of Equation of Motion for Constant Accleration with Calculus

These are valid only for constant acceleration

1. $\vec{a} = \frac{d\vec{v}}{dt}$; $d\vec{v} = \vec{a}\,dt$; $\int_{\vec{u}}^{\vec{v}} d\vec{v} = \int_0^t \vec{a}\,dt$; $[\vec{v}]_{\vec{u}}^{\vec{v}} = \vec{a}\int dt$; $\vec{v} - \vec{u} = \vec{a}\,t$; $\vec{v} = \vec{u} + \vec{a}\,t$

2. We know, $d\vec{s} = \vec{v}\,dt$

   $$\int_0^s d\vec{s} = \int_0^t \vec{v}\,dt\ ;\ \int_0^s d\vec{s} = \int_0^t (\vec{u} + \vec{a}t)\,dt\ ;\ \int_0^s d\vec{s} = \int_0^t \vec{u}\,dt + \int_0^t \vec{a}t\ dt$$

   $$\vec{s} = \vec{u}t + \vec{a}\times\frac{1}{2}t^2\ ;$$

   [Ans. $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}\,t^2$ **]**

3. Derive for one dimensional motion $v^2 - u^2 = 2as$

Example 14. Derive these equation (a) $\vec{s} = \vec{v}t - \frac{1}{2}\vec{a}\,t^2$

Solution

$$\vec{s} = \vec{v}t - \frac{1}{2}\vec{a}\,t^2,$$

we know that, $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}\,t^2$; $\vec{u} = \vec{v} - \vec{a}t$

$\therefore$ substituting the value

$$\vec{s} = (\vec{v} - \vec{a}t)t + \frac{1}{2}\vec{a}t^2 \Rightarrow \vec{s} = \vec{v}t - \vec{a}t^2 + \frac{1}{2}\vec{a}t^2 \Rightarrow \vec{s} = \vec{v}t - \frac{1}{2}\vec{a}t^2$$

Try to Derive this equation

$$\vec{s} = \left[\frac{\vec{v} + \vec{u}}{2}\right]t \qquad \text{...(1)}$$

Try to Derive this equation $\vec{v}_{avg} = \frac{\vec{v} + \vec{u}}{2}$ when equation 3rd is divided by *t* on both side, it will give

$$\frac{\vec{s}}{t} = \left[\frac{\vec{v}+\vec{u}}{2}\right]\frac{t}{t} \quad \Rightarrow \quad \vec{v}_{avg} = \frac{\vec{v}+\vec{u}}{2}$$

Displacement in nth second

$$S_n = u_n + \frac{1}{2}an^2 \qquad S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

$$S_{n^{th}} = S_n - S_{n-1} = u(1) + \frac{1}{2}a(n-n+1)(n+n-1)$$

$$s_{n^{th}} = u + \frac{1}{2}a(2n-1) \qquad \text{... (2)}$$

n and 1 in eq (2) have dimension of 't'. $s_{n^{th}}$ means average velocity in n^{th} second.

Important Points

1. These equations are used without vector sign but this does not mean they are not vectors. [as we have to use proper signs]
2. While we are following above stated method we have to use same equation for acceleration and deacceleration.
3. At any instant particle may be situated on –ve side of origin but still sign of velocity and accleration are decided by the sense of +ve x direction or from the basic definitions.

Example 15. A particle moving rectilinearly with constant acceleration is having initial velocity of 10 m/s. After some time, its velocity becomes 30 m/s. Find out velocity of the particle at the mid point of its path?

Solution Let the total distance be $2x$.

∴ distance upto midpoint = x

Let the velocity at the mid point be v and acceleration be a

From equations of motion

$$v^2 = 10^2 + 2ax \qquad \text{... (1)}$$

$$30^2 = v^2 + 2ax \qquad \text{... (2)}$$

Equation (2) – (1) gives

$$v^2 - 30^2 = 10^2 - v^2 \Rightarrow v^2 = 500 \Rightarrow v = 10\sqrt{5} \text{ m/s}$$

Example 16. A police inspector in a jeep is chasing a pickpocket an a straight road. The jeep is going at its maximum speed v (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance d away, and the motorcycle starts with a constant acceleration a. Show that the pickpocket will be caught if $v \geq \sqrt{2ad}$.

Solution Suppose the pickpocket is caught at a time t after motorcycle starts. The distance travelled by the motorcycle during this interval is

$$s = \frac{1}{2}at^2 \qquad \text{... (1)}$$

During this interval the jeep travels a distance

$$s + d = vt \qquad \text{... (2)}$$

By (1) and (2), $\frac{1}{2}at^2 + d = vt$

or, $t = \dfrac{v \pm \sqrt{v^2 - 2ad}}{a}$

The pickpocket will be caught if it is real and positive.

This will be possible if $v^2 \geq 2ad$ or, $v \geq \sqrt{2ad}$

Example 17. A car deccelerates from a speed of 20 m/s to rest in a distance of 100 m. What was its acceleration, assumed constant?

Solution [**Ans.** – 2 m/s^2]

Example 18. A 150 m long train accelerates uniformly from rest. If the front of the train passes a railway worker 50 m away from the station at a speed of 25 m/s, what will be the speed of the back part of the train as it passes the worker?

Solution [**Ans.** 50 m/s]

Example 19. Spotting a police car, you brake a Porsche from a speed of 108 km/h to a speed of 72 km/h during a displacement of 88.0 m, at a constant acceleration.

(a) What is that acceleration?

(b) How much time is required for the given decrease in speed?

Solution [**Ans.** (a) – 2.84 m/s^2; (b) 3.52 sec]

Example 20. A particle starts with velocity 10 ms^{-1} and deacceleration of 5 ms^{-2}. Find displacement and distance traversed in 6 sec.

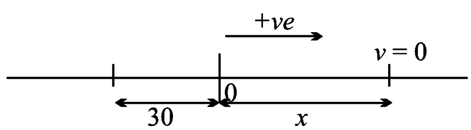
[**Ans.** displacement = – 30 m ; distance =50 m]

Solution Displacement

uniform acceleration ⇒ Newton's equations can be used

$u = 10$ ms^{-1} ; $a = -5$ ms^{-2}; $t = 6$ sec; s = ?

$s = ut + \frac{1}{2}at^2$



Ans. $s = 10 \times 6 + \frac{1}{2}(-5) \times 6 \times 6 = 60 - 90 = -30$ m

For finding distance traversed we should find displacement for $v = 0$

$u = 10, a = -5, v = 0 \qquad \therefore\ v^2 = u^2 + 2ax$

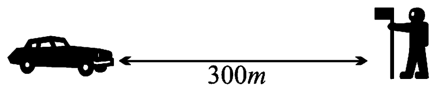
$\Rightarrow \quad x = \dfrac{v^2 - u^2}{2a} = \dfrac{(0)^2 - (10)^2}{2(-5)} = 10$ m

Ans. total distance = $2x + |s| = 2 \times 10 + 30 = 50$ m

Concept

In the equation of motion 'S' stands for displacment not for distance for calculating distance split the motion in two parts.

Example 21. A car can travel at maximum speed of 180 km/hr and can have maximum acceleration 5 m/s² and retardation of 3 m/s². How fast can it start from rest and come to rest in travelling 300 m.



Solution v_{max} = 180 km/hr = 50 m/s

$a = 5$ m/s²; $r = 3$ m/s²; $v = 0$; $u = 0$; $s = 300$ m

Part-I: Accelerating $u = 0$; $a = 5$m/s²

$$v = v_1 \quad v_1 = \sqrt{10x} \text{ m/s} \quad t_1 = \frac{\sqrt{10x}}{5} \text{ sec}$$

Part-II: Retarding $s = (300 - x)$ meter

$$u = \sqrt{10x}; \quad v = 0; \; a = -3$$

$$0 = 10x + 2(300 - x)(-3) \quad \Rightarrow \quad 10x = 6(300 - x);$$

$\therefore \quad x = 112.5$

Solving for t_1: $t_1 = 3\sqrt{5}$

$$t_2 = \frac{\sqrt{10x}}{3} = \frac{\sqrt{10 \times 112.5}}{3} = \sqrt{\frac{1125}{9}} = \sqrt{125} = 5\sqrt{5}$$

[**Ans.** Total time taken = $3\sqrt{5} + 3\sqrt{5} = 8\sqrt{5}$ sec]

Graphs in Uniformly Accelerated Motion ($a \neq 0$)

- x is a quadratic polynomial in terms of t. Hence $x - t$ graph is a parabola.

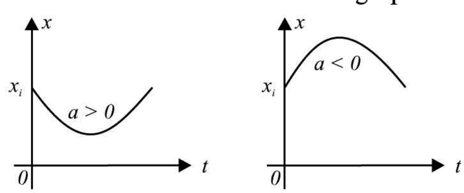


Fig 2.1: *x-t graph*

- v is a linear polynomial in terms of t. Hence $v - t$ graph is a straight line of slope a.

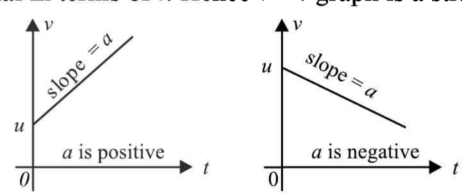


Fig 2.2: *v-t graph*

- *a–t* graph is a horizontal line because a is constant.

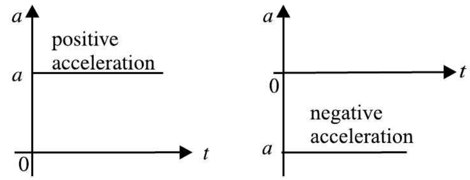


***Fig 2.3:** a-t graph*

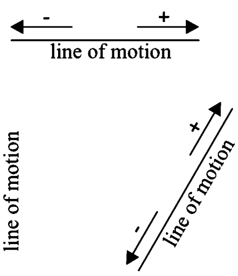
Reaction Time

When a situation demands our immediate action. It takes some time before we really respond. Reaction time is the time a person takes to observe, think and act.

Directions of Vectors in Straight Line Motion

In straight line motion, all the vectors (position, displacement, velocity and acceleration) will have only one component (along the line of motion) and there will be only two possible directions for each vector.

- For example, if a particle is moving in a horizontal line (x–axis), the two directions are right and left. Any vector directed towards right can be represented by a positive number and towards left can be represented by a negative number.
- For vertical or inclined motion, upward direction can be taken +ve and downward as –ve



- For objects moving vertically near the surface of the earth, the only force acting on the particle is its weight (mg) i.e. the gravitational pull of the earth. Hence, acceleration for this type of motion will always be $a = -g$ i.e. $a = -\ 9.8\ m/s^2$ (–ve sign, because the force and acceleration are directed downwards, If we select upward direction as positive).

Note

(a) If acceleration is in same direction as velocity, then speed of the particle increases.

(b) If acceleration is in opposite direction to the velocity then speed decreases i.e. the particle slows down. This situation is known as *retardation*.

***Example* 22.** Mr. Sharma brake his car with constant acceleration from a velocity of 25 m/s to 15 m/s over a distance of 200m.

(a) How much time elapses during this interval?

(b) What is the acceleration?

(c) If he has to continue braking with the same constant acceleration, how much longer would it take for him to stop and how much additional distance would he cover?

Solution

(a) We select positive direction for our coordinate system to be the direction of the velocity and choose the origin so that $x_i = 0$ when the braking begins. Then the initial velocity is $u_x = +25$ m/s at $t = 0$, and the final velocity and position are $v_x = +15$ m/s and $x = 200$ m at time t.

Since the acceleration is constant, the average velocity in the interval can be found from the average of the initial and final velocities.

$$\therefore \quad v_{av,\,x} = \frac{1}{2}(u_x + v_x) = \frac{1}{2}(15 + 25) = 20 \text{ m/s}.$$

The average velocity can also be expressed as $v_{av,\,x} = \frac{\Delta x}{\Delta t}$. With $\Delta x = 200$ m and $\Delta t = t - 0$, we can solve for t:

$$t = \frac{\Delta x}{v_{avx}} = \frac{200}{20} = 10 \text{ s}.$$

(b) We can now find the acceleration using $v_x = u_x + a_x t$

$$a_x = \frac{v_x - u_x}{t} = \frac{15 - 25}{10} = -1 \text{ m/s}^2.$$

The acceleration is negative, which means that the positive velocity is becoming smaller as brakes are applied (as expected).

(c) Now with known acceleration, we can find the total time for the car to go from velocity $u_x = 25$ m/s to $v_x = 0$. Solving for t, we find

$$t = \frac{v_x - u_x}{a_x} = \frac{0 - 25}{-1} = 25 \text{ } s.$$

The total distance covered is

$$x = x_i + u_x t + \frac{1}{2}a_x t^2 = 0 + (25)(25) + \frac{1}{2}(-1)(25)^2 = 625 - 312.5 = 312.5 \text{ m}.$$

Additional distance covered = 312.5 – 200 = 112.5 m.

Example 23. A particle is dropped from a tower. It is found that it travels 45 m in the last second of its journey. Find out the height of the tower?

Solution Let the total time of journey be n seconds.

Using; $s_n = u + \frac{a}{2}(2n - 1)$

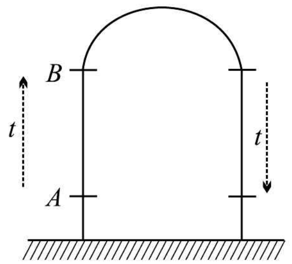
$$45 = 0 + \frac{10}{2}(2n - 1) \quad n = 5 \text{ sec}$$

Height of tower; $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 5^2 = 125$ m

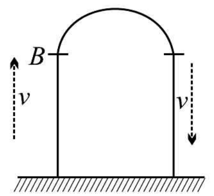
Explanatory Notes on Free Fall

If you tossed an object either up or down and could somehow eliminate the effects of air on its flight, you would find that the object accelerates downward at a certain constant rate. That rate is called the free-fall acceleration, and its magnitude is represented by g. The acceleration is independent of the object's characteristics, such as mass, density, or shape; it is the same for all objects.

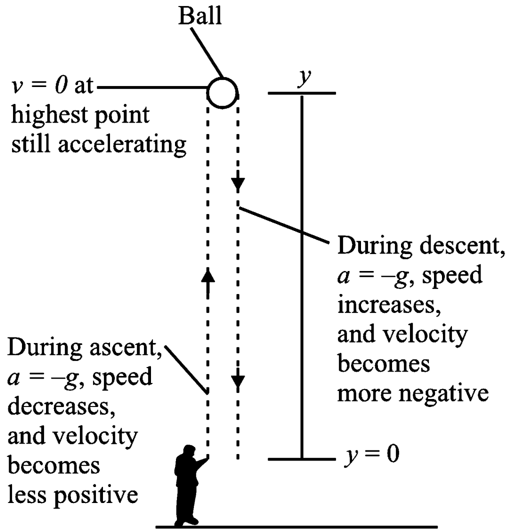
Some results:



1. Maximum Height $H = \frac{u^2}{2g}$
2. Time to reach maximum height $t = \frac{u}{g}$
3. Total time of flight $= \frac{2u}{g}$
4. Time of ascent = Time of descent for motion between two specific points.
5. If an object is dropped (means initial velocity is zero) from Height h. Its speed on reaching ground is $v = \sqrt{2gh}$ and time taken to reach ground is $t = \sqrt{\frac{2h}{g}}$
6. A particle has the same speed at a point on the path. While going vertically up and down.



Example 24. In figure a kid tosses a ball up, with an initial speed of 10 m/s. ($g = 10\text{m/s}^2$)



(a) How long does the ball take to reach its maximum height ?

(b) What is the ball's maximum height above its release point ?

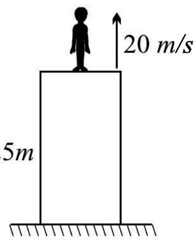
(c) How quick does the ball take to reach a point 1.8 m above its release point?

[Ans. (a) 1 sec; (b) 5m; (c) 0.2 s]

Concept

1. To understand sign. connection.
2. At heighest point velocity is zero but still acceleration.
3. a and v have opposite sign. so speed ↓ during upwards a and v have same sign. speed ↑.

Example 25. A person standing on a top of a 25 m high building throws a ball upward at the speed of 20 m/s. Find time that ball will take to come down.



Solution **Case-I**:

$u = 20;$ $g = -10;$ $v = 0;$ $t = ?$

$v = u + gt \Rightarrow 0 = 20 - 10t \Rightarrow t = 2$ sec

$$s = ut + \frac{1}{2}gt^2 = 20 \times 2 + \frac{1}{2}(-10)(4) \Rightarrow 40 - 20 = 20 \text{ m}$$

Case-II: $s = 45;$ $g = 10;$ $u = 0$

$$s = \frac{1}{2} \times 10t^2 \Rightarrow t = 3 \text{ sec} \quad \therefore \quad t = 5 \text{ sec}$$

Concept

(i) No need to brake motion into upward and downward motion.

(ii) To give the vector idea of displacement.

Explain how by using sign we can get result and how signs take care of whole path

Taking upper side as (+): $u = 20$ m/s; $a = -10$ m/s²; $s = -25$ m

$$s = ut + \frac{1}{2}at^2$$

$$-25 = 20t + \left(\frac{-10t^2}{2}\right) \Rightarrow -25 = 20t - 5t^2 \Rightarrow 5t^2 - 20t - 25 = 0$$

$t = 5, -1;$ $\therefore$ $t = 5$ sec

Note

Many answers emerged automatically with minus signs. It is important to know what these signs mean. For these two freely falling-body problems, we established a vertical axis (the y-axis) and we chose-quite arbitrarily-its upward direction to be positive.

We then chose the origin of the y-axis (that is, the y = 0 position) to suit the problem. In this example, the origin was at the top of the building, and in previous example. It was at the kid's hand. A negative value of y then means that the body is below the chosen origin. A negative velocity means that the body is moving in the negative direction of the y-axis – that is, downward. This is true no matter where the body is located.

Mathematics often generates answers that you might not have thought of as possibilities, as in this example. If you get more answers than you expect, do not automatically discard the ones that do not seem to fit. Examine them carefully for physical meaning. Like here time is our variable, then even a negative value (-1) can mean something ; negative time simply refers to time before t = 0, the (arbitrary) time at which you decided to start your stopwatch.

Concept

In kinematics we may come across situations where a moving large body drops a body. The initial velocity of the dropped body is equal to the velocity of the moving large body.

Example 26. A lift is moving up with an acceleration of $4 m/s^2$ starting from rest. 5 sec after the start, a coin is dropped from the lift. Find:

(a) The initial velocity of the dropped coin.
(b) The height attained by the lift till the time of drop.
(c) The time when the coin reaches ground.

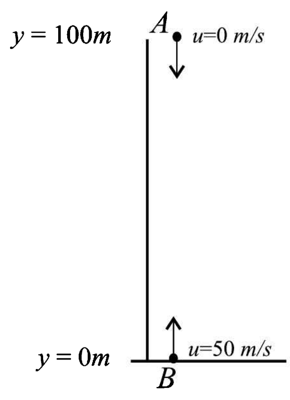
Solution (a) 20 m/s (b) 50 m (c) 5.74 sec after drop.

Example 27. A particle is dropped from height 100 m and another particle is projected vertically up with velocity 50 m/s from the ground along the same line. Find out the position where two particle will meet?

Solution Let the upward direction be positive.

Let the particles meet at a distance y from the ground.

For particle A,



$$y_0 = +100 \text{ m} \quad u = 0 \text{ m/s} \qquad g = -10 \text{ m/s}^2$$

$$y = 100 + 0(t) - \frac{1}{2} \times 10 \times t^2$$

$$[y = y_0 + ut + \frac{1}{2}at^2]$$

$$= 100 - 5t^2 \qquad \text{... (1)}$$

For particle B,

$$y_0 = 0 \text{ m} \quad \Rightarrow \quad u = +50 \text{ m/s} \quad \Rightarrow \quad a = -10 \text{ m/s}^2$$

$$y = 50(t) - \frac{1}{2} \times 10 \times t^2 = 50t - 5t^2 \qquad ...(2)$$

According to the problem;

$$50t - 5t^2 = 100 - 5t^2$$

$$t = 2 \text{ sec}$$

Putting $t = 2$ sec in eqn. (1),

$$y = 100 - 20 = 80 \text{ m.}$$

Hence, the particles will meet at a height 80 m above the ground.

***Example* 28.** A ball is thrown vertically upwards with velocity of 20 m/s towards ceiling which is 10 m high. Assume the collision be elastic in nature. Find time taken by ball to come back after collision?

Solution Motion of equation will not be valid in this case as acceleration is not constant. During the strike, time taken will be 0 and velocity in downward direction, hence acceleration will become infinite.

Now, $s = 10$ m; $u = 20$ m/s; $g = -10$ m/s^2; Time to reach ceiling, $t = ?$

$$10 = 20t - 5t^2$$

$$5t^2 - 20t + 10 = 0$$

$$t = \frac{20 \pm \sqrt{400 - 1 \times 200}}{10} = \frac{20 \pm 10\sqrt{2}}{10} = \left(2 \pm \sqrt{2}\right) \text{ sec}$$

For the level of celing, two answers will be there as it crosses it twice in causes of attaining maximum height but, ball collides with the ceiling , hence the answer will be smaller one

$$T = (2 - \sqrt{2})$$

$\therefore$ Total time will be twice of this = $2((2 - \sqrt{2}))$ sec

Concept

Elastic collision means that velocity perpendicular to the wall will reverse its direction.

***Example* 29.** A particle is thrown vertically with velocity 20 m/s . Find (a) the distance travelled by the particle in first 3 seconds, (b) displacement of the particle in 3 seconds.

Solution [**Ans.** 25m, 15m]

***Example* 30.** A stone is dropped from a balloon going up with a uniform velocity of 5 m/s. If the balloon was 50 m high when the stone was dropped, find its height when the stone hits the ground. Take g = 10 m/s^2.

Solution 68.5 m

Note

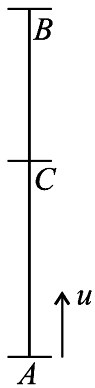
As the particle is detached from the balloon it is having the same velocity as that of balloon, but its acceleration is only due to gravity and is equal to g.

Vertical Motion Under Gravity

When a body is thrown vertically upward or dropped from a height, it moves in a vertical straight line. If the air resistance offered by air to the motion of the body is neglected, all bodies moving freely under gravity will be acted upon by its weight only. This causes a constant vertical acceleration g having value 9.8 m/s^2, so the equation for motion in a straight line with constant acceleration can be used with proper sign convantion.

Projection of a Body Vertically Upwards

Suppose a body is projected vertically upwards from a point A with velocity u.



If we take upward direction as positive

(i) At time t, its velocity $v + u = gt$

(ii) At time t, its displacement from A is given by

$$S = ut - \frac{1}{2} gt^2$$

(iii) Its velocity when it has a displacement S is given by

$$v^2 = u^2 - 2gS$$

(iv) When it reaches the maximum height from A, its velocity $v = 0$.

This happens when $t = \frac{u}{g}$. The body is instantaneously at rest at the highest point B.

(v) The maximum height reached, $H = \frac{u^2}{2g}$

(vi) Total time to go up and return to the point of projection = $\frac{2u}{g}$

Since, $S = 0$ at the point of projection,

$$S = ut - \frac{1}{2}gt^2$$

$$0 = ut - \frac{1}{2}gt^2 \quad \text{or} \quad t = \frac{2u}{g}$$

Since the time of ascent = $\frac{u}{g}$, the time of descent = $\frac{2u}{g} - \frac{u}{g} = \frac{u}{g}$

(vii) At any point C between A and B, where AC = S, the velocity v is given by,

$$v = \pm\sqrt{u^2 - 2gS}$$

The velocity of body while crossing C upwards = $+\sqrt{u^2 - 2gS}$ and while crossing C downwards is $-\sqrt{u^2 - 2gS}$. The magnitudes of the velocities are the same.

Example 31. A body is projected upwards with a velocity 98 m/s. Find:

(a) the maximum height reached.
(b) the time taken to reach the maximum height.
(c) its velocity at a height 196 m from the point of projection.
(d) velocity with which it will cross down the point of projection.
(e) the time taken to reach back the point of projection.

Solution

(a) The maximum height reached

Initial upward velocity $u = 98$ m/s

Acceleration $a = (-g) = -9.8$ m/s^2

Maximum height reached H is given by,

$$v^2 = u^2 + 2aS$$

$$0 = 98^2 + 2(-9.8)H$$

$$H = \frac{98^2}{2 \times 9.8} = 490 \text{ m}$$

(b) The time taken to reach the maximum height

$$t = \frac{u}{g} = \frac{98}{9.8} = 10 \text{ s}$$

(c) Velocity at a height of 196 m from the point of projection

$$v^2 = u^2 + 2aS$$

$$v^2 = 982 + 2(-9.8)\,196$$

$$v = \pm\sqrt{5762.4} = \pm\, 75.91 \text{ m/s}$$

+ 75.91 m/s while crossing the height upward and – 75.91 m/s while crossing it downward

(d) Velocity with which it will cross down the point of projection

$$v^2 = u^2 + 2gS$$

At the point of projection S = 0 $\quad\therefore\; v = \pm u$

While crossing the point of projection downwards, $v = -u = -98$ m/s

The velocity has the same magnitude as the initial velocity but reversed in direction

(e) The time taken to reach back the point of projection

$$t = \frac{2u}{g} = \frac{2\times 98}{9.8} = 20 \text{ s}$$

Variable Acceleration

***Example* 32.** A particle is moving on a straight line path, such that $v = 3t^2$ then find acceleration at $t = 3$ and magnitude of average velocity for first three seconds.

Solution $a = 6t$, $a = 18$ m/s²

$$v_{avg} = \frac{\int_{t_1}^{t_2} v\,dt}{t_2 - t_1}; \quad \therefore v_{avg} = \frac{\int_0^3 3t^2\,dt}{3} = \left.\frac{t^3}{3}\right|_0^3 = 9 \text{ ms}^{-1}$$

Concept

To find average speed using integration as velocity is variable.

***Example* 33.** $a = 4 - 2t$; Initial velocity at $t = 0$, $u = 5$; Find distance travelled till 12 sec?

Solution $a = 4 - 2t$

$$\int_5^v dv = \int_0^t (4 - 2t)\,dt$$

$$v = 4t - t^2 + 5 = -t^2 + 4t + 5 = -(t^2 - 4t - t) = -(t - 5)(t + 1)$$

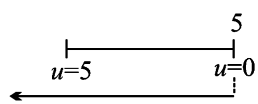
After 5 sec, velocity will be negative

$$\int_0^x dx = \int_0^t (4t - t^2 + 5)\,dt \quad\Rightarrow\quad x = 2t^2 - \frac{t^3}{3} + 5t$$

$$x_5 = x = 2(5)^2 - \frac{(5)^3}{3} + 25 = 100/3 \text{ m}$$ [In forward direction]

and $t = 12$ sec

$$x_{12} = x = 2(12)^2 - \frac{(12)^3}{3} + 5(12) = -228 \text{ m}$$



$$\text{motion} = \text{distance} = 2 \times u_5 + 228 = \frac{200}{3} + 228 \text{ m}$$

Concept

Equation of motion not applied if acceleration is variable. **If a is given as function of s or v we can use this equation $v\,dv = a\,ds$**

Example 34. If a particle accelerates with $a = kv^2$ and initial velocity = u then find velocity after s displacement.

Solution $|a| = \frac{dv}{dt}$; $kv^2 = \frac{dv}{dt}$; $k\,dt = \frac{dv}{v^2}$ on multiplying both sides of eq. by 'v'

$$\frac{ds}{dt}\cdot k\,dt = v\frac{dv}{v^2};\ \int k\,ds = \int \frac{dv}{v};\quad \ln v = ks + C \text{ at } s = 0;\ C = \ln u$$

$$\ln\left(\frac{v}{u}\right) = ks \quad \therefore\ v = ue^{ks}\,]$$

Example 35. a = – cos t;at $t = 0$; $u = 0$; $x = 1$

Position at t = π. Find distance from 0 to 2π.

Solution

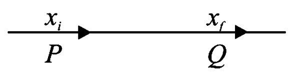
$$a = -\cos t \int_0^v dv = -\int_0^t \cos t\ dt$$

$$v = -\sin t \int_1^x dx = \int_0^\pi -\sin t\,dt$$

0 to 2π, v will change sign. velocity upto π will be negative and upto 2π it will be positive.

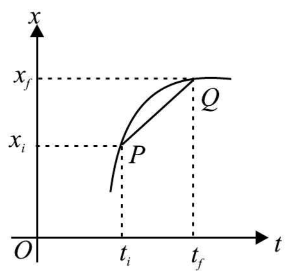
$$\int_1^x dx = \left|\int_0^\pi -\sin t\,dt\right| + \left|\int_\pi^{2\pi} \sin t\,dt\right| = 2 + 2 = 4$$

Graphical Interpretation of Some Quantities



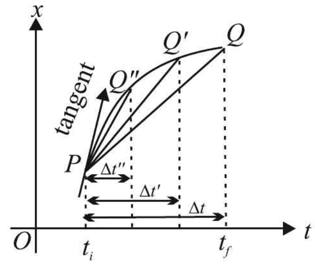
Average Velocity

If a particle passes a point P (x_i) at time $t = t_i$ and reaches Q (x_f) at a later time instant $t = t_f$, its average velocity in the interval PQ is $V_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$



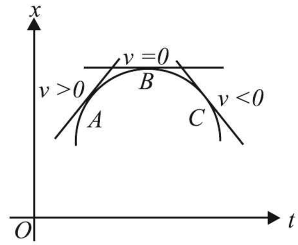
This expression suggests that the average velocity is equal to the slope of the line (chord) joining the points corresponding to P and Q on the x–t graph.

Instantaneous Velocity



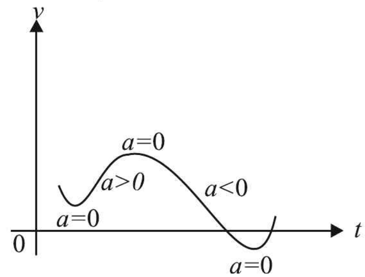
Consider the motion of the particle between the two points P and Q on the *x–t* graph shown. As the point Q is brought closer and closer to the point P, the time interval between PQ (Δ*t*, Δ*t*′, Δ*t*″,......) get progressively smaller. The average velocity for each time interval is the slope of the appropriate dotted line (PQ, PQ′, PQ″................). As the point Q approaches P, the time interval approaches zero, but at the same time the slope of the dotted line approaches that of the tangent to the curve at the point P. As $\Delta t \to 0$, V_{av} $(=\Delta x/\Delta t) \to V_{inst.}$

Geometrically, as $\Delta t \to 0$, chord PQ → tangent at P.



Hence, the instantaneous velocity at P is the slope of the tangent at P in the x-t graph. When the slope of the *x – t* graph is positive, *v* is positive (as at the point A in figure). At C, *v* is negative because the tangent has negative slope. The instantaneous velocity at point

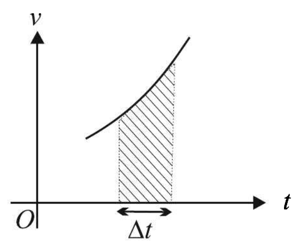
B (turning point) is zero as the slope is zero.



Instantaneous Acceleration

The derivative of velocity with respect to time is the slope of the tangent in velocity time (v–t) graph.

Displacement From v-t Graph

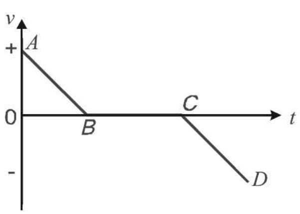


Displacement = Δ*x* = area under *v*-*t* graph.

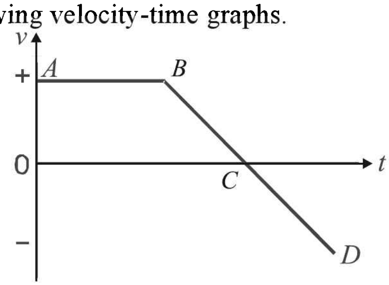
Since a negative velocity causes a negative displacement, areas below the time axis are taken negative. In similar way, we can see that $\Delta v = a\,\Delta t$ leads to the conclusion that **area under *a*-*t* graph gives the change in velocity Δ*v* during that interval.**

Example 36. Describe the motion shown by the following velocity-time graphs.

(a)



(b)



Solution

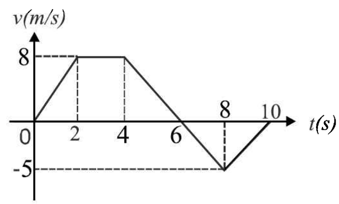
(a) **During interval AB:** velocity is +ve so the particle is moving in +ve direction, but it is slowing down as acceleration (slope of v-t curve) is negative. **During interval BC:** particle remains at rest as velocity is zero. Acceleration is also zero. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

(b) **During interval AB:** particle is moving in +ve direction with constant velocity and acceleration is zero. **During interval BC:** particle is moving in +ve direction as velocity is +ve, but it slows down until it comes to rest as acceleration is negative. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

Points to Remember

- For uniformly accelerated motion ($a \neq 0$), x–t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.
- For uniformly accelerated motion ($a \neq 0$), v–t graph is a straight line whose slope gives the acceleration of the particle.
- In general, the slope of tangent in x–t graph is velocity and the slope of tangent in v–t graph is the acceleration.
- The area under a–t graph gives the change in velocity.
- The area between the v–t graph gives the distance travelled by the particle, if we take all areas as positive.
- Area under v–t graph gives displacement, if areas below the t–axis are taken negative.

Example 37. For a particle moving along x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle?



Solution Distance travelled = Area under v-t graph (taking all areas as +ve.)

$\therefore$ Distance travelled = Area of trapezium + Area of triangle

$$= \frac{1}{2}(2+6)\times 8 + \frac{1}{2}\times 4\times 5 = 32 + 10 = 42 \text{ m}$$

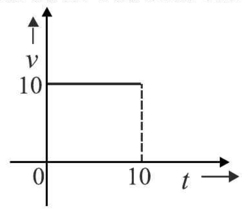
Displacement = Area under v-t graph (taking areas below time axis as -ive.)

$\therefore$ Displacement = Area of trapezium – Area of triangle

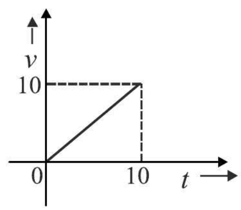
$$= \frac{1}{2}(2+6)\times 8 - \frac{1}{2}\times 4\times 5 = 32 - 10 = 22 \text{ m}$$

Hence, distance travelled = 42 m and displacement = 22 m.

Example 38. For a particle moving along x-axis, following graphs are given. Find the distance travelled by the particle in 10 s in each case?



(a)



(b)

Solution Area under the graph

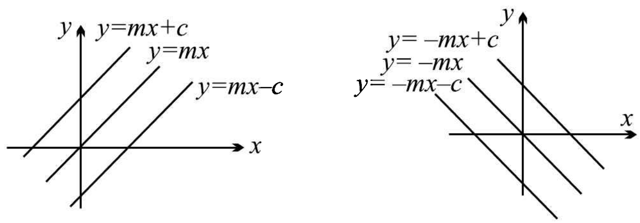
(a) Distance = 10 × 10 = 100 m

(b) Distance = $\frac{1}{2} \times 10 \times 10 = 50m$

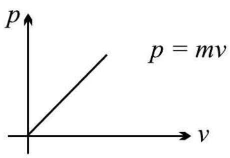
[Ans. (a) 100m; (b) 50m]

Interpretation of Some More Graphs

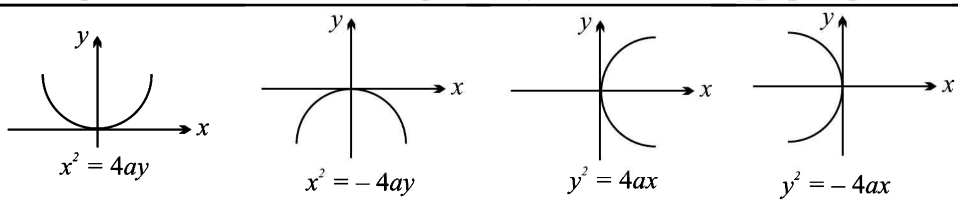
If maxiumum power of x is 1 and maximum power of y is 1, graph is straight line



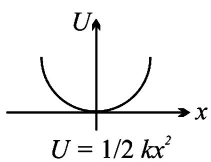
Example 39.



If maxiumum power of x is 2 and maximum power of y is 1 or vice versa, graph is parabola.



Example 40.

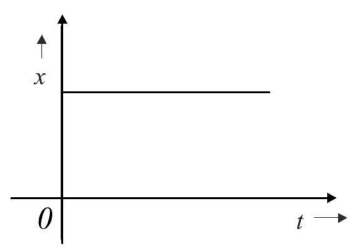


If maxiumum power of x and y is 2, graph may be circle, ellipse, two straight line tc.

Position vs. Time Graph

Zero Velocity

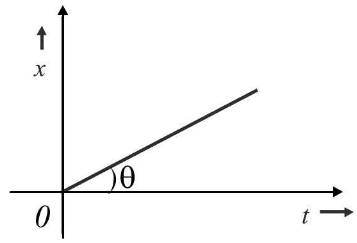
As position of particle is fix at all the time, so the body is at rest.



Slope; $\frac{dx}{dt} = \tan\theta = \tan 0^\circ = 0$

Velocity of particle is zero

Uniform Velocity

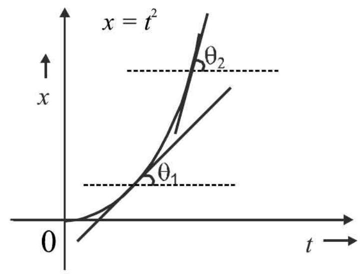


Here $\tan\theta$ is constant $\tan\theta = \frac{dx}{dt}$

$\therefore$ $\frac{dx}{dt}$ is constant.

$\therefore$ velocity of particle is constant.

Non uniform velocity (increasing with time)



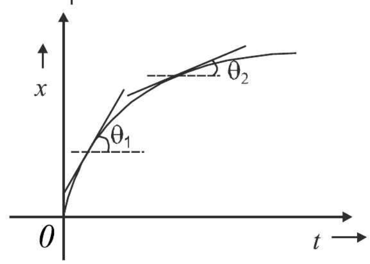
In this case;

As time is increasing, θ is also increasing.

$\therefore$ $\frac{dx}{dt} = \tan\theta$ is also increasing

Hence, velocity of particle is increasing.

Non-uniform velocity (decreasing with time)



In this case;

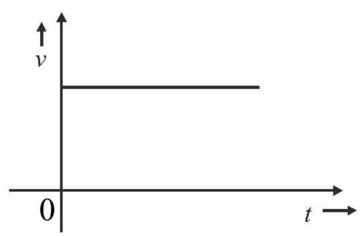
As time increases, θ decreases.

$\therefore$ $\frac{dx}{dt} = \tan\theta$ also decreases.

Hence, velocity of particle is decreasing.

Velocity vs Time Graph

Zero acceleration



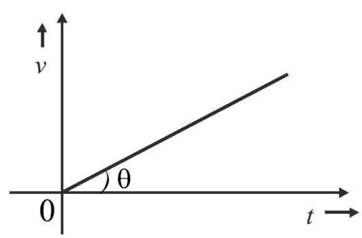
Velocity is constant.

$\tan\theta = 0$

$\therefore$ $\frac{dv}{dt} = 0$

Hence, acceleration is zero.

Uniform acceleration

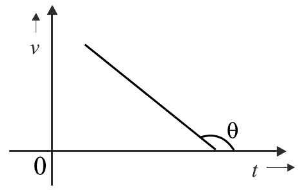


$\tan\theta$ is constant.

$\therefore$ $\frac{dv}{dt} =$ constant

Hence, it shows constant acceleration.

Uniform retardation



Since $\theta > 90^\circ$

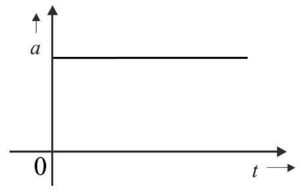
$\therefore$ $\tan\theta$ is constant and negative.

$\therefore$ $\dfrac{dv}{dt}$ = negative constant

Hence, it shows constant retardation.

Acceleration vs Time Graph

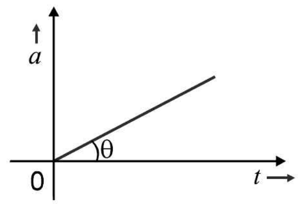
Constant Acceleration



$\tan\theta = 0$ $\qquad \therefore \dfrac{da}{dt} = 0$

Hence, acceleration is constant.

Uniformly Increasing Acceleration



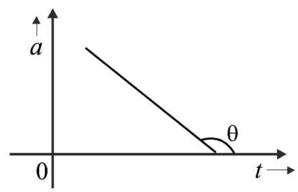
θ is constant.

$0^\circ < \theta < 90^\circ \Rightarrow \tan\theta > 0$

$\therefore$ $\dfrac{da}{dt} = \tan\theta$ = constant > 0

Hence, acceleration is uniformly increasing with time.

Uniformly Decreasing Acceleration



Since $\theta > 90^\circ$

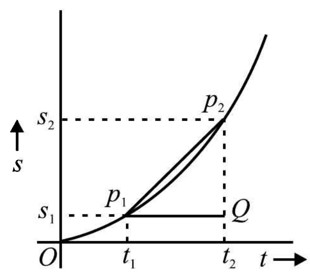
$\therefore$ $\tan\theta$ is constant and negative.

$\therefore$ $\dfrac{da}{dt}$ = negative constant

Hence, acceleration is uniformly decreasing with time

Explanatory Notes on Graph

S-t curve: If we put *s* on y-axis and *t* on x-axis for every value of *t* we have a value of *s*.



1. The average velocity from time t_1 to t_2 will be

   $$V_{avg} = \frac{s_2 - s_1}{t_2 - t_1} = \text{slope of line joining } p_1 \text{ and } p_2$$

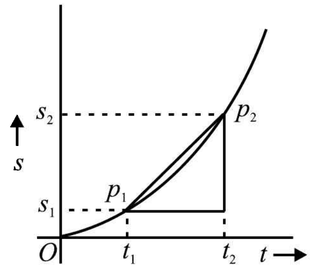
   For a particle moving along a straight line when we plot a graph of s versus t, V_{avg} is the slope of the straight line that connects two particular points on the *s*(*t*) curve: one is the point that corresponds to s_2 and t_2, and the other is the point that corresponds to s_1 and t_1. Like displacement, v_{avg} has both magnitude and direction (it is another vector quantity). Its magnitude is the magnitude of the line's slope. A positive v_{avg} (and slope) tells us that the line slants upward to the right; a negative v_{avg} (and slope), that the line slants downward to the right.

2. **Instantaneous velocity**

According to defination

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$$

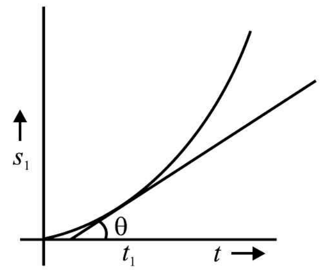
In curve if $\Delta t \to 0$ the point p_2 comes very close to point p_1.



Note

The instantaneous velocity can be found by determining the slope of the tangent to the displacement time graph at that instant. Velocity at point p_1 or time t_1 is V

$V = \tan\theta$



Cases:

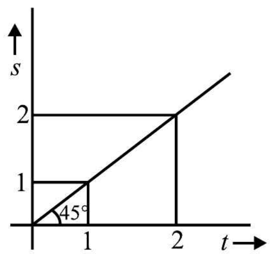
(A) Uniform velocity:

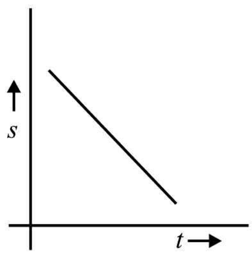
If velocity is uniform slope of curve must remain unchanged.

Curve with uniform slope is straight line

If Velocity is $1 ms^{-1} \Rightarrow S = Vt \Rightarrow s = t$

$\tan\theta = 1$





For –ve velocity

(B) Uniform acceleration: We have a particle moving with uniform acceleration a and initial velocity u. Its displacement s at any time t can be represented as

$$s = ut + \frac{1}{2} at^2$$

The curve is a parabola.

Velocity at t_1 is $\tan\theta$

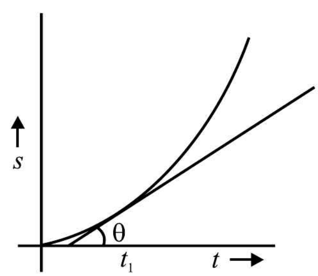
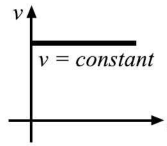
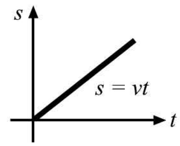
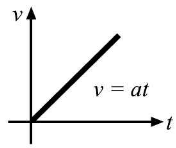
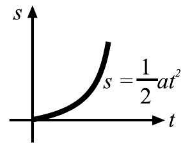
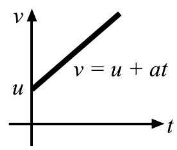
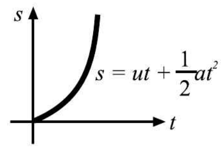
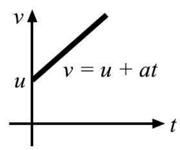
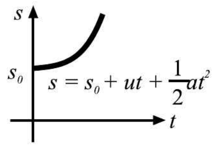
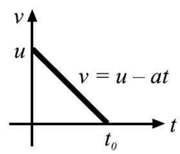
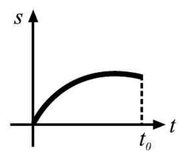
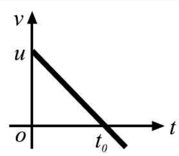
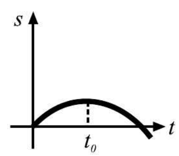
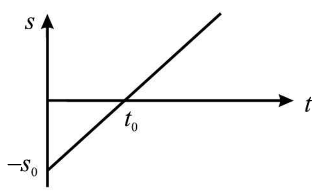


Table 2.1

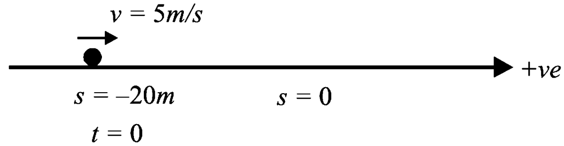
S. No.	Different Cases	*v-t Graph*	*s-t Graph*	Important Points
1.	Uniform motion			(i) Slope of v-t graph $= v =$ constant (ii) In s-t graph $s = 0$ at $t = 0$
2.	Uniformly accelerated motion with $u = 0$ and $s = 0$ at $t = 0$			(i) $u = 0$, i.e., $v = 0$ at $t = 0$ (ii) a or slope of v-t graph is constant (iii) $u = 0$, i.e., slope of s-t graph at $t = 0$, should be zero
3.	Uniformly accelerated motion with u^{1} 0 but $s = 0$ *at* $t = 0$			(i) $u = 0$, i.e., v or slope of v-t graph at $t = 0$ is not zero. (ii) s or slope of s-t graph gradually goes on increasing.
4.	Uniformly accelerated motion with u^{1} 0 and $s = s_0$ at $t = 0$			(i) $v = u$ at $t = 0$ (ii) $s = s_0$ at $t = 0$
5.	Uniformly retarded motion till velocity becomes zero			(i) Slope of s-t graph at $t = 0$ gives u (ii) Slope of s-t graph at $t = t_0$ becomes zero. (iii) In this case u can't be zero
6.	Uniformly retarded then accelerated in opposite direction			(i) At time $t = t_0$, $v = 0$ or slope of s-t graph is zero (ii) In s-t graph slope or velocity first decreases then increases with opposite sign.

***Example* 41.** Displacement-time graph of a particle moving in a straight line is shown in figure. State whether the motion is accelerated or not. Describe the motion in detail. Given $s_0 = 20$ m and $t_0 = 4$ second.



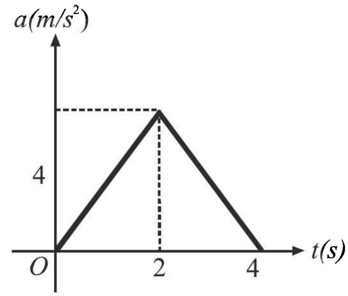
Solution Slope of s-t graph is constant. Hence, velocity of particle is constant. Further at time $t = 0$, displacement of the particle from the mean position is $-s_0$ or – 20m. Velocity of particle.

$$v = Slope = \frac{s_0}{t_0} = \frac{20}{4} = 5 \text{ m/s}$$



Motion of the particle is shown in figure. At $t = 0$ particle is at – 20 m and has a constant velocity of 5 m/s. At $t_0 = 4$ second particle will pass through its mean position.

Example 42. Acceleration-time graph of a particle moving in a straight line is shown in figure. Velocity of particle at time $t = 0$ is 2 m/s. Find velocity at the end of fourth second.



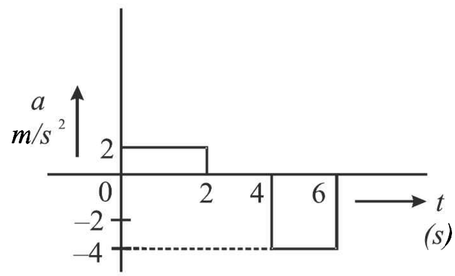
Solution $dv = adt$

or change in velocity = area under a-t graph

Hence, $v_f - v_i = \frac{1}{2}(4)(4) = 8 \text{ m/s}$

$v_f = v_i + 8 = (2 + 8) \text{ m/s} = 10 \text{ m/s}$

Example 43. The acceleration versus time graph of a particle moving along a straight line is shown in the figure. Draw the respective velocity-time graph.



Solution From $t = 0$ to $t = 2$sec, a = +2 m/s²

$\therefore \quad v = at = 2t$

or v-t graph is a straight line passing through origin with slope 2 m/s².

At the end of 2 sec,

$v = 2 \times 2 = 4$ m/s

From $t = 2$ to 4 sec, a = 0.

Hence, $v = 4$ m/s will remain constant.

From $t = 4$ to 6 sec, a = – 4 ms². Hence,

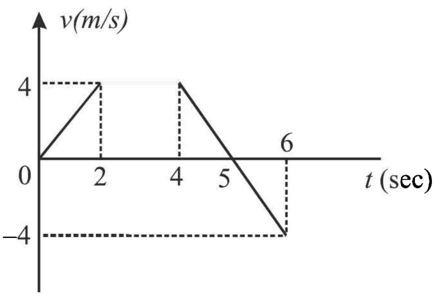
$v = u - at = 4 - 4t$ (with t = 0 at 4 sec)

$v = 0$ at $t = 1$ sec or at 5 second from origin.

At the end of 6 sec (or $t = 2$ sec) $v = -4$ m/s.

Corresponding v-t graph is shown in the figure.

Assuming at t = 0, v = 0



Example 44. A cyclist starting from a point A travels 200 m due North to a point B at constant speed of 5 ms⁻¹. He rests at B for 30 seconds and then travels 300 m due south to a point C at a constant speed of 10 ms⁻¹. Find average velocity.

Solution From A to B

Avg. velocity = –100/100 = –1 ms^{-1}

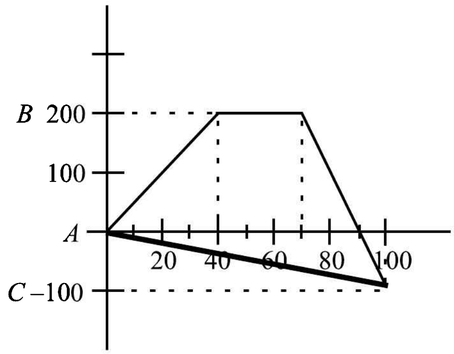
$x = 200$; $v = 5$; $t = 40$

from B to C

$v = 0$

from C to D

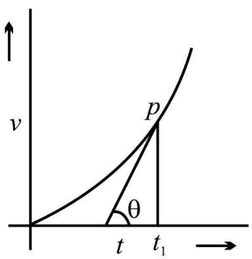
$v = -10$; $x = -300$; $t = 30$

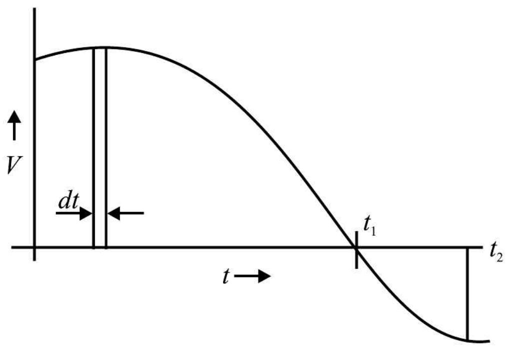


V-t curve: By using dependence of v on t we can plot a V-t graph. Slope of V-t curve at any point represents acceleration at that instant.

tan θ = acceleration at time t_1. Area under V-t graph and t-axis.

As we know $dx = V dt$ and $\int V dt = x$ = Area under V – t graph.





Thus area under curve will represent displacement in that time period.

Notes

(1) Area above t-axis +ve displacement.

(2) Area below t-axis is –ve displacement.

Thus, 1. Total displacement will be sum of areas with appropriate signs.

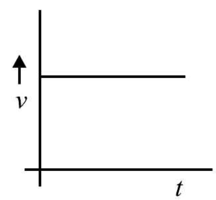
2. Total distance will be sum of areas without sign.

Cases:

(1) For uniform velocity:

acceleration = 0

slope = 0

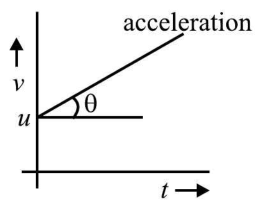


(2) For uniform straight line curve

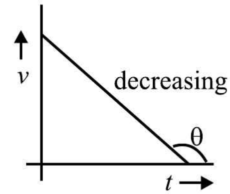
tan θ = acceleration

For increasing velocity

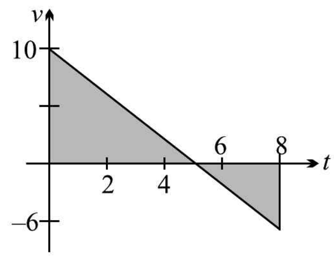
tan θ = acceleration



Note: θ is always with +ve x-axis
for decreasing velocity
(slope is –ve) i.e. $\theta > 90°$



***Example* 45.** A particle is travelling in a straight line. It has a initial velocity of 10 ms^{-1}. When it is subjected to an acceleration of –2ms^{-2} for 8 seconds. Find displacement and distance traversed in 8 seconds.



Solution $s = 10 \times 8 - \frac{1}{2} \times 2 \times 8 \times 8$

Displacement = 16 m

Displacement = $s_1 = 10 \times 5 - \frac{1}{2} \times 2 \times 25 = 25$ m

Now $u = 0$; $a = 2$; $t = 3$

$s_2 = -\frac{1}{2} \times 2 \times 9 = -9$

Alternative: $V = 10 - 2t$

Area (1) = $\frac{1}{2} \times 10 \times 5 = 25$; Area (2) = $\frac{1}{2} \times 3 \times 6 = 9$

Displacement = 25 – 9 = 16 m

Displacement = 25 + 8 = 34 m

Concept

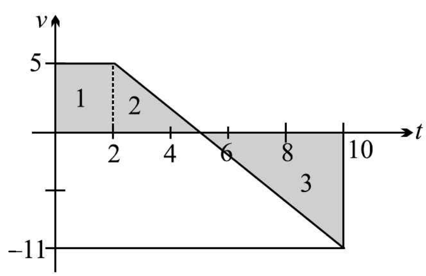
1. Using graph distance can be calculated directly.
2. Total displacement will be sum of areas with appropriate signs.
3. Total distance will be sum of areas without sign.
4. To plot straight line using equation of motion.

***Example* 46.** A particle moves in a straight line with const. velocity of 5 ms^{-1} for 2 seconds. It then moves with a constant. acceleration of –2 ms^{-2} for 8 seconds. Draw velocity–time graph for 10 seconds of motion and find.

(a) velocity

(b) displacement

(c) distance



Solution $\text{Area}_1 = 5 \times 2 = 10$

$\text{Area}_2 = \frac{1}{2} \times 5 \times 2.5 = 6.25$; $\text{Area}_3 = -\frac{1}{2} \times (11) \times 5.5 = -30.25$

Displacement = –14 m

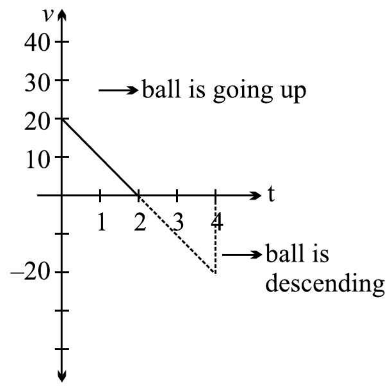
Displacement = 46.25 m

Example 47. A ball is thrown vertically upwards with velocity of 20 m/s. Using equation of motion make these graphs.

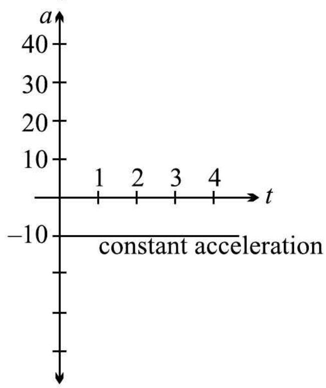
(1) *v-t* graph (2) a-t graph
(3) displacement time graph (4) speed time graph
(5) distance time graph

Solution Equation of motion $v = 20 - 10t$

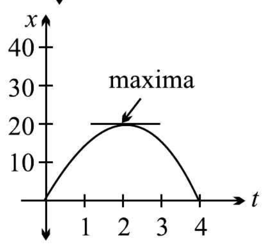
(1) *v-t* graph:



(2) *a-t* graph:



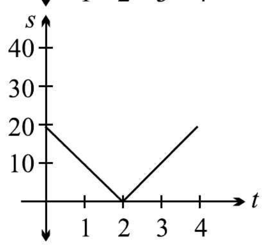
(3) Displacement-time graph:



If v change sign there is maxima or minima at that point in x-t graph

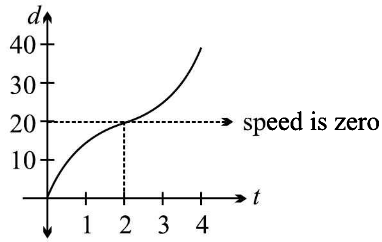
x is proportional to t^2 so it is parabola

(4) Speed-time graph:

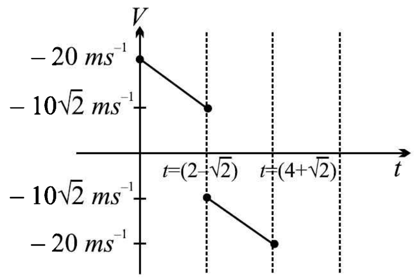


No change in sign no mark min. in distance time graph

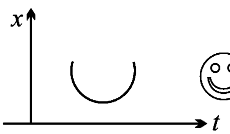
(5) Distance-time graph:



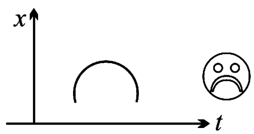
Distance time graph never decreasing so no max. min.



For fun's sake!

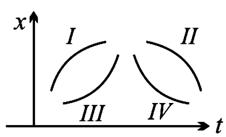


(Smiling curve)

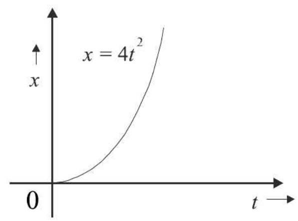


(Crying curve)

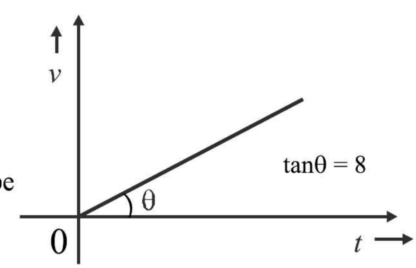
a > 0 a < 0



(I) $a < 0$ (II) $a < 0$ (III) $a > 0$(IV) $a > 0$



***Example* 48.** The displacement *vs* time graph of a particle moving along a straight line is shown in the figure. Draw velocity *vs* time and acceleration vs time graph.

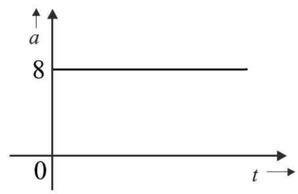


Solution $x = 4t^2$ $\Rightarrow v = \dfrac{dx}{dt} = 8t$

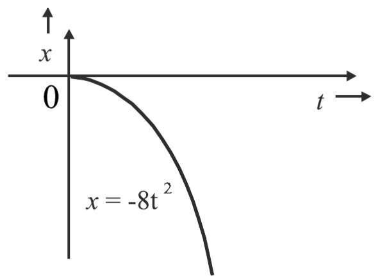
Hence, velocity-time graph is a straight line having slope i.e. $\tan\theta = 8$.

$$a = \frac{dv}{dt} = 8$$

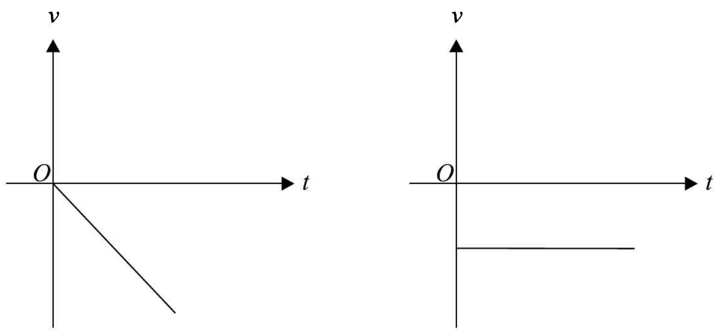
Hence, acceleration is constant throughout and is equal to 8.



Example 49. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.

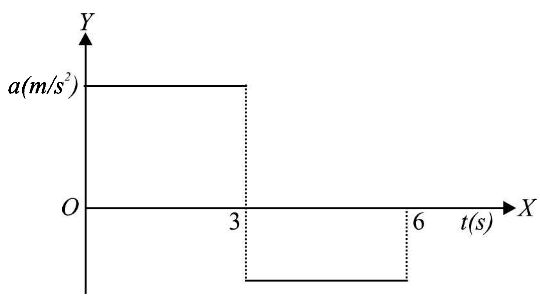


Solution



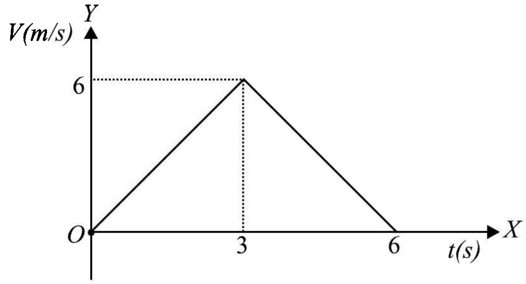
Example 50. At $t = 0$ a particle is at rest at origin. Its acceleration is 2 ms^{-2} for the first 3 s and -2 ms^{-2} for the next 3 s. Find the acceleration versus time, velocity versus time and position versus time graph.

Solution We are given that for first 3 s acceleration is 2 ms^{-2} and for next 3s acceleration is -2 ms^{-2}. Hence acceleration time graph is as shown in the figure.



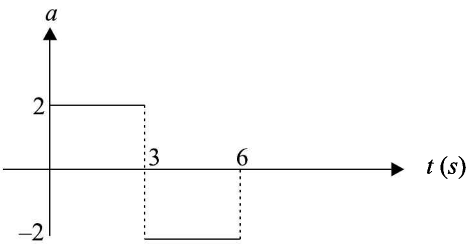
The area enclosed between a–t curve and t–axis gives change in velocity for the corresponding interval. Also at $t = 0$, $v = 0$, hence final velocity at $t = 3$s will increase to $6\,ms^{-1}$. In next 3s the velocity will decrease to zero. Hence the velocity time graph is as shown in figure.

Note that v–t curves are taken as straight line as acceleration is constant.

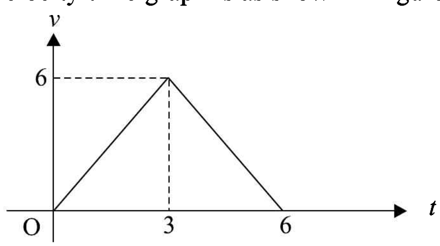


Example 51. At $t = 0$, a particle is at rest at origin. Its acceleration is $2\ ms^{-2}$ for the first 3 s and $-2\ ms^{-2}$ for the next 3 s. Find the acceleration versus time, velocity versus time and position versus time graph.

Solution We are given that for first 3 s acceleration is $2ms^{-2}$ and for next 3s acceleration is $-2ms^{-2}$. Hence, acceleration time graph is as shown in the figure.

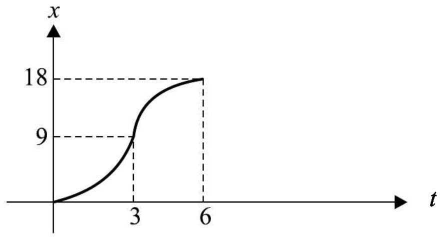


The area enclosed between a–t curve and t–axis gives change in velocity for the corresponding interval. Also at $t = 0$, $v = 0$, hence final velocity at $t = 3$s will increase to $6\,ms^{-1}$. In next 3s the velocity will decrease to zero. Hence the velocity time graph is as shown in figure.



Note that v–t curves are taken as straight line as acceleration is constant.

Now for displacement time curve, we will use the fact that area enclosed between v–t curve and time axis gives displacement for the corresponding interval. Hence displacement in first three seconds is 4.5 m and in next three seconds is 4.5 m. Also the x–t curve will be of parabolic nature as motion is with constant acceleration. Therefore, x–t curve is as shown in figure.



Motion with Non-Uniform Acceleration (use of definite integrals)

$$\Delta x = \int_{t_i}^{t_f} v(t)dt \quad \text{(displacement in time interval } t = t_i \text{ to } t_f\text{)}$$

The expression on the right hand side is called the *definite integral* of $v(t)$ between $t = t_i$ and $t = t_f$. Similarly change in velocity

$$\Delta v = v_f - v_i = \int_{t_i}^{t_f} a(t)dt$$

***Table 2.2:** Some quantities defined as derivatives and integrals*

$v(t) = \dfrac{dx}{dt}$	v = slope of x–t graph
$a(t) = \dfrac{dv}{dt}$	a = slope of v–t graphs
$F(t) = \dfrac{dp}{dt}$	F = slope of p–t graph (p = linear momentum)
$\Delta x = \int dx = \int_{t_i}^{t_f} v(t)\,dt$	Δx = area under v–t graph
$\Delta v = \int dv = \int_{t_i}^{t_f} a(t)\,dt$	Δv = area under a–t graph
$\Delta p = \int dp = \int_{t_i}^{t_f} F(t)\,dt$	Δp = area under F–t graph
$W = \int dW = \int_{x_i}^{x_f} F(x)\,dx$	W = area under F–x graph

PROBLEMS INVOLVING NON-UNIFORM ACCELERATION

Acceleration Depending on Velocity v or Time t

By definition of acceleration, we have $a = \frac{dv}{dt}$. If a is in terms of t, $\int_{v_0}^{v} dv = \int_{0}^{t} a(t)\, dt$. If a is in terms of v, $\int_{v_0}^{v} \frac{dv}{a(v)} = \int_{0}^{t} dt$. On integrating, we get a relation between v and t, and then using $\int_{x_0}^{x} dx = \int_{0}^{t} v(t)\, dt$, x and t can also be related.

ACCELERATION DEPENDING ON VELOCITY V OR POSITION X

$$a = \frac{dv}{dt} \quad \Rightarrow \quad a = \frac{dv}{dx}\frac{dx}{dt} \quad \Rightarrow \quad a = \frac{dx}{dt}\frac{dv}{dx} \quad \Rightarrow \quad a = v\frac{dv}{dx}$$

This is another important expression for acceleration.

If a is in terms of x, $\int_{v_0}^{v} v\, dv = \int_{x_0}^{x} a(x)\, dx$.

If a is in terms of v, $\int_{v_0}^{v} \frac{v\, dv}{a(v)} = \int_{x_0}^{x} dx$

On integrating, we get a relation between x and v. Using $\int_{x_0}^{x} \frac{dx}{v(x)} = \int_{0}^{t} dt$, we can relate x and t.

Example 52. An object starts from rest at $t = 0$ and accelerates at a rate given by $a = 6t$. What is
(a) its velocity and
(b) its displacement at any time t?

Solution As acceleration is given as a function of time,

$$\therefore \quad \int_{v(t_0)}^{v(t)} dv = \int_{t_0}^{t} a(t)dt$$

Here $t_0 = 0$ and $v(t_0) = 0$

$$\therefore \quad v(t) = \int_{0}^{t} 6t dt = 6\left(\frac{t^2}{2}\right)\Bigg|_0^t = 6\left(\frac{t^2}{2} - 0\right) = 3t^2$$

So, $v(t) = 3t^2$

As $\Delta x = \int_{t_0}^{t} v(t)dt \quad \therefore \Delta x = \int_{0}^{t} 3t^2 dt = 3\left(\frac{t^3}{3}\right)\Bigg|_0^t = 3\left(\frac{t^3}{3} - 0\right) = t^3$

Hence, velocity $v(t) = 3t^2$ and displacement $\Delta x = t^3$

Example 53. For a particle moving along x-axis, acceleration is given as $a = 2v^2$. If the speed of the particle is v_0 at $x = 0$, find speed as a function of x.

Solution [**Ans.** $v = v_0 e^{2x}$]

Example 54. For a particle moving along x-axis, velocity is given as a function of time as $v = 2t^2 + \sin t$. At $t = 0$, particle is at origin. Find the position as a function of time?

Solution [**Ans.** $x = \frac{2}{3}t^3 - \cos(t) + 1$]

Example 55. A particle moving in a straight line has an acceleration of $(3t - 4)$ ms^{-2} at time t seconds. The particle is initially 1 m from O, a fixed point on the line, with a velocity of 2 ms^{-1}. Find the times when the velocity is zero. Find also the displacement of the particle from O when $t = 3$.

Solution Using $s = \frac{dv}{dt}$ gives

$$\frac{dv}{dt} = 3t - 4 \Rightarrow \int_0^v dv = \int_0^t (3t - 4)\, dt$$

$$\Rightarrow v - 2 = \frac{3t^2}{2} - 4t \qquad \Rightarrow v = \frac{3t^2}{2} - 4t + 2$$

The velocity will be zero when $\frac{3t^2}{2} - 4t + 2 = 0$ i.e. when $(3t - 2)(t - 2) = 0$

$$\Rightarrow t = \frac{2}{3} \text{ or } 2$$

Using $\frac{ds}{dt} = v$, we have $\frac{ds}{dt} = \frac{3t^2}{2} - 4t + 2 \Rightarrow \int_1^s ds = \int_0^3 \left(\frac{3t^2}{2} - 4t + 2\right) dt$

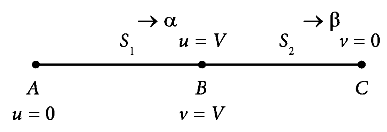
$$\Rightarrow s - 1 = \left[\frac{t^2}{2} - 2t^2 + 2t\right]_0^3 = 1.5 \Rightarrow s = 2\frac{1}{2}$$

Therefore the particle is 2.5 m from O when $t = 3$ sec

Example 56. A car accelerates from rest at a constant rate α for sometimes, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t, the maximum velocity acquired by the car is:

(A) $\frac{\alpha\beta t}{\alpha + \beta}$ (B) $\frac{(\alpha + \beta)}{\alpha\beta} t$ (C) $\frac{\alpha^2 + \beta^2}{\alpha\beta} t$ (D) $\frac{\alpha^2 - \beta^2}{\alpha\beta} t$

Solution



For motion from A to B, $V = \alpha t_1$ or $t_1 = \frac{V}{\alpha}$

For motion from B to C, $0 = V - \beta t_2$ or $t_2 = \frac{V}{\beta}$

$\therefore \quad t = t_1 + t_2 = \frac{V}{\alpha} + \frac{V}{\beta} = \frac{V(\alpha+\beta)}{\alpha\beta}$

or $\quad V = \frac{\alpha\beta t}{(\alpha+\beta)}$

$\therefore$ **(A)** is the right answer.

Example 57. Two particles P and Q start simultaneously from A with velocities 15 m/s and 20 m/s respectively. They move in the same direction with different accelerations. When P overtakes Q at B, velocity of P is 30 m/s. The velocity of Q at B is:

(A) 30 m/s (B) 25 m/s (C) 20 m/s (D) 15 m/s

Solution The average velocity is the same, when overtaking takes place.

$15 + 30 = 20 + V$ or $v = 25$ m/s

$\therefore$ **(B)** is the right answer.

Example 58. A particle is travelling with velocity of 2 m/s and moves in a straight line with a retardation of 0.1 m/s^2. The time at which the particle is 15 m from the starting point is:

(A) 10 s (B) 20 s (C) 25 s (D) 40 s

Solution $S = ut + \frac{1}{2}at^2$; $\quad 15 = 2t + \frac{1}{2}(-0.1)\,t^2$

$\Rightarrow \quad 20 \times 15 = 40t - t^2$ or $t^2 - 40t + 300 = 0$

$(t - 30)(t - 10) = 0$; $t = 30$ s or $t = 10$ s

The particle is at a distance 15 m from starting point at $t = 10$ s and also $t = 30$ s

$\therefore$ **(A)** is the right answer

Example 59. A particle moves along a straight line according to the law $S^2 = at^2 + 2bt + c$. The acceleration of the particle varies as:

(A) S^{-3} (B) $S^{2/3}$ (C) S^2 (D) $S^{5/2}$

Solution $S = (at^2 + 2bt + c)^{1/2}$

Differentiating, $\frac{dS}{dt} = \frac{1}{2}(at^2 + 2bt + c)^{-1/2} \times (2at + 2b) = \frac{at + b}{\sqrt{at^2 + 2bt + c}}$

$$\frac{d^2S}{dt^2} = \frac{\left(\sqrt{at^2 + 2bt + c}\right) \times a - \frac{(at+b)(at+b)}{\sqrt{at^2 + 2bt + c}}}{\left(at^2 + 2bt + c\right)}$$

$$= \frac{a\left(at^2 + 2bt + c\right) - (at + b)^2}{\sqrt{at^2 + 2bt + c} \times \left(at^2 + 2bt + c\right)} = \frac{\left(ac - b^2\right)}{S \,.\, S^2}$$

$\therefore \quad \frac{d^2S}{dt^2} = \frac{1}{S^3} \Rightarrow$ Acceleration S^{-3}

$\therefore$ **(A)** is the right answer.

Example 60. A stone A is dropped from rest from a height h above the ground. A second stone B is simultaneously thrown vertically up with velocity v. The value of v which would enable the stone B to meet the stone A midway between their initial positions is:

(A) $2\,gh$ (B) $2\sqrt{gh}$ (C) $\sqrt{gh}$ (D) $\sqrt{2gh}$

Solution Time of travel of each stone = t

Distance travelled by each stone = $\frac{h}{2}$

For stone A, $\frac{h}{2} = \frac{1}{2}gt^2$ i.e. $t = \sqrt{\frac{h}{g}}$

For stone B, $\frac{h}{2} = ut - \frac{1}{2}gt^2 = u\sqrt{\frac{h}{g}} - \frac{1}{2}g\left(\frac{h}{g}\right)$

$\Rightarrow \quad \frac{h}{2} = u\sqrt{\frac{h}{g}} - \frac{h}{2}$ or $u\sqrt{\frac{h}{g}} = h \quad \therefore \quad u = h\sqrt{\frac{g}{h}} = \sqrt{gh}$

$\therefore$ **(C)** is the right answer.

Example 61. A body is dropped from rest from a height h. It covers a distance $\frac{9h}{25}$ in the last second of fall. The height h is:

(A) 102.5 m (B) 112.5 m (C) 122.5 m (D) 132.5 m

Solution t is the time to reach ground.

$h = \frac{1}{2}at^2 \,;\ \left(1 - \frac{9}{25}\right)h = \frac{1}{2}a\,(t-1)^2$

$\left(1 - \frac{9}{25}\right) = \frac{(t-1)^2}{t^2} \,;\ \frac{16}{25} = \frac{(t-1)^2}{t^2}$ or $\frac{4}{5} = \frac{t-1}{t} \quad \therefore t = 5$ sec

$h = \frac{1}{2} \times 9.8 \times 5^2 = 122.5$ m

$\therefore$ **(C)** is the right answer.

Example 62. A stone is dropped from rest from the top of a cliff. A second stone is thrown vertically down with a velocity of 30 m/s two seconds later. At what distance from the top of a cliff do they meet?

(A) 60 m (B) 120 m (C) 80 m (D) 44 m

Solution The two stones meet at distance S from top of cliff t seconds after first stone is dropped.

For 1st stone $S = \frac{1}{2}gt^2$; For 2nd stone $S = u\,(t-2) + \frac{1}{2}g\,(t-2)^2$

i.e. $\frac{1}{2}gt^2 = ut - 2u + \frac{1}{2}gt^2 - 2\,gt + 2g$

$0 = (u - 2g)\,t - 2\,(u-g)\,;\ t = \frac{2\,(u-g)}{u-2g} = \frac{2(30-10)}{30-20} = 4$ s

Distance S at which they meet = $\frac{1}{2} \times gt^2 = \frac{1}{2} \times 10 \times 16$

$= 80$ m from top of cliff

∴ **(C)** is the right answer.

Example 63. A particle P is projected vertically upward from a point A. Six seconds later, another particle Q is projected vertically upward from A. Both P and Q reach A simultaneously. The ratio of maximum heights reached by P and Q = 64: 25. Find the velocity of the projection of Q in m/s.

(A) $7g$ (B) $6g$ (C) $5g$ (D) $4g$

Solution $\frac{1}{2}g(t+3)^2$; $\frac{1}{2}gt^2 = 64: 25$

or $(t+3)^2 : t^2 = 64: 25$; or $(t+3) : t = 8: 5$

$5t + 15 = 8t$ or $3t = 15$; $t = 5$ sec

$v = g \times t = 9.8 \times 5 = 49$ m/s $= 5g$ m/s

∴ **(C)** is the right answer.

Example 64. A particle moving in a straight line has an acceleration of (3t – 4) ms^{-2} at time t seconds. The particle is initially 1 m from O, a fixed point on the line, with a velocity of 2 ms^{-1}. Find the times when the velocity is zero. Find also the displacement of the particle from O when $t = 3$.

Solution Using $s = \frac{dv}{dt}$ gives

$$\frac{dv}{dt} = 3t - 4 \Rightarrow \int_0^v dv = \int_0^t (3t-4)\,dt$$

$$\Rightarrow v - 2 = \frac{3t^2}{2} - 4t \qquad \Rightarrow v = \frac{3t^2}{2} - 4t + 2$$

The velocity will be zero when $\frac{3t^2}{2} - 4t + 2 = 0$ i.e. when $(3t-2)(t-2) = 0$

$$\Rightarrow t = \frac{2}{3} \text{ or } 2$$

Using $\frac{ds}{dt} = v$, we have $\frac{ds}{dt} = \frac{3t^2}{2} - 4t + 2$

$$\Rightarrow \int_1^s ds = \int_0^3 \left(\frac{3t^2}{2} - 4t + 2\right) dt \Rightarrow s - 1 = \left[\frac{t^2}{2} - 2t^2 + 2t\right]_0^3 = 1.5$$

$$\Rightarrow s = 2\frac{1}{2}$$

Therefore the particle is 2.5 m from O when $t = 3$ sec

Example 65. Pick up the correct statements:

(A) area under $a - t$ graph gives velocity

(B) area under $a - t$ graph gives change in velocity

(C) path of projectile as seen by another projectile is parabola

(D) a body, whatever be its motion, is always at rest in a frame of reference fixed to body itself.

Solution $\frac{dv}{dt} = a \Rightarrow \int_{v_1}^{v_2} dv = \int a\, dt$

$\Rightarrow$ Δv = Area under $a - t$ graph

where Δv = magnitude of change in velocity.

The path of a projectile as seen by another projectile is a straight line becomes, the relative velocity between the particles remains constant.

$\therefore$ **(B)** and **(D)** are correct.

Example 66. A body when projected vertically up, covers a total distance D. During the time of its flight *t*. If there were no gravity, the distance covered by it during the same time is equal to

(A) 0 (B) D (C) 2D (D) 4D.

Solution The displacement of the body during the time *t* as it attains the point of projection

$\Rightarrow S = 0 \quad \Rightarrow v_0 t - \frac{1}{2} g t^2 = 0 \quad \Rightarrow t = \frac{2v_0}{g}$

During the same time *t*, the body moves in absence of gravity through a distance

$D' = v_0 t$, because in absence of gravity $g = 0$

$\Rightarrow D' = v_0 \left(\frac{2v_0}{g} \right) = \frac{2v_0^2}{g}$ ...(i)

In presence of gravity the total distance covered is

$D = 2H = 2\frac{v_0^2}{2g} = \frac{v_0^2}{g}$ ...(ii)

(i) ÷ (ii) $\Rightarrow D' = 2D$

$\therefore$ **(C)** is the right answer.

Example 67. Two particles 1 and 2 are allowed to descend on the two frictionless chord OA and OB of a vertical circle, at the same instant from point O. The ratio of the velocities of the particles 1 and 2 respectively, when they reach on the circumference will be (OB is the diameter)

(A) $\sin\alpha$ (B) $\tan\alpha$ (C) $\cos\alpha$ (D) none of these.

Solution OA = $d\cos\alpha$, $a_{OA} = g\cos\alpha$

Along OA $\Rightarrow v_A^2 = 2g\cos\alpha\, d\cos\alpha$

Along OB$v_B^2 = 2gd$ $\Rightarrow \frac{v_A}{v_B} = \cos\alpha$

Hence, **(C)** is correct.

Example 68. A particle moves along a straight line according to the law $S^2 = at^2 + 2bt + c$. The acceleration of the particle varies as

(A) S^{-3} (B) $S^{2/3}$ (C) S^2 (D) $S^{5/2}$.

Solution $S = (at^2 + 2bt + c)^{1/2}$

Differentiating, $\dfrac{dS}{dt} = \dfrac{1}{2}(at^2 + 2bt + c)^{-1/2} \times (2at + 2b) = \dfrac{at + b}{\sqrt{at^2 + 2bt + c}}$

$$\frac{d^2S}{dt^2} = \frac{(\sqrt{at^2 + 2bt + c}) \times a - \dfrac{(at+b)(at+b)}{\sqrt{at^2 + 2bt + c}}}{(at^2 + 2bt + c)}$$

$$= \frac{a(at^2 + 2bt + c) - (at - b)^2}{\sqrt{at^2 + 2bt + c} \times (at^2 + 2bt + c)} = \frac{(ac - b^2)}{S \times S^2}$$

$\therefore$ $\dfrac{d^2S}{dt^2} \propto \dfrac{1}{S^3}$ $\Rightarrow$ acceleration $\propto S^{-3}$

$\therefore$ **(A)** is the right answer.

Example 69. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t, the maximum velocity acquired by car is

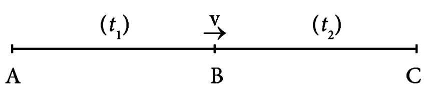
(A) $V = \dfrac{\alpha\beta}{(\alpha + \beta)}t$ (B) $V = \dfrac{\alpha\beta}{(\alpha - \beta)}t$ (C) $V = \dfrac{2\alpha\beta}{(\alpha + \beta)}t$ (D) $V = \dfrac{2\alpha\beta}{(\alpha - \beta)}t$.

Solution From motion from A to B $V = \alpha t_1$ or $t_1 = \dfrac{V}{\alpha}$

From motion from B to C $0 = V - \beta t_2$ or $t_2 = \dfrac{V}{\beta}$

$\therefore$ $t = t_1 + t_2 = \dfrac{V}{\alpha} + \dfrac{V}{\beta} = \dfrac{V(\alpha + \beta)}{\alpha\beta}$

or $V = \dfrac{\alpha\beta}{(\alpha + \beta)}t$



$\therefore$ **(A)** is the right answer.

Example 70. A stone A is dropped from rest from a height h above the ground. A second stone B is simultaneously thrown vertically up from a point on the ground with velocity v. The line of motion of both the stones is same. The values of v which would enable the stone B to meet the stone A midway between their initial positions is

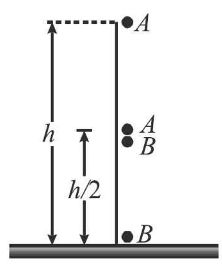
(A) $2\,gh$ (B) $2\sqrt{gh}$ (C) $\sqrt{gh}$ (D) $\sqrt{2gh}$.

Solution Time of travel of each stone

Distance travelled by each stone $= \dfrac{h}{2}$

For stone A, $\dfrac{h}{2} = \dfrac{1}{2}gt^2$ i.e., $t = \sqrt{\dfrac{h}{g}}$

For stone B, $\dfrac{h}{2} = ut - \dfrac{1}{2}gt^2 = u\sqrt{\dfrac{h}{g}} - \dfrac{1}{2}g\left(\dfrac{h}{g}\right)$



$$\Rightarrow \quad \frac{h}{2} = u\sqrt{\frac{h}{g}} - \frac{h}{2} \quad \text{or,} \quad u\sqrt{\frac{h}{g}} = h \;\therefore\; u = h\sqrt{\frac{g}{h}} = \sqrt{gh}$$

The correct option is **(C)**

Example 71. A stone is dropped from rest from the top of a cliff. A second stone is thrown vertically down with a velocity of 30 m/s two seconds later. At what distance from the top of a cliff do they meet?

(A) 60 m (B) 120 m (C) 80 m (D) 44 m

Solution The two stones meet at distance S from top of cliff t seconds after first stone is dropped.

For 1st stone $S = \frac{1}{2}gt^2$; For 2nd stone $S = u(t-2) + \frac{1}{2}g(t-2)^2$.

i.e., $\frac{1}{2}gt^2 = ut - 2u + \frac{1}{2}gt^2 - 2gt + 2g$

$0 = (u-2g)t - 2(u-g)$; $\quad t = \frac{2(u-g)}{u-2g} = \frac{2(30-10)}{30-20} = 4s$

Distance S at which they meet $= \frac{1}{2} \times gt^2 = \frac{1}{2} \times 10 \times 16$ = 80 m from top of cliff

∴ **(C)** is the right answer.

Example 72. A car accelerates from rest at a constant rate α for sometimes, after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t, the maximum velocity acquired by the car is:

(A) $\frac{\alpha\beta t}{\alpha+\beta}$ (B) $\frac{(\alpha+\beta)}{\alpha\beta}t$ (C) $\frac{\alpha^2+\beta^2}{\alpha\beta}t$ (D) $\frac{\alpha^2-\beta^2}{\alpha\beta}t$

Solution For motion from A to B, $V = \alpha t_1$ or $t_1 = \frac{V}{\alpha}$

For motion from B to C, $0 = V - \beta t_2$ or $t_2 = \frac{V}{\beta}$

∴ $t = t_1 + t_2 = \frac{V}{\alpha} + \frac{V}{\beta} = \frac{V(\alpha+\beta)}{\alpha\beta}$ or $V = \frac{\alpha\beta t}{(\alpha+\beta)}$ ∴

(A) is the right answer.

Example 73. Two particles P and Q start simultaneously from A with velocities 15 m/s and 20 m/s respectively. They move in the same direction with different accelerations. When P overtakes Q at B, velocity of P is 30 m/s. The velocity of Q at B is:

(A) 30 m/s (B) 25 m/s (C) 20 m/s (D) 15 m/s

Solution The average velocity is the same, when overtaking takes place.

$15 + 30 = 20 + V$

or $v = 25$ m/s

∴ **(B)** is the right answer.

Example 74. A particle is travelling with velocity of 2 m/s and moves in a straight line with a retardation of 0.1 m/s^2. The time at which the particle is 15 m from the starting point is:

(A) 10 s (B) 20 s (C) 25 s (D) 40 s

Solution $S = ut + \frac{1}{2} at^2$; $15 = 2t + \frac{1}{2} ' (-0.1) t^2$

$\Rightarrow$ $20 \times 15 = 40t - t^2$ or $t^2 - 40t + 300 = 0$

$(t - 30)(t - 10) = 0$; $t = 30$ s or $t = 10$ s

The particle is at a distance 15 m from starting point at t = 10 s and also t = 30 s

$\therefore$ **(A)** is the right answer.

Example 75. A particle moves along a straight line according to the law $S^2 = at^2 + 2bt + c$. The acceleration of the particle varies as:

(A) S^{-3} (B) $S^{2/3}$ (C) S^2 (D) $S^{5/2}$

Sol. $S = (at^2 + 2bt + c)^{1/2}$

Differentiating, $\frac{dS}{dt} = \frac{1}{2}(at^2 + 2bt + c)^{-1/2} \times (2at + 2b) = \frac{at + b}{\sqrt{at^2 + 2bt + c}}$

$$\frac{d^2S}{dt^2} = \frac{\left(\sqrt{at^2 + 2bt + c}\right) \times a - \frac{(at+b)(at+b)}{\sqrt{at^2 + 2bt + c}}}{(at^2 + 2bt + c)}$$

$$= \frac{a(at^2 + 2bt + c) - (at + b)^2}{\sqrt{at^2 + 2bt + c} \times (at^2 + 2bt + c)} = \frac{(ac - b^2)}{S \,.\, S^2}$$

$\therefore$ $\frac{d^2S}{dt^2} = \frac{1}{S^3}$ $\Rightarrow$ Acceleration S^{-3}

$\therefore$ **(A)** is the rigth answer.

Example 76. A body is dropped from rest from a height h. It covers a distance $\frac{9h}{25}$ in the last second of fall. The height h is:

(A) 102.5 m (B) 112.5 m (C) 122.5 m (D) 132.5 m

Solution t is the time to reach ground.

$$h = \frac{1}{2} at^2; \left(1 - \frac{9}{25}\right) h = \frac{1}{2} a (t - 1)^2$$

$$\left(1 - \frac{9}{25}\right) = \frac{(t-1)^2}{t^2}; \frac{16}{25} = \frac{(t-1)^2}{t^2} \text{ or } \frac{4}{5} = \frac{t-1}{t} \therefore t = 5 \text{ sec}$$

$$h = \frac{1}{2} \times 9.8 \times 5^2 = 122.5 \text{ m}$$

$\therefore$ **(C)** is the right answer.

Example 77. A point moves in a straight line under the retardation av^2. If the initial velocity is u, the distance covered in ' t ' seconds is:

(A) $a\,u\,t$ (B) $\frac{1}{a}\log(a\,u\,t)$ (C) $\frac{1}{a}\log(1+a\,u\,t)$ (D) $a\log(a\,u\,t)$

Solution The retardation is given by $\frac{dv}{dt} = -av^2$

integrating between proper limits

$$\Rightarrow \quad -\int_u^v \frac{dv}{v^2} = \int_0^t a\,dt \qquad \text{or} \qquad \frac{1}{v} = at + \frac{1}{u}$$

$$\Rightarrow \quad \frac{dt}{dx} = at + \frac{1}{u} \qquad \Rightarrow \qquad dx = \frac{udt}{1+aut}$$

integrating between proper limits

$$\Rightarrow \quad \int_0^s dx = \int_0^t \frac{u\,dt}{1+aut} \qquad \Rightarrow \qquad S = \frac{1}{a}\ln(1+aut)$$

$\therefore$ **(C)** is the right answer.

Example 78. A particle is thrown upwards from ground . It experiences a constant resistance force which can produce retardation 2 m/s^2 . The ratio of time of ascent to the time of descent is: [g = 10 m/s^2]

(A) 1: 1 (B) $\sqrt{\frac{2}{3}}$ (C) $\frac{2}{3}$ (D) $\sqrt{\frac{3}{2}}$

Solution Let a be the retardation produced by resistive force, t_a and t_d be the time ascent and descent respectively.

If the particle rises upto a height h

then $h = \frac{1}{2}(g+a)\,t_a^2$ and $h = \frac{1}{2}(g-a)\,t_d^2$

$$\therefore \quad \frac{t_a}{t_d} = \sqrt{\frac{g-a}{g+a}} = \sqrt{\frac{10-2}{10+2}} = \sqrt{\frac{2}{3}}$$

$\therefore$ **(B)** is the right answer.

Example 79. Velocity of a particle moving along a straight line at any time 't' is given by $V = \cos\left(\frac{\pi}{3}t\right)$. The distance travelled by the particle in the first two seconds is equal to

(A) $\frac{\sqrt{3}}{2\pi}$ (B) $\frac{3\sqrt{3}}{2\pi}$ (C) $\frac{3\sqrt{3}}{\pi}$ (D) zero

Solution $v = \cos\left(\frac{\pi}{3}t\right) \Rightarrow \frac{dx}{dt} = \cos\left(\frac{\pi}{3}t\right)$

$$\Rightarrow \quad x = \int_0^2 \cos\left(\frac{\pi}{3}t\right)dt = \int_0^{\frac{3}{2}} \cos\left(\frac{\pi}{3}t\right)dt + \int_{\frac{3}{2}}^{2} \cos\left(\frac{\pi}{3}t\right)dt$$

$$= \frac{3}{\pi}\left[\left(\sin\frac{\pi t}{3}\right)_0^{\frac{3}{2}} + \left(\sin\frac{\pi t}{3}\right)_{\frac{3}{2}}^{2}\right] = \frac{3}{\pi}\left[1 - 0 + \frac{\sqrt{3}}{2} - 1\right] \Rightarrow x = \frac{3\sqrt{3}}{2\pi}$$

∴ **(B)** is the right answer.

Example 80. The position vector of a particle is given as $\vec{r} = (t^2 - 4t + 6)\hat{i} + (t^2)\hat{j}$. The time after which the velocity vector and acceleration vector becomes perpendicular to each other is equal to:

(A) 1 sec (B) 2 sec (C) 1.5 sec (D) not possible

Solution $\vec{r} = (t^2 - 4t + 6)\hat{i} + t^2\hat{j}$; $\vec{v} = \frac{d\vec{r}}{dt} = (2t - 4)\hat{i} + 2t\hat{j}$, $\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 2\hat{j}$

if $\vec{a}$ and $\vec{v}$ are perpendicular

$\vec{a}.\vec{v} = 0$

$(2\hat{i} + 2\hat{j}).((2t - 4)\hat{i} + 2t\hat{j}) = 0$

$8t - 8 = 0$

$t = 1$ sec.

∴ **(A)** is the right answer.

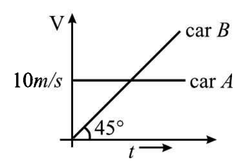
Example 81. Initially car A is 10.5 m ahead of car B. Both start moving at time $t = 0$ in the same direction along a straight line. The velocity time graph of two cars is shown in figure. The time when the car B will catch the car A, will be

(A) $t = 21$ sec (B) $t = 2\sqrt{5}$ sec (C) 20 sec. (D) None of these

Hint: $x_A = x_B$

$10.5 + 10t = \frac{1}{2}at^2 \quad a = \tan 45° = 1$

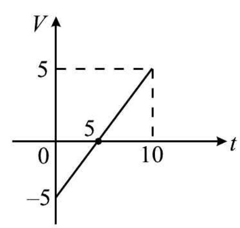
$t^2 - 20t - 21 = 0 t = \frac{20 \pm \sqrt{400 + 84}}{2}$



$t = 21$ sec.

∴ **(A)** is correct option.

Example 82. A particle moves rectilinearly with a constant acceleration 1 m/s^2. Its speed after 10 seconds is 5 m/s. Find the distance covered by the particle in this duration



Solution From the graph distance = $2\left(\frac{1}{2} \times 5 \times 5\right) = 25$ m

Example 83. The velocity of a particle moving along x-axis is given as $v = x^2 - 5x + 4$ (in m/s) where x denotes the x-coordinate of the particle in metres. Find the magnitude of acceleration of the particle when the velocity of particle is zero?

(A) 0 m/s^2 (B) 2 m/s^2 (C) 3 m/s^2 (D) none of these

Solution

(A) $v = 0 \Rightarrow x^2 - 5x + 4 = 0$

$x = 1\text{m}\ \&\ 4\text{m}$

$$\frac{dv}{dt} = (2x-5)\,v = (2x-5)\,(x^2-5x+4)$$

at $x = 1$ m and 4m ; $\dfrac{dv}{dt} = 0$

Example 84. A particle of mass m moves along a curve $y = x^2$. When particle has x-coordinate as 1/2 and x-component of velocity as 4m/s then.

(A) the position coordinate of particle are (1/2, 1/4)
(B) the velocity of particle will be along the line $4x - 4y - 1 = 0$.
(C) the magnitude of velocity at that instant is $4\sqrt{2}$ m/s
(D) the magnitude of angular momentum of particle about origin at that position is 0.

Solution **[Ans. (A,B,C)]** On the curvey $= x^2$ at $x = 1/2$

$y = \dfrac{1}{4}$ Hence the coordinate $\left(\dfrac{1}{2}, \dfrac{1}{4}\right)$

Differentiating: $y = x^2$

$v_y = 2xv_x$

$$v_y = 2\left(\frac{1}{2}\right)(4) = 4 \text{ m/s}$$

Which satisfies the line

$4x - 4y - 1 = 0$ (tangent to the curve)

& magnitude of velocity:

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = 4\sqrt{2} \text{ m/s}$$

As the line $4x - 4y - 1$ does not pass through the origin, therefore (D) is not correct.

Example 85. A particle is moving in xy-plane at 2 m/s along x-axis. 2 seconds later, its velocity is 4 m/s in a direction making 60° with positive x-axis. Its average acceleration for this period of motion is:

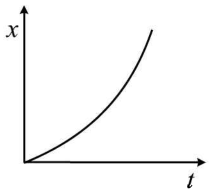
(A) $\sqrt{5}$ m/s^2, along y-axis (B) $\sqrt{3}$ m/s^2, along y-axis
(C) $\sqrt{5}$ m/s^2, along at 60° with positive x-axis (D) 3 m/s^2, at 60° with positive x-axis.

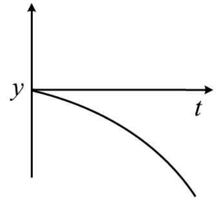
Solution

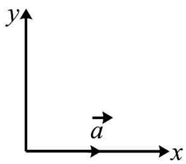
$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{[4\cos 60\hat{i} + 4\sin 60\hat{j}] - [2\hat{i}]}{2} = \frac{(2\hat{i} + 2\sqrt{3}\,\hat{j}) - 2\hat{i}}{2} = \sqrt{3}\,\hat{j} \text{ m/s}^2$$

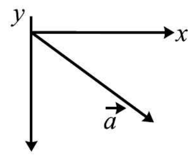
$\therefore$ **(B)** is the correct option.

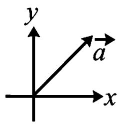
***Example* 86.** Graphs I and II give coordinates $x(t)$ and $y(t)$ of a particle moving in the x-y plane Acceleration of the particle is constant and the graphs are drawn to the same scale. Which of the vector shown in options best represents the acceleration of the particle:

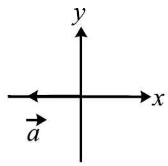




(A) 

(B) 

(C) 

(D) 

∴ **(B)** is the right answer.

***Example* 87.** Position of a particle moving along x-axis is given by $x = 3t - 4t^2 + t^3$, where x is in meters and t in seconds.

(A) Find the position of the particle at $t = 2s$.
(B) Find the displacement of the particle in the time interval from $t = 0$ to $t = 4s$.
(C) Find the average velocity of the particle in the time interval from $t = 2$s to $t = 4s$.
(D) Find the velocity of the particle at $t = 2s$.

Solution (A) $x_{(t)} = 3t - 4t^2 + t^3$

$\Rightarrow$ $x_{(2)} = 3 \times 2 - 4 \times (2)^2 + (2)^3 = 6 - 4 \times 4 + 8 = -2\text{m}$.

(B) $x_{(0)} = 0$

$\Rightarrow$ $x_{(4)} = 3 \times 4 - 4 \times (4)^2 + (4)^3 = 12$ m.

Displacement $= x_{(4)} - x_{(0)} = 12$ m

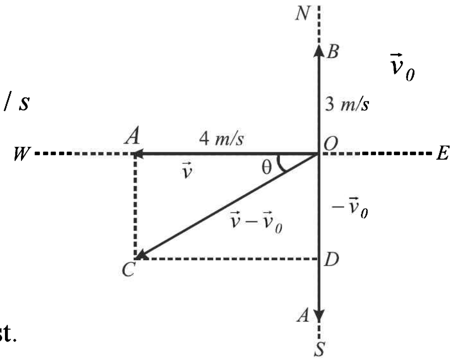
(C) $$\langle v \rangle = \frac{x_{(4)} - x_{(2)}}{(4-2)} = \frac{12-(-2)}{2} m/s = 7 m/s$$

(D) $$\frac{dx}{dt} = 3 - 8t + 3t^2$$

$\Rightarrow$ $$v_{(2)} = \left(\frac{dx}{dt}\right)_{(2)} = 3 - 8 \times 2 + 3 \times (2)^2 = -1 m/s$$

***Example* 88.** A body moving in a curved path possesses a velocity 3 m/s towards north at any instant of its motion. After 10s, the velocity of the body was found to be 4 m/s towards west. Calculate the average acceleration during this interval.

Solution To solve this problem the vector nature of velocity must be taken into account. In the figure, the initial velocity v_0 and the final velocity v are drawn from a common origin. The vector difference of them is found by the parallelogram method.



The magnitude of difference is

$$|v - v_0| = OC = \sqrt{OA^2 + AC^2} = \sqrt{4^2 + 3^2} = 5m/s$$

The direction is given by

$$\tan\theta = \frac{3}{4} = 0.75 \ , \ \theta = 37^\circ$$

∴ Average acceleration

$$= \frac{|\vec{v} - \vec{v}_0|}{t} = \frac{5}{10} = 0.5 \text{ m/s}^2 \text{ at } 37^\circ \text{ South of West.}$$

Example 89. The velocity time graph of a moving object is given in the figure. Find the maximum acceleration of the body and distance travelled by the body in the interval of time in which this acceleration exists.

Solution Acceleration is maximum when slope is maximum.

$$a_{\max} = \frac{80 - 20}{40 - 30} = 6m/s^2$$

Displacement is the area under the curve.

$$\Rightarrow S = \frac{1}{2}(20 + 80)(10) = 500m$$

Example 90. A steel ball is dropped from the roof of a building. An observer standing in front of a window 1.5m high notes that the ball takes $\frac{1}{10}$ s to fall from the top to the bottom of the window. The ball reappears at the bottom of the window 2s after passing it on the way down. If the collision between the ball and the ground is perfectly elastic, then find the height of the building. Take g = 10 m/s².

Solution Since collision is perfectly elastic, the speed of the ball just before collision is equal to the speed of the ball just after collision. Hence time of descent is equal to the time of ascent. Therefore time taken by the ball to reach the ground from the bottom of the window is 1 sec.

Let u be the speed of the ball when it is at the top of the window

$$\Rightarrow \quad 1.5 = u\frac{1}{10} + \frac{1}{2} \times 10 \times \frac{1}{100} . \left(\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2\right)$$

$\Rightarrow$ u = 14.5 m/s

∴ ball is dropped hence its initial speed is 0

It t be the time taken by the ball to acquire the speed of 14.5 m/s, then

$14.5 = 0 + 10 \times t, \ (\vec{v} = \vec{u} + \vec{a}t)$ $\Rightarrow$ t = 1.45 sec

Hence total time of descent is given by

$$T = 1.45 + \frac{1}{10} + 1 = 2.55s$$

If H be the height of the building,

then $H = 0 + \frac{1}{2}gt^2 \quad \Rightarrow \quad H = \frac{1}{2} \times 10 \times (2.55)^2 \quad \Rightarrow \quad H = 32.5$ m

Example 91. A block of ice starts sliding down from the top of an inclined roof of a house along a line of the greatest slope. The inclination of the roof with the horizontal is 30°. The heights of the highest and lowest points of the roof are 8.1 m and 5.6 m respectively. At what horizontal distance from the lowest point will the block hit the ground? Neglect any friction. [$g = 9.8$ m/s^2]

Solution Acceleration of the block along the greatest slope is equal to $a = g \sin 30^\circ$

Distance travelled by the block along the greatest slope is equal to

$$S = \frac{(8.1 - 5.6)}{\sin 30^\circ} = 5m.$$

If u be the speed of the block when it is just about to leave the roof then

$u^2 = 0 + 2g \sin 30^\circ \times 5$

$u = 7$ m/s

If t be the time taken to hit the ground then

$$5.6 = u \sin 30^\circ t + \frac{1}{2}gt^2 = \frac{7}{2}t + \frac{1}{2} \times 9.8t^2$$

$\Rightarrow \quad 7t^2 + 5t - 8 = 0$

$$t = \frac{-5 \pm \sqrt{25 - (4)(7) \times (-8)}}{2 \times 7} \quad \Rightarrow \quad t = \frac{-5 \pm 15.78}{14} s,$$

-ve value is to be rejected.

i.e., $t = \frac{-5 + 15.78}{14} = 0.77$ sec.

Horizontal distance travelled is equal to

$$x = u \cos 30^\circ t = \frac{7\sqrt{3}}{2} \times \frac{10.78}{14} m \quad \Rightarrow x = 4.67\,\text{m}$$

Example 92. A particle moving with uniform acceleration in a straight line covers a distance of 3 m in the 8th second and 5 m in the 16th second of its motion. What is the displacement of the particle from the beginning of the 6th second to the end of 15th second?

Solution The distance travelled during the nth second of motion of a body is given by

$$S_n = u + \frac{1}{2}a(2n - 1) \quad S = un + \frac{1}{2}an^2$$

For the motion during the 8th second, $u(n-1) - \frac{1}{2}a(n-1)^2$

$$3 = u + \frac{1}{2}a(16-1) = u + \frac{15a}{2} \qquad ...(i)$$

For the motion during the 16th second,

$$5 = u + \frac{1}{2}a(32-1) = u + \frac{31a}{2}$$

Subtracting equations (i) from (ii),

$$8a = 2$$

or acceleration $a = \frac{1}{2} ms^2$

From equation (i), $u = 3 - \left(\frac{15}{2} \times \frac{1}{4}\right) = \frac{9}{8} ms^{-1}$

Now, the velocity at the end of 5 s (velocity at the beginning of 6th second)

$$v_1 = u + 5a$$

The velocity at the end of 15th s, $v_2 = u + 15a$

Average velocity during this interval of 10 seconds

$$= \frac{v_1 + v_2}{2} = \frac{(u-5a) + (u+15a)}{2} = u + 10a$$

Distance travelled during this interval

S = average velocity x time $= (u + 10a) \times t = \left(\frac{9}{8} + \frac{10}{4}\right) \times 10 = \frac{290}{8} = 36.25m$.

Example 93. An automobile can accelerate or decelerate at a maximum value of $\frac{5}{3} m/s^2$ and can attain a maximum speed of 90 km/hr. If it starts from rest, what is the shortest time in which can travel one kilometre, if it is to come to rest at the end of the kilometre run?

Solution In order that the time of motion be shortest, the car should attain the maximum velocity with the maximum acceleration after the start, maintain the maximum velocity for as along as possible and then decelerate with the maximum retardation possible, consistent with the condition that, the automobile should come to rest immediately after covering a distance of 1 km.

Let t_1 be the time of acceleration, t_2 be the time of uniform velocity and t_3 be the time of retardation.

Now, maximum velocity possible = 90 km/hr $= 90 \times \frac{5}{18}$ m/s = 25 m/s

$$t_1 = \frac{v-u}{a} = \frac{25-0}{\frac{5}{3}} = 15s$$

Similarly, the time of retardation is also given by

$$t_3 = \frac{0-25}{-\frac{5}{3}} = 15s$$

During the period of acceleration, the distance covered

$= \text{average velocity } \times \text{time } = \dfrac{25+0}{2} \times 15 = 187.5m$

During the period of deacceleration, the distance covered is the same, hence = 187.5 m the total distance covered under constant velocity = 1000 – 375 = 625 m

Time of motion under constant velocity $t_2 = \dfrac{625}{25} = 25s$

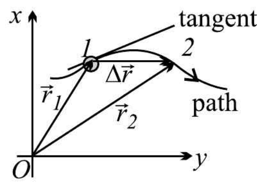
the shortest time of motion = $t_1 + t_2 + t_3$ = 15 + 25 + 15 = 55 seconds.

MOTION IN TWO AND THREE DIMENSIONS

When a particle is moving in space then its motion can be broken up in three co-ordinate axes(x, y& z). The motion in these three directions is governed only by velocity and acceleration in that particular direction and is totally independent of the velocities and acceleration in other directions.

Lets say a particle is moving in in space $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

Gives position of particle on space.



VELOCITY

Using the language of calculus, we may write $\vec{v}$ as the derivative

$$\vec{v} = \frac{d\vec{r}}{dt} \quad \text{....(1)}$$

To write Equation in a unit vector form, we substitute for $\vec{r}$ from equation (1)

$$\vec{v} = \frac{d}{dx}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

This equation can be simplified somewhat by writing it is

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \quad \text{....(2)}$$

where the scalar components of $\vec{v}$ are

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \; v_z = \frac{dz}{dt} \quad \text{....(3)}$$

Differentiating $\vec{r}$ w.r.t. time gives us velocity vector of particle at that time.

ACCELERATION

Similarly, if we differentiate $\vec{V}$ w.r.t. time we get acceleration of particle $\vec{a} = \dfrac{d\vec{V}}{dt}$

$$\vec{a} = \frac{d\vec{v}}{dt} \quad \text{....(4)}$$

If the velocity changes in either magnitude or direction (or both), the particle must have an accleration.

We can write equation (4) in a unit vector form by substituting for $\vec{v}$ from equaion (2) to obtain

$$\vec{a} = \frac{d}{dt}\left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right) = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

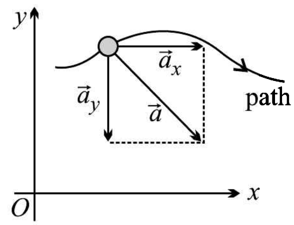
we can rewrite this as

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} \qquad ...(5)$$

where the scalar components of $\vec{a}$ are

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt} \qquad ...(6)$$

Thus, we can find the scalar components of $\vec{a}$ by differentiating the scalar components of $\vec{v}$. Figure shows an acceleration vector $\vec{a}$ and its scalar components for a particle moving in two dimensions.



Now, collecting equations of motion relating to *x* & *y* axes separately

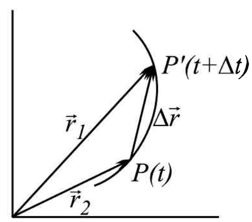
x-axis	*y*-axis
$V_x = \frac{dx}{dt}$	$V_y = \frac{dy}{dt}$
$a_x = \frac{dV_x}{dt}$	$a_y = \frac{dV_y}{dt}$

Thus we can see that motion in plane is composed of two straight line motions. **These motions are completely independent of each other**. Only thing connecting them is fact that they are occuring simultaneously.

VELOCITY IS ALONG TANGENT OF PATH

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle position. $\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$

$\Delta\vec{r}$ will be along tangent.



The result is the same in three dimensions:

Problem Solving Strategy

For solving any problem of two dimensional motion, following is standard algorithm.

1. Break the motion in two separate one dimensional motions
2. Solve both motions separately.
3. Connect the final equations throughly 'time' (because it is the only common thing).

Example 94. A particle with velocity $\vec{v}_0 = -2\hat{i} + 4\hat{j}$ (in meters per second) at $t = 0$ undergoes a constant acceleration $\vec{a}$ of magnitude $a = 3$ m/s^2 at an angle $\theta = 127°$ from the positive direction of the x-axis. What is the particle's velocity $\vec{v}$ at $t = 5$ sec, in unit vector notation ?

Solution We know that $v = v_0 + at$

now $v_x = v_{0x} + a_x t$ and $v_y = v_{0y} + a_y t$

$a_x = a\cos\theta = (3\text{ m/s}^2)(\cos 127°) = -1.80\text{ m/s}^2$

$a_y = a\sin\theta = (3\text{ m/s}^2)(\sin 127°) = +2.40\text{ m/s}^2$

at time $t = 5$ sec

$v_x = -2\text{ m/s} + (-1.80\text{ m/s}^2)(5\text{ sec}) = -11\text{ m/s}$

$v_y = 4\text{ m/s} + (2.40\text{ m/s}^2)(5\text{ sec}) = 16\text{ m/s}$

Thus, at $t = 5$ sec,

$$\vec{v} = (-11 m/s)\hat{i} + (16 m/s)\hat{j}$$

Example 95. A particle is moving such that its velocity $\vec{v}$ is equal to $a\hat{i} + bt\hat{j}$. Then find its acceleration and equation of trajectory (at $t = 0$, $(x, y) \equiv 0, 0$)

Solution $V_x = a$ $\qquad V_y = bt$

$a_x = 0$ $\qquad a_y = b$

$\frac{dx}{dt} = a$ $\qquad \frac{dy}{dt} = bt$

$x = at + c \rightarrow 0$ $\qquad y = \frac{bt^2}{2} + c \rightarrow 0$

$x = at$ $\qquad x = \frac{bt^2}{2}$

Thus $y = \frac{bx^2}{2a^2}$, path is parabola and acceleration = $b\hat{j}$

Example 96. A particle moves in the x–y plane according to the law $x = at$; $y = at(1-\alpha t)$ where a and α are positive constants and t is time. Find the velocity and acceleration vector. The moment t_0 at which the velocity vector forms angle of 90° with acceleration vector.

Solution $V_x = a$; $V_y = a - 2a\alpha t$ $\qquad \Rightarrow \vec{V} = a\hat{i} + (a - 2a\alpha t)\hat{j}$

$a_x = 0$; $a_y = -2a\alpha$ $\qquad \Rightarrow \vec{a} = -2a\alpha\hat{j}$

for 90°,

$\vec{V}\cdot\vec{a} = 0$

$-2a\alpha(a - 2a\alpha t) = 0$

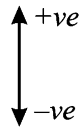
$1 - 2\alpha t = 0 \Rightarrow t = 1/(2\alpha)$ sec.

Example 97. A particle is projected vertitically upwards with velocity 40 m/s. Find the displcement and distance travelled by the particle in

(a) 2 sec (b) 4 sec (c) 6 sec

Take $g = 10$ m/s^2

Solution Here u is positive (upwards) and a is negative (downwards). So, first we will find t_0, the time when velocity becomes zero.



$$t_0 = \left|\frac{u}{a}\right| = \frac{40}{10} = 4\,\text{sec}$$

(a) $t < t_0$ Therfore, distance and displcement are equal.

$$d = s = ut + \frac{1}{2}at^2 = 40 \times 2 - \frac{1}{2} \times 10 \times 4 = 60m$$

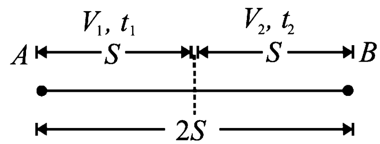
(b) $t = t_0$. So, again distance and displcement are equal

$$d = s = 40 \times 4 - \frac{1}{2} \times 10 \times 16 = 80\,m$$

Example 98. A particle starts from A and moves the half of the total distance covered with speed V_1 and another half with constant speed V_2. Find the average speed during whole journey.

Solution

$$V_{aV} = \frac{\text{Total distance}}{\text{total time interval}}$$



$$= \frac{2S}{t_1 + t_2} = \frac{2S}{\frac{S}{V_1} + \frac{S}{V_2}} = \frac{2}{\frac{1}{V_1} + \frac{1}{V_2}} \quad V_{aV} = \frac{2V_1V_2}{V_1 + V_2}$$

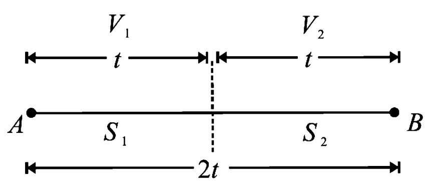
If whole distance is divided in n equal part and particle covers each part with velocities V_1, V_2, V_3, ...V_n. then

$$V_{aV} = \frac{n}{\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3} + \ldots + \frac{1}{V_n}} \quad \text{or} \quad V_{aV} = \frac{1}{n}\left[\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3} + \ldots + \frac{1}{V_n}\right]$$

Example 99. A particle starts its journey from A and moves with speed V_1 during 1st half time interval and with speed V_2 during IInd half time interval of whole journey. Find the average speed during whole time interval.

Solution Average speed

$$V_{aV} = \frac{S_1 + S_2}{2t}$$



$$V_{aV} = \frac{V_1 \times t + V_2 \times t}{2t}$$

$$V_{aV} = \frac{V_1 + V_2}{2}$$

If the whole journey is divided in n equal time interval and particle covers each time interval with velocity V_1, V_2,......V_n then average speed.

$$V_{aV} = \frac{V_1 + V_2 + \ldots\ldots + V_n}{n}$$

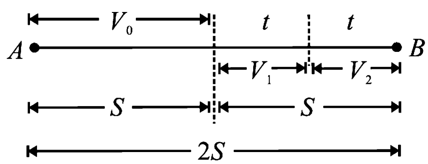
Example 100. A point transversed half the distance with speed V_0. The remaining part of the distance was covered with speed V_1 for half the time and with speed V_2 for the other half of the time. Find the mean velocity of the point averaged over the whole time interval.

Solution This problem is the combination of above two illustration.

Let the average speed during IInd half distance be V'.

$$\therefore \quad V_{aV} = \frac{2V_0V'}{V_0 + V'} \qquad \ldots (1)$$

Since the IInd half distance is divided in two half time interval hence average speed during IInd half is;

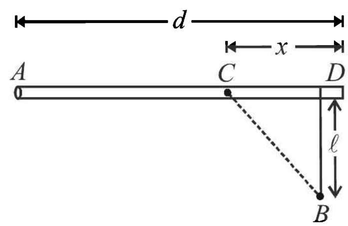


$$V' = \frac{V_1 + V_2}{2}$$

Thus, from equation (1)

$$V_{aV} = \frac{2V_0(V_1 + V_2)/2}{V_0 + \frac{V_1 + V_2}{2}}; \quad V_{aV} = \frac{2V_0(V_1 + V_2)}{2V_0 + V_1 + V_2}$$

Example 101. From point 'A' located on a high way one has to get by car as soon as possible to point 'B' located in the field at a distance 'ℓ' from the highway. It is known that the car moves in the field η times slower than on the highway? At what distance from point D one must turn off the highway?



Solution Let the car turn off from point 'C' which is situated at a distance x from the point 'D'. The velocityof the car on highway is V, thus velocity on the field becomes V/η.

Total time $$t = t_{AC} + t_{CB} = \frac{AC}{V} + \frac{CB}{V/\eta}$$

$$t = \frac{d - x}{V} + \frac{\eta}{V}\sqrt{x^2 + \ell^2}$$

For time to be minimum $\frac{dt}{dx}$ should be zero. i.e., $\frac{dt}{dx} = 0$

$$\Rightarrow \frac{d}{dx}\left[\frac{d - x}{V} + \frac{\eta}{V}\sqrt{x^2 + \ell^2}\right] = 0 \qquad \Rightarrow \frac{1}{V}\left[0 - 1 + \eta\frac{d}{dx}\sqrt{x^2 + \ell^2}\right]$$

$$\Rightarrow \frac{1}{V}\left[-1 + \eta\frac{1}{2}\frac{2x}{\sqrt{x^2 + \ell^2}}\right] = 0 \qquad \Rightarrow -1 + \frac{\eta x}{\sqrt{x^2 + \ell^2}} = 0$$

$$\Rightarrow \quad \eta x = \sqrt{x^2 + \ell^2} \Rightarrow \quad \eta^2 x^2 = x^2 + \ell^2$$

$$\Rightarrow \quad (\eta^2 - 1)x^2 = \ell^2 \Rightarrow \quad x = \frac{\ell}{\sqrt{\eta^2 - 1}}$$

Summary

Rectilinear Motion: Rectilinear motion is motion, along a straight line or in one dimension.

Displacement: The vector joining the initial position of the particle to its final position during an interval of time.

Distance: The length of the actual path travelled by a particle during a given time interval

Average Velocity $= \dfrac{\text{displacement}}{\text{time interval}} = \dfrac{x_f - x_i}{t_f - t_i}$

Average Speed $= \dfrac{\text{distance travelled}}{\text{time interval}}$

Instantaneous Velocity: $V_{\text{inst.}} = \lim\limits_{\Delta t \to 0} \left(\dfrac{\Delta x}{\Delta t}\right) = \dfrac{dx}{dt}$

Average Acceleration $= \dfrac{\text{change in velocity}}{\text{time interval}} = \dfrac{v_f - v_i}{t_f - t_i}$

Instantaneous Acceleration:

$$a = \frac{dv}{dt} = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t}\right)$$

Equations of Motion

(a) $v = u + at$

(b) $s = ut + 1/2\ at^2$

$s = vt - 1/2\ at^2$

$x_f = x_i + ut + 1/2\ at^2$

(c) $v^2 = u^2 + 2as$

(d) $s = 1/2\ (u + v)\ t$

(e) $s_n = u + a/2\ (2n - 1)$

Important Points to Remember

- For uniformly accelerated motion ($a \neq 0$), x–t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.
- For uniformly accelerated motion ($a \neq 0$), v–t graph is a straight line whose slope gives the acceleration of the particle.

- In general, the slope of tangent in x–t graph is velocity and the slope of tangent in v–t graph is the acceleration.
- The area under a–t graph gives the change in velocity.
- The area between the v–t graph gives the distance travelled by the particle, if we take all areas as positive.
- Area under v–t graph gives displacement, if areas below the t–axis are taken negative.

Maxima and Minima

Conditions for maxima are:–

$\dfrac{dy}{dx} = 0$ (b) $\dfrac{d^2y}{dx^2} < 0$

Conditions for minima are:–

$\dfrac{dy}{dx} = 0$ (b) $\dfrac{d^2y}{dx^2} > 0$

Motion with Non-Uniform Acceleration

$$\Delta x = \int_{t_i}^{t_f} v(t)\,dt$$

$$\Delta v = v_f - v_i = \int_{t_i}^{t_f} a(t)\,dt$$

Solving Problems which Involves Non-uniform Acceleration

If a is in terms of t, $\displaystyle\int_{v_0}^{v} dv = \int_0^t a(t)\,dt$

If a is in terms of v, $\displaystyle\int_{v_0}^{v} \frac{dv}{a(v)} = \int_0^t dt$

If a is in terms of x, $\displaystyle\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx$.

If a is in terms of v, $\displaystyle\int_{v_0}^{v} \frac{v\,dv}{a(v)} = \int_{x_0}^{x} dx$

EXERCISE

Exercise–1: Subjective Problems

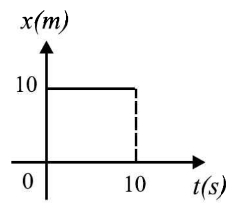
1. A particle covers each 1/3 of the total distance with speed v_1, v_2 and v_3 respectively. Find the average speed of the particle ?
2. The position of a particle moving on x-axis is given by $x = 4t^3 + 3t^2 + 6t + 4$. Find
   (a) The velocity and acceleration of particle at $t = 5$ s.

(b) The average velocity and average acceleration during the interval $t = 0$ to $t = 5$ s, $x = 4t^3 + 3t^2 + 6t + 4$

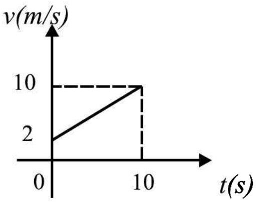
(a) The velocity and acceleration of particle at $t = 5$ s.

(b) The average velocity and average acceleration during the interval t = 0 to t = 5 s.

3. A train starts from rest and moves with a constant acceleration of 2.0 m/s^2 for half a minute. The brakes are then applied and the train comes to rest in one minute. Find (a) the total distance moved by the train, (b) the maximum speed attained by the train and (c) the position(s) of the train at half the maximum speed.
4. A particle starts from rest with a constant acceleration. At a time t second, the speed is found to be 100 m/s and one second later the speed becomes 150 m/s. Find (a) the acceleration and (b) the distance travelled during the $(t+1)^{th}$ second.
5. For a particle moving along x-axis, following graphs are given. Find the distance travelled by the particle in 10 s in each case.

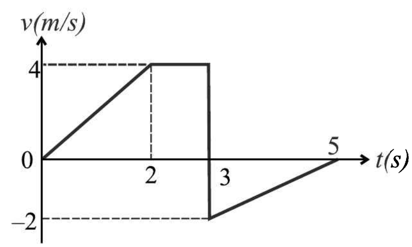


(A)

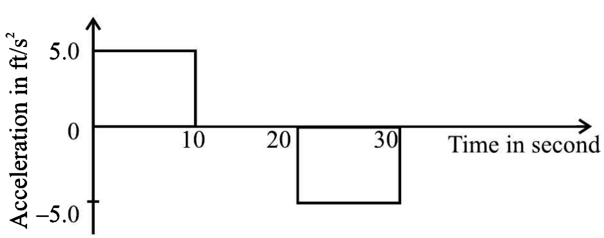


(B)

6. For a particle moving along x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle?Also find the average velocity of the particle?



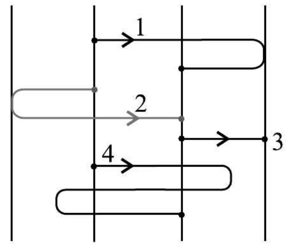
7. The acceleration of a cart started at $t = 0$, varies with time as shown in figure. Find the distance travelled in 30 seconds and draw the position-time graph.



8. For a particle moving rectilinearly, acceleration as a function of speed is given as $a = 8v^2$. Find the speed as a function of x if the particle is having a speed of v_0 at $x = 0$?
9. Under what conditions does the magnitude of the average velocity equal to the average speed.

10. Can an object have increasing speed but its acceleration decreases? If yes, give an example; if not, explain why?

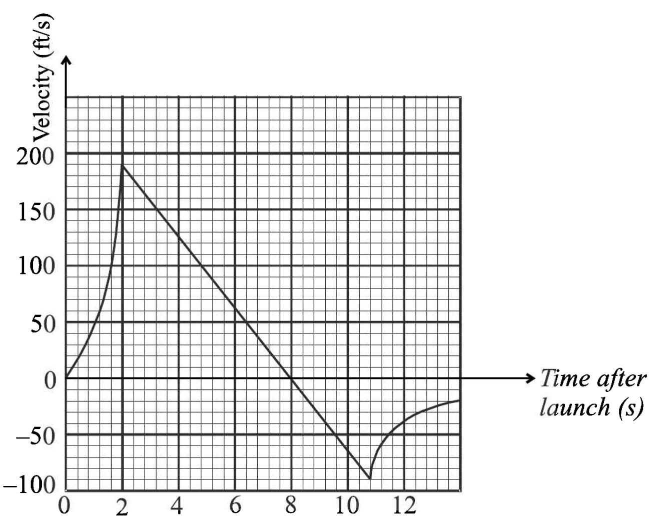
11. Figure shows four paths along which obejcts move from a starting point to a final point (particle is moving along the same straight line), all in the same time. The paths pass over a grid of equally spaced straight lines. Rank the paths according to
    (a) the average velocity of the objects and
    (b) the average speed of the objects, greatest first.



12. A man walking with a speed 'v' constant in magnitude and direction passes under a lantern hanging at a height H above the ground. Find the velocity with which the edge of the shadow of the man's head moves over the ground, if his height is 'h'.

13. An elevator is descending with uniform acceleration. To measure the acceleration, a person in the elevator drops a coin at the moment the elavator starts. The coin is 6 ft above the floor of the elevator at time it is dropped. The person observes that the coin strikes the floor in 1 second. Calculate from these data the acceleration of the elevator. [Take g = 32 ft/s^2]

14. When a model rocket is launched, the propellant burns for a few seconds, accelerating the rocket upward. After burnout, the rocket moves upward for a while and then begins to fall. A parachute opens shortly after the rocket starts down. The parachute slows the rocket to keep it from breaking when it lands. The figure here shows velocity data from the flight of the model rocket. Use the data to answer the following.
    (a) How fast was the rocket climbing when the engine stopped?
    (b) For how many seconds did the engine burn?
    (c) When did the rocket reach its highest point? What was its velocity then?
    (d) When did the parachute open up ? How fast was the rocket falling then?



    (e) How long did the rocket fall before the parachute opened?
    (f) When was the rocket's acceleration greatest?
    (g) When was the acceleration constant? What was its value then (to the nearest interger)?

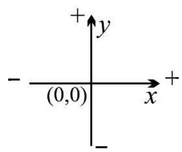
15. At a distance L = 400m from the traffic light brakes are applied to a locomotive moving at a velocity v = 54 km/hr. Determine the position of the locomotive relative to the traffic light 1 minute after the application of the brakes if its acceleration is –0.3m/sec^2.

16. A particle goes from A to B with a speed of 40km/h and B to C with a speed of 60km/h. If $AB = 6BC$, the average speed in km/h between A and C is ____

[**Hint:** Average speed $= \dfrac{\text{total distance travelled}}{\text{time taken}}$]

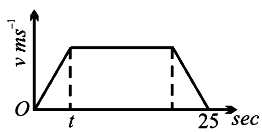
17. An object moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is – 5.00 cm, what is its acceleration?

18. A particle is moving along x-axis. Initially it is located 5 m left of origin and it is moving away from the origin and slowing down. In this coordinate system, what are the signs of the initial velocity and acceleration.

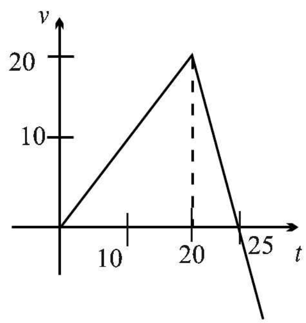


V_0	a

19. The velocity-time graph of the particle moving along a straight line is shown. The rate of acceleration and deceleration is constant and it is equal to 5 ms^{-2}. If the average velocity during the motion is 20 ms^{-1}, then find the value of t.



20. The figure shows the v–t graph of a particle moving in straight line. Find the time when particle returns to the starting point.



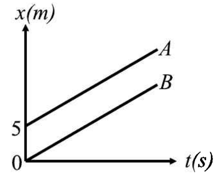
21. A speeder in an automobile passes a stationary policeman who is hiding behind a bill board with a motorcycle. After a 2.0 sec delay (reaction time) the policeman accelerates to his maximum speed of 150 km/hr in 12 sec and catches the speeder 1.5 km beyond the billboard. Find the speed of speeder in km/hr.

22. The position coordinate of a particle that is confined to move along a straight line is given by $x = 2t^3 - 24t + 6$ where x is measured from a convenient origin and t is in seconds. Determine the distance travelled by the particle during the interval from t=1 sec to t = 4 sec.

Exercise–2: Objective Problems

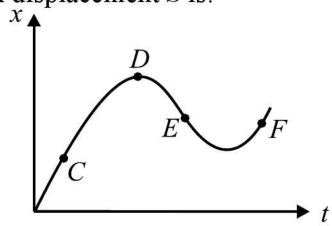
Single Correct Option

1. A motor car is going due north at a speed of 50 km/h. It makes a 90º left turn without changing the speed. The change in the velocity of the car is about
   (A) 50 km/h towards west (B) $50\sqrt{2}$km/h towards south-west
   (C) $50\sqrt{2}$ km/h towards north-west (D) zero
2. A particle has a velocity u towards east at $t = 0$. Its acceleration is towards west and is constant, Let x_A and x_B be the magnitude of displacements in the first 10 seconds and the next 10 seconds.
   (A) $x_A < x_B$
   (B) $x_A = x_B$
   (C) $x_A > x_B$
   (D) the information is insufficient to decide the relation of x_A with x_B.
3. A ball takes t seconds to fall from a height h_1 and $2t$ seconds to fall from a height h_2. Then h_1/h_2 is
   (A) 2 (B) 4 (C) 0.5 (D) 0.25
4. A body starts from rest and is uniformly acclerated for 30 s. The distance travelled in the first 10 s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then x_1: x_2: x_3 is the-same-as
   (A) 1: 2: 4 (B) 1: 2: 5 (C) 1: 3: 5 (D) 1: 3: 9
5. A stone is released from an elevator going up with an acceleration a. The acceleration of the stone after the release is
   (A) *a* upward (B) (*g*-*a*) upward
   (C) (*g*-*a*) downward (D) *g* downward
6. A person standing near the edge of the top of a building throws two balls A and B. The ball A is thrown vertically downward and the ball B is thrown vertically upward with the same speed. The ball A hits the ground with a speed υ_A and the ball B hits the ground with a speed υ_B. We have
   (A) $\upsilon_A > \upsilon_B$ (B) $\upsilon_A < \upsilon_B$
   (C) $\upsilon_A = \upsilon_B$
   (D) the relation between A and B depends on height of the building above the ground.
7. Figure shows position-time graph of two cars A and B.
   (A) Car A is faster than car B.
   (B) Car B is faster than car A.
   (C) Both cars are moving with same velocity.
   (D) Both cars have positive acceleration.

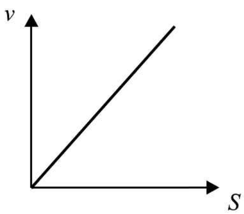


8. The displacement–time graph of a moving particle is shown below. The instantaneous velocity of the particle is negative at the point:
   (A) C (B) D
   (C) E (D) F

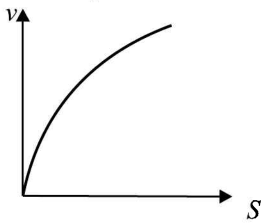
9. A particle starts from rest and moves along a straight line with constant acceleration. The variation of velocity v with displacement S is:



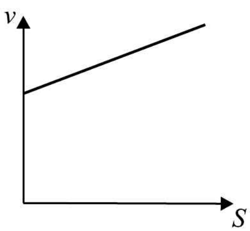
(A)



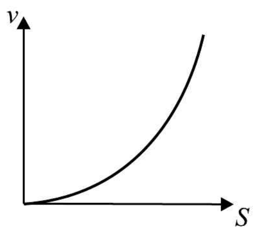
(B)



(C)



(D)



10. The displacement time graphs of two particles A and B are straight lines making angles of respectively 30^0 and 60^0 with the time axis. If the velocity of A is v_A and that of B is v_B, then the value of $\frac{v_A}{v_B}$ is

(A) 1/2 (B) $1/\sqrt{3}$
(C) $\sqrt{3}$ (D) 1/3

11. The initial velocity of a particle is u (at $t = 0$) and the acceleration f is given by ($f = at$). Which of the following relations is valid?

(A) $v = u + at^2$ (B) $v = u + \dfrac{at^2}{2}$
(C) $v = u + at$ (D) $v = u$

12. A stone is dropped into a well in which the level of water is h below the top of the well. If v is velocity of sound, the time T after which the splash is heard is given by

(A) $T = 2h/v$ (B) $T = \sqrt{\dfrac{2h}{g}} + \dfrac{h}{v}$

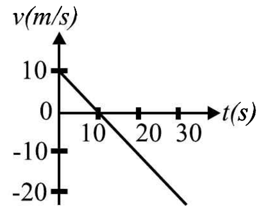
(C) $T = \sqrt{\dfrac{2h}{g}} + \dfrac{h}{2v}$ (D) $T = \sqrt{\dfrac{h}{2g}} + \dfrac{2h}{v}$

13. A body is released from the top of a tower of height h metre. It takes T seconds to reach the ground. Where is the ball at the time $T/2$ seconds ?
(A) At $h/4$ metre from the ground (B) At h/2 metre from the ground
(C) At $3h/4$ metre from the ground (D) Depend upon the mass of the ball
14. A stone is thrown vertically upward with an initial velocity u from the top of a tower, reaches the ground with a velocity $3u$. The height of the tower is:
(A) $\frac{3u^2}{g}$ (B) $\frac{4u^2}{g}$ (C) $\frac{6u^2}{g}$ (D) $\frac{9u^2}{g}$
15. A particle starts from rest with uniform acceleration a. Its velocity after n seconds is v. The displacement of the body in the last two seconds is:
(A) $\frac{2v(n-1)}{n}$ (B) $\frac{v(n-1)}{n}$ (C) $\frac{v(n+1)}{n}$ (D) $\frac{2v(2n+1)}{n}$
16. Consider the motion of the tip of the minute hand of a clock. In one hour
(A) The displacement is zero
(B) The distance covered is zero
(C) The average speed is zero
(D) The average velocity is zero
17. A particle moves along the x-axis as $x = u(t-2) + a(t-2)^2$
(A) The initial velocity of the particle is u
(B) The acceleration of the particle is a
(C) The acceleration of the particle is $2a$
(D) At $t = 2s$ particle is at the origin.
18. Mark the correct statements for a particle going on a straight line:
(A) If the velocity and acceleration have opposite sign, the object is slowing down.
(B) If the position and velocity have opposite sign, the particle is moving towards the origin.
(C) If the velocity is zero at an instant, the acceleration should also be zero at that instant.
(D) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.
19. The velocity of a particle is zero at $t = 0$
(A) The acceleration at $t = 0$ must be zero
(B) The acceleration at $t = 0$ may be zero.
(C) If the acceleration is zero from $t = 0$ to $t = 10$ s, the speed is also zero in this interval.
(D) If the speed is zero from $t = 0$ to $t = 10$ s the acceleration is also in the interval.

Mark the Correct Statements.

(A) The magnitude of the velocity of a particle is equal to its speed.
(B) The magnitude of average velocity in an interval is equal to its average speed in that interval.
(C) It is possible to have a situation in which the speed of a particle is always zero but the average speed is not zero
(D) It is possible to have a situation in which the speed of the particle is never zero but the average speed in an interval is zero.

21. The velocity-time plot for a particle moving on a straight line is shown in fig.



(A) The particle has constant acceleration
(B) The particle has never turned around.
(C) The particle has zero displacement
(D) The average speed in the interval 0 to 10s is the same as the average speed in the interval 10s to 20s.

22. A particle is projected vertically upwards from a point A on the ground. It takes t_1 time to reach a point B but it still continues to move up. If it takes further t_2 time to reach the ground from point B then height of point B from the ground is

(A) $\frac{1}{2}g(t_1+t_2)^2$ (B) $g\,t_1\,t_2$

(C) $\frac{1}{8}g(t_1+t_2)^2$ (D) $\frac{1}{2}\,g\,t_1\,t_2$

23. Balls are thrown vertically upward in such a way that the next ball is thrown when the previous one is at the maximum height. If the maximum height is 5m, the number of balls thrown per minute will be
(A) 40 (B) 50
(C) 60 (D) 120

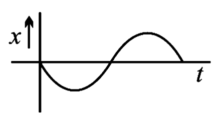
24. The co-ordinates of a moving particle at a time t, are given by, $x = 5 \sin 10\,t$, $y = 5 \cos 10t$. The speed of the particle is:
(A) 25 (B) 50
(C) 10 (D) None

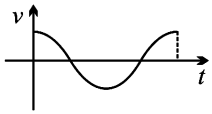
25. An object is moving along the x-axis with position as a function of time given by $x = x(t)$. Point O is at $x = 0$. The object is definitely moving toward O when
(A) $dx/dt < 0$ (B) $dx/dt > 0$
(C) $d(x^2)/dt < 0$ (D) $d(x^2)/dt > 0$

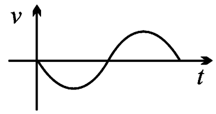
26. A particle starts moving rectilinearly at time $t = 0$ such that its velocity 'v' changes with time 't' according to the equation $v = t^2 - t$ where t is in seconds and v is in m/s. The time interval for which the particle retards is
(A) $t < 1/2$ (B) $1/2 < t < 1$
(C) $t > 1$ (D) $t < 1/2$ and $t > 1$

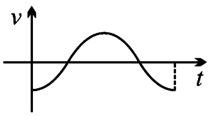
27. An object is tossed vertically into the air with an initial velocity of 8 m/s. Using the sign convention upwards as positive, how does the vertical component of the acceleration a_y of the object (after leaving the hand) vary during the flight of the object?
(A) On the way up $a_y > 0$, on the way down $a_y > 0$
(B) On the way up $a_y < 0$, on the way down $a_y > 0$
(C) On the way up $a_y > 0$, on the way down $a_y < 0$
(D) On the way up $a_y < 0$, on the way down $a_y < 0$

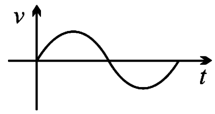
28. If position time graph of a particle is sine curve as shown, what will be its velocity-time graph.



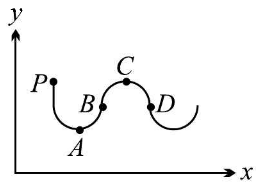
(A) 

(B) 

(C) 

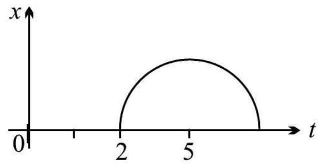
(D) 

29. A man moves in x-y plane along the path shown. At what point is his average velocity vector in the same direction as his instantaneous velocity vector. The man starts from point P.



(A) A
(B) B
(C) C
(D) D

30. Position-time graph is shown which is a semicircle from $t=2$ to $t=8$ sec. Find time t at which the instantaneous velocity, is equal to average velocity over first t seconds

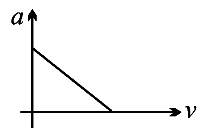


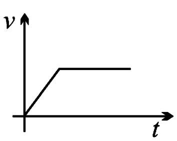
(A) 4.8 sec
(B) 3.2 sec
(C) 2.4 sec
(D) 5 sec

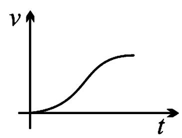
31. The greatest acceleration or deceleration that a train may have is a. The minimum time in which the train may reach from one station to the other separated by a distance d is

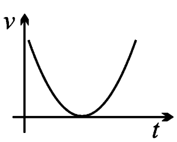
(A) $\sqrt{\frac{d}{a}}$
(B) $\sqrt{\frac{2d}{a}}$
(C) $\frac{1}{2}\sqrt{\frac{d}{a}}$
(D) $2\sqrt{\frac{d}{a}}$

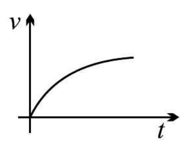
32. Acceleration versus velocity graph of a particle moving in a straight line starting from rest is as shown in figure. The corresponding velocity-time graph would be



(A) 

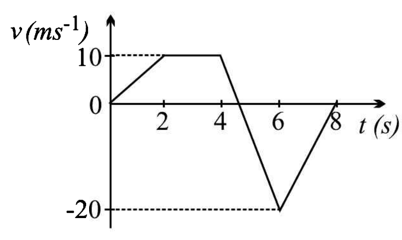
(B) 

(C) 

(D) 

Question No. 33 to 38 (6 questions)

The figure shows a velocity-time graph of a particle moving along a straight line.



33. Choose the incorrect statement.
The particle comes to rest at
(A) $t = 0$ s (B) $t = 5$ s
(C) $t = 8$ s (D) none of these

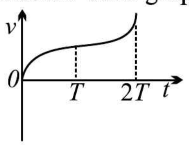
34. Identify the region in which the rate of change of velocity $\left|\frac{\Delta\vec{v}}{\Delta t}\right|$ of the particle is maximum
(A) 0 to 2s (B) 2 to 4s
(C) 4 to 6 s (D) 6 to 8 s

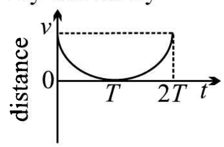
35. If the particle starts from the position $x_0 = -15$ m, then its position at t = 2s will be
(A) –5 m (B) 5 m
(C) 10 m (D) 15 m

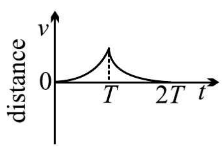
36. The maximum displacement of the particle is
(A) 33.3 m (B) 23.3 m
(C) 18.3 m (D) zero

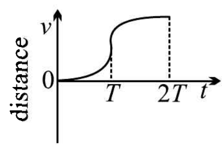
37. The total distance travelled by the particle is
(A) 66.7 m (B) 51.6 m
(C) zero (D) 36.6 m

38. The distance-time graph of the particle is correctly shown by

(A)


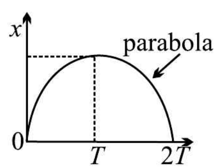
(B)


(C)


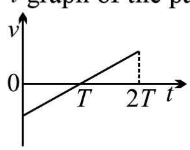
(D)


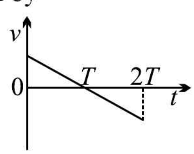
Question No. 39 to 43 (5 questions)

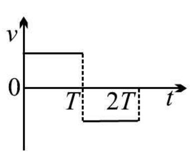
The x-t graph of a particle moving along a straight line is shown in figure

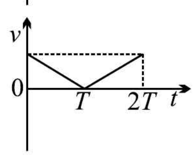


39. The v-t graph of the particle is correctly shown by

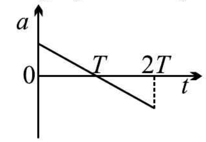
(A)


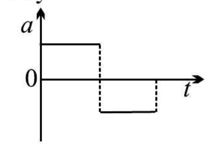
(B)


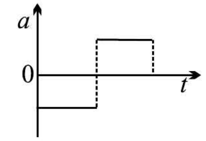
(C)


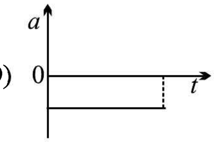
(D)


40. The *a-t* graph of the particle is correctly shown by

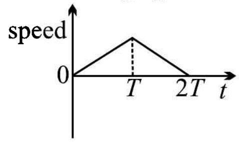
(A) 

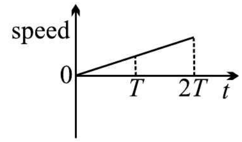
(B) 

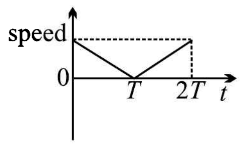
(C) 

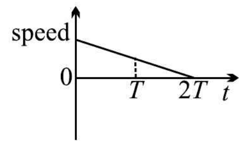
(D) 

41. The speed-time graph of the particle is correctly shown by

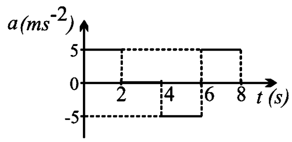
(A) 

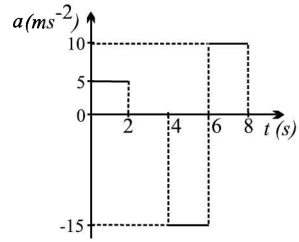
(B) 

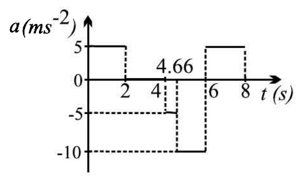
(C) 

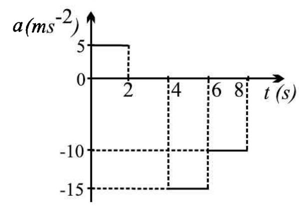
(D) 

42. The correct acceleration-time graph of the particle is shown as

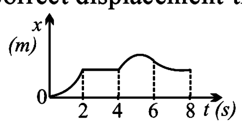
(A) 

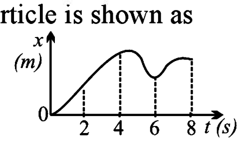
(B) 

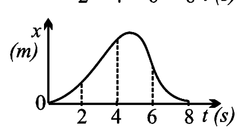
(C) 

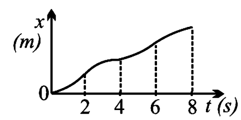
(D) 

43. The correct displacement-time graph of the particle is shown as

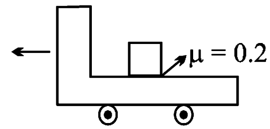
(A) 

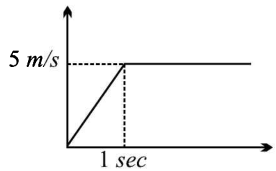
(B) 

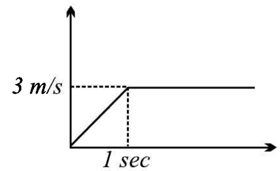
(C) 

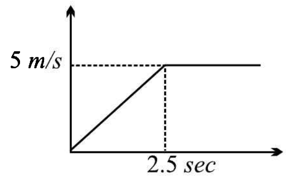
(D) 

44. A truck starting from rest moves with an acceleration of 5 m/s² for 1 sec and then moves with constant velocity. The velocity w.r.t ground v/s time graph for block in truck is
(Assume that block does not fall off the truck)



(A) 

(B) 

(C) 

(D) None of these

45. If angular velocity of a disc depends an angle rotated θ as $\omega = \theta^2 + 2\theta$, then its angular acceleration α at $\theta = 1$ rad is:
(A) 8 rad/sec² (B) 10 rad/sec²
(C) 12 rad/sec² (D) None

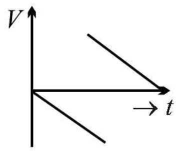
46. If a particle takes t second less and acquires a velocity of v ms^{-1} more in falling through the same distance (starting from rest) on two planets where the accelerations due to gravity are $2g$ and $8g$ respectively then:
(A) $v = 2gt$ (B) $v = 4gt$
(C) $v = 5gt$ (D) $v = 16gt$

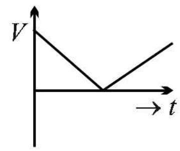
47. Tangential acceleration of a particle moving in a circle of radius 1 m varies with time t as (initial velocity of particle is zero). Time after which total acceleration of particle makes and angle of 30° with radial acceleration is
(A) 4 sec (B) 4/3 sec
(C) $2^{2/3}$ sec (D) $\sqrt{2}$ sec

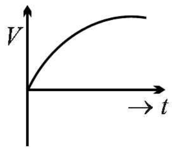
48. A particle moves along a straight line in such a way that it's acceleration is increasing at the rate of 2 m/s³. It's initial acceleration and velocity were 0, the distance covered by it in $t = 3$ second is.
(A) 27 m (B) 9 m
(C) 3 m (D) 1 m

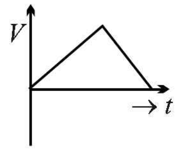
49. A ball is thrown vertically down with velocity of 5m/s. With what velocity should another ball be thrown down after 2 seconds so that it can hit the 1st ball in 2 seconds
(A) 40 m/s (B) 55 m/s
(C) 15 m/s (D) 25 m/s

50. The velocity- time graph of a body falling from rest under gravity and rebounding from a solid surface is represented by which of the following graphs?

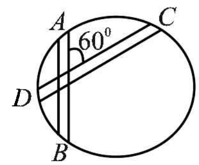
(A) 

(B) 

(C) 

(D) 

51. A disc arranged in a vertical plane has two groves of same length directed along the vertical chord AB and CD as shown in the fig. The same particles slide down along AB and CD. The ratio of the time t_{AB}/t_{CD} is



(A) 1: 2
(B) 1: $\sqrt{2}$
(C) 2: 1
(D) $\sqrt{2}:1$

52. The magnitude of displacement of a particle moving in a circle of radius a with constant angular speed ω varies with time t as

(A) $2\,a\sin\omega t$
(B) $2a\sin\dfrac{\omega t}{2}$
(C) $2a\cos\omega t$
(D) $2a\cos\dfrac{\omega t}{2}$

53. A body moves with velocity $v = \ell\, n\, x$ m/s where x is its position. The net force acting on body is zero at

(A) 0 m
(B) $x = e^2$ m
(C) $x = e$ m
(D) $x = 1$ m

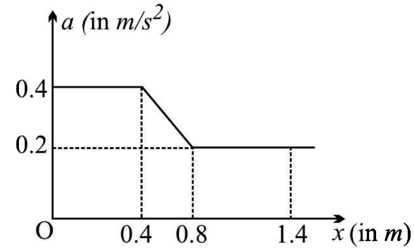
54. A body of mass 1 kg is acted upon by a force $\vec{F} = 2\sin 3\pi t\,\hat{i} + 3\cos 3\pi t\,\hat{j}$ find its position at $t = 1$ sec if at $t = 0$ it is at rest at origin.

(A) $\left(\dfrac{3}{3\pi^2}, \dfrac{2}{9\pi^2}\right)$
(B) $\left(\dfrac{2}{3\pi^2}, \dfrac{2}{3\pi^2}\right)$
(C) $\left(\dfrac{2}{3\pi}, \dfrac{2}{3\pi^2}\right)$
(D) none of these

55. A force $F = Be^{-Ct}$ acts on a particle whose mass is m and whose velocity is 0 at $t = 0$. It's terminal velocity is:

(A) $\dfrac{C}{mB}$
(B) $\dfrac{B}{mC}$
(C) $\dfrac{BC}{m}$
(D) $-\dfrac{B}{mC}$

56. The acceleration of a particle which moves along the positive x-axis varies with its position as shown. If the velocity of the particle is 0.8 m/s at $x = 0$, the velocity of the particle at $x = 1.4$ is(in m/s)

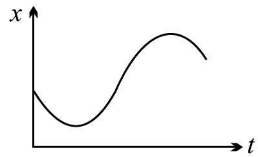


(A) 1.6
(B) 1.2
(C) 1.4
(D) none of these

57. A ball is thrown vertically downwards with velocity $\sqrt{2gh}$ from a height h. After colliding with the ground it just reaches the starting point. Coefficient of restitution is

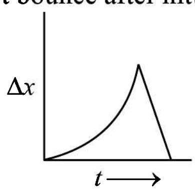
(A) $1/\sqrt{2}$
(B) 1/2
(C) 1
(D) $\sqrt{2}$

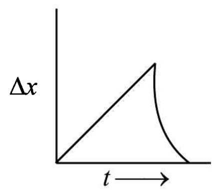
58. The graph of position x versus time t represents the motion of a particle. If b and c are both positive constants, which of the following expressions best describes the acceleration a of the particle?

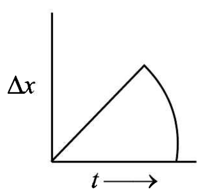


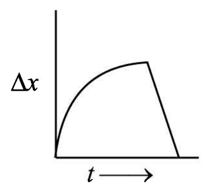
(A) $a = b - ct$
(B) $a = +b$
(C) $a = -c$
(D) $a = b + ct$

59 Two stones are thrown up vertically and simultaneously but with different speeds. Which graph correctly represents the time variation of their relative positions Δx. Assume that stones do not bounce after hitting ground.

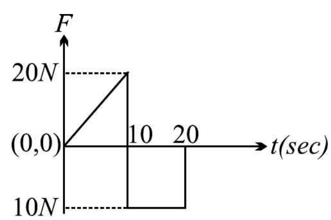
(A) 

(B) 

(C) 

(D) 

60. A particle of mass 1 kg is acted upon by a force 'F' which varies as shown in the figure. If initial velocity of the particle is 10 ms^{-1}, the maximum velocity attained by the particle during the period is



(A) 210 ms^{-1}
(B) 110 ms^{-1}
(C) 100 ms^{-1}
(D) 90 ms^{-1}

Question No. 61 to 65 (5 questions)

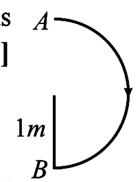
Two balls A and B are thrown with same velocity u from the top of a tower. Ball A is thrown vertically upwards and the ball B is thrown vertically downwards.

61. Choose the correct statement
    (A) Ball B reaches the ground with greater velocity
    (B) Ball A reaches the ground with greater velocity
    (C) Both the balls reach the ground with same velocity
    (D) Cannot be interpreted
62. If t_A and t_B are the respective times taken by the balls A and B respectively to reach the ground, then identify the correct statement
    (A) $t_A > t_B$ (B) $t_A = t_B$
    (C) $t_A < t_B$ (D) Cannot be interpreted
63. If $t_A = 6$ s and $t_B = 2$ s, then the height of the tower is
    (A) 80 m (B) 60 m (C) 45 m (D) none of these
64. The velocity u of each ball is
    (A) 10 ms^{-1} (B) 15 ms^{-1} (C) 20 ms^{-1} (D) none of these
65. If a ball C is thrown with the same velocity but in the horizontal direction from the top of the tower, then it will reach the ground in time t_c equal to
    (A) 4 s (B) 3.46 s (C) 4.2 s (D) none of these

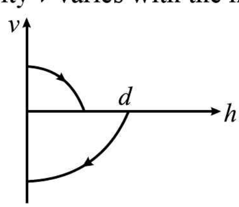
Exercise–3

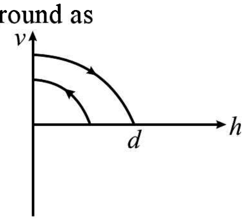
(JEE/REE QUESTIONS OF PREVIOUS YEARS)

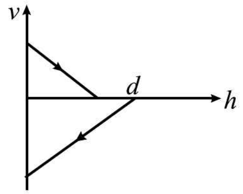
1. A particle of mass 10^{-2} kg is moving along the positive x–axis under the influence of a force $F(x) = -\dfrac{K}{2x^2}$ where $K = 10^{-2}$ N m^2. At time $t = 0$ it is at $x = 1.0$ m and its velocity is $v = 0$. Find
    (i) its velocity when it reaches $x = 0.50$ m
    (ii) the time at which it reaches $x = 0.25$ m. **[JEE, 1998, 2008]**
2. In 1.0 sec. a particle goes from point A to point B moving in a semicircle of radius 1.0 m. The magnitude of average velocity is: **[JEE, 1999, 2002]**
    (A) 3.14 m/sec (B) 2.0 m/sec
    (C) 1.0 m/sec (D) zero

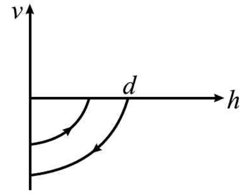


3. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height $d/2$. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as **[JEE, 2000, 2003]**

(A)


(B)


(C) 

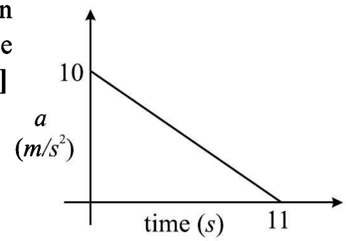
(D) 

4. A block is moving down a smooth inclined plane starting from rest at time t = 0. Let S_n be the distance travelled by the block in the interval $t = n - 1$ to $t = n$. The ratio $\frac{S_n}{S_{n+1}}$ is

[JEE Scr., 2003, 2004]

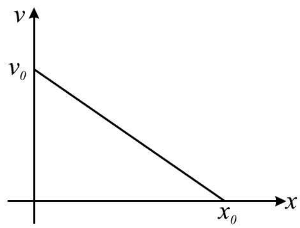
(A) $\frac{2n-1}{2n}$ (B) $\frac{2n-1}{2n+1}$ (C) $\frac{2n+1}{2n-1}$ (D) $\frac{2n}{2n-1}$

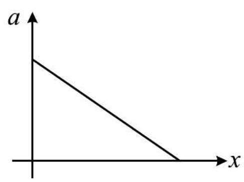
5. A particle is initially at rest, It is subjected to a linear acceleration a, as shown in the figure. The maximum speed attained by the particle is **[JEE Scr., 2003, 2004]**

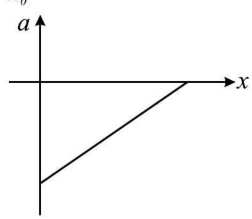


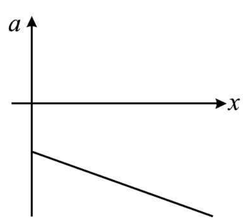
(A) 605 m/s (B) 110 m/s
(C) 55 m/s (D) 550 m/s

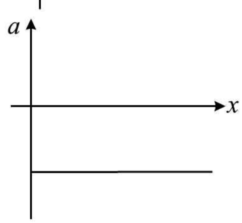
6. The velocity displacement graph of a particle moving along a straight line is shown. The most suitable acceleration-displacement graph will be **[JEE Scr., 2003, 2005]**



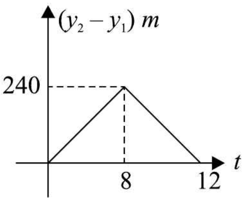
(A) 

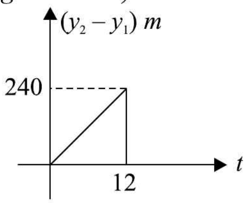
(B) 

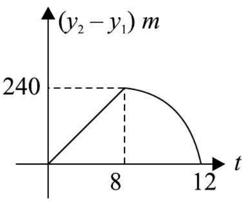
(C) 

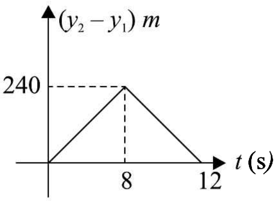
(D) 

7. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground, is n times that taken by it to reach the heighest point of its path. The relation between H, u and n is **[JEE Main, 2014]**
   (A) $gH = (n-2)^2u^2$ (B) $2gH = nu^2(n-2)$
   (C) $gH = (n-2)u^2$ (D) $2gH = n^2u^2$
   [Ans. (A)**]**
8. Two stones are thrown up simultaneously from the edge of a cliff 240m high with initial speed of 10 *m/s* and 40 *m/s* respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? (Assome stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\ m/s^2$) **[JEE Main, 2015]**

(A) 

(B) 

(C) 

(D) 

[Ans. (A)**]**

Previous Years' AIEEE Questions

1. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest? **[AIEEE - 2002, 4/300]**
   (A) 1 cm (B) 2 cm (C) 3 cm (D) 4 cm
2. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If V_A and V_B are their respective velocities on reaching the ground, then **[AIEEE - 2002, 4/300]**
   (A) $v_B > v_A$ (B) $v_A = v_B$
   (C) $v_A > v_B$ (D) their velocities depends on their masses
3. Speeds of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances at which the two cars are stopped from that instant is: **[AIEEE - 2002, 4/300]**
   (A) 1: 1 (B) 1: 4 (C) 1: 8 (D) 1: 1
4. The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by: **[AIEEE - 2003, 4/300]**
   (A) $\sqrt{\alpha^2 + \beta^2}$ (B) $3t^2\sqrt{\alpha^2 + \beta^2}$ (C) $t^2\sqrt{\alpha^2 + \beta^2}$ (D) $\sqrt{\alpha^2 + \beta^2}$
5. A car moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. if the same car is moving at a speed of 100 km/hr, the minimum stopping distance is: **[AIEEE - 2003, 4/300]**
   (A) 12 m (B) 18 m (C) 24 m (D) 6 m
6. A ball is released from the top of a tower of height h metres. It takes T seconds to reach the ground. What is the position of the ball in T/3 seconds? **[AIEEE - 2004, 4/300]**
   (A) h/9 metre from the ground (B) 7h/9 metre from the ground
   (C) 8h/9 metre from the ground (D) 17h/9 metre from the ground

7. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast, ie. 120 km/h, the stopping distance will be
(A) 20 m (B) 40 m (C) 60 m (D) 80 m

8. The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is: **[AIEEE 2005, 4.300]**
(A) $-2abv^2$ (B) $2bv^2$ (C) $-2av^3$ (D) $2av^3$

9. A car, starting from rest, accelerates at the rate f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance travelled is 15 S, then: **[AIEEE 2005, 4/300]**
(A) S = ft (B) $S = \frac{1}{6}ft^2$ (C) $S = \frac{1}{72}ft^2$ (D) $S = \frac{1}{4}ft^2$

10. A particle is moving eastwards with a velocity of 5 ms^{-1}. In 10 second the velocity changes to 5 ms^{-1} northwards. The average acceleration in this time is: **[AIEEE 2005, 4/300]**
(A) $\frac{1}{\sqrt{2}}$ ms^{-1} towards north-west (B) $\frac{1}{2}$ ms^{-2} towards north
(C) zero (D) $\frac{1}{2}$ ms^{-2} towards north-west.

11. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s². He reaches the ground with a speed of 3 m/s. At what height, did he bail out? **[AIEEE 2005, 4/300]**
(A) 91 m (B) 182 m (C) 293 m (D) 111 m

12. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$ The displacement of the particle varies with time as **[AIEEE-2006, 3/180]**
(A) $t^{1/2}$ (B t^3 (C) t^2 (D) t

13. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is **[AIEEE 2007, 3/120]**
(A) $v_0 + 2g + 3f$ (B) $v_0 + \frac{g}{2} + \frac{f}{3}$ (C) $v_0 + g + f$ (D) $v_0 + \frac{g}{2} + f$

14. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is: **[AIEEE 2009, 4/144]**
(A) $7\sqrt{2}$ units (B) 7 units (C) 8.5 units (D) 10 units

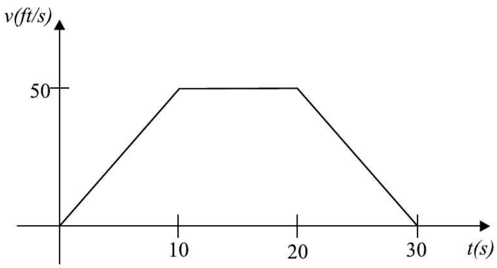
15. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is: **[AIEEE 2010, 4/144]**
(A) $y = x^2$ + constant (B) $y^2 = x$ + constant
(C) xy = constant (D) $y^2 = x^2$ + constant

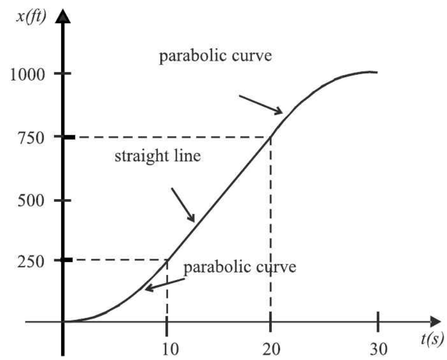
16. At time $t = 0$ s particle starts moving along the x-axis. If its kinetic energy increases uniformly with time t, the net force acting on it must be proportional to **[AIEEE 2011]**
(A) $\sqrt{t}$ (B) constant (C) t (D) $\frac{1}{\sqrt{t}}$

ANSWER KEY

Exercise - 1

1. $\dfrac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_1v_3}$
2. (a) $v_{t=5\,s} = 336$ units, $a_{t=5\,s} = 126$ units ; (b) $<v> = 121$ units, $<a> = 66$ units
3. (a) 2.7 km , (b) 60 m/s, (c) 225 m and 2.25 km
4. (a) 50 m/s^2; (b) 125 m
5. (A) 0; (B) 60m
6. distance travelled = 10 m; displacement = 6 m; average velocity = 1.2 m/s
7. 1000 ft. ,



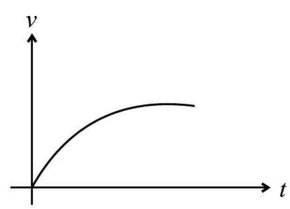


8. $v = v_0 e^{8x}$
9. $<\vec{v}> = <v> \therefore \dfrac{\text{displacment}}{\text{time}} = \dfrac{\text{distance travelled}}{\text{time}}$

   When the distance travelled is equal to the displacement of the particle, i.e. particle moves along the straight line in the same direction without reversing the direction of motion.
10. Yes. Speed of object will increase if acceleration is in the direction of velocity, yet its magnitude may decrease.

   $a = \dfrac{dv}{dt}$ = slope of the curve.

   acceleration (slope of the curve) is decreasing with time yet the speed is increasing.



11. (a) $<\vec{v}_1> = <\vec{v}_2> = <\vec{v}_3> = <\vec{v}_4>$ (b) $v_{4,\,avg} > v_{1,\,avg} = v_{2,\,avg} > v_{3,\,avg}$
12. $\left(\dfrac{H}{H-h}\right)v$
13. 20 ft/s^2
14. (a) 190 ft/s (b) 2 s (c) 8 s, 0 ft/s (d) 10.8 s, 90 ft/s

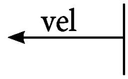
   (e) 2.8 s (f) greatest acceleration happens 2 s after launch

   (g) constant acceleration between 2 and 10.8 s, – 32 ft/s^2.

15. 25m　　16. 42km/hr　　17. $-16\ \text{cm/s}^2$

18.

V_0	a
−	+



Because particle is slowing down so velocity & acceleration are in opposite direction.

19. 5 s　　20. 36.2 sec.　　21. 122.7 km/hr　　22. 74m

Exercise - 2

1.	B	2.	D	3.	D	4	C
5.	D	6.	C	7.	C	8	C
9.	B	10.	D	11.	B	12	B
13.	C	14.	B	15.	A	16	A,D
17.	C,D	18	A,B,D	19	B,C,D	20	A
21.	A,D	22.	B	23.	C	24	B
25.	C	26.	B	27.	D	28	C
29.	C	30.	B	31.	D	32	D
33.	B	34.	C	35.	A	36	A
37.	A	38.	A	39.	B	40	D
41.	C	42.	B	43.	C	44	C
45.	C	46.	B	47.	C	48	B
49.	A	50.	A	51.	B	52	B
53.	D	54.	C	55.	B	56	B
57.	A	58.	A	59	C	60	B
61.	C	62.	A	63.	B	64	C
65.	B						

Exercise - 3

1 (i) $\vec{v} = -1\,\hat{i}$ m/s (ii) $t = \frac{\pi}{3} + \frac{\sqrt{3}}{4}$

2	B	3.	A	4.	B	5	C
6	B						

Previous Years' AIEEE Questions

1.	(A)	2.	(B)	3.	(B)	4.	(D)
5.	(C)	6.	(C)	7.	(D)	8.	(C)
9.	(C)	10.	(A)	11.	(C)	12.	(C)
13.	(B)	14.	(A)	15.	(D)	16.	(D)

PROJECTILE MOTION

Basic Concept

Projectile

Any object that is given an initial velocity obliquely and that subsequently follows a path determined by the gravitational force acting on it, is called a **Projectile**. A projectile may be a football, a cricket ball, or any other object.

Trajectory

The path followed by a particle (here projectile) during its motion is called its **Trajectory**.

Note

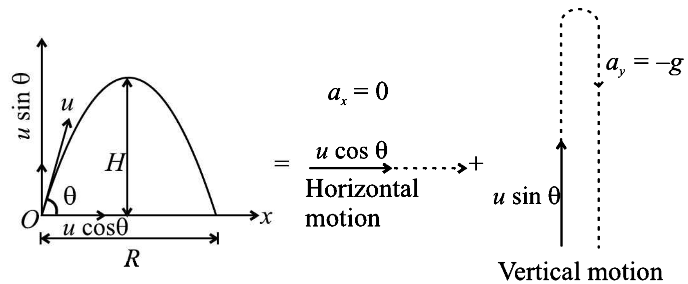
1. We shall consider only trajectories that are of sufficiently short range so that the gravitational force can be considered constant in both magnitude and direction.
2. All effects of air resistance will be ignored; thus our results are precise only for motion in a vacuum on flat, non rotating Earth.

Projectile Motion

(i) The motion of projectile is known as projectile motion.

(ii) It is an example of two dimensional motion with constant acceleration.

(iii) Projectile motion is considered as combination of two simultaneous motions in mutually, perpendicular directions which are completely independent from each other i.e., horizontal motion and vertical motion.



Parabolic path = vertical motion + horizontal motion

If a constant force (and hence constant acceleration) acts on a particle at an angle θ (≠ 0 or 180ŗ) with the direction of its initial velocity (≠ zero) the path followed by the particle is a parabola and the motion of the particle is called as projectile motion. Projectile motion is a two dimensional motion i.e., motion of the particle is constrained in a plane.

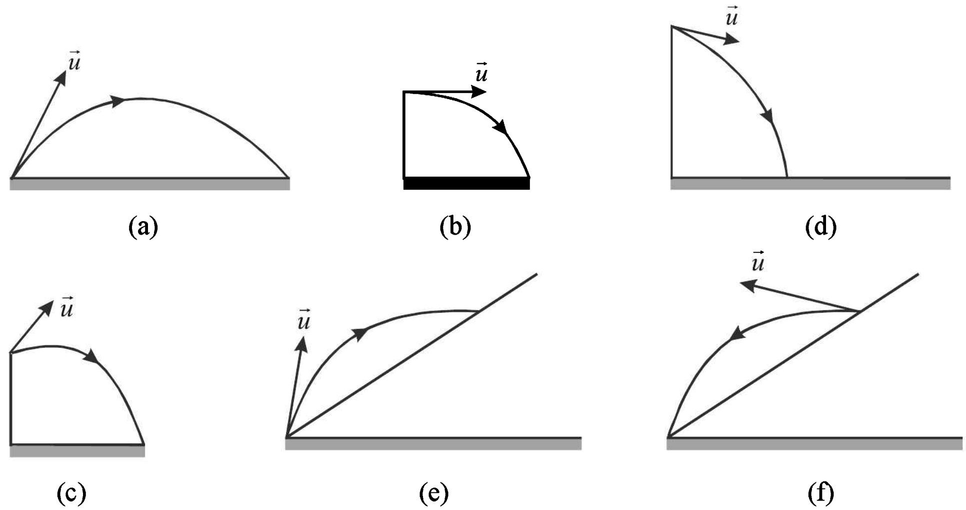


***Fig 2.1:** Types of Projectile Motion*

When a particle is thrown obliquely near the earth's surface it moves in a parabolic path, provided the particle remains close to the surface of earth and the air resistance is negligible. This is an example of projectile motion. The different types of projectile motion we come across are shown in figure 2.1.

In all the above cases acceleration of the particle is g downwards.

In any problem of projectile motion we usually follow the three steps given below:
Step 1. Select two mutually perpendicular directions x and y.
Step 2. Write down the proper values of u_x, a_x, u_y and a_y with sign.
Step 3. Apply those equations from the six listed above which are required in the problem.

Projectile Thrown at an Angle with Horizontal

(i) Consider a projectile thrown with a velocity u making an angle θ with the horizontal.

(ii) Initial velocity u is resolved in components in a coordinate system in which horizontal direction is taken as x-axis, vertical direction as y-axis and point of projection as origin.

$$u_x = u\cos\theta, \quad u_y = u\sin\theta$$

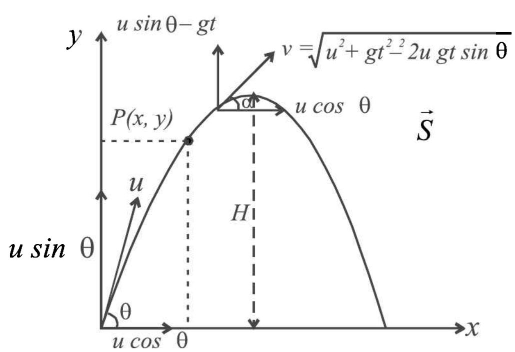


Fig 2.2

(iii) Again this projectile motion can be considered as the combination of horizontal and vertical motion. Therefore,

	Horizontal direction	**Vertical direction**
(a)	Initial velocity $u_x = u \cos \theta$	Initial velocity $u_y = u \sin \theta$
(b)	Acceleration $a_x = 0$	Acceleration $a_y = g$
(c)	Velocity after time t, $v_x = u \cos \theta$	Velocity after time $t, v_y = u \sin \theta - gt$

Trajectory Equation

If we consider the horizontal direction,

$$x = u_x . t$$

$$x = u \cos \theta . t \qquad ...(1)$$

For vertical direction:

$$y = u_y . t - 1/2\, gt^2 = u \sin \theta . t - 1/2\, gt^2 \qquad ...(2)$$

Substituting the value x in equation (1)

$$\boxed{y = u \sin \theta \cdot \frac{x}{u \cos \theta} - \frac{1}{2} g \left(\frac{x}{u \cos \theta} \right)^2}$$

This is an equation of parabola called as **trajectory equation of projectile motion**.

Time of Flight

Since the displacement along vertical direction does not occur. So, Net displacement = 0

$$(u \sin \theta)\, T - \frac{1}{2}\, gT^2 = 0$$

$$\boxed{T = \frac{2u \sin \theta}{g}}$$

Horizontal Range

$$R = u_x . T$$

$$R = u \cos \theta . \frac{2u \sin \theta}{g} \quad \boxed{R = \frac{u^2 \sin 2\theta}{g}}$$

Here two points are important regarding the range of a projectile.

(i) Range is maximum where $\sin 2\theta = 1$ or $\theta = 45$ţ and this maximum range is;

$$R_{max} = \frac{u^2}{g} \qquad (\text{at } \theta = 45ţ)$$

(ii) For given value of u range at α and range at 90ţ $- \theta$ are equal although times of flight and maximum heights may be different. Because

$$R_{90° - \theta} = \frac{u^2 \sin 2(90° - \theta)}{g}$$

$$= \frac{u^2 \sin 2(180^\circ - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g} = R_\theta$$

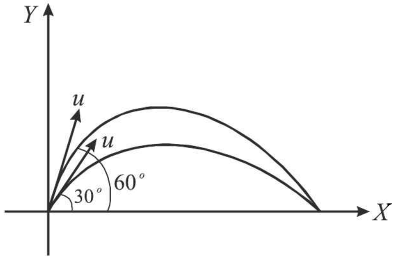


Fig 2.3

So, $R_{30^\circ} = R_{60^\circ}$ or $R_{20^\circ} = R_{70^\circ}$

This is shown in figure 2.3.

Maximum Height

Using 3rd equation of motion i.e., $v^2 = u^2 + 2as$

we have for vertical direction $0 = u^2 \sin^2 \theta - 2gH$

$$\boxed{H = \frac{u^2 \sin^2 \theta}{2g}}$$

Resultant Velocity

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}$$

where $|\vec{v}| = \sqrt{u^2 \cos^2 \theta + (u \sin \theta - gt)^2}$ and $\tan \theta = v_y / v_x$.

Note

- Results of above text are valid only for complete flight, that is when proejctile lands at same horizontal level from which it has been projected.
- Vertical component of velocity is zero when particle moves horizontally i.e., at the highest point of trajectory.
- Vertical component of velocity is positive when particle is moving up and vertical component of velocity is negative when particle is coming down if vertical upwards direction is taken as positive. Any direction upward or downward can be taken as positive and if downward direction is taken as positive then vertical component of velocity coming down is positive.

General Result

(i) For maximum range $\theta = 45^\circ$

$R_{max} = \frac{u^2}{g}$ In this situation $H_{max} = \frac{R_{max}}{2}$

(ii) We get the same range for two angle of projections α and $(90 - \alpha)$ but in both cases, maximum heights attained by the particles are different.

(iii) If $$R = H$$

i.e. $$\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g} \Rightarrow \tan \theta = 4$$

(iv) Range can also be expressed as

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u \sin \theta . u \cos \theta}{g} = \frac{2u_x u_y}{g}$$

(v) Change in momentum :

(a) Initial velocity $\vec{u}_i = u \cos \theta \ \hat{i} + u \sin \theta \ \hat{j}$

(b) Final velocity $\vec{u}_f = u \cos \theta \hat{i} - u \sin \theta \hat{j}$

Change in velocity for complete motion

$\Delta \vec{u} = \vec{u}_f - \vec{u}_i = -2u \sin \theta \hat{j}$

(c) Change in momentum for complete motion

$\Delta \vec{P} = \vec{P}_f - \vec{P}_i = m(\vec{u}_f - \vec{u}_i) = m(-2u \sin \theta) \hat{j} = -2mu \sin \theta \hat{j}$

(d) Velocity at the highest point $= \vec{u}_f = u \cos \theta \ \hat{i}$

Change in momentum at highest point

$m(\vec{u}_f - \vec{u}_i) = m[u \cos \theta \hat{i} - (u \cos \theta \hat{i} + u \sin \theta \hat{j})] = -mu \sin \theta \hat{j}$

Explanatory Notes on Projectile Motion General Idea

Special Case of Two-Dimensional Motion

A particle moves in a vertical plane with some initial velocity $\vec{v}_0$ but its acceleration is always the freefall acceleration $\vec{g}$, which is downward. Such a particle is called a projectile (meaning that it is projected or launched) and its motion is called **projectile motion**.

Assumptions:

- Particle remains close to earth's surface, so acceleration due to gravity remains constant.
- Air resistance is neglected.
- Distance that projectile travels is small so that earth can be treated as plane surface.

Two Straight Line Motions

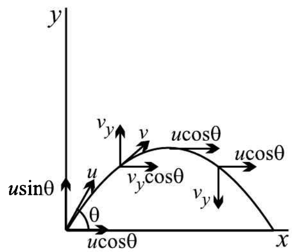
Our goal here is to analyse projectile motion using the tools for two dimensional motion. This feature allows us to break up a problem involving two dimensional motion into two separate and easier one-dimensional problems,

(a) The horizontal motion is motion with uniform velocity (no effect of gravity)

(b) The vertical motion is motion of uniform acceleration, or freely falling bodies.

Note

In projectile motion, the horizontal motion and the vertical motion are independent of each other, that is either motion does not affects the other.



Treating as Two Straight Line Motions

The horizontal Motion(x axis) : Because there is no acceleration in the horizontal direction, the horizontal component v_x of the projectile's velocity remains unchanged from its initial value v_{0x} throughout the motion,

The vertical motion(y axis) : The vertical motion is the motion we discussed for a particle in free fall.

As is illustrated in figure and equation, the vertical component behaves just as for a ball thrown vertically upward. It is directed upward initially and its magnitude steadily decreasing to zero, which marks the maximum height of the path. The vertical velocity component then reverses direction, and its magnitude becomes larger with time.

x-axis	y-axis
Initial velocity(u_x) = $u\cos\theta$	Initial velocity(u_y) = $u\sin\theta$
acceleration(a_x) = 0	acceleration(a_y) = $-g$
Thus, velocity after time t	Thus, velocity after time t
$v_x = u\cos\theta$	$v_y = u\sin\theta - gt$
Displacement after time t	Displacement after time t
$x = u\cos\theta t$	$y = u\sin\theta t - gt^2/2$

Resultant velocity

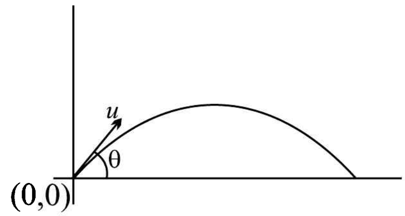
$$(\vec{V}_R) = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$$

$$|\vec{V}_R| = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2} \quad \& \quad \tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta}$$

where α is angle that velocity vector makes with horizontal. Also known as direction or angle of motion

Vectorial Treatment (Optional)

Lets say a particle is projected at an angle θ from horizontal with a velocity. Now if we take point of projection as origin and take vertically upward as positive y-axis and direction of projection horizontally as x-axis.



$$\vec{u} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$$

$$\vec{a} = -g\hat{j}$$

Now since acceleration is uniform

Velocity after time t : $\vec{V} = \vec{u} + \vec{a}t \quad \Rightarrow \quad \vec{V} = u\cos\theta\hat{i} + u\sin\theta\hat{j} + (-g\hat{j})t$

$$\Rightarrow \quad \vec{v} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{i}$$

Displacement after time t : $\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \quad \Rightarrow \quad \vec{s} = \vec{u}(\cos\theta\hat{i})t + (u\sin\theta\hat{j})t + \frac{1}{2}(-g\hat{j})t^2$

$$\Rightarrow \quad \vec{S} = u(u\cos\theta\hat{i})t + (u\sin\theta t - \frac{1}{2}gt^2\hat{j})$$

LEVEL GROUND PROJECTION

Lets say we project a particle with velocity u at an angle θ from horizontal. Now let us derive these quantitites:

Time of Flight (T)

$$T = \frac{2u\sin\theta}{g}$$

Considering vertical motion $s_y = 0$; $u_y = \text{v}\sin\theta$; $a_y = -\text{g}$

$$0 = u\sin\theta T - gT^2/2 \quad \Rightarrow \quad T = \frac{2u\sin\theta}{g}$$

Maximum Height (H)

$$H = \frac{u^2\sin^2\theta}{2g}$$

Vertical velocity at maximum height $v_y = 0$

$$0 = u^2\sin^2\theta - 2gH \quad \Rightarrow \quad H = \frac{u^2\sin^2\theta}{2g}$$

Horizontal Range (R)

$$R = \frac{u^2\sin 2\theta}{2g} = \frac{2u_x u_y}{g}$$

Total time $T = \dfrac{2u\sin\theta}{g}$

Velocity in horizontal direction $u_x = u\cos\theta$

Total displacement in horizontal direction $R = u\cos\theta T$

$$R = \frac{u^2 \sin 2\theta}{2g}$$

Example 1. A body is thrown with initial velocity 10m/sec. at an angle 37° from horizontal. Find

(i) Time of flight (ii) Maximum height

(iii) Range (iv) Position vector after $t = 1$ sec.

Solution (i) 1.2 sec (ii) 1.8 m (iii) 8.6 m (iv) $(16\hat{i} - 8\hat{j})$]

Caution: This equation does not given the horizontal distance travelled by a projectile when the final height is not the launch height.

Maximum Range

$$R = \frac{u^2 \sin 2\theta}{g} \quad \text{for } \theta = 45°,$$

$$R \text{ is maximum} R_{max} = \frac{u^2}{g}$$

Note

For complementry angles i.e. $\theta + \alpha = 90°$, the range is same for same projection speed but maximum height and time of flight are different.

Example 2. A person can throw a ball vertically upto maximum height of 20 m. How far can he throw the ball.

Solution $H = \frac{u^2}{2g}$ $\therefore\ u = 20$ m/s $R_{max} = \frac{u^2}{g} = 40$ m

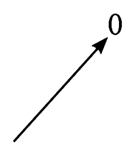
Objective : Max height does not always stand for $\left(\frac{u^2 \sin^2\theta}{2g}\right)$

Example 3. A particle is projected with a speed u at an angle θ with horizontal. Find the average velocity of projectile for the period during which it crosses half of maximum height.

Solution ucos θ along horizontal

avg. velocity is a vector

First we will find vertical component



$$\vec{V}_y = \frac{\text{Total Displacement}}{\text{Total time}} = 0$$

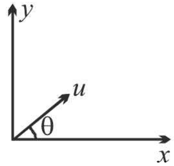
Horizontal $V_{1x} = V_{2x} = u\cos\theta$

$$\bar{V}_x = u\cos\theta$$

Concept: Horizontal component of velocity of projectile remains uncharged.

Equation of Trajectory

Lets say point of projection is our origin and horizontal direction is x-axis and vertically upwards is positive y-axis.



positive y-axis.

We know $x = u \cos\theta t$

$\therefore$ $t = \dfrac{x}{u\cos\theta}$

...(1)

also $y = u \sin\theta t - \dfrac{1}{2} gt^2$ .....(2)

Putting value of 't' from eq. (1) in eq. (2), we get

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

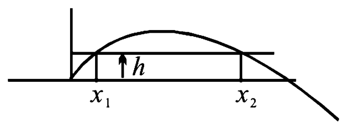
Important Points

- This is an equation of a parabola, this implies that particle moves on parabolic path.
- This equation can be considered to be quadratic in x as

$$\left(\frac{g}{2u^2\cos^2\theta}\right)x^2 - (\tan\theta)\,x + y = 0$$

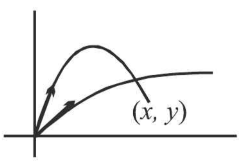
Thus, for every value of y we can have maximum two values of x. This means simply that projectile can pass through same height for two different x coordinates.

- This equation can be considered to be quadratic in $\tan\theta$ as



$$\left(\frac{gx^2}{2u^2}\right)\tan^2\theta - x\tan\theta + \left(y + \frac{gx^2}{2u^2}\right) = 0$$

Thus, for every value of (x, y) we can have two angles of projection. This means we can hit a given target by using two different angles of projection.



- The equation of trajectory can also be modified to give following result.

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Taking tan θ common and doing some manipulation we can write equation as

$$y = x \tan\theta \left[1 - \frac{x}{R}\right]$$

Example 4. A body is projected with a speed of 30 ms^{-1} at an angle of 30ţ with the vertical. Find the maximum height, time of flight and the horizontal range of the motion. **[Take $g = 10$ m/s^2]**

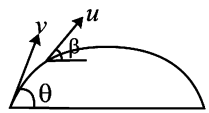
Solution Here $u = 30$ ms^{-1}, Angle of projection, θ = 90 – 30 = 60ţ

Maximum height, $H = \dfrac{u^2 \sin^2\theta}{2g} = \dfrac{30^2 \sin^2 60^\circ}{10} = \dfrac{900}{20} \times \dfrac{3}{4} = \dfrac{135}{4}$ m

Time of flight, $T = \dfrac{2u\sin\theta}{g} = \dfrac{2 \times 30 \times \sin 60^0}{10} = 3\sqrt{3}$ sec.

Example 5. A stone is thrown with a velocity v at angle θ with horizontal. Find its speed when it makes an angle β with the horizontal.

Solution Since horizontal component of velocity remains constant. Therefore,



$v \cos\theta = u \cos\beta$ $\qquad u = \dfrac{v\cos\theta}{\cos\beta}$

Example 6. A projectile is thrown in the upward direction making an angle of 60^0 with the horizontal with a speed of 147 m/s. Find the time after which its inclination with the horizontal is 45$^{0?}$

Solution $u_x = 147 \times \cos 60^0 = \dfrac{147}{2}$ $\qquad u_y = 147 \times \sin 60^0 = \dfrac{147\sqrt{3}}{2}$

$v_y = u_y + a_y t = \dfrac{147\sqrt{3}}{2} - gt$ $\qquad v_x = u_x = \dfrac{147}{2}$

When angle is 45^0, $\quad \tan 45^0 = \dfrac{v_y}{v_x}$

$\Rightarrow \quad v_y = v_x \qquad \Rightarrow \quad \dfrac{147\sqrt{3}}{2} - gt = \dfrac{147}{2}$

$\Rightarrow \quad \dfrac{147}{2}(\sqrt{3} - 1) = gt \qquad \Rightarrow \quad t = \dfrac{147}{2 \times 10}(\sqrt{3} - 1)$ s

Example 7. A large number of bullets are fired in all directions with the same speed v. What is the maximum area on the ground on which these bullets will spread?

Solution Maximum distance upto which a bullet can be fired is its maximum range, therefore

$R_{max} = \dfrac{v^2}{g}$ Maximum area $= \pi(R_{max})^2 = \dfrac{\pi v^4}{g^2}$.

Example 8. The velocity of projection of a projectile is given by : $\vec{u} = 5\hat{i} + 10\hat{j}$. Find

(a) Time of flight, $\qquad$ (b) Maximum height,

(c) Range

Solution We have $u_x = 5$ $u_y = 10$

(a) Time of flight $= \dfrac{2u\sin\theta}{g} = \dfrac{2u_y}{g} = \dfrac{2\times 10}{10} = 2$ s

(b) Maximum height $= \dfrac{u^2\sin^2\theta}{2g} = \dfrac{u_y^{\,2}}{2g} = \dfrac{10\times 10}{2\times 10} = 5$ m

(c) Range $= \dfrac{2u\sin\theta . u\cos\theta}{g} = \dfrac{2\times 10\times 5}{10} = 10$ m

Example 9. There are two angles of projection for which the horizontal range is the same. Show that the sum of the maximum heights for these two angles is independent of the angle of projection.

Solution There are two angles of projection α and 90ţ $- \alpha$ for which the horizontal range R is same.

Now, $H_1 = \dfrac{u^2\sin^2\alpha}{2g}$ and $H_2 = \dfrac{u^2\sin^2(90^\circ - \alpha)}{2g} = \dfrac{u^2\cos^2\alpha}{2g}$

Therefore, $H_1 + H_2 = \dfrac{u^2}{2g}(\sin^2\alpha + \cos^2\alpha) = \dfrac{u^2}{2g}$

Clearly the sum of the heights for the two angles of projection is independent of the angles of projection.

Example 10. A particle is projected with initial velocity u, making an angle 'θ' with the horizontal. There are two values of time for which projectile is at the same height. Find the sum of these two time interval.

Solution For vertically upward motion of a projectile,

$$y = (u\sin\alpha)t - \frac{1}{2}gt^2 \quad \text{or} \quad \frac{1}{2}gt^2 - (u\sin\alpha)t + y = 0$$

This is a quadratic equation in t. Its roots are

$$t_1 = \frac{u\sin\alpha - \sqrt{u^2\sin^2\alpha - 2gy}}{g} \quad \text{and} \quad t_2 = \frac{u\sin\alpha + \sqrt{u^2\sin^2\alpha - 2gy}}{g}$$

$$\therefore \quad t_1 + t_2 = \frac{2u\sin\alpha}{g} = T \text{ (time of flight of the projectile)}$$

Example 11. A particle moves in the x-y plane with velocity $v_x = 8t - 2$ and $v_y = 2$. If it passes through the point $x = 14$ and $y = 4$ at $t = 2$s. Find the equation (x-y relation) of the path.

Solution $v_x = 8t - 2$ or $\dfrac{dx}{dt} = 8t - 2$

or $\int_{14}^{x} dx = \int_{2}^{t} (8t - 2)dt$

or $x - 14 = [4t^2 - 2t]_2^t = 4t^2 - 2t - 12$

or $x = 4t^2 - 2t + 2$ ...(i)

Further, $v_y = 2$

or $\dfrac{dy}{dt} = 2$ $\therefore \int_4^y dy = \int_2^t 2dt$

or $y - 4 = [2t]_2^t = 2t - 4$ or $y = 2t$

or $t = \dfrac{y}{2}$ ...(ii)

Substituting value of 't' from Equation (ii) in Equation (i) we have,

$$x = y^2 - y + 2$$

This is the desired trajectory (x-y relation)

Example 12. A batter hits a baseball so that it leaves the bat with an initial speed $v_0 = 37.0$ m/s at an initial angle $\alpha_0 = 53ţ$, at a location where $g = 10.0$ m/s^2

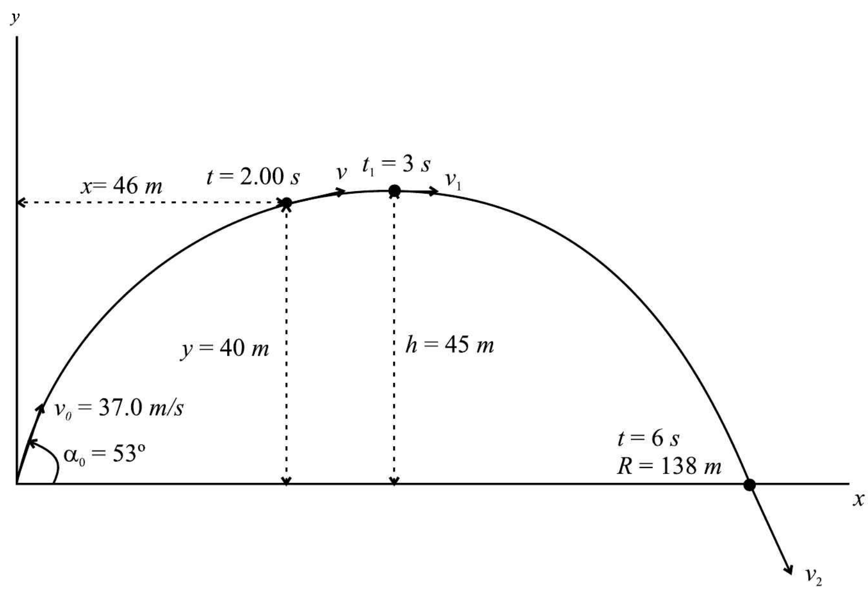
(a) Find the position of the ball, and the magnitude and direction of its velocity, when $t = 2.0$ s.

(b) Find the time when the ball, reaches the highest point of flight and the find its height h at this point

(c) Find the horizontal range R - that is, the horizontal distance from the starting point to the point at which the ball hits the ground. For each part, treat the baseball as a projectile

Solution The initial velocity of the ball has components

$$v_{0x} = v_0 \cos \alpha_0 = (37.0 \text{ m/s}) \cos 53ţ = 22.3 \text{ m/s}$$
$$v_{0y} = v_0 \sin \alpha_0 = (37.0 \text{ m/s}) \sin 53ţ = 29.5 \text{ m/s}$$



(a) $x = v_{0x} t = (22.3 \text{ m/s})(2.00 \text{ s}) = 44.6$ m

$$y = v_{0y} t - \frac{1}{2} gt^2 = (29.5 \text{ m/s})(2 \text{ s}) - \frac{1}{2}(10 \text{ m/s}^2)(2 \text{ s})^2 = 59.0 - 20 = 39.0 \text{ m}$$

$$v_x = v_{0x} = 22.3 \text{ m/s}$$

$v_y = v_{0y} - gt = 29.5$ m/s – (10 m/s²) (2.00 s) = 9.5 m/s

The y-component of velocity is positive, which means that the ball is still moving upward at this time (Figure). The magnitude and direction of the velocity are

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(22.3 m/s)^2 + (9.5 m/s)^2} = 24.2 \text{ m/s}$$

$$\alpha = \tan^{-1}\left(\frac{10.0 m/s}{22.3 m/s}\right) = \tan^{-1} 0.4$$

(b) At the highest point, the vertical velocity v_y is zero at time t_1 ; then

$$v_y = 0 = v_{0y} - gt_1$$

$$t_1 = \frac{v_{0y}}{g} = \frac{29.5 \text{ m/s}}{10 \text{ m/s}^2} = 3.0 \text{ s}$$

The height h at this time is the value of y when $t = t_1 = 3$ s;

$$h = v_{0y}\, t_1 - \frac{1}{2} g t_1^{\,2} = (29.5 \text{ m/s})(3.0 \text{ s}) - \frac{1}{2}(10.0 \text{ m/s}^2)(3.0 \text{ s})^2$$

$$= 88.5 - 45 = 43.5 \text{ m}$$

(c) $y = 0 = v_{0y}\, t_2 = \frac{1}{2} g t_2^{\,2} = t_2\left(v_{0y} - \frac{1}{2} g t_2\right)$

This is a quadratic equation for t_2. It has two roots,

$$t_2 = 0 \qquad \text{and} \qquad t_2 = \frac{2v_{0y}}{g} = \frac{2(29.5\, m/s)}{10\, m/s^2} = 5.9 \text{ s}$$

There are two times at which $y = 0$; $t_2 = 0$ is the time the ball leaves the ground, and $t_2 = 6$ s is the time of its return. This is exactly twice the time to reach the highest point, so the time of descent equals the time of ascent. (This is always true if the starting and end points are at the same elevation and air resistance can be neglected).

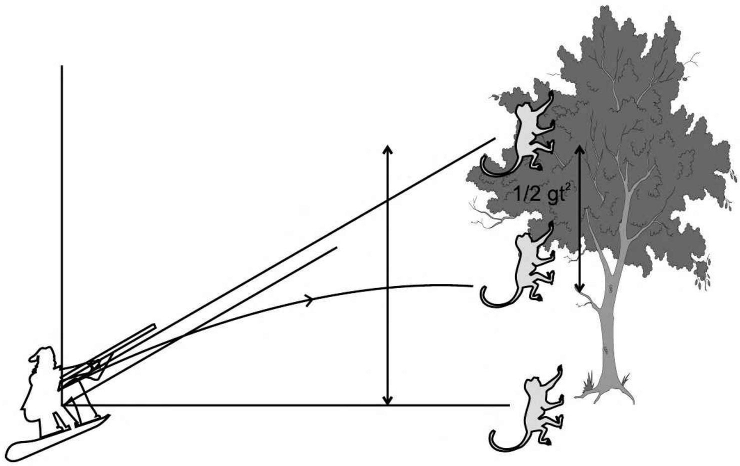
The horizontal range R is the value of x when the ball returns to the ground, that is, at $t = 6.64$ s;R = $v_{0x} t_2$ = (22.3 m/s) (5.9 s) = 131.6 m

The vertical component of velocity when the ball hits the ground is

$$v_y = v_{0y} - gt_2 = 29.5 \text{ m/s} - (10 \text{ m/s}^2)(5.9 \text{ s}) = -29.5 \text{ m}$$

That is, v_y has the same magnitude as the initial vertical velocity v_{0y} but the opposite direction (down). Since v_x is constant, the angle $\alpha = -53ȿ$)(below the horizontal) at the point is the negative of the initial angle $\alpha_0 = 53ȿ$.

***Example* 13.** A clever monkey escapes from the zoo. The zoo keeper finds him in a tree. After failing to entice the monkey, the keeper shoots. The clever monkey lets go at the same instant the dart leaves the gun barrel, intending to land on the ground and escape. Show that the dart always hits the monkey, regardless of the dart's muzzle velocity (provided that it gets to the monkey before he hits the ground).



Solution The monkey drops straight down, so x_{monkey} = d at all times.

For the dart, $x_{dart} = (v_0 \cos a_0)t$. When these x-coordinates are equal, $d = (v_0 \cos \alpha_0)t$,

or $$t = \frac{d}{v_0 \cos\alpha_0}$$

To have the dart hit the monkey, it must be true that $y_{monkey} = y_{dart}$ at this same time. The monkey is in one-dimensional free fall $y_{monkey} = d \tan \alpha_0 - \frac{1}{2} gt^2$.

For the dart $$y_{dart} = (v_0 \sin \alpha_0)\, t - \frac{1}{2}\, gt^2$$

So if $d \tan \alpha_0 = (v_0 \sin \alpha_0)$ t at the time when the two x-coordinates are equal, then $y_{monkey} = y_{dart}$ and we have a hit.

Example 14. A ball is thrown from ground level so as to just clear a wall 4 m high at a distance of 4 m and falls at a distance of 14 m from the wall. Find the magnitude and direction of the ball.

Solution The ball passes through the point P(4, 4). So its range = 4 + 14 = 18 m.

The trajectory of the the ball is,

$$y = x \tan \theta \left(1 - \frac{x}{R}\right)$$

Now x = 4m, y = 4m and R = 18 m

$$\therefore \quad 4 = 4 \tan \theta \left[1 - \frac{4}{18}\right] = 4 \tan \theta \cdot \frac{7}{9}$$

or $$\tan\theta = \frac{9}{7} \quad \Rightarrow \quad \theta = \tan^{-1}\frac{9}{7}$$

And $$R = \frac{2u^2 \sin\theta\cos\theta}{g}$$

or $$18 = \frac{2}{9.8} \times u^2 \times \frac{9}{\sqrt{130}} \times \frac{7}{\sqrt{130}}$$

or $$u^2 = \frac{18 \times 9.8 \times 130}{2 \times 9 \times 7} = 182$$

or $$u = \sqrt{182} \text{ and } \theta = \tan^{-1}\frac{9}{7}$$

Example 15. Two projectiles are thrown with different speeds and at different angles so as the cover the same maximum height. Find out the sum of the times taken by each to the reach to highest point, if time of flight is T.

Solution Total time taken by either of the projectile.

Example 16. A particle is projected with speed 10 m/s at an angle 60^0 with horizontal. Find :

(a) Time of flight
(b) Range
(c) Maximum height
(d) Velocity of particle after one second.
(e) Velocity when height of the particle is 1 m

Solution

(a) $\sqrt{3}$ sec.
(b) $5\sqrt{3}$ m
(c) $\frac{15}{4}$ m
(d) 5.17 m/s
(e) $\vec{v} = 5\hat{i} \pm \sqrt{55}\,\hat{j}$

Example 17. A particle is projected up from the ground with a velocity of 147 m/s at an angle of projection 30° with the horizontal. Find

(a) The time of flight,
(b) The greatest height reached,
(c) The horizontal range and
(d) The velocity at a height of 98 m.

Solution

(a) The time of flight $T = \frac{2u\sin\theta}{g} = \frac{2 \times 147 \times \sin 30°}{9.8} = 15\,\text{s}$

(b) The greatest height reached $H = \frac{u^2\sin^2\theta}{2g} = \frac{(147/2)^2}{2 \times 9.8} = 275.6$ m

(c) The horizontal range $R = \frac{u^2\sin 2\theta}{g} = \frac{147^2 \times \sin 60°}{9.8} = 1909.6$ m

(d) The horizontal velocity at a height of 98 m = $u\cos\theta$

i.e., $$v_x = 147\cos 30° = 147 \times \frac{\sqrt{3}}{2} = \frac{147\sqrt{3}}{2} = 127.3 \text{ m/s}$$

The vertical velocity at this height

$$v_y^2 = (u\sin\theta)^2 - 2 \times 9.8 \times 98 = (147\sin 30°) - 19.6 \times 98 = 3481.45$$

$$v_y = 59 \text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = 140.3 \text{ m/s}$$

in a direction *a* with horizontal where $\alpha = \tan^{-1}(0.4635) = 24.86°$

The velocity at 98 m height has two values (i) while going up and (ii) while coming down.

Both have the same magnitude except for the difference in directions.

Example 18. Find the maximum horizontal range when the velocity of projection is 30 m/s. Find the two directions of projection to give a range of 45 m. Take $g = 10$ m/s^2.

Solution

(i) Maximum range $R_m = \dfrac{u^2}{g} = \dfrac{30^2}{10} = 90$ m

(ii) Now $\dfrac{u^2 \sin 2\theta}{g} = 45$ or, $\sin 2\theta = \dfrac{45 \times 10}{30 \times 30} = \dfrac{1}{2}$

or, $2\theta = 30°$ or $150°$ $[\because (180 - \theta) = \sin\theta]$

or $\theta = 15°$ or $75°$

Therefore for a given velocity of projection and for a given range, two directions of projection are possible.

Example 19. What is the least velocity with which a cricket ball can be thrown through a distance of 100 m?

Solution Since the range is given, the least velocity of projection is that value when the angle of projection is 45°. For velocity *u* to be least

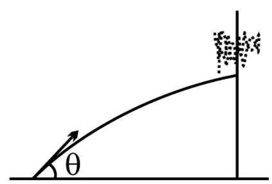
$$\frac{u^2 \sin 2\theta}{g} = 100 \text{ where } \theta = 45° \quad \text{or,} \quad \frac{u^2}{g} = 100$$

$$u^2 = 100 \times 9.8 = 980 \qquad u = 31.3 \text{ m/s}$$

Example 20. We have a hose pipe which disposes water at the speed of 10 ms^{-1}. The safe distance from a building on fire, on ground is 5 m. How high can this water go? (take g = 10 ms^{-2})

Solution Here we must understand that taking range of projectile as 10m and making projectile hit the building when it is at maximum height is wrong. By doing this we are not acheiving maximum *y* for given *x* = 5m. This just makes highest pt. of path to like on *x* = 5, But there may be other path for which *y* will be maximum for given *x*. This problem will be solved by using equation of trajectory by putting *x* = 5m and maximising *y* by varying θ.

$$y = x \tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$



Putting we get *x* = 5m

$$y = 5\tan\theta - \frac{10 \times 25 \sec^2\theta}{2 \times 100}$$

$$5\tan^2\theta - 20\tan\theta + (4y + 5) = 0$$

for real roots discriminant must be positive.

$$400 - 4 \times 5\,(4y + 5) > 0$$

Solving $3.75 \geq y$

hence maximum $y = 3.75$ m

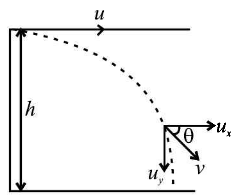
If we have taken range as 10 m then angle of projection will be θ = 45ţ corsponding maximum hight $H = 2.5$m which is smaller than our answer.

Concept

Find the maximum height by using wrong method it comes out to be 2.5.

Projectile Thrown Parallel to the Horizontal

Consider a projectile thrown from point O at some height h from the ground with a velocity u. Now we shall deal the characteristics of projectile motion with the help of horizontal and vertical direction motions.



	Horizontal direction	Vertical direction
(i)	Initial velocity $u_x = u$	Initial velocity $u_y = 0$
(ii)	Acceleration $a_x = 0$	Acceleration $a_y = g$ (downward)

Trajectory Equation

The path traced by projectile is called the trajectory.

After time t, $\quad x = ut$ ...(1)

$$y = \frac{-1}{2} gt^2 \quad ...(2)$$

From equation (1) $t = x/u$

Put value of t in equation (2)

$$\boxed{y = -\frac{1}{2} g . \frac{x^2}{u^2}}$$

This is **trajectory equation of the projectile.**

Velocity at a General Point P(x, y)

$$v = \sqrt{u_x^2 + u_y^2}$$

Here horizontal velocity of the projectile after time t

$$v_x = u$$

velocity of projectile in vertical direction after time t

$v_y = 0 + (-g)t = -gt = gt$ (downward)

$\therefore \quad v = \sqrt{u^2 + g^2t^2}$ and $\tan\theta = v_y/v_x$

Displacement

The displacement of the particle is expressed by

$$S = x\hat{i} + y\hat{j} = (ut)\hat{i} + (\frac{1}{2}gt^2)\hat{j} \qquad \text{where} \qquad |S| = \sqrt{x^2 + y^2}$$

Time of Flight

This is equal to the time taken by the projectile to return to ground. From equation of motion

$$S = ut + \frac{1}{2}at^2$$

Therefore for vertical direction– $h = v_y t + \frac{1}{2}(-g)t^2$

At highest point $\quad v_y = 0 \quad \Rightarrow \quad h = \frac{1}{2}gt^2$

$$t = \pm\sqrt{\frac{2h}{g}} \quad \Rightarrow \quad t = \sqrt{\frac{2h}{g}}$$

Horizontal Range

Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground.

$$R = u_x \, . \, t$$

$$\boxed{R = u\sqrt{\frac{2h}{g}}}$$

Velocity at Vertical Depth h

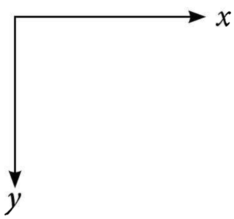
Along vertical direction $v_y^2 = 0^2 + 2 \, . \, (-h)\,(-g)$

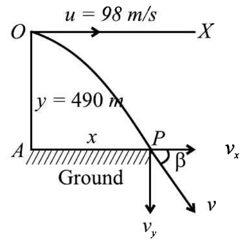
$$\boxed{v_y = \sqrt{2gh}}$$

Example 21. A projectile is fired horizontally with a speed of 98 ms^{-1} from the top of a hill 490 m high. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the velocity with which the projectile hits the ground. (take $g = 9.8 \, m/s^2$)

Solution

(i) The projectile is fired from the top O of a hill with speed $u = 98 \, ms^{-1}$ along the horizontal as shown as OX. It reaches the target P at vertical depth OA, in the coordinate system as shown, $OA = y = 490$m





As, $y = \frac{1}{2}gt^2$ $\quad \therefore \quad 490 = \frac{1}{2} \times 9.8\, t^2$

or $t = \sqrt{100} = 10$ s.

(ii) Distance of the target from the hill is given by,

AP = x = Horizontal velocity × time = 98 × 10 = 980 m.

(iii) The horizontal and vertical components of velocity v of the projectile at point P are

$$v_x = u = 98\ \text{ms}^{-1}$$

$$v_y = u_y + gt = 0 + 9.8 \times 10 = 98\ \text{ms}^{-1}$$

$$V = \sqrt{v_x^2 + v_y^2} = \sqrt{98^2 + 98^2} = 98\sqrt{2} = 139\ \text{ms}^{-1}$$

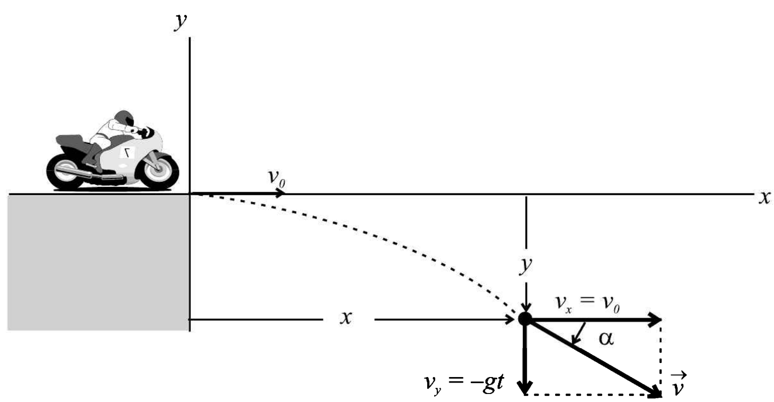
Now if the resultant velocity v makes an angle β with the horizontal, then

$$\tan\beta = \frac{v_y}{v_u} = \frac{98}{98} = 1$$

$\therefore$ β = 45ŗ

Example 22. A motorcycle stunt rider rides off the edge of a cliff. Just at the edge his velocity is horizontal, with magnitude 9.0 m/s. Find the motorcycle's position, distance from the edge of the cliff and velocity after 0.5 s.

Solution



At $t = 0.50$ s, the x and y-coordinates are

$$x = v_{0x}t = (9.0\ \text{m/s})\ (0.50\ \text{s}) = 4.5\ \text{m}$$

$$y = -\frac{1}{2}gt^2 = -\frac{1}{2}(10\text{ m/s}^2)(0.50\text{ s})^2 = -1.2\text{ m}$$

The negative value of y shows that this time the motorcycle is below its starting point.

The motorcycle's distance from the origin at this time

$$r = \sqrt{x^2 + y^2} = \sqrt{(4.5m)^2 + (-1.2m)^2} = \sqrt{\left(\frac{45}{10}m\right)^2 + \left(-\frac{12}{10}m\right)^2}$$

$$= \frac{3}{10}\sqrt{(15)^2 + (4)^2} \simeq 5\text{ sec.}$$

The components of velocity at this time are

$$v_x = v_{0x} = 9.0\text{ m/s}$$

$$v_y = -gt = (-10\text{ m/s}^2)(0.50\text{ s}) = -5\text{ m/s.}$$

The speed (magnitude of the velocity) at this time is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9.0m/s)^2 + (-5m/s)^2} = 10.2\text{ m/s}$$

The angle α of the velocity vector is

$$= \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(\frac{-5\ m/s}{9.0\ m/s}\right)$$

Example 23. Two tall buildings face each other and are at a distance of 180 m from each other. With what velocity must a ball be thrown horizontally from a window 55 m above the ground in one building, so that it enters a window 10.9 m above the ground in the second building.

Solution [**Ans.** 60 m/s]

Example 24. Two paper screens A and B are separated by a distance of 100 m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. Neglect the resistance of paper and air.

Solution [**Ans.** 700 m/s]

RELATIVE MOTION BETWEEN TWO PROJECTILES

Let us now discuss the relative motion between two projectiles or the path observed by one projectile of the other. Suppose that two particles are projected from the ground with speed u_1 and u_2 at angles α_1 and α_2 as shown in figure 2.4.

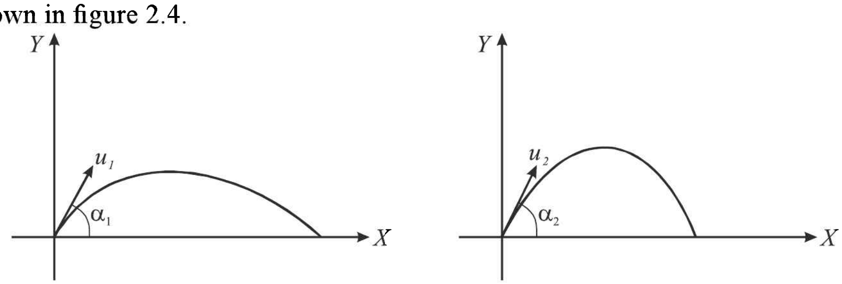


Fig 2.4

Acceleration of both the particles is g downwards. So, relative acceleration between them is zero because.

$$a_{12} = a_1 - a_2 = g - g = \text{zero}$$

i.e., the relative motion between the two particles is uniform.

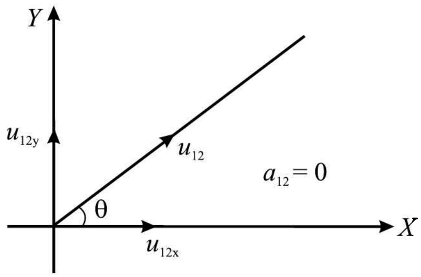
Now, $u_{1x} = u_1\cos\alpha_1$, $u_{2x} = u_2\cos\alpha_2$

$u_{1y} = u_1\sin\alpha_1$ and $u_{2t} = u_2\sin\alpha_2$

Therefore, $u_{12x} = u_{1x} - u_{2x} = u_1\cos\alpha_1 - u_2\cos\alpha_2$

and $u_{12y} = u_{1y} - u_{2y} = u_1\sin\alpha_1 - u_2\sin\alpha_2$

u_{12x} and u_{12y} are the x and y components of relative velocity of 1 with respect to 2.



Hence, relative motion of 1 with respect to 2 is a straight line at an angle

$$\theta = \tan^{-1}\left(\frac{u_{12y}}{u_{12x}}\right) \text{ with positive x-axis.}$$

Now, if $u_{12x} = 0$ or $u_1\cos\alpha_1 - u_2\cos\alpha_2$, the relative motion is along y-axis or in vertical direction (as θ = 90ţ). Similarly, if $u_{12y} = 0$ or $u_1\sin\alpha_1 - u_2\sin\alpha_2$, the relative motion is along x-axis or in horizontal direction (as θ = 0ţ).

Projectile from a Tower

Case (i) : Horizontal projection

$u_x = u$; $u_y = 0$; $a_y = -g$

Case (ii) : Projection at an angle θ above horizontal

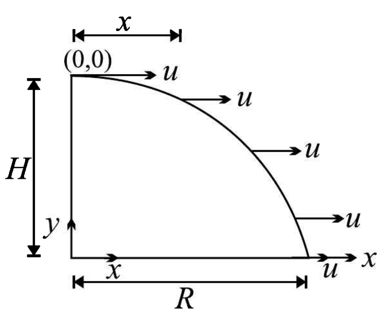
$u_x = u\cos\theta$; $u_y = u\sin\theta$; $a_y = -g$

Case (iii) : Projection at an angle θ below horizontal

$u_x = u\cos\theta$; $u_y = -u\sin\theta$; $a_y = -g$

In all the above three cases, we can calculate the velocity of projectile at the instant of striking the ground by using $v = \sqrt{v_x^2 + v_y^2}$ and $\tan\phi = \frac{v_y}{v_x}$, where ϕ is the angle at which the projectile strikes the ground.

Horizontal Projection (Projection From Height)



(Take it as an example rather than a topic)

Lets say that a particle is projected from some height H in horizontal direction with velocity u. We consider point of projection as origin (0, 0). Considering horizontal direction as + ve x-axis and vertically upwards as +ve y-axis.

x-axis	**y-axis**
$u_x = u$	$u_y = 0$
$a_x = 0$	$a_y = 0$
vel. after time t	vel. after time t
$v_x = u$	$v_y = -gt$
Disp. after time t	Displacement after time t
$x = ut$	$y = \frac{-gt^2}{2}$

Velocity

$$\vec{v} = u\hat{i} - gt\hat{j} \qquad |\vec{v}| = \sqrt{u^2 + g^2t^2}$$

Displacement

$$\vec{s} = u + \hat{i} - \frac{gt^2}{2}\hat{j} \qquad \vec{s} = \sqrt{u^2t^2 + \left(\frac{1}{2}gt^2\right)^2}$$

Equation of Trajectory

$$t = \frac{x}{u} \qquad \therefore \quad y = -\frac{gx^2}{2u^2}$$

Time of Flight (T)

$$s_y = -H \qquad a_y = -2 \qquad u_y = 0$$

$$-x = -\frac{1}{2}gT^2 \qquad \Rightarrow \quad T = \sqrt{\frac{2H}{g}}$$

Range (R)

$$R = u_xT = uT = u\sqrt{\frac{2H}{g}}$$

Note

From some height if we drop a particle and simultaneously project another particle horizontaly then:

(a) Both will reach surface together.

(b) Vertical velocity will be same.

(c) Particle projected horizontally will have larger total velocity compared to particle projected horizontally at all times.

Example 25. A projectile is fired horizontaly with a velocity of 98 m/s from the top of a hill 490 m high. Find

(i) Time to reach ground

(ii) The horizontal distance from foot of hill to ground.

(iii) The velocity with which it hills the ground.

Solution [**Ans.** (i) 10 sec, (ii) 980 m, (iii) 98√2 m/sec]

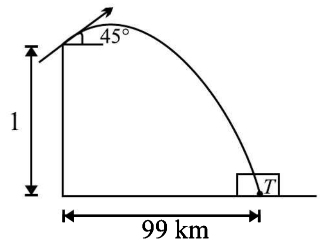
***Example* 26.** A ball is thrown from the top of a tower with an initial velocity of 10m/s at an angle 37° above the horizontal, hits the ground at a distance 16m from the base of tower. Calculate height of tower. [g = 10 m/s^2]

Solution **[Ans.** 8m.]

***Example* 27.** Prithvi missile is fired to destroy an enemy military base situated on same horizontal level, situated 99 km away. The missile rises vertically for 1 km and then for remainder of flight, it followes parbolic path like a free body under earth's gravity, at an angle of 45°. Calculate its velocity at beginning of parabolic path. (g= 10 ms^{-2})

Solution For horizontal motion time t

$$t = \frac{99\times10^3}{u\cos 45°}$$



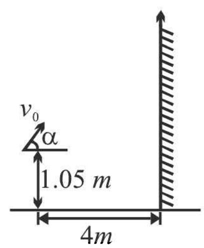
for vertical $-1 \times 10^3 = u \sin 45°\, t - \frac{1}{2}\times 10 \times t^2$

$$1 \times 10^3 + \frac{u\sin 45°}{u\sin 45°} \times 99 \times 10^3 = \frac{10}{2} \times \frac{(99\times10^3)^2\times 2}{u^2}$$

$$u^2 = \frac{(99\times10^3)^2\times10}{100\times10^3}\; u = 99 \times 10^3 \sqrt{\frac{1}{10^4}} = 990 \text{ ms}^{-1}$$

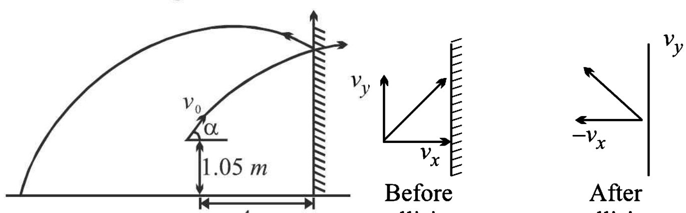
If we project some body from a height, at an oblique angle, then we can solve the problem simply like level ground projectile just allowing for negative value of y cordinate. Since the particle will go below the level of projection.

***Example* 28.** A boy stands 4 m away from a vertical wall and throws a ball leaves the boy's hand at h = 1.05 m above the ground with initial velocity $v_0 = 10\sqrt{2}$ ms^{-1} at an angle of 45ŗ from the horizontal. After striking the wall elastically the ball rebounds. Where does the ball hit the ground. (g = 10 ms^{-1})



Solution Elatic Collision with the wall will reverse direction of its horizontal component without effecting vertical component. (Explain it to students)

Hence time of flight will be.



$$u_y = 10\sqrt{2} \text{ sin 45ŗ} = 10 \text{ m/s}; \; a_y = -10 \text{ m/s}^2$$

$$s = -1.05 \text{ m} - 1.05 = 10\, t - 5\, t^2$$

$$5\, t^2 - 10\, t\; 1.05 = 0$$

Solving we get time of flight = 2.1 sec.

Consdering horizontal of ball before colliding with wall.

Time before collision = $\frac{4}{u_x} = 0.4$

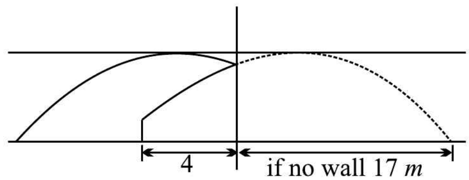
Thus remaining time to return is 1.7 sec.

Hence horizontal distance travelled in this time is = 1.7 × 10 = 17 m

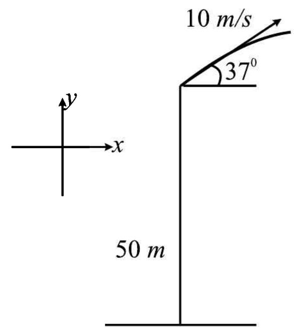
Note

Change in Y-does not affect X. They can be treated individually.

If there had been no wall the ball would have gone 21 m away from the feet of the boy. The only effect of wall is to reverse the direction of horizontal velocity and 'T' maximum height will remain same in both cases.



Example 29. From the top of a 50m high tower a stone is projected with speed 10 m/s, at an angle of 37ţ as shown in figure. Find out (a) velocity after 3s (b) time of flight. (c) horizontal range. d) the maximum (height attained by the particle.



Solution

(a) Initial velocity in horizontal direction = 10 cos 37 = 8 m/s

Initial velocity in vertical direction = 10 sin 37ţ = 6 m/s

Velocity after 3 seconds

$v = v_x\hat{i} + v_y\hat{j} = 8\hat{i} + (u_y + a_y t)\hat{j} = 8\hat{i} - 24\hat{j}$

(b) $S_y = u_y t + \frac{1}{2} a_y t^2 \quad \Rightarrow \quad -50 = 6 \times t + \frac{1}{2} \times (-10)\, t^2$

$5t^2 - 6t - 50 = 0 \quad \Rightarrow \quad t = \frac{6 \pm \sqrt{1036}}{10}$

(c) Range = $8 \times \left(\frac{6 \pm \sqrt{1036}}{10}\right)$

(d) $v_y = u_y + a_y t \qquad 0 = 6 - 10t \qquad t = 0.6$

or $\quad 0 = 6 - 2 \times 10 \times h \qquad h = 1.8$

maximum height = 50 + 1.8 = 51.8 m.

Example 30. Two stones A and B are projected simultaneously from the top of a 100 m high tower. Stone B is projected horizontally with speed 10 m/s, and stone A is dropped from the tower. Find out the following

(a) time of flight of the two stone.
(b) distance between two stones after 3 sec.
(c) angle of strike with ground.
(d) horizontal range of particle B.

Solution [**Ans.** (a) $2\sqrt{5}$ sec. (b) $x_B = 30$ m, $y_B = 45$ (c) $\tan^{-1} 2\sqrt{5}$ (d) $20\sqrt{5}$ m]

Projection from a Moving Body

Consider a boy standing on a trolley who throws a ball with speed u at an angle θ with the horizontal. Trolley moves horizontally with constant speed v .

Case (i): When ball is projected in the direction of motion of the trolley, horizontal component of ball's velocity $= u \cos\theta + v$ Initial vertical component of ball's velocity $= u \sin\theta$

Case (ii) : The ball is projected opposite to the direction of motion of the trolley

Horizontal component of ball's velocity $= u \cos\theta - v$

Initial vertical component of ball's velocity $= u \sin\theta$

Case (iii): The ball projected upwards from a platform moving with speed v upwards.

Horizontal component of ball's velocity $= u \cos\theta$

Initial vertical component of ball's velocity $= u \sin\theta + v$

Case (iv) : The ball projected upwards from a platform moving with speed v downwards.

Horizontal component of ball's velocity $= u \cos\theta$

Initial vertical component of ball's velocity $= u \sin\theta - v$

Example 12. A particle is projected at an angle of 30^0 with speed 20 m/s :

(i) Find out position vector of the particle after 1s

(ii) Find out angle betwen velocity vector and position vector at $t = 1$s

Sol. (i) $s_x = u \cos\theta\, t = 20 \times \dfrac{\sqrt{3}}{2} \times t = 10\sqrt{3}$ m

$s_y = u \sin\theta t + \dfrac{1}{2} \times 10 \times t^2 = 20 \times \dfrac{1}{2} \times 1 - 5\,(1)^2 = 5$m

Position vector $= 10\sqrt{3}\,\hat{i} + 5\hat{j}$

(ii) $v_x = 10\sqrt{3}\,\hat{i}$

$v_y = u_y + a_y t = 10 - 10 = 0$

$\vec{v} = 10\sqrt{3}\,\hat{i}$ $\qquad\qquad \vec{v}\,.\,\vec{s} = |v|\,|s| \cos\theta$

$$\cos\theta = \frac{10\sqrt{3} \times 10\sqrt{3}}{10\sqrt{3} \times \sqrt{325}} = \frac{10\sqrt{3}}{\sqrt{325}} = 2\sqrt{\frac{3}{13}}$$

$$\theta = \cos^{-1}\left(2\sqrt{\frac{3}{13}}\right)$$

Example 31. A boy standing on a long railroad car throws a ball straight upwards. The car is moving on the horizontal road with an acceleration of 1 m/s^2 and the projection speed in the vertical direction is 9.8 m/s. How far behind the boy will the ball fall on the car?

Solution Let the initial velocity of car be 'u'. $t = \dfrac{2u_\perp}{g} = 2$

where $u_\perp$ = component of velocity in vertical direction

$$x_c = u \times 2 + \frac{1}{2} \times 1 \times 2^2 = 2u + 2$$

where x_c = distance travelled by car; x_b = distance travelled by ball

$x_b = 2u$; $x_c - x_b = 2u + 2 - 2u = 2\text{m}$

Example 32. A person is standing on a truck moving with a constant velocity of 14.7 m/s on a horizontal road. The man throws a ball in such a way that it returns to the truck after the truck has moved 58.8m. Find the speed and the angle of projection (a) as seen from the truck , (b) as seen from the road.

Solution [**Ans.** (a) 19.6 m/s upward (b) 24.5 m/s at 53ş with horizontal]

PROJECTION ON AN INCLINED PLANE

To solve the problem of projectile motion on an incline plane we can adopt two types of axis system as shown in the figures 2.5 and 2.6.

Case (i) : Up the incline

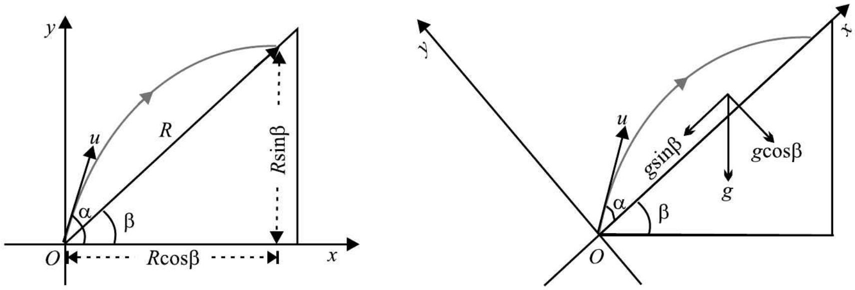


Fig 2.5

axis system 1
Here α is angle of projection with the horizontal.
In this case :

$a_x = 0 \quad u_x = u\cos\alpha$

$a_y = -g \quad u_y = u\sin\alpha$

axis system 2
Here α is angle of projection with the inclined plane
In this case :

$a_x = -g\sin\beta \quad u_x = u\cos\alpha$

$a_y = -g\cos\beta \quad u_y = u\sin\alpha$

Time of flight (T) :

when the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2$$

$$\Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Maximum height (H):

when half of the time is elasped y coordinate is equal to maximum height of the projectile

$$H = u\ \sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\sin\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u^2_\perp}{2g_\perp}$$

Range along the inclined plane (R):

When the particle strikes the inclined plane x coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2}a_x t^2$$

$$\Rightarrow \quad R = u\cos\theta\left(\frac{2u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\cos\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$$

Case (ii) : Down the incline

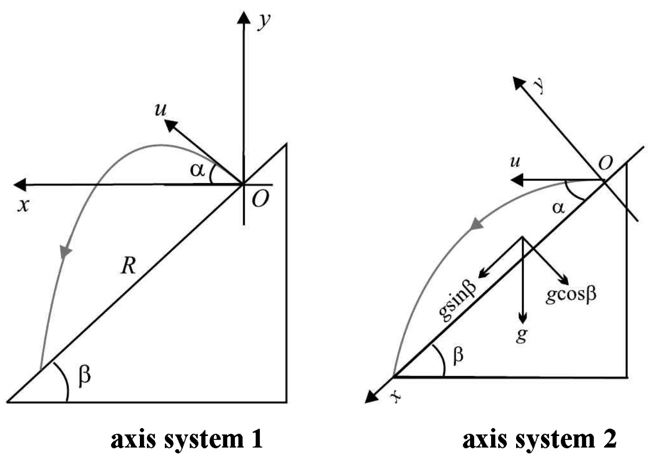


Fig 2.6

In this case :

$a_x = 0$	$u_x = u\cos\alpha$	$a_x = g\sin\beta$	$u_x = u\cos\alpha$
$a_y = -g$	$u_y = u\sin\alpha$	$a_y = -g\cos\beta$	$u_y = u\sin\alpha$

Time of flight (T):

when the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2$$

$$\Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_\perp}{g_\perp}$$

Maximum height (H):

when half of the time is elasped y coordinate is equal to maximum height of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2} g\sin\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u_\perp^2}{2g_\perp}$$

Range along the inclined plane (R):

When the particle strikes the inclined plane *x* coordinate is equal to range of the particle

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow \quad R = u\cos\theta\left(\frac{2u\sin\alpha}{g\cos\beta}\right) + \frac{1}{2} g\cos\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad R = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$$

***Table:** Standard results for projectile motion on an inclined plane*

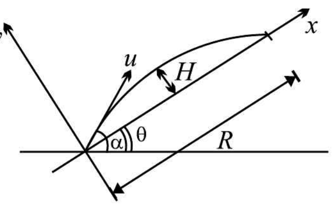
	Up the Incline	Down the Incline
Range	$\frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}$	$\frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}$
Time of flight	$\frac{2u\sin\alpha}{g\cos\beta}$	$\frac{2u\sin\alpha}{g\cos\beta}$
Angle of projection for maximum range	$\frac{\pi}{4}-\frac{\beta}{2}$	$\frac{\pi}{4}+\frac{\beta}{2}$
Maximum Range	$\frac{u^2}{g(1+\sin\beta)}$	$\frac{u^2}{g(1-\sin\beta)}$

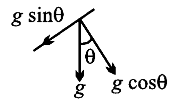
Note

Here α is the angle of projection with the incline and β is the angle of incline. For a given speed, the direction - which gives the maximum range of the projectile on an incline, bisects the angle between the incline and the vertical, for upward or downward projection.

Explanatory Notes on Projection on Inclined Plane

There is an inclined plane making an angle θ with horizontal. A particle is projected at an angle α from horizontal.





x-axis

$u_x = u \cos(\alpha - \theta)$

$a_x = -g \sin\theta$

vel. at any time t

$v_x = u \cos(\alpha - \theta) - g \sin\theta\, t$

y-axis

$u_y = u \sin(\alpha - \theta)$

$a_y = -g \cos\theta$

vel. at any time t

$v_y = u \sin(\alpha - \theta) - g \cos\theta\, t$

Time of Flight

Displacement in y direction $s_y = 0$

$$0 = u \sin(a - \theta)\, T - \frac{1}{2} g \cos\theta\, T^2$$

$$T = \frac{2u \sin(\alpha - \theta)}{g \cos\theta}$$

Maximum Height (Relative to Inclined Plane)

Point where $v_y = 0$ is max. height

$$(0)^2 = u^2 \sin^2(\alpha - \theta) - 2g \cos\theta\, H$$

$$H = \frac{u^2 \sin^2(\alpha - \theta)}{2g \cos\theta}$$

Range Along the Inclined Plane

$$s_x = u_x T + \frac{1}{2} a_x T^2$$

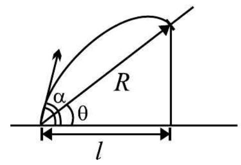
$$R = \frac{u \cos(\alpha - \theta) \times 2u \sin(\alpha - \theta)}{g \cos\theta} - \frac{2 \sin\theta \times 2 \times 2u^2 \sin^2(\alpha - \theta)}{2g^2 \cos^2\theta}$$

$$= \frac{2u^2 \sin(\alpha - \theta)[\cos(\alpha - \theta)\cos\theta - \sin\theta \sin(\alpha - \theta)]}{g \cos^2\theta}$$

$$R = \frac{2u^2 \sin(\alpha - \theta)\cos\alpha}{g\cos^2\theta}; \qquad R = \frac{u^2[\sin(2\alpha - \theta) - \sin\theta]}{g\cos^2\theta}$$

Alternate Method:

$l = u \cos \alpha \, T$



$$R = \frac{l}{\cos\theta}$$

$$R = \frac{u\cos\alpha}{\cos\theta} \times \frac{2u\sin(\alpha - \theta)}{g\cos\theta}$$

$$R = \frac{2u^2\sin(\alpha - \theta)\cos\alpha}{g\cos^2\theta}$$

Important: *Presence of incline plane does not affect the path of projectile in any way.*

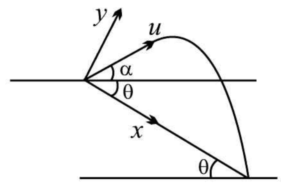
Maximum Range

$$R = \frac{u^2[\sin(2\alpha - \theta) - \sin\theta]}{g\cos^2\theta}$$

For max. range $2\alpha - \theta = \frac{\pi}{2} \quad \Rightarrow \quad \alpha = \frac{\pi}{4} + \frac{\theta}{2}$ so $R_{max} = \frac{u^2}{g(1 + \sin\theta)}$

Projection from Top of Incline Plane

Incline plane is at an angle θ with horizontal and a particle is projected at an angle α from horizontal. In all formulae replace θ with –θ



$$\mu = \frac{u^2\sin^2(\alpha + \theta)}{2g\cos\theta} \qquad T = \frac{2u\sin(\alpha + \theta)}{g\cos\theta}$$

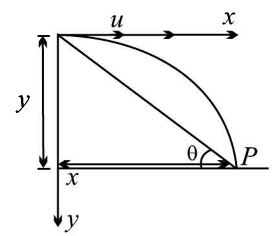
$$R = \frac{2u^2\sin(\alpha + \theta)\cos\alpha}{g\cos^2\theta} \qquad R_{max} = \frac{u^2}{g(1 - \sin\theta)} \text{ and } \alpha = \frac{\pi}{4} - \frac{\theta}{2}$$

Note

If a particle strikes the incline plane ⊥ then its comp. of velocity along incline must be zero.

Example 33. A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far along the plane, from the point of projection will particle strike the plane?

Solution



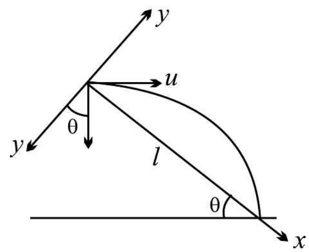
***x*-axis**	***y*-axis**
$u_x = u$	$u_x = 0$
$a_x = 0$	$a_x = g$
$x = ut$	$y = \frac{gt^2}{2}$

$$\Rightarrow \qquad y = \frac{gx^2}{2u^2}$$

also $\dfrac{y}{x} = \tan\theta$ $\Rightarrow$ $x\tan\theta = \dfrac{gx^2}{2u^2}$

$x = 0,\ \dfrac{2u^2\tan\theta}{g}$ $\quad x = \dfrac{2u^2\tan\theta}{g}$ $\Rightarrow$ $y = \dfrac{2u^2\tan^2\theta}{g}$

dist. $l = \sqrt{x^2 + y^2}$ $\quad l = \dfrac{2u^2\tan\theta\sec\theta}{g}$



Alternate Method:

$R = \dfrac{2u^2\sin(\alpha+\theta)\cos\alpha}{g\cos^2\theta}$ $\quad R = 2u^2\tan\theta\sec\theta$

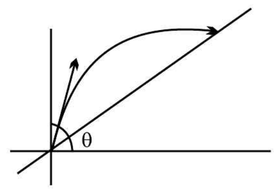
Concept

To analyse problem with different choices of coordinate axis.

Example 34. A particle is projected up an inclined plane. Plane is inclined at an angle θ with horizontal and particle is projected at an angle α with horizontal. If particle strikes the plane horizontally prove that tan α = 2 tan θ

Solution We know time of flight

$$T = \frac{2u\sin(\alpha+\theta)}{g\cos\theta}$$



considering vertical motion

$u = v\sin\alpha \qquad a = -g \qquad v = 0$

$$\therefore \quad T = \frac{u\sin\alpha}{g} = \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

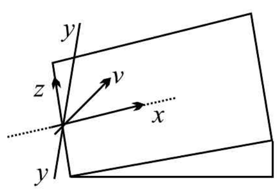
$\sin\alpha\cos\theta = 2\sin\alpha\cos\theta - 2\cos\alpha\sin\theta$

$2\cos\alpha\sin\theta = \sin\alpha\cos\theta$

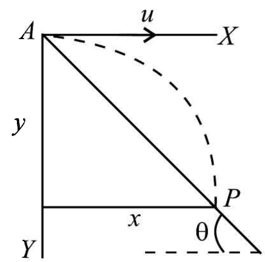
Important: Even on an inclined projection, projectile can be treated as projection from ground.

Note

Line of greatest slope: When projection is not along line of greatest slope then 3–D motion and three comp. One will not be affected by gravity (v_z).



Example 35. A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane?



Solution Take X,Y-axes as shown in figure. Suppose that the particle strikes the plane at a point P with coordinates (x, y). Consider the motion between A and P.

Motion in x direction :

Initial velocity = u

Acceleration = 0

$$x = ut \qquad ...(i)$$

Motion in y direction :

Initial velocity = u

Acceleration = g

$$y = \frac{1}{2}gt^2 \qquad ...(ii)$$

Eliminating t from (i) and (ii)

$$y = \frac{1}{2}g\frac{x^2}{u^2} \qquad \text{Also } y = x\tan\theta$$

Thus , $\frac{gx^2}{gu^2} = x\tan\theta$ giving $x = 0$ or, $\frac{2u^2\tan\theta}{g}$

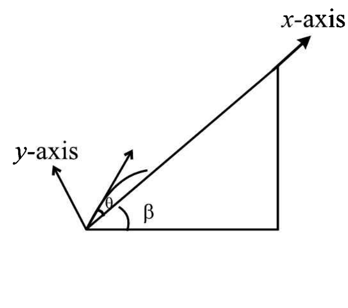
Clearly the point P corresponds to $x = \frac{2u^2\tan\theta}{g}$ then $y = x\tan\theta = \frac{2u^2\tan^2\theta}{g}$

The distance AP = $\sqrt{x^2+y^2} = \frac{2u^2}{g}\tan\theta\sqrt{1+\tan^2\theta} = \frac{2u^2}{g}\tan\theta\sec\theta$

Example 36. A projectile is thrown at an angle θ with an inclined plane of inclination β as shown in figure. Find the relation between β and θ if :

(a) projectile strikes the inclined plane perpendicularly,

(b) projectile strikes the inclined plane horizontal.



Solution

(a) If projectile strikes perpendicularly.

$v_x = 0$ when projectile strikes

$v_x = u_x + a_x t$

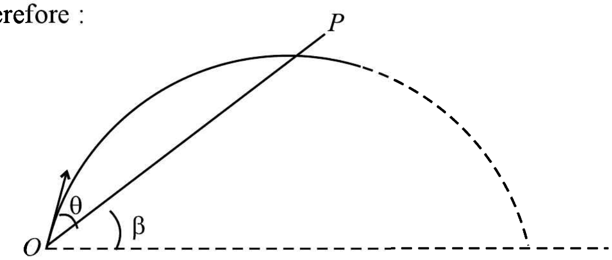
$0 = u\cos\theta - g\sin\beta\, T$

$$T = \frac{u\cos\theta}{g\sin\beta}$$

we also know that $T = \frac{2u\sin\theta}{g\cos\beta}$

$$\Rightarrow \quad \frac{u\cos\theta}{g\sin\beta} = \frac{2u\sin\theta}{g\cos\beta} \qquad \Rightarrow 2\tan\theta = \cot\beta$$

(b) If projectile strikes horizontally, then at the time of striking the projectile will be at the maximum height from the ground. Therefore :



$$t_{OP} = \frac{2u\sin\theta}{g\cos\beta}$$

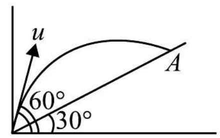
$$t_{OP} = \frac{2u\sin(\theta+\beta)}{2\times g}$$

$$\Rightarrow \quad \frac{2u\sin\theta}{g\cos\beta} = \frac{2u\sin(\theta+\beta)}{2g}$$

$$\Rightarrow \quad 2\sin\theta = \sin(\theta+\beta)\cos\beta\,.$$

Example 37. A shell is fired from a gun from the bottom of a hill along its slope. The slope of the hill is $\alpha = 30°$ and the angle of barrel to the horizontal is $\beta = 60°$ the initial velocity of shell is 21 m/s. Find the distance from the gun to the point at which the shell falls.

Solution We can write the equation of motion as



$$x = ut\cos\beta$$

$$y = ut\sin\beta - \frac{gt^2}{2}$$

$OA = \ell$

At the moment of the shell falls to the ground

$x = \ell\cos\alpha = \ell\cos30°$

$y = \ell\sin\alpha = \ell\sin30°$

$\ell\cos\alpha = ut\cos\beta$ ...(i)

$\ell\sin\alpha = ut\sin\beta - \dfrac{gt^2}{2}$ ...(ii)

$$\therefore \quad t = \frac{\ell\cos\alpha}{u\cos\beta}$$

$$\ell\sin\alpha = \frac{\ell\cos\alpha\sin\beta}{\cos\beta}\frac{g\ell^2\cos^2\alpha}{2u^2\cos^2\beta}$$

$$\ell = \frac{2u^2\sin(\beta-\alpha)\cos\beta}{g\cos^2\alpha}$$

Substituting $u = 21$ m/s, $\alpha = 30°$, $\beta = 60°$ and $g = 9.8$ m/s^2, we get ℓ = **30 metres**

Example 38. A particle moving with uniform acceleration in a straight line covers a distance of 3 m in the 8th second and 5 m in the 16th second of its motion. What is the displacement of the particle from the beginning of the 6th second to the end of 15th second?

Solution The distance travelled during the nth second of motion of a body is given by

$$S_n = u + \frac{1}{2}a(2n-1)$$

For the motion during the 8th second,

$$3 = u + \frac{1}{2}a(16-1) = u + \frac{15a}{2} \quad \text{...(i)}$$

For the motion during the 16th second,

$$5 = u + \frac{1}{2}a(32-1) = u + \frac{31a}{2} \quad \text{...(ii)}$$

Subtracting equation (i) from (ii)

$$8a = 2$$

or acceleration $a = 1/4 \text{ m/s}^2$

From equation (i), $u = 3 - \left(\frac{15}{2} \times \frac{1}{4}\right) = \frac{9}{8} m/s$

Now, the velocity at the end of 5 *s* (velocity at the beginning of 6th second)

$$v_1 = u + 5a$$

Average velocity during this interval of 10 seconds $= \frac{v_1 + v_2}{2}$

$$= \frac{(u+5a)+(u+15a)}{2} = u + 10a$$

Distance travelled during the interval

$S =$ average velocity × time $= (u + 10a) \times$ t

$$= \left(\frac{9}{8} + \frac{10}{4}\right) \times 10 = \frac{290}{8} = 36.25 \text{ m}$$

Elastic Collision of a Projectile with a Wall

Suppose a projectile is projected with speed u at an angle θ from point O on the ground. Range of the projectile is R. If a wall is present in the path of the projectile at a distance x from the point O. The collision with the wall is elastic, path of the projectile changes after the collision as described below.

Case I : If $x \geq \frac{R}{2}$

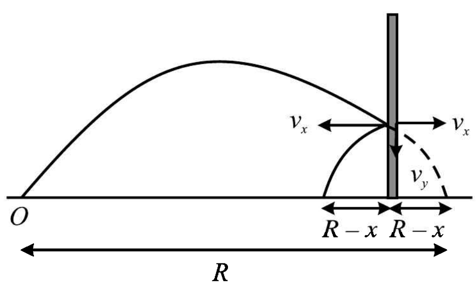


Fig 2.7

Direction of x component of velocity is reversed but its magnitude remains the same and y component of velocity remains unchanged, therefore the remaining distance $(R - x)$ is covered in the backward direction and projectile falls a distance $(R - 2x)$ ahead of the point O as shown in figure 2.7.

Case II : If $x < \frac{R}{2}$

Direction of x component of velocity is reversed but its magnitude remains the same and y component of velocity remains unchanged, therefore the remaining distance $(R - x)$ is covered in the backward direction and projectile falls a distance $(R - 2x)$ behind the the point O as shown in figure 2.8.

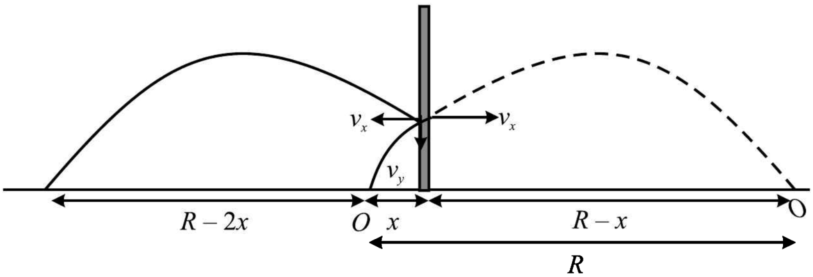


Fig 2.8

***Example* 39.** A particle is projected from point P with velocity $5\sqrt{2}$ ms^{-1} perpendicular to the surface of a hollow right angle cone whose axis is vertical. It collides at Q normally. The time of the flight of the particle is

(A) 1 sec. (B) $\sqrt{2}$ sec. (C) $2\sqrt{2}$ sec (D) 2 sec.

Solution $t = \frac{u}{g\sin\theta} = \frac{5\sqrt{2}\times\sqrt{2}}{10} = 1\,\text{sec}$

Hence (A) is the right answer.

***Example* 40.** A particle is projected from a point O with velocity u in a direction making an angle α upward with the horizontal. At P, it is moving at right angles to its initial direction of projection. Its velocity at P is

(A) $u \tan\alpha$ (B) $u \cot\alpha$ (C) $u\,\text{cosec}\,\alpha$ (D) $u \sec\alpha$

Solution $v\cos(90 - \alpha) = v\sin\alpha = u\cos\alpha;\ v = u\cot\alpha$

$\therefore$ (B) is the right answer

***Example* 41.** A particle is projected at an angle α with horizontal from the foot of a plane whose inclination to horizontal is β. Show that it will strike the plane at right angles if, $\cot\beta = 2\tan(\alpha - \beta)$.

Solution Let u be the velocity of projection so that $u\cos(\alpha - \beta)$ and $u\sin(\alpha - \beta)$ are the initial velocities respectively parallel to perpendicular to the inclined plane. The acceleration in these two directions are $(-g\sin\beta)$ and $(-g\cos\beta)$.

The initial component of velocity perpendicular to PQ is $u\sin(\alpha - \beta)$ and the acceleration in this direction is $(-g\cos\beta)$. If T is the time the particle takes to go from P to Q then in time T the space described in a direction perpendicular to PQ is zero.

$$0 = u\sin(\alpha - \beta).T - \frac{1}{2}g\cos\beta.T^2$$

$$T = \frac{2u\sin(\alpha - \beta)}{g\cos\beta}$$

If the direction of motion at the instant when the particle hits the plane be perpendicular to the plane, then the velocity at that instnat parallel to the plane must be zero.

$\therefore \quad u\cos(\alpha - \beta) - g\sin\beta\, T = 0$

$\therefore \quad \dfrac{u\cos(\alpha - \beta)}{g\sin\beta} = T = \dfrac{2u\sin(\alpha - \beta)}{g\cos\beta}$

$\therefore \quad \cot\beta = 2\tan(\alpha - \beta)$

Example 42. A stone is projected from the point on the ground in such a direction so as to hit a bird on the top of a telegraph post of height h and then attain the maximum height $2h$ above the ground. If at the instnat of projection the bird were to fly away horizontally with uniform speed, find the ration between horizontal velocities of the bird and stone if the stone still hits the bird while descending.

Solution The situation is shown in figure. Let θ be the angle of projection and u the velocity of projection.

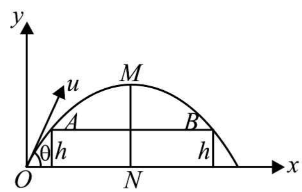
Maximum height MN = $2h$

$$MN = 2h = \frac{u^2\sin^2\theta}{2g}$$

$\therefore \quad u\sin\theta = 2\sqrt{gh}$ ...(i)

Let t be the time taken by stone to attain the vertical height h above the ground.

$\therefore \quad h = (u\sin\theta)t - \frac{1}{2}gt^2$



$$t^2 - \left(\frac{2u\sin\theta}{g}\right)t + \frac{2h}{g} = 0$$

$$t = \frac{u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} \frac{2h}{g}}$$

Substituting the value of usinθ from (i),

$$t = \frac{2\sqrt{gh}}{g} \pm \sqrt{\frac{4gh}{g^2} \frac{2h}{g}} = \sqrt{\frac{4h}{g}} \pm \sqrt{\frac{2h}{g}}$$

$$t_1 = \sqrt{\frac{4h}{g}} - \sqrt{\frac{2h}{g}}, \quad t_2 = \sqrt{\frac{4h}{g}} + \sqrt{\frac{2h}{g}}$$

where t_1 and t_2 are time to reach A and B respectively shown in the figure. If v is the horizontal velocity of bird, then AB = vt_2.

AB is also equal to ucos$\theta(t_2 - t_1)$, where ucosθ is constant horizontal velocity of stone

$t_2 - t_1 = 2\sqrt{\dfrac{2h}{g}}$ $\quad\quad \therefore \quad u\cos\theta . 2\sqrt{\dfrac{2h}{g}} = vt_2$

$$\frac{v}{u\cos\theta} = \frac{2\sqrt{\frac{2h}{g}}}{t_2} = \frac{2\sqrt{\frac{2h}{g}}}{\sqrt{\frac{2h}{g}}(\sqrt{2}+1)} = \frac{2}{\sqrt{2}+1} = 2(\sqrt{2}-1)$$

Example 43. A man can swim with a velocity V_1 relative to water in a river flowing with speed V_2. Show that it will take him $\frac{V_1}{\sqrt{V_1^2 - V_2^2}}$ times as long to swim a certain distance upstream and back as to swim the same distance and back perpendicular to the direction of the stream ($V_1 > V_2$).

Solution Suppose the man swims a distance x up and the same distance down the stream.

Velocity of man upstream relative to the ground = $V_1 - V_2$.

Time taken for this, $t_1 = \frac{x}{V_1 - V_2}$

Velocity of man downstream relative to the ground = $V_1 + V_2$

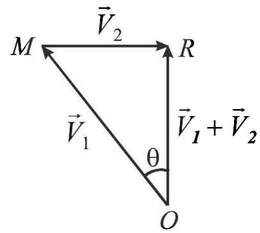
Time taken for this, $t_2 = \frac{x}{V_1 + V_2}$

Total time taken $t_1 + t_2 = \frac{x}{V_1 - V_2} + \frac{x}{V_1 + V_2} = \frac{2V_1 x}{V_1^2 - V_2^2}$

Next the man intends crossing the river perpendicular to the direction of the stream. If he wants to cross the river straight across he must swim in a direction OM such that the vector sum of velocity of man + velocity of river will give him a velocity relative to the ground in a direction perpendicular to the direction of the stream. In the Figure the velocity relative to the ground is $\overrightarrow{OR}$ and the magnitude of $\overrightarrow{OR} = \sqrt{V_1^2 - V_2^2}$

Now the man swims a distance x up and x down perpendicular to the river flow. Time taken for this,

$$t = \frac{2x}{\sqrt{V_1^2 - V_2^2}}$$



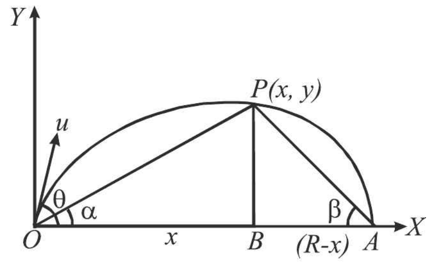
Then the ratio,

$$\frac{t_1 + t_2}{t} = \frac{\frac{2V_1 x}{(V_1^2 - V_2^2)}}{\frac{2x}{\sqrt{v_1^2 - v_2^2}}}$$

$$= \frac{2V_1 x}{V_1^2 - V_2^2} \times \frac{\sqrt{V_1^2 - V_2^2}}{2x} = \frac{V_1}{\sqrt{V_1^2 - V_2^2}}$$

Example 44. A particle thrown over a triangle from one end of a horizontal base of a vertical triangle gazes the vertex and falls on the other end of the base. If α and β be the base angle and θ the angle of projection, prove that tan θ= tan α + tan β.

Solution The situation is shown in the figure.



From figure,

$$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{(R-x)}$$

where R is the range.

$$\therefore \quad \tan\alpha + \tan\beta = \frac{y(R-x) + xy}{x(R-x)}$$

or $$\tan\alpha + \tan\beta = \frac{y}{x} \times \frac{R}{(R-x)} \qquad \text{...(i)}$$

we known

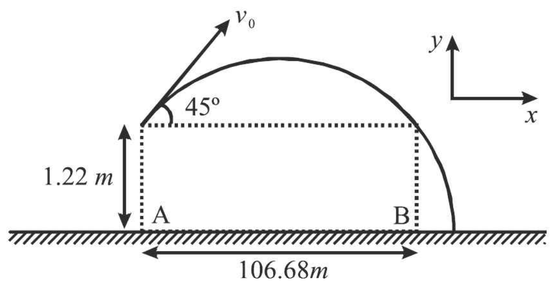
$$y = x\tan\theta\left(1 - \frac{x}{R}\right)$$

or $$\tan\theta = \frac{y}{x} \times \frac{R}{(R-x)} \qquad \text{...(ii)}$$

from equation (i) and (ii), we have

$\tan\theta = \tan\alpha + \tan\beta$

Example 45. A batsman hits a ball at a height of 1.22 m above the ground so that ball leaves the bat an angle 45ţ with the horizontal. A 7.31 m high wall is situated at a distance of 97.53 m from the position of the batsman. Will the ball clear the wall if its maximum horizontal distance from point of projection is 106.68 m. Take $g = 10$ m/s².



Solution $$\text{R(range)} = \frac{V_0^2 \sin 2\theta}{g}$$

$$\Rightarrow \quad v_0^2 = \frac{Rg}{\sin 2\theta} = Rg \text{ as } \theta = 45^\circ$$

$$\Rightarrow \quad v_0 = \sqrt{Rg}$$

Equation of trajectory

$$Y = x\tan 45^\circ - \frac{gx^2}{2v_0^2\cos^2 45^\circ} = x - \frac{gx^2}{2Rg\frac{1}{2}} \quad \text{using (1)}$$

Puttingx = 97.53, we get

$$y = 97.53 - \frac{10 \times (97.53)^2}{106.68 \times 10} = 8.35$$

Hence, height of the ball from the ground level is $h = 8.35 + 1.22 = 9.577$ m.

as height of the wall is 7.31 m so the ball will clear the wall.

Example 46. A particle projected with velocity v_0, strikes at right angles a plane through the point of projection and of inclination β with the horizontal. Find the height of the point struck, from horizontal plane through the point of projection.

Solution Let α be the angle between the velocity of projection and the inclined plane.

$v_{0x'} = v_0 \cos\alpha, \quad v_{0y'} = v_0 \sin\alpha$

$a_{x'} = -g\sin\beta, \quad a_{y'} = -g\cos\beta \qquad \Rightarrow \qquad v_x'(t) = v_0\cos\alpha - g\sin\beta\, t$

At the point of impact $v_{x'} = 0$

$$\Rightarrow \quad t = \frac{v_0\cos\alpha}{g\sin\beta} \qquad \text{...(i)}$$

Also y' at the point is zero.

$$\Rightarrow \quad v_0\sin\alpha t - \frac{1}{2}g\cos\beta t^2 = 0$$

$$\Rightarrow \quad t = \frac{2v_0\sin\alpha}{g\cos\beta} \qquad \text{...(ii)}$$

From (i) and (ii) $\dfrac{v_0\cos\alpha}{g\sin\beta} = \dfrac{2v_0\sin\alpha}{g\cos\beta}$

$$\tan\alpha = \frac{1}{2}\cot\beta \qquad \text{...(iii)}$$

$$x = v_0\cos(\alpha+\beta)t$$

$$= v_0[\cos\alpha\cos\beta - \sin\alpha\sin\beta]\frac{v_0\cos\alpha}{g\sin\beta} = \frac{v_0^{\,2}}{g}[\cos^2\alpha\cot\beta - \sin\alpha\cos\alpha]$$

$$= \frac{v_0^2}{g}\left[\left(\frac{2}{\sqrt{4+\cot^2\beta}}\right)^2\cot\beta - \frac{\cot\beta}{\sqrt{4+\cot^2\beta}}\frac{2}{\sqrt{4+\cot^2\beta}}\right] \quad \left(\text{using } \tan\alpha = \frac{1}{2}\cot\beta\right)$$

$$= \frac{v_0^{\,2}}{g}\frac{2\cot\beta}{4+\cot^2\beta}$$

From figure

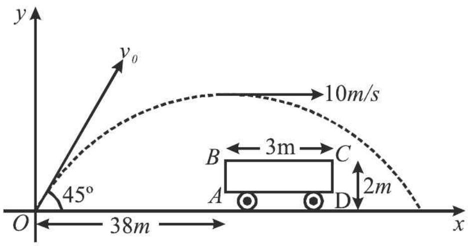
$$\therefore \quad y = x\tan\beta = \frac{v_0^{\,2}}{g}.\frac{2\cot\beta}{4+\cot^2\beta}\tan\beta \Rightarrow y = \frac{2v_0^{\,2}}{g(4+\cot^2\beta)}.$$

Example 47. A ball is thrown from the origin in the $x-y$ plane with velocity 28.28 m/s at an angle 45ţ to the x-axis. At the same instant a trolley also starts moving with uniform velocity of 10 m/s along the positive x-axis. Initially rear end of the trolley is located at 38m from the origin. Determine the time and position at which the ball hits the trolley.

Solution Let t be the instant at which the ball hits rear face AB of the trolley.

Then $(v_0\cos 45° - u_0)t = 38$

$$\text{or } t = \frac{38}{v_0\cos 45° - u_0} = \frac{38}{28.28\cos 45° - 10} = 3.8s$$



At $t = 3.8s$, the y – coordinate of the ball is

$$y = (v_0 \sin 45°)t - \frac{1}{2}gt^2 = 20t - 5t^2$$

or $\quad y = 20(3.8) - 5(3.8)^2 = 3.8$ m

Since 3.8 m > 2m, therefore, the ball cannot hit the rear face of the trolley. Now, we assume that the ball hits the top face BC of the trolley, and let t' be that instant. Then,

$$y = 2 = 20t' - 5t'^2 \quad \text{or} \quad t'^2 - 4t' + 0.4 = 0$$

$t' = 3.9$s

Let d be the distance from the point B at which the ball hits the trolley.

Then, $d = (v_0 \cos 45° - u0)\,(t' - t) = (20 - 10)\,(3.9 - 3.8) = $ lm

Example 48. Find the radius of a rotating wheel if the linear velocity v_1 of a point on the rim is 2.5 times greater than the linear velocity v_2 of a point 5 cm closer to wheel axle.

Solution Let the radius of the disc = r (in cm.)

$$\Rightarrow \quad \frac{v_2}{v_1} = \frac{(r-5)\omega}{r\omega}$$

v_1 is 2.5 times greater than v_2

$$\Rightarrow \quad \frac{v}{2.5v_2} = \frac{r-5}{r} \Rightarrow \quad r = 2.5\,r - 12.5$$

$$\Rightarrow \quad 1.5\,r = 12.5 \Rightarrow r = \frac{12.5}{1.5} = 8.33 \text{ cm.}$$

Example 49. Two men A and B, A standing on the extended floor nearby a building and B is standing on the roof of the building. Both throw a stone each towards each other. Then which of the following will be correct.

(A) stone will hit A, but not B
(B) stone will hit B, but not A
(C) stone will not hit either of them, but will collide with each other
(D) none of these.

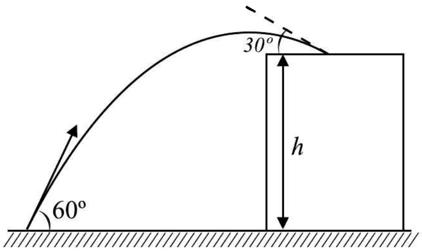
Solution Path will not be straight line but parabolic hence neither stone will hit any person. Condition of collision will depend upon direction as well as velocities of projection that are not given. Hence, (D) is the correct option.

Example 50. A particle is projected up the inclined such that its component of velocity along the incline is 10 m/s. Time of flight is 2 sec and maximum height above the incline is 5 m. Then velocity of projection will be:

(A) 10 m/s (B) $10\sqrt{2}$ m/s (C) $5\sqrt{5}$ m/s (D) none

Solution **[Ans.** (B)]

Example 51. A stone projected at an angle of 60ş from the ground level strikes at an angle of 30ş on the roof of a building of height 'h'. Then the speed of projection of the stone is :



(A) $\sqrt{2gh}$ (B) $\sqrt{6gh}$ (C) $\sqrt{3gh}$ (D) $\sqrt{gh}$

Solution Let initial and final speeds of stone be u and v.

$\therefore$ $v^2 = u^2 - 2gh$ ...(1)

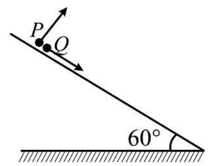
and $v \cos 30° = u \cos 60°$ ...(2)

solving 1 and 2 we get

$u = \sqrt{3gh}$

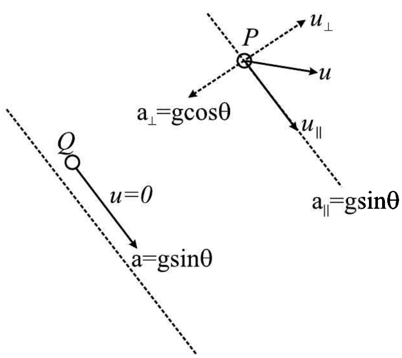
$\therefore$ (C) is the correct option.

Example 52 A particle P is projected from a point on the surface of smooth inclined plane (see figure). Simultaneously another particle Q is released on the smooth inclined plane from the same position. P and Q collide after $t = 4$ second. The speed of projection of P is



(A) 5 m/s (B) 10 m/s (C) 15 m/s (D) 20 m/s

Solution It can be observed from figure that P and Q shall collide if the initial component of velocity of P along incline. $u_{\parallel} = 0$ that is particle is projected perpendicular to incline.



$\therefore$ Time of flight $T = \dfrac{2u_{\perp}}{g\cos\theta} = \dfrac{2u}{g\cos\theta}$

$\therefore$ $u = \dfrac{gT\cos\theta}{2} = 10$ m/s.

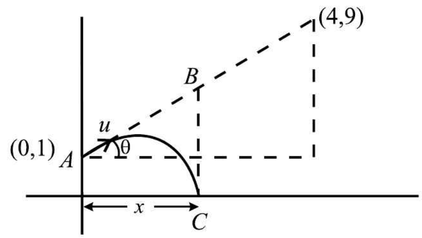
$\therefore$ (B) is the right option.

Example 53. A particle is projected from a point (0, 1) on y-axis (assume + Y direction vertically upwards) aiming towards a point (4, 9). It fell on ground along x-axis in 1 sec. Taking $g = 10$ m/s^2 and all coordinate in metres. Find the X-coordinate where it fell.

(A) (3, 0) (B) (4, 0) (C) (2, 0) (D) $(2\sqrt{5}, 0)$

Solution

$$\tan\theta = \frac{9-1}{4-0} = 2$$



$$\text{now, } -1 = u\sin\theta\,(1) - \frac{1}{2}g\,(1)^2$$

$$u\sin\theta = 4 \quad \Rightarrow u = \frac{4}{(2\sqrt{5})} = 2\sqrt{5}$$

$$\text{now, } x = 4\cos\theta\,(1) = (2\sqrt{5}) \times \frac{1}{\sqrt{5}} = 2\text{m}$$

$\therefore$ (C) is the right answer.

Example 54. A car starts with constant acceleration $a = 2$m/s^2 at $t = 0$. Two coins are released from the car at $t = 3$ & $t = 4$. Each coin takes 1 second to fall on ground. Then the distance between the two coins will be (Assume coin sticks to the ground.)

(A) 9 m (B) 7 m (C) 15 m (D) 2m

Solution

$v = at = 2t$

Velocity of car at $t = 3$ $v_1 = 6$ m/s

at $t = 4$ $v_2 = 8$ m/s

Coin 1 will fall with horizontal velocity 6 m/s and second coin will fall with horizontal velocity 8 m/s. Both will travel 6 m & 8 m horizontally before they fall from the point of release.

Car moves $\frac{(6+8)}{2} \times 1 = 7$ m. In fourth second position of first coin $x_1 = 6$, $x_2 = 7+8=15$

$\Rightarrow$ $x_2 - x_1 = 15 - 6 = 9$m **[Ans. (A)]**

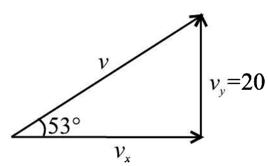
Example 55. Velocity of a stone projected, 2 second before it reaches the maximum height, makes angle 53° with the horizontal then the velocity at highest point will be

(A) 20 m/s (B) 15 m/s (C) 25 m/s (D) 80/3 m/s

Solution

Two second before maximum height $v_y = g \times 2 = 20$ cm/s

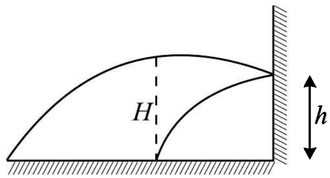
$$\tan 53° = \frac{20}{v_x} \quad v_x = 15 \text{ m/s}$$



at maximum height $v = v_x = 15$ m/s

[Ans. (B)]

Example 56. A stone is projected from a horizontal plane. It attains maximum height 'H' & strikes a stationary smooth wall and falls on the ground vertically below the maximum height. Assume the collision to be elastic the height of the point on the wall where ball will strike is :

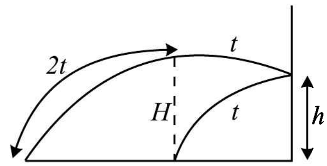


(A) $\frac{H}{2}$ (B) $\frac{H}{4}$ (C) $\frac{3H}{4}$ (D) None of these

Solution

$$H = \frac{1}{2}g(2t)^2 = 2gt^2 \quad \text{...(1)}$$

$$h = H - \frac{1}{2}gt^2 \quad \text{...(2)}$$



By (1) & (2)

$$h = H - \frac{H}{4} = \frac{3H}{4}$$

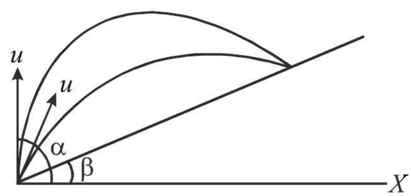
[Ans. (C)]

Example 57. Two bodies are projected from the same point with equal speeds in such directions that they both strike the same point on a plane whose inclination is β. If α be the angle of projection of the first body with the horizontal show that the ratio of their times of flight is;

$$\frac{\sin(\alpha-\beta)}{\cos\alpha}$$

Solution Let α′ be the angle of projection of the second body.

$$R = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha-\beta) - \sin\beta]$$



Range of both the bodies is same. Therefore,

$$\sin(2\alpha-\beta) = \sin(2\alpha'-\beta)$$

or $$2\alpha'-\beta = \pi-(2\alpha-\beta)$$

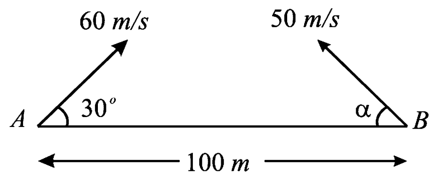
$$\alpha' = \frac{\pi}{2} - (\alpha-\beta)$$

Now, $T = \dfrac{2u\sin(\alpha-\beta)}{g\cos\beta}$ and $T' = \dfrac{2u\sin(\alpha'-\beta)}{g\cos\beta}$

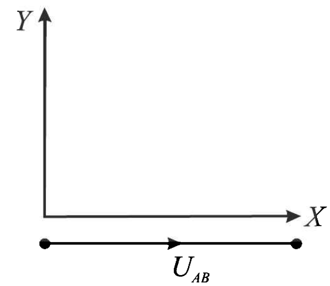
$$\therefore \quad \frac{T}{T'} = \frac{\sin(\alpha-\beta)}{\sin(\alpha'-\beta)} = \frac{\sin(\alpha-\beta)}{\sin\left\{\frac{\pi}{2}-(\alpha-\beta)-\beta\right\}}$$

$$= \frac{\sin(\alpha-\beta)}{\sin\left(\frac{\pi}{2}-\alpha\right)} = \frac{\sin(\alpha-\beta)}{\cos\alpha}$$

Example 58. A particle A is projected with an initial velocity of 60 m/s at an angle 30° to the horizontal. At the same time a second particle B is projected in opposite direction with initial speed of 50 m/s from a point at a distance of 100 m from A. If the particles collide in air, find (a) the angle of projection á of particle B (b) time when the collision takes place and (c) the distance of P from A. Where collision occurs (g= 10 m/s²).



Solution



(a) Taking x and y directions as shown in figure.

Here,
$$\vec{a}_A = -g\hat{j}$$
$$\vec{a}_B = -g\hat{j}$$
$$u_{Ax} = 60\cos 30° = 30\sqrt{3}\, m/s$$
$$u_{Ay} = 60\sin 30° = 30\, m/s$$
$$u_{Bx} = -50\cos\alpha \quad \text{or} \quad u_{By} = 50\sin\alpha$$

Relative acceleration between the two is zero as $\vec{a}_A = \vec{a}_B$. Hence, the relative motion between the two is uniform. It can be assumed that B is at rest and A is moving with $\vec{u}_{AB}$. Hence, the two particles will collide if $\vec{u}_{AB}$ is along AB. This is possible only when

$$u_{Ay} = u_{By}$$

i.e., component of relative velocity along y-axis should be zero.

or $$30 = 50\sin\alpha \qquad \therefore \alpha = \sin^{-1}(3/5)$$

(b) Now, $|\vec{u}_{AB}| = u_{Ax} - u_{Bx} = (30\sqrt{3} + 50\cos\alpha)\, m/s$

$$= \left(30\sqrt{3} + 50 \times \frac{4}{5}\right) m/s = (30\sqrt{3} + 40)\, m/s$$

Therefore, time of collision is

$$t = \frac{AB}{|\vec{u}_{AB}|} = \frac{100}{30\sqrt{3} + 40}$$

or t = 1.09 s

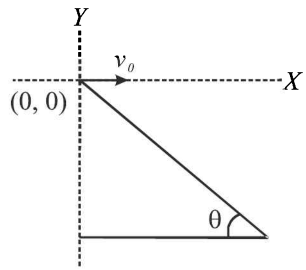
(c) Distance of point P from A where collision takes place is

$$s = \sqrt{(u_{Ax}t)^2 + \left(u_{Ay}t - \frac{1}{2}gt^2\right)^2}$$

$$=\sqrt{(30\sqrt{3}\times1.09)^2+\left(30\times1.09-\frac{1}{2}\times10\times1.09\times1.09\right)^2}$$

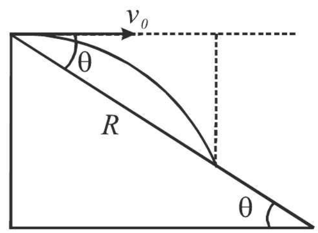
or $s = 62.64$ m

Example 59. A man standing on a hill top projects a stone horizontally with speed v_0 as shown in figure. Taking the co-ordinate system as given in the figure. Find the co-ordinates of the point where the stone will hit the hill surface.



Solution Range of the projectile on an inclined plane (down the plane) is,

$$R=\frac{u^2}{g\cos^2\beta}[\sin(2\alpha+\beta)+\sin\beta]$$



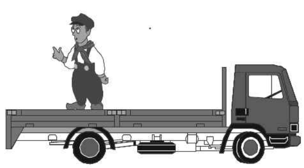
Here, $u = v_0$, $\alpha=0$ and $\beta=\theta$

$$\therefore \quad R=\frac{2v_0^2\sin\theta}{g\cos^2\theta}$$

Now, $x=R\cos\theta=\dfrac{2v_0^2\tan\theta}{g}$

and $y=-R\sin\theta=-\dfrac{2v_0^2\tan^2\theta}{g}$

Example 60. A car accelerating at the rate of 2 m/s² from rest from origin is carrying a man at the rear end who has a gun in his hand. The car is always moving along positive x-axis. At t = 4s, the man fires from the gun and the bullet hits a bird at t = 8 s. The bird has a position vector $40\hat{i}+80\hat{j}+40\hat{k}$. Find velocity of projection of the bullet. Take the y-axis in the horizontal plane. (g = 10 m/s²).



Solution Let velocity of bullet be,

$$\vec{v}=v_x\hat{i}+v_y\hat{j}+v_z\hat{z}$$

At t = 4 sec, x-co-ordinate of car is;

$$x_c=\frac{1}{2}at^2=\frac{1}{2}\times2\times16=16m$$

x-coordinate of bird is $x_b = 40$ m

$\therefore \quad x_b = x_c + v_x (8-4)$ or $40 = 16 + 4v_x$

$\therefore \quad v_x = 6$m/s

Similarly, $y_b = y_c + v_y (8-4)$

or $80 = 0 + 4v_y$ or $v_y = 20$m/s

and $z_b = z_c + v_z (8-4) - \frac{1}{2} g (8-4)^2$ or $40 = 0 + 4v_z - \frac{1}{2} \times 5 \times 16$

or $v_z = 20$ m/s²

$\therefore$ Velocity of projection of bullet

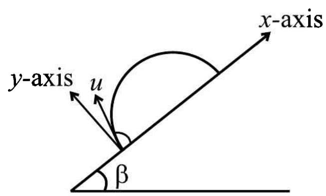
$\vec{v} = (6\hat{i} + 20\hat{j} + 20\hat{k})\, m/s$

EXERCISE

Exercise–1: Subjective Problems

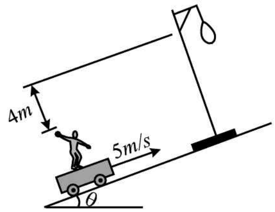
1. In order to project a body for maximum range, what is the condition?
2. What is the angle between the directions of velocity and acceleraiton at the highest point of a projectile path?
3. At what point of the projectile path the speed is minimum?
4. Two bodies are projected at angles θ and $(90 - \theta)$ to the horizontal with the same speed. Find the ratio of their times of flight?
5. In above question find the ratio of the maximum vertical heights?
6. What should be the angles of projection to obtain maximum height and maximum time of flight?
7. A bob of mass 0.1 kg hung from the ceiling of a room by a string 2 m long is set into oscillation. The speed of the bob at its mean position is 1 ms^{-1}. What is the trajectory of the bob; if the string is cut
   (a) when the bob is at one of its extreme position
   (b) at its mean position
8. A projectile can have the same range R for two angles of projections. If t_1 and t_2 be the times of flight in two cases, then find out relation between t_1t_2 and R?
9. The height y and the distance x along the horizontal plane of a projectile on a certain planet (without no surrounding atmosphere) are given by $y = (8t - 5t^2)$ meter and $x = 6$t m. Then what will be the velocity of projection?
10. A glass marble projected horizontal from the top of a table falls at a distance x from the edge of the table. If h is the height of the table, find the velocity of projection?
11. A projectile is fired horizontally with a velocity of 98 m/s from the top of a hill 490 m high. Find:
    (i) the time taken to reach the ground
    (ii) the distance of the target from the hill
    (iii) the velocity with which the particle hits the ground

12. The equation of a projectile is $y = \sqrt{3}x - \frac{gx^2}{2}$, find the angle of projection?
13. Consider a boy on a trolley who throws a ball with speed 20 m/s at an angle 37ţ w.r.t. trolley which moves horizontally with speed 10 m/s.
    (a) Find horizontal and vertical components of initial velocity of ball when ball is projected in direction of motion of trolley.
    (b) Find horizontal and vertical components of initial velocity of ball when ball is projected opposite to direction of motion of trolley
14. Consider a boy on a platform who throws a ball with speed 20 m/s at an angle 37^0 w.r.t. platform which moves upwards with speed 10 m/s. (a) Find the horizontal and vertical component of balls velocity. (b) Find horizontal and vertical components of balls velocity when ball is projected downwards from the platform.
15. A bomb is dropped from an aeroplane moving horizontally at a certain height from the ground. Does the time taken by the bomb to reach the ground depend on the velocity of the aeroplane?
16. A particle is projected at an angle θ with an inclined plane making an angle β with the horizontal as shown in figure, speed of the particle is u, after time t find :
    (a) x component of acceleration?
    (b) y component of acceleration?
    (c) x component of velocity?
    (d) y component of velocity?
    (e) x component of displacement?
    (f) y component of displacement?
    (g) y component of velocity when particle is at maximum distance from the incline plane?

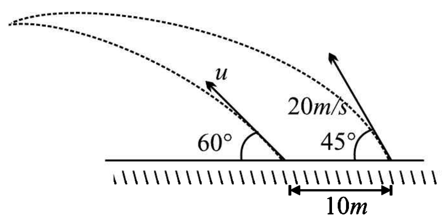


17. On an inclined plane of inclination 30ţ, a ball is thrown at an angle of 60ţ with the horizontal from the foot of the incline with a velocity of $10\sqrt{3}$ ms^{-1}. If $g = 10$ ms^{-2}, then find the time in which ball will hit the inclined plane?
18. The direction of motion of a projectile at a certain instant is inclined at an angle α to the horizon. After t seconds it is inclined an angle β. Find the horizontal component of velocity of projection in terms of g, t, α and β.
19. A radius vector of a point A relative to the origin varies with time t as $\vec{r} = at\hat{i} - bt^2\hat{j}$, where a and b are positive constants and $\hat{i}$ *and* $\hat{j}$ are the unit vectors of the x and y axes. Find:
    (i) the equation of the point's trajectory $y(x)$; plot this function
    (ii) the time dependence of the velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ vectors as well as of the moduli of these quantities .
20. Two particles are projected simultaneously with the same speed V in the same vertical plane with angles of elevation θ and 2θ, where $\theta < 45$ţ. At what time will their velocities be parallel.
21. If 4 seconds be the time in which a projectile reaches a point P of its path and 5 seconds the time from P till it reaches the horizontal plane through the point of projection . Find the height of P above the horizontal plane. [$g = 9.8$ m/sec^2]
22. A man is travelling on a flat car which is moving up a plane inclined at $\cos\theta = 4/5$ to the horizontal with a speed 5 m/s. He throws a ball towards a stationary hoop located perpendicular to the incline in such a way that the ball moves parallel to the slope of the incline while going

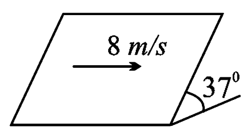
through the centre of the hoop. The centre of the hoop is 4 m high from the man's hand calculate the time taken by the ball to reach the hoop.



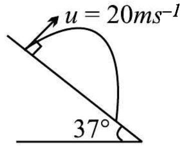
23. A rifle with a muzzle velocity of 100m/s shoots a bullet at small target 30m away in the same horizontal line. How high above the target must the gun be aimed so that the bullet will hit the target.
(**Hint:** Use small angle approximation.)
24. A man can throw a stone with initial speed of 10 m/s. Find the maximum horizontal distance to which he can throw the stone in a room of height h m for : (i) $h = 2$ m & (ii) $h = 4$ m
25. A particle is projected in x-y plane with y-axis along vertical, the point of projection being origin. The equation of a projectile is $y = \sqrt{3}\,x - \frac{gx^2}{2}$. The angle of projectile is ________ and initial velocity is __________.
26. A particle is projected in the X-Y plane with y-axis along vertical. 2 sec after projection the velocity of the particle makes an angle 45° with the X-axis. 4 sec after projection, it moves horizontally. Find the velocity of projection.
27. A particle is projected upwards with a velocity of 100 m/sec at an angle of 60° with the vertical. Find the time when the particle will move perpendicular to its initial direction, taking $g = 10$ m/sec^2.
28. A ball is projected at an angle of 30° above with the horizontal from the top of a tower and strikes the ground in 5 sec at an angle of 45° with the horizontal . Find the height of the tower and the speed with which it was projected.
29. In the figure shown, the two projectiles are fired simultaneously. What should be the initial speed of the left side projectile for the two projectiles to hit in mid-air?



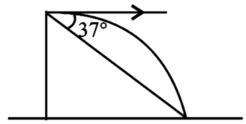
30. A ball starts from rest and accelerates at 0.500 m/s^2 while moving down an inclined plane 9.00 m long. When it reaches the bottom, the ball rolls up another plane, where, after moving 15.0 m, it comes to rest. (a) What is the speed of the ball at the bottom of the first plane? (b) How long does it take to roll down the first plane? (c) What is the acceleration along the second plane? (d) What is the ball's speed after moving through 8.00 m up the second plane?
31. A ball is projected on smooth inclined plane in direction perpendicular to line of greatest slope with velocity of 8m/s. Find it's speed after 1 sec.



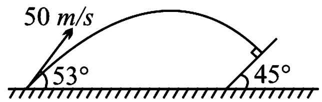
32. Find range of projectile on the inclined plane which is projected perpendicular to the incline plane with velocity 20m/s as shown in figure.



33. A ball is thrown horizontally from a cliff such that it strikes ground after 5 sec. The line of sight from the point of projection to the point of hitting makes an angle of 37° with the horizontal. What is the initial velocity of projection.



34. A rocket is launched at an angle 53ţ to the horizontal with an initial speed of 100 ms^{-1}. It moves along its initial line of motion with an acceleration of 30 ms^{-2} for 3 seconds. At this time its engine falls & the rocket proceeds like a free body. Find :
(i) the maximum altitude reached by the rocket
(ii) total time of flight.
(iii) the horizontal range.



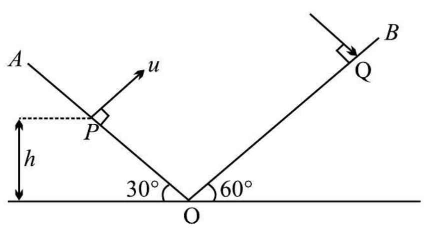
35. The speed of a particle when it is at its greatest height is √2/5 of its speed when it is at its half the maximum height. The angle of projection is ______ and the velocity vector angle with the horizontal at half the maximum height is ______ .

36. A projectile is thrown with velocity of 50 m/s towards an inclined plane from ground such that it strikes the inclined plane perpendicularly. The angle of projection of the projectile is 53° with the horizontal and the inclined plane is inclined at an angle of 45° to the horizontal.
(a) Find the time of flight.
(b) Find the distance between the point of projection and the foot of inclined plane.

37. A projectile is thrown from a platform at a height 10 m above the ground with velocity of 20 m/sec. At what angle should the projectile be thrown to reach the farthest point from O, which is vertically below the point from which it is thrown. [$g = 10\ m/s^2$]

38. Two guns, situated at the top of a hill of height 10 m, fire one shot each with the same speed 5√3 m/s at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at a point P. Find
(a) the time interval between the firings, and
(b) the coordinates of the point P. Take origin of the coordinates system at the foot of the hill right below the muzzle and trajectories in X-Y plane.

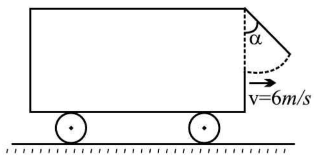
39. Two inclined planes OA and OB having inclination (with horizontal) 30° and 60° respectively, intersect each other at O as shown in fig. A particle is projected from point P with velocity $u = 10\sqrt{3}$ m s^{-1} along a direction perpendicular to plane OA. If the particle strikes plane OB perpendicularly at Q, calculate
(a) time of flight,
(b) velocity with which particle strikes the plane OB,
(c) vertical height h of P from O,
(d) maximum height from O attained by the particle
(e) distance PQ



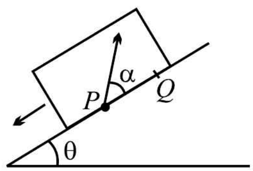
40. A hunter is riding an elephant of height 4m moving in straight line with uniform speed of 2m/sec. A deer running with a speed V in front at a distance of $4\sqrt{5}$m moving perpendicular to the direction of motion of the elephant. If hunter can throw his spear with a speed of 10m/sec. relative to the elephant, then at what angle θ to it's direction of motion must he throw his spear horizontally for a successful hit. Find also the speed 'V' of the deer.

41. A, B & C are three objects each moving with constant velocity. A's speed is 10 m/sec in a direction $\overrightarrow{PQ}$. The velocity of B relative to A is 6 m/sec at an angle of, $\cos^{-1}(15/24)$ to PQ. The velocity of C relative to B is 12 m/sec in a direction $\overrightarrow{QP}$, then find the magnitude of the velocity of C.

42. A glass wind screen whose inclination with the vertical can be changed, is mounted on a cart as shown in figure. The cart moves uniformly along the horizontal path with a speed of 6 m/s. At what maximum angle α to the vertical can the wind screen be placed so that the rain drops falling vertically downwards with velocity 2 m/s, do not enter the cart?



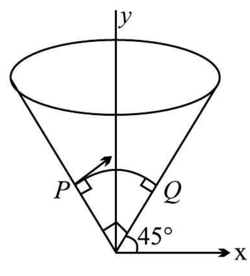
43. A large heavy box is sliding without friction down a smooth plane of inclination θ. From a point P on the bottom of a box, a particle is projected inside the box. The initial speed of the particle with respect to box is u and the direction of projection makes an angle α with the bottom as shown in figure.



(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands. (Assume that the particle does not hit any other surface of the box. Neglect air resistance).

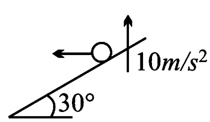
(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

44. A particle is projected from point P with velocity $5\sqrt{2}$ m/s perpendicular to the surface of a hollow right angle cone whose axis is vertical. It collides at Q normally. Find the time of the flight of the particle.



45. The horizontal range of a projectiles is R and the maximum height attained by it is H. A strong wind now begins to blow in the direction of motion of the projectile, giving it a constant horizontal acceleration = $g/2$. Under the same conditions of projection, find the horizontal range of the projectile.

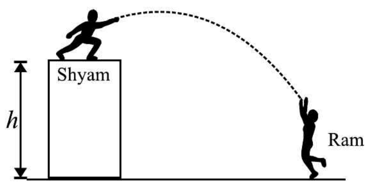
46. A particle is thrown horizontally with relative velocity 10 m/s from an inclined plane, which is also moving with acceleration 10 m/s^2 vertically upward. Find the time after which it lands on the plane (g = 10 m/s^2)



47. A particle is projected with a velocity $2\sqrt{ag}$ so that it just clears two walls of equal height 'a' which are at a distance '$2a$' apart. Show that the time of passing between the walls is $2\sqrt{a/g}$.

48. A stone is projected from the point of a ground in such a direction so as to hit a bird on the top of a telegraph post of height h and then attain the maximum height $2h$ above the ground. If at the instant of projection, the bird were to fly away horizontally with a uniform speed, find the ratio between the horizontal velocities of the bird and the stone, if the stone still hits the bird while descending.

49. Two persons Ram and Shyam are throwing ball at each other as shown in the figure. The maximum horizontal distance from the building where Ram can stand and still throw a ball at Shyam is d_1. The maximum horizontal distance of Ram from the building where Shyam can throw a ball is d_2. If both of them can throw ball with a velocity of $\sqrt{2gk}$, find the ratio of d_1/d_2. Neglect the height of each person.



Exercise–2: Objective Problems

1. Two stones are projected from the same point with same speed making angles $45^\circ + \theta$ and $45^\circ - \theta$ with the horizontal respectively. If $\theta < 45^\circ$, then the horizontal ranges of the two stones are in the ratio of
   (A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 1 : 4

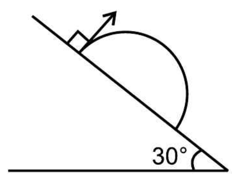
2. A hunter takes an aim at a monkey sitting an a tree and fires a bullet. Just when the bullet leaves barrel of the gun, it so happens that the monkey begins to fall freely. The bullet will
   (A) Go above the monkey
   (B) Go below the monkey
   (C) Hit the monkey
   (D) May or may not hit the monkey. It will depend upon the velocity of the bullet

3. It was calculated that a shell when fired form a gun with a certain velocity and at an angle of elevation $\frac{5\pi}{36}$ rad should strike a given target. In actual practice, it was found that a hill just prevented the trajectory. At what angle of elevation should the gun be fired to hit the target
   (A) $\frac{5\pi}{36}$ rad (B) $\frac{11\pi}{36}$ rad (C) $\frac{7\pi}{36}$ rad (D) $\frac{13\pi}{36}$ rad

4. A projectile is thrown with a speed v at an angle θ with the vertical. Its average velocity between the instants it crosses half the maximum height is
   (A) $v \sin \theta$, horizontal and in the plane of projection
   (B) $v \cos \theta$, horizontal and in the plane of projection
   (C) $2v \sin \theta$, horizontal and perpendicular to the plane of projection
   (D) $2v \cos \theta$, vertical and in the plane of projection

5. Two bullets are fired horizontally, simultaneously and with different velocities from the same place. Which bullet will hit the ground earlier?
   (A) It would depend upon the weights of the bullets
   (B) The slower one
   (C) The faster one
   (D) Both will reach simultaneously
6. A stone is thrown upwards. It returns to ground describing a parabolic path. Which of the following remains constant?
   (A) Speed of the ball (B) Kinetic energy of the ball
   (C) Vertical component of velocity (D) Horizontal component of velocity.
7. A body is thrown horizontally with a velocity $\sqrt{2gh}$ from the top of a tower of height h. It strikes the level ground through the foot of the tower at a distance x from the tower. The value of x is
   (A) h (B) $\frac{h}{2}$ (C) $2h$ (D) $\frac{2h}{3}$
8. A particle, with an initial velocity v_0 in a plane, is subjected to a constant acceleration in the same plane. Then, in general, the path of the particle could be
   (A) A circle (B) A straight line (C) A parabola (D) A hyperbola
9. A ball is projected from a certain point on the surface of a planet at a certain angle with the horizontal surface. The horizontal and vertical displacement x and y vary with time t in second as : $x = 10\sqrt{3}t$ and $y = 10t - t^2$
   The maximum height attained by the ball is
   (A) 100 m (B) 75 m (C) 50 m (D) 25 m
10. A bag is dropped from an aeroplane flying horizontally at a constant speed. If air resistance is ignored, where will the aeroplane be when the bag hits the ground?
   (A) Ahead of the bag (B) Directly above the bag
   (C) Far behind the bag (D) Data is not sufficient
11. The path of one projectile in motion as seen from another moving projectile is
   (A) A straight line (B) A circle
   (C) An ellipse (D) A parabola
12. A plane surface is inclined making an angle θ with the horizontal. From the bottom of this inclined plane, a bullet is fired with velocity v. The maximum possible range of the bullet on the inclined plane is
   (A) $\frac{v^2}{g}$ (B) $\frac{v^2}{g(1+\sin\theta)}$
   (C) $\frac{v^2}{g(1-\sin\theta)}$ (D) $\frac{v^2}{g(1+\cos\theta)}$
13. A ball is projected horizontal with a speed v from the top of a plane inclined at an angle 45ţ with the horizontal. How far from the point of projection with the ball strike the plane?
   (A) $\frac{v^2}{g}$ (B) $\sqrt{2}\,\frac{v^2}{g}$ (C) $\frac{2v^2}{g}$ (D) $\sqrt{2}\left[\frac{2v^2}{g}\right]$

14. The time of flight of a projectile on an upward inclined plane depends upon
    (A) Angle of inclination of the plane
    (B) Angle of projection
    (C) The value of acceleration due to gravity
    (D) All of these
15. A ball rolls of the top of a stairway horizontally with a velocity of 4.5 m s^{-1}. Each step is 0.2 m high and 0.3 m wide. If g is 10 ms^{-1}, then the ball will strike the nth step where n is equal to
    (A) 9 (B) 10 (C) 11 (D) 12
16. The velocity of projection of a projectile is $(6\hat{i} + 8\hat{j})$ ms^{-1}. The horizontal range of the projectile is
    (A) 4.9 m (B) 9.6 m (C) 19.6 m (D) 14 m
17. If R and h represent the horizontal range and maximum height respectively of an obliquie projectile, then $\frac{R^2}{8h} + 2h$ represents
    (A) Maximum horizontal range (B) Maximum vertical range
    (C) Time of flight (D) Velocity of projectile at highest point
18. A particle move along the parabolic path $x = y^2 + 2y + 2$ in such a way that the y-component of velocity vector remain 5m/s during the motion. The magnitude of the accleration of the particle is
    (A) 50 m/s^2 (B) 100 m/s^2
    (C) $10\sqrt{2}$ m/s^2 (D) 0.1 m/s^2
19. A ball is projected from point A with a velocity 10 m/s perpendicular to the inclined plane as shown in figure. Range of the ball on the inclined plane is

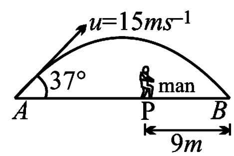


    (A) $\frac{40}{3}$ m (B) $\frac{20}{13}$ m
    (C) $\frac{13}{20}$ m (D) $\frac{13}{40}$ m
20. Two projectiles A and B are thrown with the same speed such that A makes angle θ with the horizontal and B makes angle θ with the vertical, then
    (A) Both must have same time of flight
    (B) Both must achieve same maximum height
    (C) A must have more horizontal range than B
    (D) Both may have same time of flight
21. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?
    (A) The one with the farthest range.
    (B) The one which reaches maximum height.
    (C) The one with the greatest initial velocity.
    (D) The one leaving the bat at 45° with respect to the ground.

Question No. 22 to 24 (3 questions)

A projectile is thrown with a velocity of 50 ms^{-1} at an angle of 53° with the horizontal

22. Choose the incorrect statement
    (A) It travels vertically with a velocity of 40 ms^{-1}
    (B) It travels horizontally with a velocity of 30 ms^{-1}
    (C) The minimum velocity of the projectile is 30 ms^{-1}
    (D) None of these
23. Determine the instants at which the projectile is at the same height
    (A) $t = 1$s and $t = 7$s (B) $t = 3$s and $t = 5$s
    (C) $t = 2$s and $t = 6$ s (D) all the above
24. The equation of the trajectory is given by
    (A) $180y = 240x - x^2$ (B) $180y = x^2 - 240x$
    (C) $180y = 135x - x^2$ (D) $180y = x^2 - 135x$
25. A particle is projected from a horizontal plane (x-z plane) such that its velocity vector at time t is given by $\vec{V} = a\hat{i} + (b - ct)\hat{j}$. Its range on the horizontal plane is given by
    (A) $\frac{ba}{c}$ (B) $\frac{2ba}{c}$ (C) $\frac{3ba}{c}$ (D) None
26. A ball is thrown from a point on ground at some angle of projection. At the same time a bird starts from a point directly above this point of projection at a height h horizontally with speed u. Given that in its flight ball just touches the bird at one point. Find the distance on ground where ball strikes
    (A) $2u\sqrt{\frac{h}{g}}$ (B) $u\sqrt{\frac{2h}{g}}$ (C) $2u\sqrt{\frac{2h}{g}}$ (D) $u\sqrt{\frac{h}{g}}$
27. A projectile is fired with a speed u at an angle θ with the horizontal. Its speed when its direction of motion makes an angle 'α' with the horizontal is
    (A) $u \sec\theta \cos\alpha$ (B) $u \sec\theta \sin\alpha$
    (C) $u \cos\theta \sec\alpha$ (D) $u \sin\theta \sec\alpha$
28. A ball is hit by a batsman at an angle of 37° as shown in figure. The man standing at P should run at what minimum velocity so that he catches the ball before it strikes the ground. Assume that height of man is negligible in comparison to maximum height of projectile.

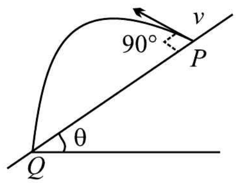


    (A) 3 ms^{-1} (B) 5 ms^{-1}
    (C) 9 ms^{-1} (D) 12 ms^{-1}
29. A ball is projected from top of a tower with a velocity of 5 m/s at an angle of 53ĩ to horizontal. Its speed when it is at a height of 0.45 m from the point of projection is
    (A) 2 m/s (B) 3 m/s
    (C) 4 m/s (D) Data insufficient.
30. Particle is dropped from the height of 20m from horizontal ground. A constant force acts on the particle in horizontal direction due to which horizontal acceleration of the particle becomes 6 ms^{-2}. Find the horizontal displacement of the particle till it reaches ground.
    (A) 6 m (B) 10 m (C) 12 m (D) 24 m

31. Find time of flight of projectile thrown horizontally with speed 10 ms^{-1} from a long inclined plane which makes an angle of $\theta = 45°$ from horizontal.
(A) $\sqrt{2}$ sec (B) $2\sqrt{2}$ sec (C) 2 sec (D) None

32. A projectile is fired with a velocity at right angle to the slope which is inclined at an angle θ with the horizontal. The expression for the range R along the incline is
(A) $\frac{2v^2}{g}\sec\theta$ (B) $\frac{2v^2}{g}\tan\theta$ (C) $\frac{2v^2}{g}\tan\theta\sec\theta$ (D) $\frac{v^2}{g}\tan^2\theta$

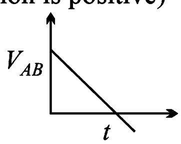
33. If time taken by the projectile to reach Q is T, than PQ =

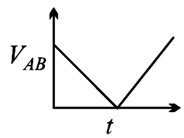


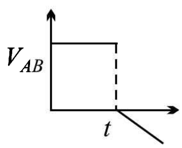
(A) $Tv\sin\theta$ (B) $Tv\cos\theta$ (C) $Tv\sec\theta$ (D) $Tv\tan\theta$

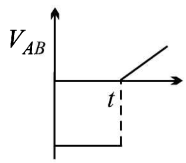
34. It takes one minute for a passenger standing on an escalator to reach the top. If the escalator does not move it takes him 3 minute to walk up . How long will it take for the passenger to arrive at the top if he walks up the moving escalator?
(A) 30 sec (B) 45 sec (C) 40 sec (D) 35 sec

35. A body A is thrown vertically upwards with such a velocity that it reaches a maximum height of h. Simultaneously another body B is dropped from height h. It strikes the ground and does not rebound. The velocity of A relative to B v/s time graph is best represented by : (upward direction is positive)

(A) 

(B) 

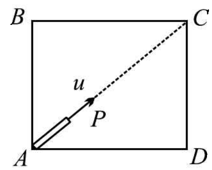
(C) 

(D) 

36. A body is thrown up in a lift with a velocity u relative to the lift and the time of flight is found to be t. The acceleration with which the lift is moving up is
(A) $\frac{u - gt}{t}$ (B) $\frac{2u - gt}{t}$ (C) $\frac{u + gt}{t}$ (D) $\frac{2u + gt}{t}$

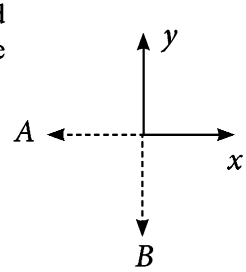
37. A hunter tries to hunt a monkey with a small, very poisonous arrow, blown from a pipe with initial speed v_0. The monkey is hanging on a branch of a tree at height H above the ground. The hunter is at a distance L from the bottom of the tree. The monkey sees the arrow leaving the blow pipe and immediately loses the grip on the tree, falling freely down with zero initial velocity. The minimum initial speed v_0 of the arrow for hunter to succeed while monkey is in air :
(A) $\sqrt{\frac{g(H^2+L^2)}{2H}}$ (B) $\sqrt{\frac{gH^2}{\sqrt{H^2+L^2}}}$ (C) $\sqrt{\frac{g(H^2+L^2)}{H}}$ (D) $\sqrt{\frac{2gH^2}{\sqrt{H^2+L^2}}}$

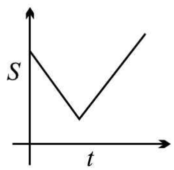
38. A large rectangular box moves vertically downward with an acceleration a. A toy gun fixed at A and aimed towards C fires a particle P.

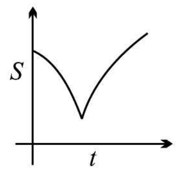


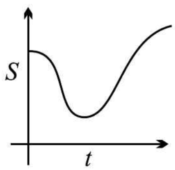
(A) P will hit C if $a = g$
(B) P will hit the roof BC, if $a > g$
(C) P will hit the wall CD if $a < g$
(D) May be either (A), (B) or (C), depending on the speed of projection of P

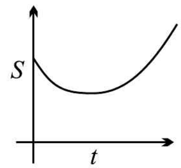
39. Particles A and B are moving with constant velocities along x and y axis respectively, the graph of separation between them with time is



(A) 

(B) 

(C) 

(D) 

Question No. 40 to 42 (3 questions)

Two projectiles are thrown simultaneously in the same plane from the same point. If their velocities are v_1 and v_2 at angles θ_1 and θ_2 respectively from the horizontal, then answer the following questions

40. The trajectory of particle 1 with respect to particle 2 will be
(A) A parabola
(B) A straight line
(C) A vertical straight line
(D) A horizontal straight line

41. If $v_1\cos\theta_1 = v_2\cos\theta_2$, then choose the incorrect statement
(A) One particle will remain exactly below or above the other particle
(B) The trajectory of one with respect to other will be a vertical straight line
(C) Both will have the same range
(D) None of these

42. If $v_1\sin\theta_1 = v_2\sin\theta_2$, then choose the incorrect statement
(A) The time of flight of both the particles will be same
(B) The maximum height attained by the particles will be same
(C) The trajectory of one with respect to another will be a horizontal straight line
(D) None of these

43. Average velocity of a particle is projectile motion between its starting point and the highest point of its trajectory is : (projection speed = u, angle of projection from horizontal= θ)
(A) $u\cos\theta$
(B) $\frac{u}{2}\sqrt{1+3\cos^2\theta}$
(C) $\frac{u}{2}\sqrt{2+\cos^2\theta}$
(D) $\frac{u}{2}\sqrt{1+\cos^2\theta}$

44. A ball is dropped from height 5m. The time after which ball stops rebounding if coefficient of restitution between ball and ground $e = 1/2$, is
(A) 1 sec
(B) 2 sec
(C) 3 sec
(D) infinite

45. A particle is projected vertically upwards from O with velocity v and a second particle is projected at the same instant from P (at a height h above O) with velocity v at an angle of projection θ. The time when the distance between them is minimum is

(A) $\dfrac{h}{2v\sin\theta}$ (B) $\dfrac{h}{2v\cos\theta}$ (C) h/v (D) $h/2v$

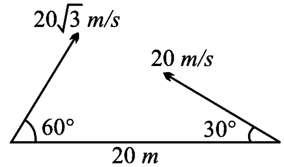
46. A ball is projected from ground with a velocity V at an angle θ to the vertical. On its path it makes an elastic collison with a vertical wall and returns to ground. The total time of flight of the ball is

(A) $\dfrac{2v\sin\theta}{g}$ (B) $\dfrac{2v\cos\theta}{g}$ (C) $\dfrac{v\sin 2\theta}{g}$ (D) $\dfrac{v\cos\theta}{g}$

47. Two particles are moving along two long straight lines, in the same plane, with the same speed = 20 cm/s. The angle between the two lines is 60°, and their intersection point is O. At a certain moment, the two particles are located at distances 3m and 4m from O, and are moving towards O. Subsequently, the shortest distance between them will be

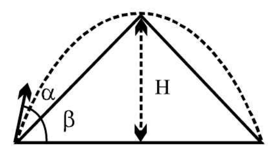
(A) 50 cm (B) $40\sqrt{2}$ cm (C) $50\sqrt{2}$ cm (D) $50\sqrt{3}$ cm

48. In the figure shown, the two projectiles are fired simultaneously. The minimum distance between them during their flight is



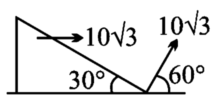
(A) 20 m (B) $10\sqrt{3}$ m
(C) 10 m (D) None

49. A shell fired from the base of a mountain just clears it. If α is the angle of projection then the angular elevation of the summit β is



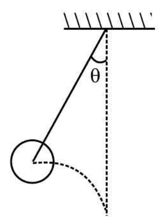
(A) $\dfrac{1}{2}\alpha$ (B) $\tan^{-1}(1/2)$
(C) $\tan^{-1}(1/2 \tan\alpha)$ (D) $\tan^{-1}(2\tan\alpha)$

50. A particle is projected at angle 60° with speed $10\sqrt{3}$, from the point 'A' as shown in the figure. At the same time the wedge is made to move with speed $10\sqrt{3}$ towards right as shown in the figure. Then the time after which particle will strike with wedge is



(A) 2 sec (B) $2\sqrt{3}$ sec
(C) $\dfrac{4}{\sqrt{3}}$ sec (D) None

51 A ball is held in the position shown with string of length 1 m just taut and then projected horizontally with a velocity of 3 m/s. If the string becomes taut again when it is vertical, angle θ is given by



(A) 53° (B) 30°
(C) 45° (D) 37°

52. An aeroplane flying at a constant velocity releases a bomb. As the bomb drops down from the aeroplane,

(A) It will always be vertically below the aeroplane
(B) It will always be vertically below the aeroplane only if the aeroplane is flying horizontally

(C) It will always be vertically below the aeroplane only if the aeroplane is flying at an angle of 45° to the horizontal

(D) It will gradually fall behind the aeroplane if the aeroplane is flying horizontally

53. Two particles are projected simultaneously in the same vertical plane, from the same point on ground, but with same speeds but at different angles (< 90°) to the horizontal. The path followed by one, as seen by the other, is

(A) A vertical straight line

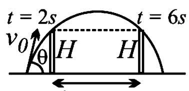
(B) A straight line making a constant angle with the horizontal

(C) A parabola

(D) A hyperbola

Question No. 54 to 59 (6 questions)

A projectile crosses two walls of equal height H symmetrically as shown in the figure



54. The time of flight T is given by

(A) 8 s (B) 9 s (C) 7 s (D) 10 s

55. The height of each wall is

(A) 240 m (B) 120 m (C) 60 m (D) 30 m

56. The maximum height of the projectile is

(A) 120 m (B) 80 m

(C) 160 m (D) cannot be obtained

57. If the horizontal distance between the two walls is d = 120 m, then the range of the projectile is

(A) 240 m (B) 160 m

(C) 300 m (D) cannot be obtained

58. The angle of projection of the projectile is

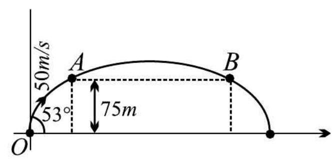
(A) $\tan^{-1}(3/4)$ (B) $\tan^{-1}(4/3)$ (C) $\tan^{-1}(4/5)$ (D) $\tan^{-1}(3/5)$

59. The velocity of projection is

(A) 30 ms^{-1} (B) 40 ms^{-1} (C) 50 ms^{-1} (D) none of these

Question No. 60 and 61 (2 questions)

At t = 0 a projectile is fired from a point O (taken as origin) on the ground with a speed of 50 m/s at an angle of 53° with the horizontal. It just passes two points A and B each at height 75 m above horizontal as shown in the figure.



60. The horizontal separation between the points A and B is

(A) 30 m (B) 60 m (C) 90 m (D) None

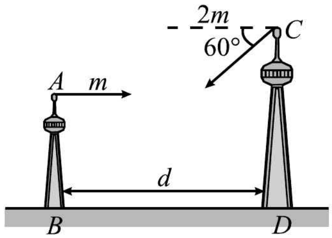
61. The distance (in metres) of the particle from origin at t = 2 sec.

(A) $60\sqrt{2}$ (B) 100 (C) 60 (D) 120

Exercise–3

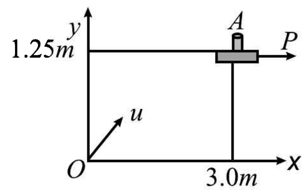
JEE/REE Questions of Previous Years

1. A ship is approaching a cliff of height 105 m above sea level. A gun fitted on the ship can fire shots with a speed of 110 ms^{-1}. Find the maximum distance from the foot of the cliff from where the gun can hit an object on the top of the cliff. [$g = 10\ m/s^2$] **[REE 1994, 2006]**
2. Two towers *AB* and *CD* are situated a distance ' d ' apart as shown in the fig. *AB* is 20 m high and *CD* is 30 m high from the ground. An object of mass ' m ' is thrown from the top of *AB* horizontally with a velocity of 10 m/s towards CD. Simultaneously another object of mass 2 m is thrown from the top of *CD* at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid air and stick to each other.



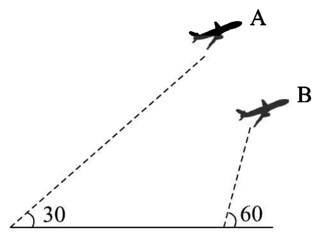
   (i) calculate the distance '*d*' between the towers.
   (ii) find the position where the objects hit the ground. **[JEE 1994, 2006]**
3. A building 4.8 m high 2*b* meters wide has a flat roof. A ball is projected from a point on the horizontal ground 14.4 m away from the building along its width. If projected with velocity 16 m/s at an angle of 45° with the ground, the ball hits the roof in the middle, find the width 2*b*. Also find the angle of projection so that the ball just crosses the roof if projected with velocity $10\sqrt{3}$ m/s. **[REE 1995, 2006]**
4. Two guns situated on the top of a hill of height 10 m, each fired shots with the same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontally and other fires upwards at an angle of 60° with the horizontal. The shot collide in air at a point P. Find: **[JEE 1996, 2005]**
   (i) the time interval between the firings and
   (ii) the coordinates of the point P.
   Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in *x*-*y* plane.
5. A vertical pole has a red mark at some height. A stone is projected from a fixed point on the ground. When projected at an angle of 45° it hits the pole orthogonally 1 m above the mark. When projected with a different velocity at an angle of $\tan^{-1}(3/4)$, it hits the pole orthogonally 1.5 m below the mark. Find the velocity and angle of projection so that it hits the mark orthogonally to the pole. [$g = 10\ m/sec^2$] **[REE 1996, 2006]**
6. The trajectory of a projectile in a vertical plane is $y = ax - bx^2$, where *a*, *b* are constants and *x* and *y* are respectively the horizontal and vertical distances of the projectile from the point of projection. The maximum height attained is _______ and the angle of projection from the horizontal is _______. **[JEE 1997, 2002]**
7. The coordinates of a particle moving in a plane are given by $x(t) = a\cos(pt)$ and $y(t) = b\sin(pt)$, where *a*, *b* ($< a$) and *p* are positive constants of appropriate dimensions. **[JEE 1999, 2002]**
   (A) The path of the particle is an ellipse
   (B) The velocity and acceleration of the particle are normal to each other at $t = \pi/2p$
   (C) The acceleration of the particle is always directed towards a focus
   (D) The distance travelled by the particle in time interval $t = 0$ to $= \pi/2p$ is *a*.

8. An object A is kept fixed at the point $x = 3$ m and $y = 1.25$ m on a plank P raised above the ground. At time $t = 0$ the plank starts moving along the + x direction with an acceleration 1.5 m/s^2. At the same instant a stone is projected from the origin with a velocity u as shown. A stationary person on the ground observes the stone hitting the object during its downward motion at an angle of 45ţ to the horizontal. All the motions are in x-y plane. Find 'u' and the time after which the stone hits the object. [Take $g = 10$ m/s^2] **[JEE 2000, 2005]**

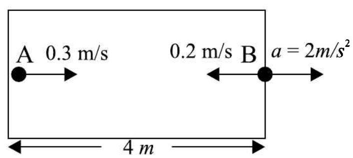


9. Shots fired simultaneously from the top and foot of a vertical cliff at elevations of 30^0 and 60^0 respectively, strike an object simultaneously which is at a height of 100 meters from the ground and at a horizontal distance of $200\sqrt{3}$ meters from the cliff. Find the height of the cliff, the velocities of projection of the shots and the time taken by the shots to hit the object. **[REE 2000, 2005]**

10. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ m/s. At time $t = 0$, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$. A just escape being hit by B, t_0 in second is **[JEE Advanced, 2014]**
[Ans. (5)**]**



11. A rocket is moving in a gravity free space with a constant acceleration of 2m/s^2 along +x direction (see figure). The length of a chamber inside the rocket is 4m. A bll is thrown from the left end of the chamber in +x direction with a speed of 0.3 m/s relative to the rocket. At the same time, another ball is thron in –x direction with a speed of 0.2 m/s from its right end relative to the rocket. The time in second when the two balls hit each other is



[JEE Advanced, 2014 (Integer Type)]

[Ans. (2)**]**

Previous Years' AIEEE Questions

1. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal.The kinetic energy of the ball at the highest point of its flight will be : **[AIEEE-2002, 4/300]**
   (A) E (B) $E/\sqrt{2}$ (C) $E/2$ (D) zero

2. A body playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30º with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground? [g = 10 m/s^2, sin 30º = 1/2, cos 30º = $\frac{\sqrt{3}}{2}$]
   [AIEEE-2002, 4/300]
   (A) 5.20 m (B) 4.33 m (C) 2.60 m (D) 8.66 m

3. A projectile can have the same range R for two angles of projection. If T_1 and T_2 be the time of flights in the two cases, then the product of the two times of flights is directly proportional to : **[AIEEE-2004, 05, 4/300]**
   (A) $1/R^2$ (B) $1/R$ (C) R (D) R^2
4. A ball is thrown from a point with a speed v_0 at angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $v_0/2$ to catch the ball? If yes, what should be the angle of projection? **[AIEEE-2004, 4/300]**
   (A) Yes, 60° (B) Yes, 30° (C) No (D) Yes, 45°
5. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[AIEEE-2007, 3/120]**
   (A) K (B) Zero (C) $K/4$ (D) $K/2$
6. A particle has an initial velocity of $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is : **[AIEEE-2009, 4/144]**
   (A) $7\sqrt{2}$ units (B) 7 units (C) 8.5 units (D) 10 units
7. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone upto will be **[AIEEE 2012]**
   (A) $20\sqrt{2}$ m (B) 10 m (C) $10\sqrt{2}$ m (D) 20 m
8. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If $g = 10\,\text{m/s}^2$, the equation of its trajectory is **[JEE Main 2013]**
   (A) $y = x - 5x^2$ (B) $y = 2x - 5x^2$ (C) $4y = 2x - 5x^2$ (D) $4y = 2x - 25x^2$

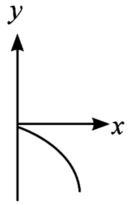
ANSWER KEY

Exercise - 1

1. Angle of projection = 45ţ
2. 90ţ
3. At the highest point.
4. sin θ : cos θ
5. $\sin^2\theta : \cos^2\theta$
6. (a) θ = 90ţ , (b) θ = 90^0 .
7. (a) vertically downwards (b) parabolic path
8. $t_1t_2 = 2R/g$
9. 10 m/s
10. $x\sqrt{\dfrac{g}{2h}}$
11. (i) 10 sec. (ii) 980 m; (iii) 138.59 m/s
12. tan θ = √3
13. (a) 26, 12 (b) –6, 12
14. (a) 16, 22 (b) 16, 2
15. No
16. (a) $-g\sin\beta$, (b) $-g\cos\beta$, (c) $u\cos\theta - g\sin\beta \times t$, (d) $u\sin\theta - g\cos\beta \times t$, (e) $u\cos\theta \times t - \dfrac{1}{2}g\sin\beta \times t^2$, (f) $u\sin\theta \times t - \dfrac{1}{2}g\cos\beta \times t^2$, (g) zero.

17. 2 s

18. $\dfrac{gt}{\tan\alpha - \tan\beta}$

19. (i) $y = -\dfrac{bx^2}{a^2}$ 

;

(ii) $\vec{v} = a\hat{i} - 2bt\hat{j}$, $\vec{\omega} = -2b\hat{j}$,

$|\vec{v}| = \sqrt{a^2 + 4b^2t^2}$, $|\vec{\omega}| = 2b$

20. $\dfrac{v}{g}\cos\left(\dfrac{\theta}{2}\right)\operatorname{cosec}\left(\dfrac{3\theta}{2}\right)$

21. 98 meters

22. 1 s

23. $h = 0.45$m

24. (i) $4\sqrt{6}$, (ii) 10m

25. 60, 2 m/sec.

26. $20\sqrt{5}$ m/s

27. 20 sec

28. $u = 50(\sqrt{3} - 1)$ m/sec., $H = 125(-\sqrt{3} + 2)$m

29. $20 \times \sqrt{2/3}$ m/s

30 (a) 3m/s (b) 6 sec. (c) –0.3 m/s^2 (d) 2.05 m/s

31. 10 m/s

32. 75 m

33. 100/3 m/s

34. (i) 1503.2m (ii) 35.54 sec (iii) 3970.56m

35. 60°, $\tan^{-1}\left(\sqrt{3/2}\right)$

36. (a) $t = 7$ sec, (b) 175 m

37. $\tan^{-1}\left(\sqrt{2/3}\right)$

38. (a) 1 sec, (b) ($5\sqrt{3}$ m, 5 m)

39. (a) 2 sec, (b) 10 ms^{-1}, (c) 5 m, (d) 16.25 m, (e) 20 m

40. $q = 37°$, $v = 6$m/s

41. 5 m/sec

42. 2 $\tan^{-1}(1/3)$

43. (a) $\dfrac{u^2\sin 2\alpha}{g\cos\theta}$, (b) $v = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$

44. 1 sec

45. $R + 2H$

46. $\dfrac{1}{\sqrt{3}}$ sec

48. $\dfrac{2}{\sqrt{2}+1}$

49. $\sqrt{\dfrac{k-h}{k+h}}$

Exercise - 2

1. A	2. C	3. D	4. A
5. D	6. D	7. C	8. B,C

9.	D	10.	B	11.	A	12.	B
13.	D	14.	D	15.	A	16.	B
17.	A	18.	A	19.	A	20.	D
21.	B	22.	A	23.	D	24.	A
25.	B	26.	C	27.	C	28.	B
29.	C	30.	C	31.	C	32.	C
33.	D	34.	B	35.	C	36.	B
37.	A	38.	A,B	39.	D	40.	B
41.	C	42.	D	43.	B	44.	C
45.	D	46.	B	47.	D	48.	C
49.	C	50.	A	51.	D	52.	A
53.	B	54.	A	55.	C	56.	B
57.	A	58.	B	59.	C	60.	B
61.	A						

Exercise - 3

1. 1100 m
2. (i) 17.32 m

   (ii) combined mass strikes at 11.55 m from B and 5.77 m from D
3. width of the roof is 9.6 m; $\theta = \tan^{-1}\left(\frac{3}{2}\right)$
4. (i) 1 s (ii) $(5\sqrt{3}, 5)$
5. $\frac{\sqrt{3620}}{3}$ m/s , $\tan^{-1}\left(\frac{9}{10}\right)$
6. $\frac{a^2}{4b}$, $\tan^{-1} a$
7. A, B
8. $u = 7.29$ m/s, $t = 1$ s.
9. 400 m, $V_T = 40\sqrt{3}$ m/s, $V_F = 40$ m/s, $T = 10$ s.

Previous Years' AIEEE Questions

1.	(C)	2.	(D)	3.	(C)	4.	(A)
5.	(C)	6.	(A)	7.	(D)	8.	(B)

RELATIVE MOTION

A person travelling by train ask his co-passenger 'Dilli aa gaya kya?' (Has Delhi come?). This statement seems to claim that Delhi is moving towards the passenger. We may say that his understanding of world is wrong as all of us know that 'actually' it is the train which is moving not the huge city of Delhi.

If we observe from passenger's point of view we will realise that relative to him all parts of train are at rest, which means relative to him train is at rest. Still when passenger gets down from train he is in different city, thus it must be the cities which are moving.

The point is that when a person standing on ground says that actually it is the train (hence passengers sitting in train) which is moving and reaching to Delhi (which of course is stationory), he is also right. The difference here is of refrence frame.

Motion is a combined property of the object under study and the observer. Motion is always relative, there is no such term like absolute motion or absolute rest. Motion is always defined with respect to an observer or reference frame.

Reference frame :

Reference frame is an axis system from which motion is observed. A clock is attached to measure time. Reference frame can be stationary or moving. There are two types of reference frame:

(i) **Inertial reference frame :** A frame of reference in which Newton's first law is valid is called as inertial reference frame.

(ii) **Non-inertial reference frame :** A frame of reference in which Newton's first law is not valid is called as non-inertial reference frame.

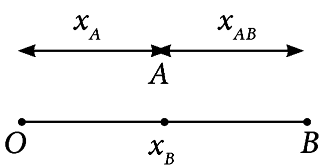
Note

Earth is by definition a non-inertial reference frame because of its centripetal acceleration towards sun. But, for small practical applications earth is assumed stationary hence, it behaves as an inertial reference frame.

Relative Velocity

Definition : Relative velocity of a particle (object) A with respect to B is defined as the velocity with which A appears to move is B if considered to be at rest. In other words, it is the velocity with which A appears to move as seen by the B considering itself to be at rest.

Relative Motion Along Straight Line:



$$v_A = \frac{dx_A}{dt}, \quad v_B = \frac{dx_B}{dt}$$

$$x_{BA} = x_B - x_A$$

$$v_{BA} = \frac{dx_B}{dt} - \frac{dx_A}{dt}$$

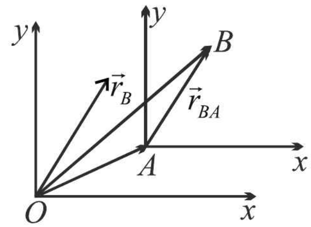
$$v_{BA} = v_B - v_A$$

$$\Rightarrow\ v_{AA} = v_A - v_A = 0 \text{ (velocity of A with respect to A)}$$

Note

Velocity of an object w.r.t. itself is always zero.

Explanatory Notes on Relative Motion Analysis



In two dimensional situation, consider three observers at rest O, A & B. Position of B from refrence frame of A is $\vec{r}_{BA}$. We must understand that if A wants to go and meet B he needs to look only at $\vec{r}_{BA}$ and decide direction of his velocity which will take him to B. For A, origin of refrence frame is not O it is A itself. From our knowledge of vectors we can deduce that

$$\vec{r}_B = \vec{r}_A + \vec{r}_{BA} \text{ thus } \vec{r}_{BA} = \vec{r}_B - \vec{r}_A \qquad ...(1)$$

Position vector of B w.r.t A is defined as $\vec{r}_{BA}$. Differentiating this equation w.r.t. time we get

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A \qquad ...(2)$$

On further differentiating we get

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A \qquad ...(3)$$

Lets see how we use this concept in our daily life.

Suppose that a car A travelling on a straight road at 80 km h^{-1} passes a car B going in the same direction at 60 km h^{-1}, (Fig 27(a)). Then, velocity of A relative to B is given by

$$\upsilon_A - \upsilon_B = 80 \text{ km h}^{-1} - 60 \text{ km h}^{-1} = 20 \text{ km h}^{-1} \text{ (to the right in (fig. 2.7(b))}$$

We know that if somebody passes us by in same direction we dont find them moving very fast.

If A and B are travelling in opposite directions, Fig. 2.7(c), we show this by giving one velocity + sign, say υ_A, and the other – sign. Hence we can write

$$\upsilon_A - \upsilon_B = +80 \text{ km h}^{-1} - (-60 \text{ km h}^{-1}) = (80 + 60) \text{ km h}^{-1}$$

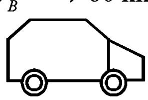
$$= 140 \text{ km h}^{-1} \text{ (to the right in Fig 2.7(d))}$$

We know that if somebody passes us by in opposite direction we find them moving really fast.

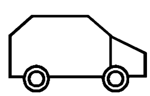
In effect, in both cases, to find the velocity of A relative to B, we have applied B's velocity reversed to both cars. It is then just as if B is at rest and A has two velocities υ_A and υ_B, which are subtracted when υ_A and υ_B are in the same direction and added when they are in opposite directions.

World as seen by observer on ground and World as seen by B

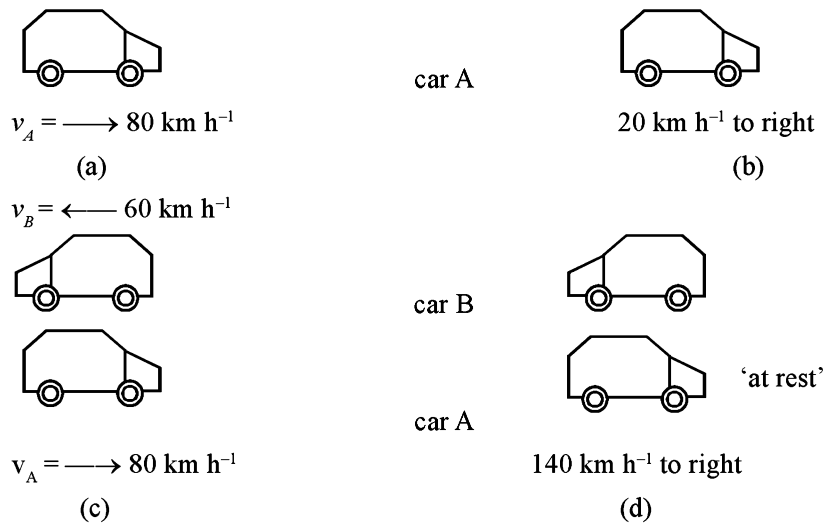
$v_B \longrightarrow$ 60 km h^{-1}



Car B



at rest



Concept

Relative acet. of bodies during motion under zero gravity .

Example 1. An object A is moving with 5 m/s and B is moving with 20 m/s in the same direction. (Positive x-axis)

(i) Find velocity of B with respect to A.

Solution $v_B = 20\hat{i}$ m/s $\Rightarrow$ $v_A = 5\hat{i}$ m/s $\Rightarrow$ $v_B - v_A = 15\hat{i}$ m/s

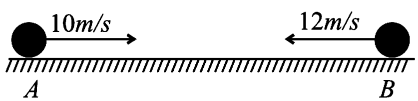
(ii) Find velocity of A with respect to B

Solution $v_B = 20\hat{i}$ m/s, $v_A = 15\hat{i}$ m/s $\Rightarrow$ $v_{AB} = v_A - v_B = -15\hat{i}$ m/s

Note

$v_{BA} = -v_{AB}$

Example 2. Two objects A and B are moving towards each other with velocities 10 m/s and 12 m/s respectively a shown.



(i) Find out velocity of A with respect to B.

Solution $v_{AB} = v_A - v_B = (10) - (-12) = 22$ m/s towards right.

(ii) Find out velocity of B with respect to A

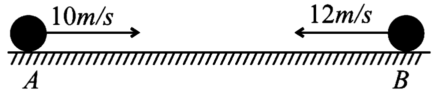
Solution $v_{BA} = v_B - v_A = (-12) - (10) = -22$ m/s towards left.

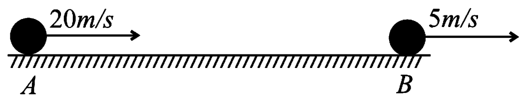
Velocity of Approach:

It is the rate at which a separation between two moving particles decreases.

If separation decreases velocity of approach is positive,

Velocity of approach = 22 m/s





Velocity of approach = 15 m/s

If separation increases, velocity of approach is negative. It is mainly called velocity of separation.

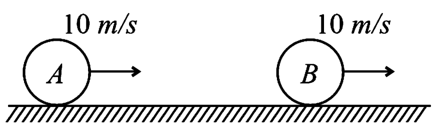
Velocity of separation:

It is the rate with which separation between two moving object increases.

Velocity of separation = 2 m/s

Velocity of separation = 15 m/s

Example 3. Two balls A and B are moving in the same direction with equal velocities, find out their relative velocity.



Velocity of A with respect to $B\left(\vec{v}_{AB}\right) = 0$

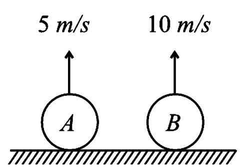
Example 4. A and B are thrown vertically upward with velocity, 5 m/s and 10 m/s respectively (g = 10 m/s². Find separation between them after one second.

Solution

$$S_A = ut - \frac{1}{2}gt^2 = 5t - \frac{1}{2} \times 10 \times t^2$$

$$= 5 \times 1 - 5 \times 1^2 = 5 - 5 = 0$$

$$S_B = ut - \frac{1}{2}gt^2 = 10 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 10 - 5 = 5$$



$\therefore \quad S_B - S_A$ = separation = 5m.

Alter:

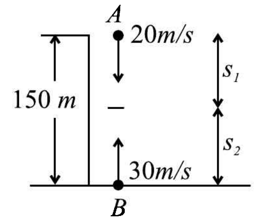
By relative $\quad \vec{a}_{BA} = \vec{a}_B - \vec{a}_A = (-10) - (-10) = 0$

Also $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 10 - 5 = 5$ m/s

∴ $\vec{s}_{BA}$ (in 1 sec) $= \vec{v}_{BA} \times t = 5 \times 1 = 5$ m

∴ Distance between A and B after 1 sec = 5 m.

Example 5. A ball is thrown downwards with a speed of 20 m/s from top of a building 150 m high and simultaneously another ball is thrown vertically upwards with a speed of 30 m/s from the foot of the building. Find the time when both the balls will meet. **(g= 10 m/s²)**



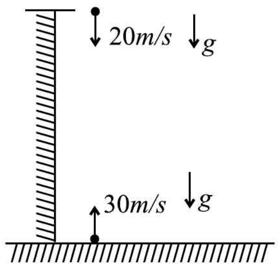
Solution

(I) $S_1 = 20t + 5t^2$

$+ \quad S_2 = 30t - 5t^2$

$150 = 50t$

⇒ $t = 3$ s.



(II) Relative acceleration of both is zero since both have acceleration in downward direction

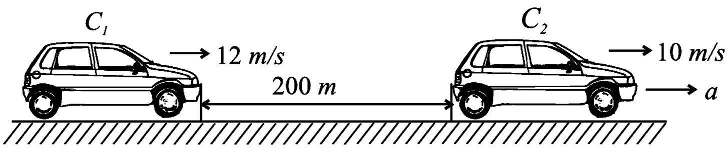
$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = g - g = 0$$

$$\vec{v}_{BA} = 30 - (-20) = 50$$

$$s_{BA} = v_{BA} \times t$$

$$t = \frac{s_{BA}}{v_{BA}} = \frac{150}{50} = 3 \text{ s}$$

Example 6. Two cars C_1 and C_2 moving in the same direction on a straight road with velocities 12 m/s and 10 m/s respectively. When the separation between the two was 200 m C_2 started accelerating to avoid collision. What is the minimum acceleration of car C_2 so that they don't collide.



Solution By relative

$$\vec{a}_{C_1C_2} = \vec{a}_{C_1} - \vec{a}_{C_2} = 0 - a = (-a)$$

$$\vec{v}_{C_1C_2} = \vec{v}_{C_1} - \vec{v}_{C_2} = 12 - 10 = 2 \text{ m/s.}$$

So by relativity we want the car to stop.

∴ $v^2 - u^2 = 2as$.

⇒ $0 - 2^2 = -2 \times a \times 200$ ⇒ $a = \frac{1}{100}$ m/s² = 0.1 m/s² = 1 cm/s².

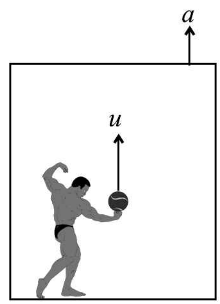
∴ Minimum acceleration needed by car C_2 = 1 cm/s²

Relative Motion in Lift

Example 7 A lift is moving up with acceleration a. A person inside the lift throws the ball upwards with a velocity u relative to hand.

(A) What is the time of flight of the ball?

(B) What is the maximum height reached by the ball in the lift?



Solution

(a) $\vec{a}_{BL} = \vec{a}_B - \vec{a}_L = (g + a)$ downwards

$$\vec{s} = \vec{u}\,t + \frac{1}{2}\vec{a}_{BL}t^2 \qquad 0 = uT - \frac{1}{2}(g+a)T^2$$

$$\therefore \qquad T = \frac{2u}{(g+a)}$$

(b) $v^2 - u^2 = 2\ as \qquad 0 - u^2 = -2(g + a)\,H$

$$H = \frac{u^2}{2(g+a)}$$

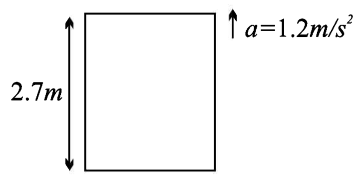
Example 8. An elevator car whose floor-to-ceiling distance is equal to 2.7 m starts ascending with constant acceleration 1.2 m/s^2 ; 2.0 s after the start a bolt begins falling from the ceiling of the car. Find:

(a) the bolt's free fall time;

(b) the displacement and the distance covered by the bolt during the free fall in the reference frame fixed to the ground.

{Explain accln of bolt is not same as that of the lift.}

[**Ans.** (A) 0.7 s; (B) 0.7 and 1.3 m respectively]



Solution

(a) $v = u + at$ (velocity after 2sec)

$v = +\,1.2 \times 2 \quad \because \quad u_{rel} = 0$

$v = 2.4$ m/s

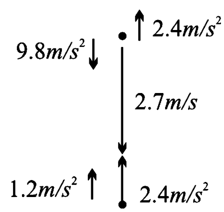
$S_{rel} = u_{rel}t + 1/2\, a_{rel}t^2$

$a_{rel} = 9.8 - (-\,1.2) = 11$ m/s

$+\,2.7 = u_{rel}\,t + 1/2\ 11\ t^2$

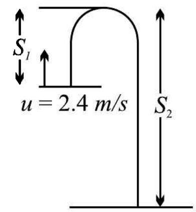
$\sqrt{\dfrac{2.7 \times 2}{11}} = t$

$t = 0.7$ sec



(b) (i) Displacement $s = ut + 1/2\ at^2$

$$2.4 \times 0.7 - \frac{1}{2} \times 9.8 \times 0.7^2 = 0.72 \text{ m}$$



(ii) Distance

$$v = u + at$$

$$0 = 2.4 - 9.8 \times t$$

$$t = 0.24 \text{ s.}$$

$$s_1 = 2.4 \times 0.24 - \frac{1}{2} \times 9.8 \times 0.24^2 = 0.3 \text{ m}$$

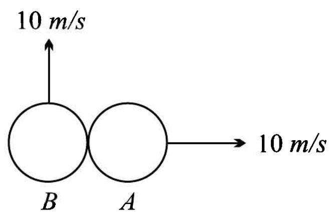
$$s_2 = \frac{1}{2} \times 9.8 \times 0.46^2 = 1.03 \text{ m}$$

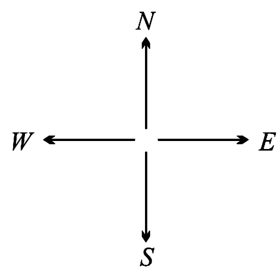
distance $= s_1 + s_2 = 1.3$ m]

Example 9. A transparent lift A is going upwards with velocity 20 ms^{-1} and retarding at the rate of 8 ms^{-2}. Second transparent lift B is located in front of it and is going down at 10 ms^{-1} with retardation of 2 ms^{-2}. At the same instant a bolt from the ceiling of lift A drops inside lift A. If height of car of lift A is 16 m then find the distance travelled by bolt as observed by a person in lift B till the time it collides with floor of lift A.

Solution [**Ans.** 51 m]

Example 10 Object A and B has velocities 10 m/s. A is moving along East while B is moving towards North from the same point as shown. Find velocity of A relative to $B(\vec{v}_{AB})$.

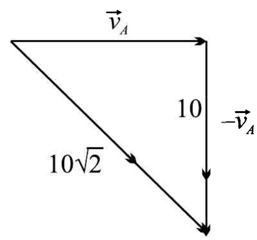




Solution

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

$$\therefore \quad |\vec{v}_{AB}| = 10\sqrt{2}$$



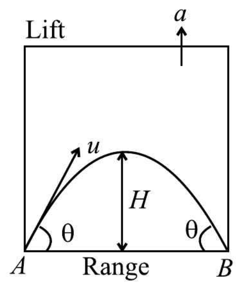
Note

$$|\vec{v}_A - \vec{v}_B| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

Projectile Motion in a Lift Moving with Acceleration a Upwards

(1) Initial velocity = u

(2) Velocity at maximum height = $u \cos\theta$

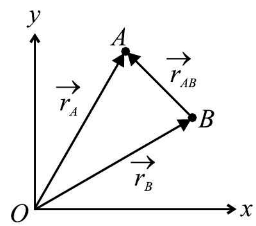


(3) $T = \dfrac{2u\sin\theta}{g+a}$

(4) Maximum height $(H) = \dfrac{u^2\sin^2\theta}{2(g+a)}$

(5) Range $= \dfrac{u^2\sin 2\theta}{g+a}$

Relative Motion in Two Dimensions



$\vec{r}_A$ = position of A with respect to O

$\vec{r}_B$ = position of B with respect to O

$\vec{r}_{AB}$ = position of A with respect to B.

$\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$

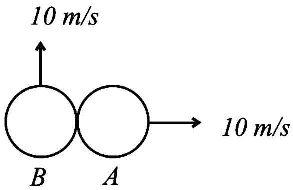
$\therefore \quad \dfrac{d(\vec{r}_{AB})}{dt} = \dfrac{d(\vec{r}_A)}{dt} - \dfrac{d(\vec{r}_B)}{dt}. \qquad \Rightarrow \vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

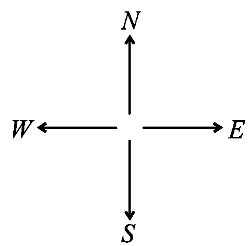
$\dfrac{d(\vec{v}_{AB})}{dt} = \dfrac{d(\vec{v}_A)}{dt} - \dfrac{d(\vec{v}_B)}{dt} \qquad \Rightarrow \vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

Note

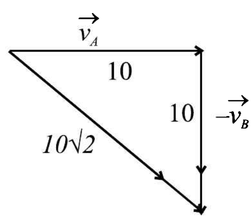
These formulae are not applicable for light.

Example 11. Object A and B has velocities 10 m/s. A is moving along East while B is moving towards North from the same point as shown. Find velocity of A relative to B ($\vec{v}_{AB}$)



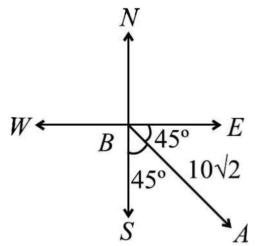


Solution $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B \therefore \qquad |\vec{v}_{AB}| = 10\sqrt{2}$

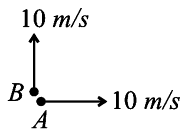


Note

$|\vec{v}_A - \vec{v}_B| = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$



Example 12 Two particles A and B are projected in air. A is thrown horizontally, B is thrown vertically up. What is the separation between them after 1 sec.



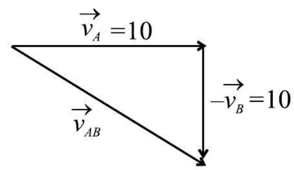
Solution

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = 0$

$\therefore \quad \vec{v}_{AB} = \sqrt{10^2 + 10^2} = 10\sqrt{2}$

$\therefore \quad s_{AB} = v_{AB}t = (10\sqrt{2})\, t = 10\sqrt{2}$ m

Consider the situation, shown in figure



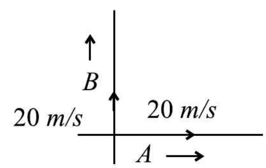
Example 13.

(i) Find out velocity of B with respect to A

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 20\hat{j} - 20\hat{i}$

(ii) Find out velocity of A with respect to B

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 20\hat{i} - 20\hat{j}$



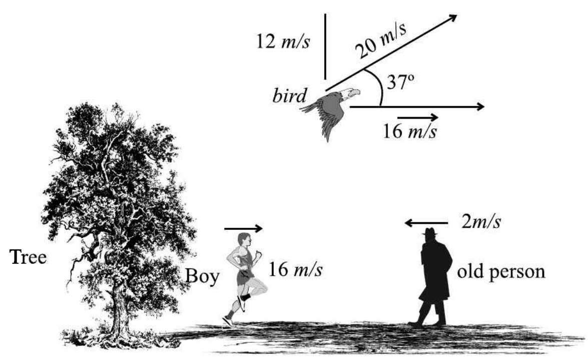
Example 14.

(1) Find out motion of tree, bird and old man as seen by boy.

(2) Find out motion of tree, bird, boy as seen by old man

(3) Find out motion of tree, boy and old man as seen by bird.

Solution



(1) With respect to boy :

$v_{\text{tree}} = 16$ m/s (←)

$v_{\text{bird}} = 12$ m/s (↑)

$v_{\text{old man}} = 18$ m/s (←)

(2) With respect to old man :

$v_{\text{Boy}} = 18$ m/s (→)

$v_{\text{Tree}} = 2$ m/s(→)

$v_{\text{Bird}} = 18$ m/s (→) and 12 m/s (↑)

(3) With respect to bird :

v_{Tree} = 12 m/s (↓) and 16 m/s (←)

$v_{\text{old man}}$ = 18 m/s (←) and 12 m/s (↓).

v_{Boy} = 12 m/s (↓).

Example 15.

(a) Find the out motion of tree, bird and old man as seen by boy.
(b) Find out motion of tree, bird, boy as seen by old man
(c) Find out motion of tree, boy and old man as seen by bird.

Solution

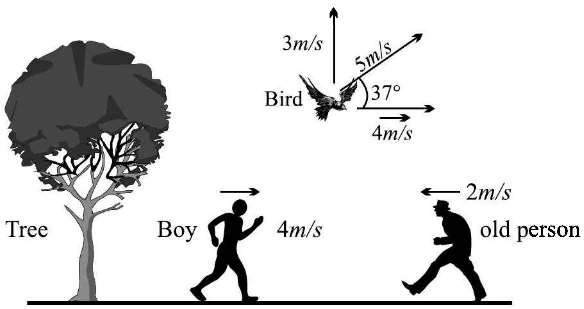
(a) With respect to boy :

V_{tree} = 4 m/s (←)

V_{bird} = 3 m/s (↑) & 0 m/s (→)

$V_{\text{old man}}$ = 6 m/s (←)

(b) With respect to old man :



V_{boy} = 6 m/s (→)

V_{tree} = 2 m/s (→)

V_{bird} = 6 m/s (→) and 3 m/s (↑)

(c) With respect to Bird :

V_{tree} = 3 m/s (↓) and 4 m/s (←)

$V_{\text{old man}}$ = 6 m/s (←) and 3 m/s (↓)

V_{boy} = 3 m/s (↓)

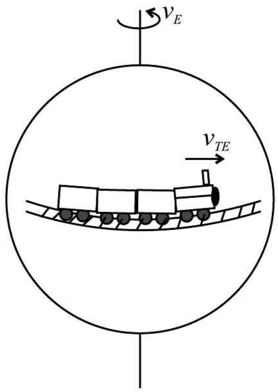
Motion of a Train Moving through Equator

If a train is moving at equator on the earth's surface with a velocity v_{TE} relative to earth's surface and a point on the surface of earth with velocity v_E relative to its centre, then

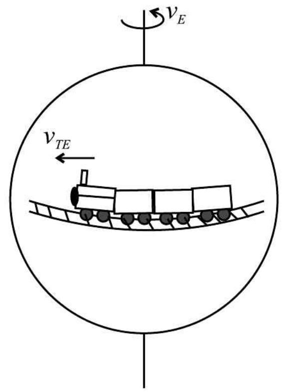
$\vec{v}_{TE} = \vec{v}_T - \vec{v}_E$ or $\vec{v}_T = \vec{v}_{TE} + \vec{v}_E$

So, if the train moves from west to east and if the train moves from east to west

(the direction of motion of earth on its axis) (i.e. opposite to the motion of earth)



$v_T = v_{TE} + v_E$



$v_T = v_{TE} - v_E$

Relative Motion of a Moving Train

If a boy is running with speed $\vec{v}_{BT}$ on a train moving with velocity $\vec{v}_T$ relative to ground, the speed of the boy relative to ground $\vec{v}_B$ will be given by:

$\vec{v}_{BT} = \vec{v}_B - \vec{v}_T$ or $\vec{v}_B = \vec{v}_{BT} + \vec{v}_T$

so, if the boy is running in the direction of train

$$v_B = u + v$$

and if the boy is running on the train in a direction opposite to the motion of train

$$v_B = u - v$$

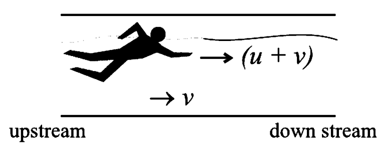
Relative Motion in River Flow

If a man can swim relative to water with velocity $\vec{v}_{mR}$ and water is following relative to ground with velocity $\vec{v}_R$, velocity of man relative to ground $\vec{v}_m$ will be given by :

$\vec{v}_{mR} = \vec{v}_m - \vec{v}_R$ or $\vec{v}_m = \vec{v}_{mR} + \vec{v}_R$

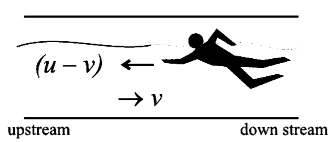
So, if the swimming is in the direction of flow of water,

$$v_m = v_{mR} + v_R$$



and if the swimming is opposite to the flow of water,

$$v_m = v_{mR} - v_R$$



Example 16. A swimmer capable of swimming with velocity v relative to water jumps in a flowing river having velocity u. The man swims a distance d down stream and returns back to the original position. Find out the time taken in complete motion.

Solution

$$t = t_{\text{down}} + t_{\text{up}}$$

$$= \frac{d}{v+u} + \frac{d}{v-u} = \frac{2dv}{v^2 - u^2}$$

Crossing River

A boat or man in a river always moves in the direction of resultant velocity of velocity of boat (or man) and velocity of river flow.

1. Shortest Time

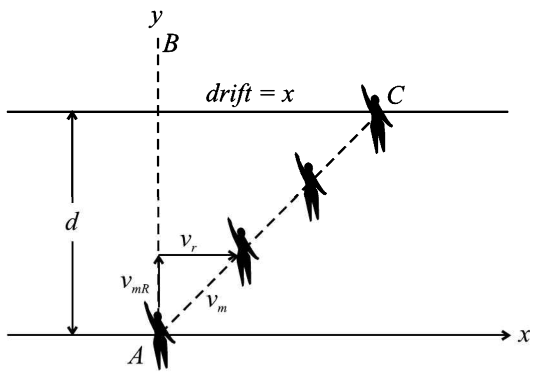
The person swims perpendicular to the river flow crossing a river : consider a river having flow velocity $\vec{v}_R$ and swimmer jump into the river from a point A, from one bank of the river, in a direction perpendicular to the direction of river current. Due to the flow velocity of river the swimmer is drifted along the river by a distance BC and the net velocity of the swimmer will be $\vec{v}_m$ along the direction AC.

If we find the components of velocity of swimmer along and perpendicular to the flow these are.

Velocity along the river, $v_x = v_R$.

Velocity perpendicular to the river, $v_f = v_{mR}$

The net speed is given by $v_m = \sqrt{v_{mR}^2 + v_R^2}$



at an angle of $\tan\theta = \frac{v_{mR}}{v_R}$ (down stream with the direction of flow).

Velocity of v_y is used only in crossing the river, time taken to cross the river is $t = \frac{d}{v_y} = \frac{d}{v_{mR}}$.

Velocity v_x is only used to drift the motion of the swimmer in the river, drift is along the river flow,

$x = (v_x)(t)$ or $x = v_R \frac{d}{v_{mR}}$

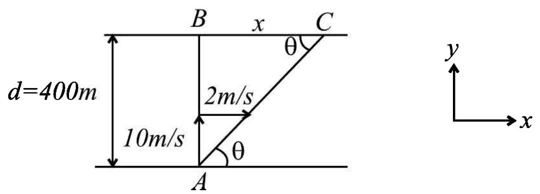
Example 17. A river 400 m wide is flowing at a rate of 2.0 m/s. A boat is sailing at a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river.

(a) Find the time taken by the boat to reach the opposite bank.

(b) How far from the point directly opposite to the starting point does the boat reach the opposite bank.

(c) In what direction does the boat actually move.

Solution



(a) Time taken to cross the river

$$t = \frac{d}{v_y} = \frac{400m}{10m/s} = 40\text{ s}$$

(b) Drift $(x) = (v_x)(t) = (2\text{m/s})(40\text{s}) = 80$ m

(c) Actual direction of boat,

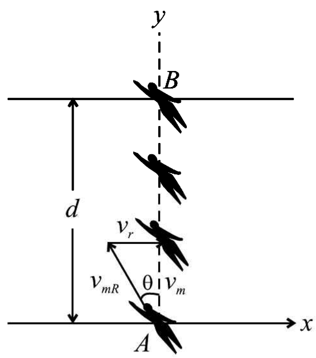
$$\theta = \tan^{-1}\left(\frac{10}{2}\right) = \tan^{-1} 5, \text{ (downstream) with the river flow.}$$

2. Shortest Path

When the person crosses the river perpendicularly (along the shortest path). It should swim up stream making an angle θ with AB such that the resultant velocity $\vec{v}_m$, of man must be perpendicular to the flow of river along AB.

If we find the components of velocity of swimmer along and perpendicular to the flow, these are,

velocity along the river, $v_x = 0$ and velocity perpendicular to river $v_y = \sqrt{v_{mR}^2 - v_R^2}$



The net speed is given by $v_m = \sqrt{v_{mR}^2 - v_R^2}$ at an angle of 90ŗ with the river direction.

velocity v_y is used only to cross the river, therefore time to cross the river,

$$t = \frac{d}{v_y} = \frac{d}{\sqrt{v_{mR}^2 - v_R^2}}$$

and velocity v_x is zero, therefore, in this case the drift (x) should be zero.

$$x = 0$$

or $v_x = v_R - v_{mR}\sin\theta = 0$

or $v_R = v_{mR}\sin\theta$

or $\theta = \sin^{-1}\left(\dfrac{v_R}{v_{mR}}\right)$

Hence, to cross the river perpendicular (along the shortest path) the man should swim at an angle of $\dfrac{\pi}{2} + \sin^{-1}\left(\dfrac{v_R}{v_{mR}}\right)$ upstream from the direction of river flow.

further, since $\sin\theta < 1$,

Swimmer can cross the river perpendicularly only when $v_{mR} > v_R$ ie

Practically it is not possible to reach at B if the river velocity (v_R) is too high.

Example 18. A man can swim at the rate of 5 km/h in still water. A river 1 km wide flows at the rate of 3 km/h. The man wishes to swim across the river directly opposite to the starting point.

(a) Along what direction must the man swim?

(b) What should be his resultant velocity?

(c) How much time the would take to cross?

Solution The velocity of man with respect to river v_{mR}= 5 km/hr, this is greater than the river flow velocity, therefore, he can cross the river directly (along the shortest path). The anlge of swim must be

$$\theta = \frac{\pi}{2} + \sin^{-1}\left(\frac{v_R}{v_{mR}}\right) = 90ţ + \sin^{-1}\left(\frac{v_r}{v_{mR}}\right)$$

$= 90ţ + \sin^{-1} = 90ţ + 37ţ = 127ţ$, with the river flow (upstream)

(b) Resultant velocity will be $v_m = \sqrt{v_{mR}^2 - v_R^2} = \sqrt{5^3 - 3^2} = 4$ km/hr

along the direction perpendicular to the river flow.

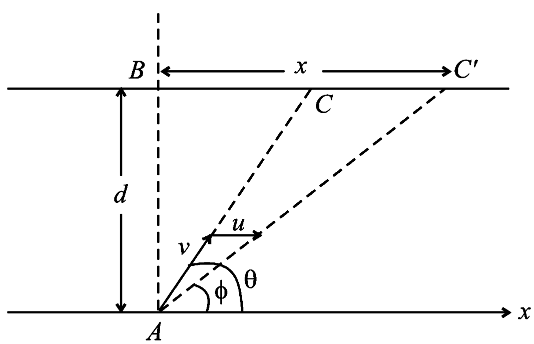
(c) Time taken to cross the

$$t = \frac{d}{\sqrt{v_{mR}^2 - v_R^2}} = \frac{1\,km}{4\,km/hr} = \frac{1}{4}\text{h} = 15 \text{ min}$$

Example 19. The velocity of about in still water is 5 km/h it crosses 1 km wide river in 15 minutes along the shortest possible path. Determine the velocity of water in the river in km/h.

Solution [**Ans.** 3km/h]

Example 20. A man wishes to cross a river flowing with velocity u jumps at an anlge θ with the river flow. Find out the net velocity of the man with respect to ground if he can swim with speed v. Also find how far from the point directly opposite to the starting point does the boat reach the opposite bank. In what direction does the boat actually move, if the width of the river is d.



Solution

(i) Velocity of man

$= v_M = \sqrt{u^2 + v^2 + 2vu\cos\theta}$

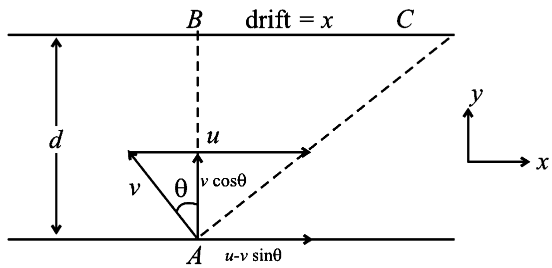
(ii) $\tan\phi = \dfrac{v\sin\theta}{u + v\cos\theta}$

$(v \sin\theta)\, t = d \quad \Rightarrow t = \dfrac{d}{v\sin\theta}$

(iii) $x = (u + v\cos\theta)\, t = (u + v\cos\theta)\dfrac{d}{v\sin\theta}$

Example 21. A boat moves relative to water with a velocity which is n times less than the river flow velocity. At what anlge to the stream direction must the boat move to minimize drifting?

Solution In this problem, one thing should be carefully meted that the velocity of boat is less than the river flow velocity. In such a case, boat cannot reach the point directly opposite to its starting point. i.e. drift can never be zero. Thus, to minimize the drift, boat starts at an angle θ from the normal direction up stream as shown.



Now, again if we find the components of velocity of boat along and perpendicular to the flow, these are, velocity along the river, $v_x = u - v\sin\theta$.

and velocity perpendicular to the river, $v_y = v\cos\theta$.

time taken to cross the river is $t = \dfrac{d}{v_y} = \dfrac{d}{v\cos\theta}$.

In this time, drift $x = (v_x)t = (u - v\sin\theta)\dfrac{d}{v\cos\theta}$

or $\quad x = \dfrac{ud}{v}\sec\theta - d\tan\theta$

The drift x is minimum, when $\dfrac{dx}{d\theta} = 0$,

or $\quad \left(\dfrac{ud}{v}\right)(\sec\theta \,.\, \tan\theta) - d\sec^2\theta = 0$

or $\quad \dfrac{u}{v}\sin\theta = 1$

or $\quad \sin\theta = \dfrac{v}{u} = \dfrac{1}{n}$ (as $v = \dfrac{u}{n}$)

so, for minimum drift, the boat must move at an angle $\theta = \sin^{-1}\left(\dfrac{v}{u}\right)$ from normal direction or an angle $\dfrac{\pi}{2} + \sin^{-1}\left(\dfrac{v}{u}\right)$ from stream direction.

Net Motion

Walking on a Conveyor belt {students will find it easier than river swimmer problem you can read river swimmer after this}

Example 22. A person crossing a 2.5m wide conveyor belt moves with a speed of 1.6 m/s. The conveyor belt moves at uniform speed of 1.2 m/s.

(A) If the Ram walks straight across the belt, determine the velocity of the Ram relative to an observer standing on ground.

Solution If you walk across a conveyor belt while the conveyor belt takes you along the length, you will not be able to move directly across the conveyor belt, but will end up down the length.

Here the velocity of the Ram will be net effect of his own motion and due to motion of conveyor belt

The velocity of the Ram relative to the conveyor belt v_{rc}, is same as velocity of Ram if conveyor belt was still ,

v_c is the velocity of the conveyor belt

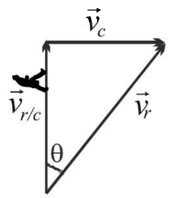
we need to find v_r, the velocity of the Ram relative to the Earth.

Writing Equation of net motion $v_r = v_{rc} + v_c$.

three vectors are shown in Figure (a).

The quantity v_{rc} is due y ; v_c is due x ; and the vector sum of the two, v_r, is at an angle θ as defined in Figure (a).

(a)

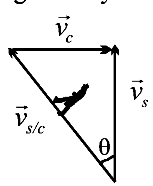


the speed υ_r of the Ram relative to the Earth is

$$\upsilon_r = \sqrt{\upsilon_{r/c}^2 + \upsilon_c^2}$$

(B) If Shyam has same speed on a still conveyor belt, and is to reach directly across the same moving conveyor belt. At what angle it should he walk?

(b)



Solution To go straight across the conveyor belt he have to walk he should walk at some angle.

Writing Equation of net motion $\boldsymbol{v}_s = \boldsymbol{v}_{s/c} + \boldsymbol{v}_c$.

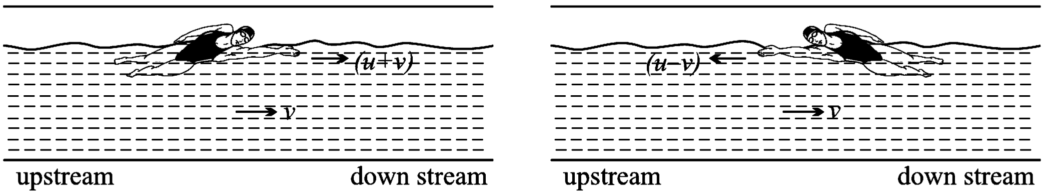
three vectors are shown in Figure (b)

As in part (b), we know v_c and the magnitude of the vector $v_{s/c}$, and we want v_c to be directed across the conveyor belt. Note the difference between the triangle in Figure (A) and the one in Figure (b),

$$\upsilon_s = \sqrt{v_{s/c}^2 - v_c^2}$$

When a man or a boat is swimming in water, he generates a velocity relative to water by his own efforts. Actual velocity of man in water will be a resultant of man's effort and the river velocity.

Down stream : Man makes efforts in direction of flow



Up stream : Man makes efforts opp. to the direction of flow

Example 23. A man whose velocity in still water is 5m/s swims from point A to B (100m downstream of A) and back to A. velocity of river is 3m/s. Find the time taken in going down stream and up stream and the average speed of the man during the motion ?

Solution In down stream velocity of man = $\vec{v}_m = \vec{v}_{mw} + \vec{v}_w = 3 + 5 = 8m/s$

In down stream time : 100/8 = 12.5 sec

In upstream velocity of man = $\vec{v}_m = \vec{v}_w + \vec{v}_w = -5 + 3 = -2$ m/s.

In up stream time : 100/2 = 50 sec

average speed = 200/62.5 = $\vec{s} = 100m$

In a similar manner, when a boat is rowed across a river, the river tries to carry it down stream whereas the boatman makes an effort at an angle to the river bank. The natural consequence is that he reaches somewhere in between. Here also, the velocity of man in still water refers to velocity due to his own efforts. This is fixed in magnitude, but the direction can be changed at will.

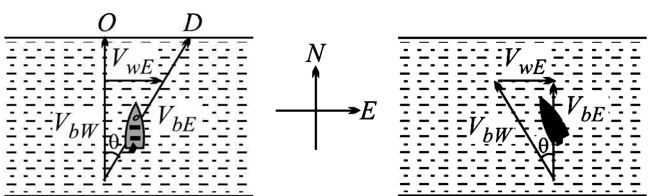


Fig. (a) Fig. (b)

For example, in the figure (a), the boat is rowed directly across in the north direction, but it will reach somewhere in the northeast direction due to the river flow. Similarly in figure (b), the boat is rowed in the north west direction, whereas it will reach in the north direction due to the effect of river flow.

Drift is the distance down stream from the point exactly opposite to the starting point where a person finally reaches. In figure (a) DO = drift. In figure (b) drift = 0

Concept

The motion in x direction does not affect the motion in the y direction and vice versa.

Example 24. A man has velocity of 2m/s in still water. He swims at an angle of 30ŗ with the river flow. River has the velocity of 1 m/s. Width of river is 50 m. Calculate the time taken to cross river and drift of the man.

Solution $x = 50\ (\sqrt{3}+1)$ and $t = 50$ sec.

Note carefully that we are breaking the velocity and the and the displacement in two parts: x direction and y direction. The motion in x direction does not affect the motion in the y direction and vice versa. This pattern we will follow in the entire kinematics.

Example 25. What should be the angle θ at which the man should swim so that the time taken to cross the river be minimum ?

Solution $\vec{v}_m = \vec{v}_{mr} + \vec{v}_r = v_{mr}\,(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}) + v_r\,\hat{i}$

$= (v_{mr}\cos\theta + v_r)\,\hat{i} + v_{mr}\sin\theta\,\hat{j}$

$\Rightarrow \dfrac{x\hat{i}}{t} + \dfrac{d}{t}\hat{j} = \vec{v}_m \qquad \Rightarrow \dfrac{d}{t} = v_{mr}\sin\theta \qquad \Rightarrow t = \dfrac{d}{v_{mr}\sin\theta}$

t minimum at sinθ maximum. ⇒ θ = 90ŗ

$t_{minimum} = d/v_{mr}$. So the man should try to swim perpendicular to the river flow to minimize the time in each case.

Example 26. What should be the angle θ at which the man should swim so that the length of path be minimum? for minimum length of the path, drift x should be minimum.

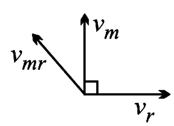
Solution $x = (v_{mr}\cos\theta + v_r)$ $\qquad t = \dfrac{d}{v_{mr}\sin\theta}(v_{mr}\cos\theta + v_r)$

$$x = d\left[\cot\theta + \frac{v_r}{v_{mr}}\text{cosec}\,\theta\right]$$

Case I. $v_{mr} > v_r$ or the river flow is mush less then the effort of the man.

In such case the minimum possible drift will be zero. So the man should has swim at the angle.

$\cos\theta = -v_r/v_{mr}$



Case II. $v_{mr} < v_r$ or the river flow is greater than velocity then velocity of man's effort.

In such case one thing should be carefully meted that the velocity of boat is less than the river flow velocity. In such a case, boat cannot reach the point directly opposite to its starting point. i.e. drift can never be zero.

$\therefore \quad x = \dfrac{d}{v_{mr}\sin\theta}(v_r + v_{mr}\cos\theta)$

$\therefore$ for x to be minimum

$\dfrac{dx}{d\theta} = 0$

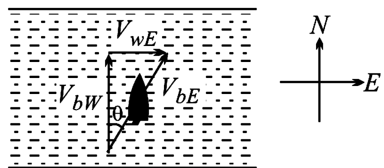
$\therefore \quad \dfrac{dx}{d\theta} = d\left[\dfrac{v_{mr}\sin\theta(-v_{mr}\sin\theta) - v_{mr}\cos\theta(v_r + v_{mr}\cos\theta)}{(v_{mr}\sin\theta)^2}\right]$

$\therefore \quad \cos\theta = \dfrac{-v_{mr}}{v_r}$

Thus, to minimize the drift, boat starts at an angle θ from the river flow.

Example 27. A man is trying to cross the river 100m wide by a boat. The river is flowing with the velocity is 5m/s and the boat's velocity in still water is 3m/s. Find the minimum time in which he can cross the river and the drift in this case?

Solution For minimum time, the man should row perpendicular to the river flow (North direction in the figure).



$$\frac{x\hat{i} + 100\hat{j}}{t} = \frac{\vec{s}}{t} = \vec{v}_m = 3\hat{i} + 5\hat{j}$$

$$\Rightarrow \quad \frac{x}{t}\hat{i} + \frac{100}{t}\hat{j} = 3\hat{i} + 5\hat{j}$$

$$\Rightarrow \quad \frac{100}{3} = t \quad \Rightarrow \quad t = 33.33 \text{ sec.}$$

$$x = 3t \quad \Rightarrow \quad x = 100 \text{ m.}$$

Example 28. Find the direction in which he should row so as to have minimum drift. Also find the minimum possible drift and the time taken to cross the river in this case ?

Solution For minimum drift

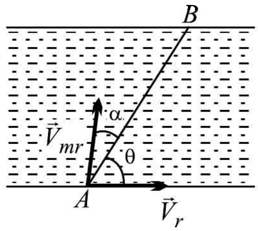
$$\cos\theta = \frac{-v_{mr}}{v_r}\{\because v_r > v_{mr}\}$$

$$\cos\theta = \frac{3}{5} \quad \Rightarrow \quad \theta = 53° \therefore 53° \text{ north of west.}$$

$$\Rightarrow \quad x = \frac{d}{v_{mr}\sin\theta}(v_{mr}\cos\theta + v_r) = \frac{100}{3\times 4/5}\left[3\times\frac{-3}{5} + 5\right] = \frac{500}{12}\times\frac{16}{5} = \frac{400}{3} \text{ m}$$

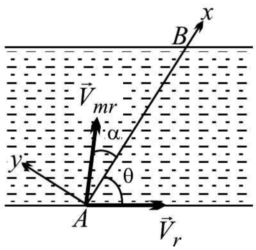
$$t = \frac{d}{v_{mr}\sin\theta} = \frac{100}{3\times 4/5} = \frac{125}{3}\text{sec}$$

Swiming in a desired direction: Many times the person is not interested in mimimizing the time or drift. But he has to reach a particular place. This is common in the cases of an airplane or motor boat.



The man desires to have this final velocity along AB in other words he has to move from A to B. We wish to find the direction in which he should make an effort so that his actual velocity is along line AB.

In this method we assume AB to be the reference line the resultant of v_{mr} and v_r is along line AB. Thus the components of v_{mr} and v_r in a direction perpendicular to line AB should cancel each other.



$$\vec{v}_m = \vec{v}_{mr} + \vec{v}_r$$

$$\vec{v}_m = [v_{mr} \cos\alpha\,\hat{i} + v_{mr} \sin\alpha\,\hat{j}] + [v_r \cos\theta\,\hat{i} - v_r \sin\theta\,\hat{j}]$$

$$v_{mr} \sin\alpha - v_r \sin\theta = 0$$

$$\Rightarrow \quad v_{mr} \sin\alpha = v_r \sin\theta$$

Concept

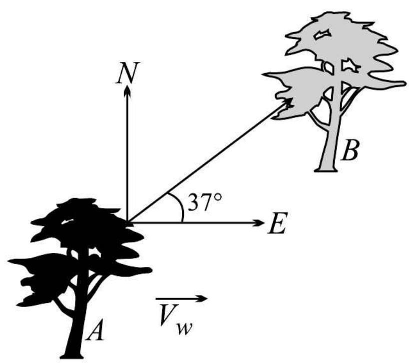
To obtain motion in desired direction.

Example 29. Wind is blowing in the east direction with a speed of 2m/s. A bird wishes to travel from tree A to tree B. Tree B is 100m away from A in a direction 37ţ north of east the velocity of bird in still air is 4m/s.

(a) Find the direction in which bird should fly so that it can reach from A to B directly.

(b) Find the actual velocity of the bird during the flight ?

(c) Find the time taken by the bird to reach B ?



Solution (a) 4 sin α = 2 sin 37ţ. $\Rightarrow \alpha = \sin^{-1}\left(\frac{3}{10}\right)$

$\Rightarrow$ 37ţ + $\sin^{-1}\left(\frac{3}{10}\right)$ with east.

(b) $\vec{v}_b = \vec{v}_{bw} + \vec{v}_w = v_w \cos 37ţ + 4 \cos\alpha.$

$$= 2 \times \frac{4}{5} + 4 \times \frac{\sqrt{9}}{10} = \frac{8}{10} + \frac{2\sqrt{91}}{5} = \frac{8+2\sqrt{91}}{5}$$

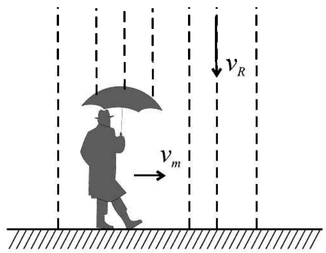
(c) $t = \dfrac{100 \times 5}{8+2\sqrt{91}} = \dfrac{250}{4+\sqrt{91}}$ sec.

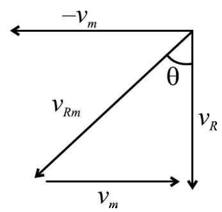
Rain Problems

If rain is falling vertically with a velocity $\vec{v}_R$ and on observer is moving horizontally with velocity $\vec{v}_m$, the velocity of rain relative to observer will be :

$\vec{v}_{Rm} = \vec{v}_R - \vec{v}_m$ or $v_{\text{Rm}} = \sqrt{v_R^2 + v_m^2}$

and direction $\theta = \tan^{-1}\left(\dfrac{v_m}{v_R}\right)$ with the vertical as shown in figure.



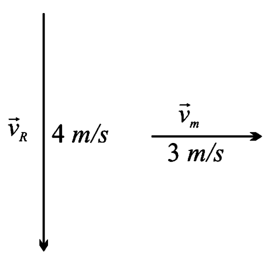


Example 30. A man is running through rain at a speed of 3 m/s. Rain is falling vertically at a speed of 4 m/s.

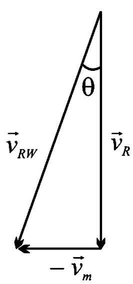
(a) What is the velocity of the rain relative to the man?

(b) How far in front of him would an umbrella have to extend to keep the rain off if he holds the umbrella 1.60 m above her feet?

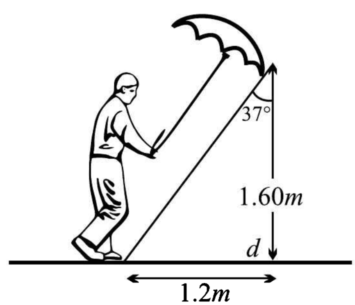
Solution



(a)



(b)



We assign the following letters ; *M*, man *R*, rain *E*, earth. We have to find $\vec{v}_{RM}$.

(a) $\vec{v}_{RW} = \vec{v}_R + (-\vec{v}_M)$

From vector diagram,

$$|\vec{\upsilon}_{RW}| = \sqrt{(v_{RE})^2 + (v_{ME})^2} = 5 \text{ m/s}$$

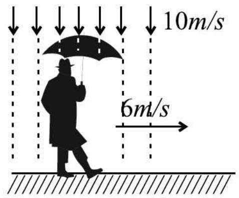
and $\tan\theta = \dfrac{|-\vec{\upsilon}_{ME}|}{|\vec{\upsilon}_{RE}|} = \left(\dfrac{3}{4}\right)$ or $\theta = \tan^{-1}\left(\dfrac{3}{4}\right) = 37°$

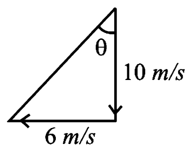
(b) From figure, $d = (1.60)(\tan\theta)$

$$= (1.60) \times \left(\frac{3}{4}\right) = 1.2 \text{ m}$$

Example 31. Rain is falling vertically and a man is moving with velocity 6 m/s. Find the angle with which umbrella should be hold by man to avoid getting wet.

Solution





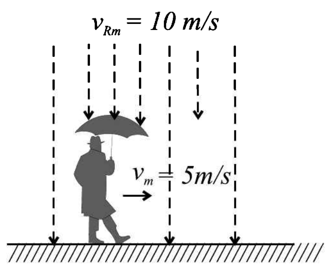
$\vec{v}_{\text{rain}} = -10\hat{j}$ $\quad \vec{v}_{\text{man}} = 6\hat{i}$

Velocity of rain with respect to man $= \vec{v}_{\text{rm}} = -10\hat{j} - 6\hat{i}$

$$\tan\theta = \frac{6}{10} \quad \therefore \theta = \tan^{-1}\left(\frac{3}{5}\right)$$

Where θ is angle with vertical

Example 32. A man moving with 5m/s observes rain falling vertically at the rate of 10 m/s. Find the speed and direction of the rain with respect to ground.



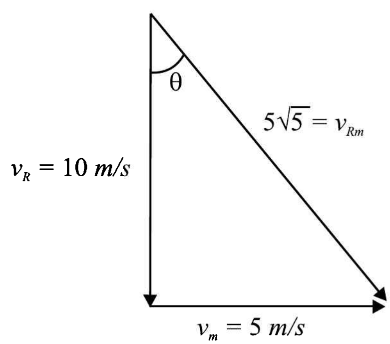
$v_{RM} = 10$ m/s, $v_M = 5$ m/s

$$\vec{v}_{RM} = \vec{v}_{Ru} - \vec{v}_M$$

$$\Rightarrow \vec{v}_{Ru} = \vec{v}_{RM} - \vec{v}_M$$

$$\Rightarrow \vec{v}_R = 5\sqrt{5}$$

$$\tan\theta = \frac{1}{2}, \theta = \tan^{-1}\frac{1}{2}$$

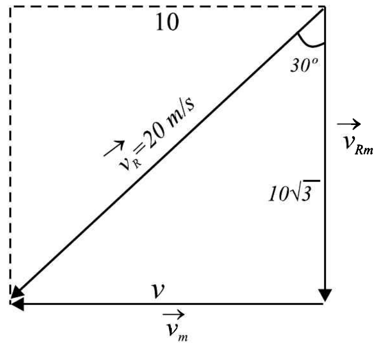


Example 33. A man standing, observes rain falling with velocity of 20 m/s at an angle of 30° with the vertical.

(1) Find out velocity of man so that rain appears to fall vertically.

(2) Find out velocity of man so that rain again appears to fall at 30° with the vertical.

Solution



(1) $\vec{v}_m = -v\hat{i}$ (let)

$\vec{v}_R = -10\hat{i} - 10\sqrt{3}\,\hat{j}$

$\vec{v}_{RM} = -(10 - v)\hat{i} - 10\sqrt{3}\,\hat{j}$

$\Rightarrow -(10 - v) = 0$ (for vertical fall, horizontal component must be zero)

or $v = 10$ m/s

(2) $\vec{v}_R = -10\hat{i} - 10\sqrt{3}\,\hat{j}$

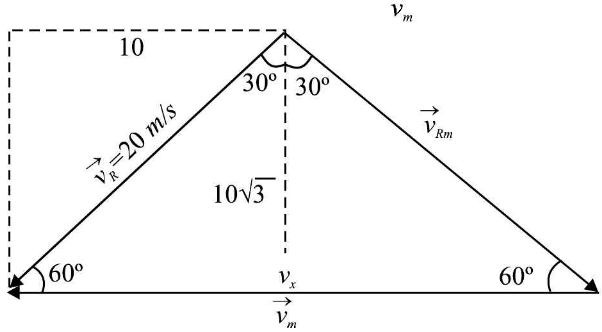
$\vec{v}_m = -v_x\,\hat{i}$

$\vec{v}_{RM} = -(10 - v_x)\hat{i} - 10\sqrt{3}\,\hat{j}$

Angle with the vertical = 30°

$\Rightarrow$ $\tan 30° = \dfrac{10 - v_x}{-10\sqrt{3}}$

$\Rightarrow$ $v_x = 20$ m/s



Wind Aeroplane

This is very similar to boat river flow problems the only difference is that boat is replaced by also plane and river is replaced by wind.

Thus, velocity of aeroplane with respect to wind

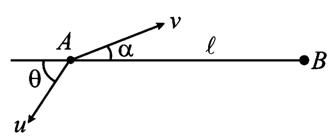
$\vec{v}_{aw} = \vec{v}_a - \vec{v}_w$ or $\vec{v}_a = \vec{v}_{aw} + \vec{v}_w$

where, $\vec{v}_a$ = absolute velocity of aeroplane and, $\vec{v}_w$ = velocity of wind.

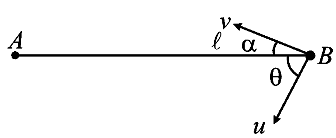
Ex.34 An aeroplane flies along a straight path *A* to *B* and returns back again. The distance between A and *B* is ℓ and the aeroplane maintains the constant speed *v*. There is a steady wind with a speed *u* at an angle θ with line *AB*. Determine the expression for the total time of the trip.

Sol. A to B :

Velocity of plane along *AB* = $v\cos\alpha - u\cos\theta$, and for no-drift from line *AB* : $v\sin\alpha = u\sin\theta$ $\Rightarrow \sin\alpha = \dfrac{u\sin\theta}{v}$



time taken from *A* to *B* : $t_{AB} = \dfrac{\ell}{v\cos\alpha - u\cos\theta}$



***B* to *A* :**

velocity of plane along $BA = v\cos\alpha + u\cos\theta$

and for no drift from line AB : $v\sin\alpha = u\sin\theta$

$$\Rightarrow \qquad \sin\alpha = \frac{u\sin\theta}{v}$$

time taken from *B* to *A* : $t_{BA} = \dfrac{\ell}{v\cos\alpha + u\cos\theta}$

total time taken $= t_{AB} + t_{BA} = \dfrac{\ell}{v\cos\alpha + u\cos\theta} + \dfrac{\ell}{v\cos\alpha + u\cos\theta}$

$$= \frac{2v\ell\cos\alpha}{v^2\cos^2\alpha + u^2\cos^2\theta} = \frac{2v\ell\sqrt{1-\dfrac{u^2\sin^2\theta}{v^2}}}{v^2 - u^2}.$$

***Example* 35.** Find the time an aeroplane having velocity v, take to fly around a square with side a and the wind blowing at a velocity u, in the two cases,

(a) if the direction of wind is along one side of the square,

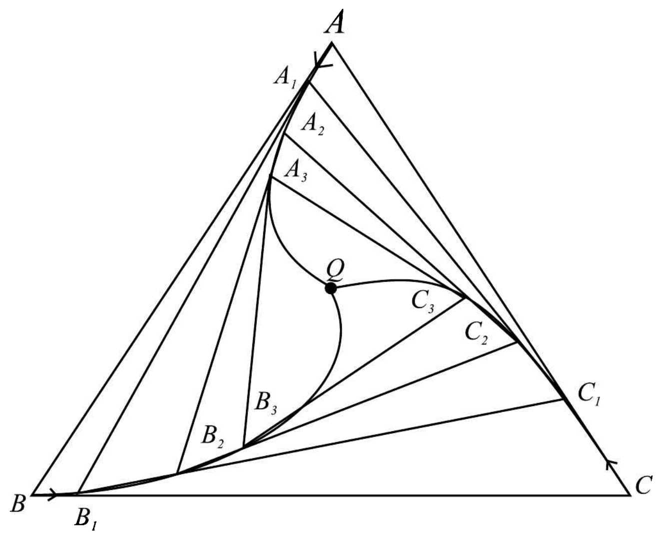
(b) If the direction of wind is along one of the diagonals of the square

[Ans.] (A) $\dfrac{2a}{v^2-u^2}(v+\sqrt{v^2-u^2})$ (B) $2\sqrt{2}a\left(\dfrac{\sqrt{2v^2-u^2}}{v^2-u^2}\right)$

Condition to Collide or to Reach at the Same Point

When the relative velocity of one particle w.r.t. to other particle is directed towards each other then they will collide. (If there is a zero relative acceleration).

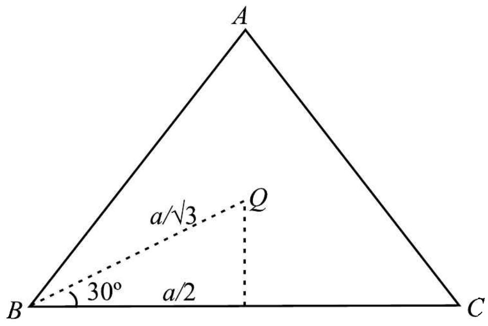
***Example* 36.** There are particles *A*, *B* and *C* are situated at the vertices of an equilateral triangle *ABC* of side a at $t = 0$. Each of the particles moves with constant speed v. A always has its velocity along *AB*, *B* along *BC* and *C* along *CA*. At what time will the particle meet each other?



Solution The motion of the particles is roughly sketched in figure. By symmetry they will meet at the centroid *O* of the triangle. At any instant the particles will from an equilateral triangle *ABC* with the same Centroid *O*. All the particles will meet at the centre. Concentrate on the motion of any one particle, say *B*. At any instant its velocity makes angle 30ŗ with *BO*.

The component of this velocity along *BO* is $v\cos 30$ŗ. This component is the rate of decrease of the distance *BO*. Initially.

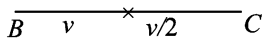
$$BO = \frac{a/2}{\cos 30^\circ} = \frac{a}{\sqrt{3}} = \text{displacement of each particle.}$$

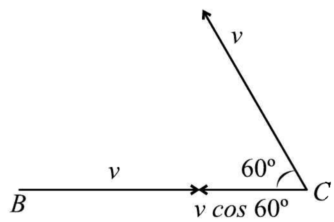


Therefore, the time taken for BO to become zero

$$= \frac{d/\sqrt{3}}{v\cos 30^\circ} = \frac{2d}{\sqrt{3}v \times \sqrt{3}} = \frac{2d}{3v}$$

Alternative : Velocity of B is v along BC. The velocity of C is along CA. Its component along BC is $v\cos 60$ŗ $= v/2$. Thus, the separation BC decreases at the rate of approach velocity.





$\therefore$ approach velocity $= v + \dfrac{v}{2} = \dfrac{3v}{2}$

Since, the rate of approach is constant, the time taken in reducing the separation BC from a to zero is

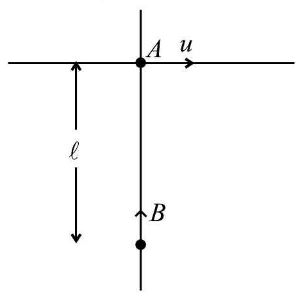
$$t = \frac{a}{\frac{3v}{2}} = \frac{2a}{3v}$$

Example 37. Six particles situated at the corners of a regular hexagon of side a move at a constant speed v. Each particle maintains a direction towards the particle at the next corner. Calculate the time the particles will take to meet each other.

Solution [**Ans.** 2 a/v.]

Example 38. 'A' moves with constant velocity u along then 'x' axis. B always has velocity towards A. After how much time will B meet A if B moves with constant speed v. What distance will be travelled by A and B.

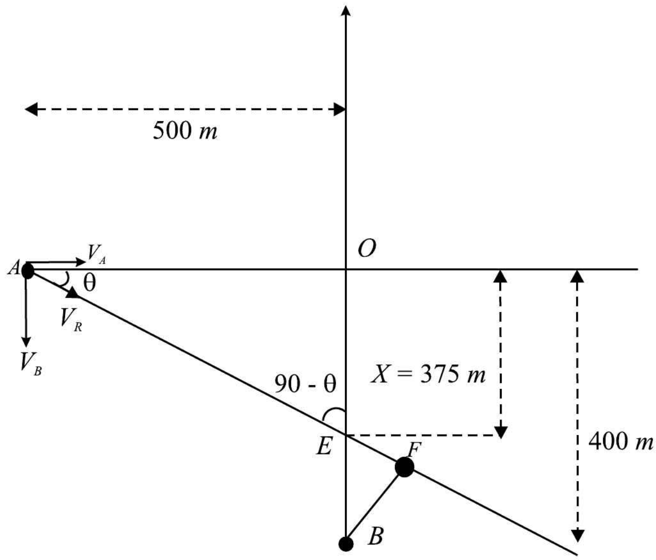
Solution Distance travelled by $A = \dfrac{v^2\ell}{v^2 - u^2}$,

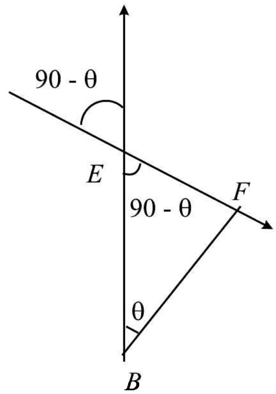


Distance travelled by $B = \dfrac{uv\ell}{v^2 - u^2}$

Example 39. Two cars A and B are moving west to east and south to north respectively along crossroads. A moves with a speed of 72 kmh^{-1} and is 500 m away from point of intersection of cross roads and B moves with a speed of 54 kmh^{-1} and is 400 m away from point of intersection of cross roads. Find the shortest distance between them ?

Solution **Method – I** (Using the concept of relative velocity)





In this method we watch the velocity of A w.r.t. B. To do this we plot the resultant velocity V_r. Since the accelerations of both the bodies is zero, so the relative acceleration between them is also zero. Hence the relative velocity will remain constant. So the path of A with respect to B will be straight line and along the direction of relative velocity of A with respect to B. The shortest distance between A & B is when A is at point F (i.e. when we drop a perpendicular from B on the line of motion of A with respect to B).

From figure

$$\tan\theta = \frac{V_B}{V_A} = \frac{15}{20} = \frac{3}{4} \qquad \text{...(i)}$$

This θ is the angle made by the resultant velocity vector with the x-axis.

Also we know that from figure

$$OE = \frac{x}{500} = \frac{3}{4} \qquad \text{...(ii)}$$

From equation (i) & (ii) we get

$x = 375$ m

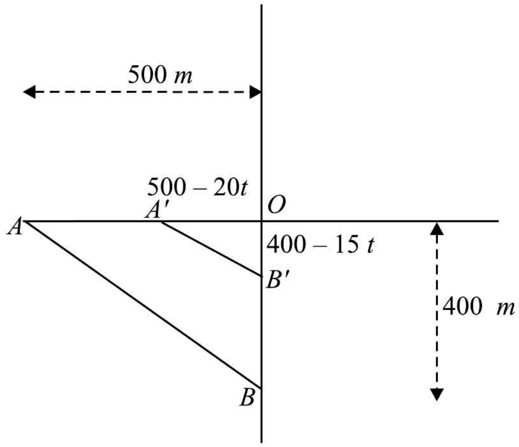
$\therefore \quad EB = OB - OE = 400 - 375 = 25$ m

But the shortest distance is BF.

From magnified figure we see that $BF = EB \cos\theta = 25 \times \frac{4}{5}$

$\therefore \quad BF = 20$ m

Method II (Using the concept of maxima – minima)



A & B be are the initial positions and A',B' be the final positions after time t.

B is moving with a speed of 15 m/sec so it will travel a distance of $BB' = 15t$ during time t.

A is moving with a speed of 20 m/sec so it will travel a distance of $AA' = 20t$ during time t.

So $OA' = 500 - 20t$

$OB' = 400 - 15t$

$\therefore \quad A'B'^2 = OA'^2 + OB'^2 = (500 - 20t)^2 + (400 - 15t)^2 \quad \text{...(i)}$

For $A'B'$ to be minimum $A'B'^2$ should also be minimum

$$\therefore \quad \frac{d(A'B'^2)}{dt} = \frac{d(400 - 15t)^2 + (500 - 20t)^2}{dt} = 0$$

$$= 2(400 - 15t)(-15) + 2(500 - 20t)(-20) = 0$$

$$= -1200 + 45t = 2000 - 80t$$

$\therefore \quad 125t = 3200$

$\therefore \quad t = \dfrac{128}{5}$ s. Hence A and B will be closest after $\dfrac{128}{5}$ s.

Now $\dfrac{d^2 A'B'}{dt^2}$ comes out to be positive hence it is a minima.

On substituting the value of t in equation (i) we get

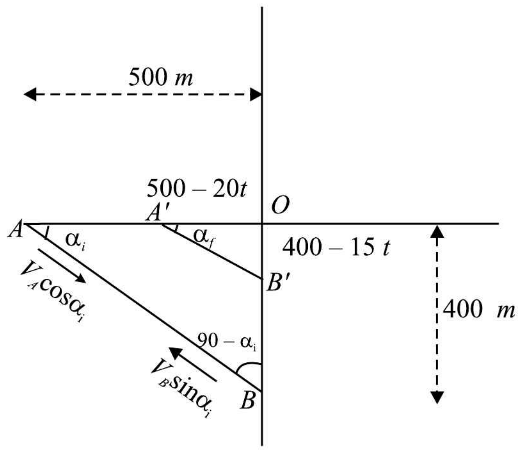
$$\therefore \quad A'B'^2 = \left(400 - 15 \times \frac{128}{5}\right)^2 + \left(500 - 20.\frac{128}{5}\right)^2$$

$$= \sqrt{16^2 + (-12)^2} = 20 \text{ m}$$

$\therefore$ Minimum distance $A'B' = 20$ m.

Method III (Using the concept of relative velocity of approach)

After time t let us plot the components of velocity of A & B in the direction along AB. When the distance between the two is minimum, the relative velocity of approach is zero.



$\therefore \quad V_A \cos\alpha_f + V_B \sin\alpha_f = 0$

(where α_f is the angle made by the line $A'B'$ with the x-axis)

$20 \cos\alpha_f = -15 \sin\alpha_f$

$$\therefore \quad \tan\alpha_f = -\frac{20}{15} = -\frac{4}{3}$$

Here do not confuse this angle with the angle θ in method (I)

because that θ is the angle made by the resultant with x-axis.

Here α_f is the angle made with x-axis when velocity of approach in zero,

$$\therefore \quad \frac{400-15t}{500-20t} = -\frac{4}{3}$$

$$\therefore \quad t = \frac{128}{5}$$

So, $OB' = 16$ m and $OA' = -12$m

$$A'B' = \sqrt{16^2 + (-12)^2} = 20 \text{ m}$$

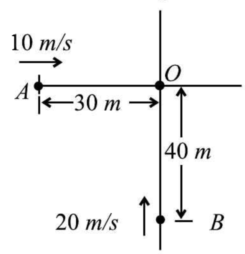
Example 40. Two ships are 10 km apart on a line running south to north. The one farther north is steaming west at 20 $km\ h^{-1}$. The other is steaming north at 20 $km\ h^{-1}$. What is their distance of closest approach ? How long do they take to reach it ?

Solution $5\sqrt{2}$ km/h; 1/4 h = 15 min consider the situation shown in figure for the two particle A and B.

Example 41

(1) Will the two particle will collide

(2) Find out shortest distance between two particles



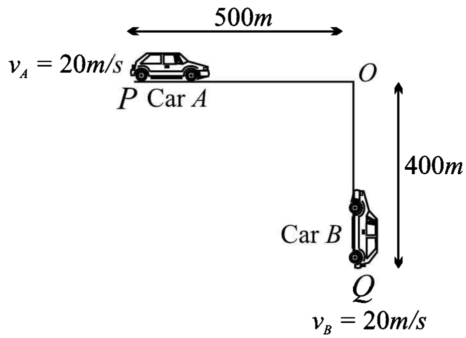
Ans. (1) The particles will not collide

(2) $4\sqrt{5}$ m.

Note

Muzzle Velocity is the velocity of bullet with respect to the gun i.e. it is Relative Velocity.

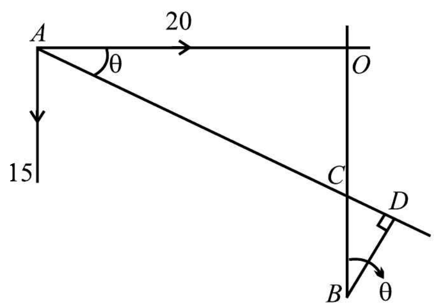
Example 42 Two roads intersect at right angles. Car A is situated at P which is 500m from the intersection O on one of the roads. Car B is situated at Q which is 400m from the intersection on the other road. They start out at the same time and travel towards the intersection at 20m/s and 15m/s respectively. What is the minimum distance between them? How long do they take to reach it.



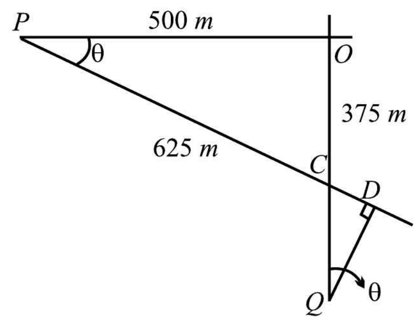
Solution First we find out the velocity of car B relative to A

As can be seen from (fig.), the magnitude of velocity of B with respective

$v_A = 20$ m/s, $V_B = 15$ m/s, $OP = 500$ m; $OQ = 400$m



$\tan\theta = \dfrac{15}{20} = \dfrac{3}{4}$; $\cos\theta = \dfrac{4}{5}$; $\sin\theta = \dfrac{3}{5}$



$OC = AD\tan\theta = 500 \times \dfrac{3}{4} = 375$ m

$BC = OB - OC = 400 - 375 = 25$ m

$BD = BC(\cos\theta) = 25 \times \dfrac{4}{5} = 20$ m

shortest distance = 20 m

$PD = PC + CD = 625 + 15 = 640$

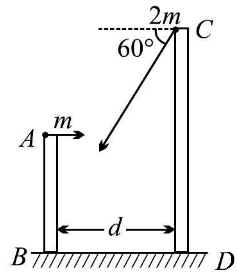
$|\vec{v}_{AB}| = 25$ m/s

$t = \dfrac{640}{25} = 25.6$ sec.

Concept

Collision of two bodies with help of relative motion.

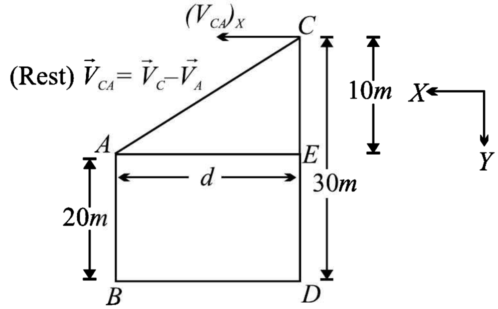
Example 43. Two towers AB and CD are situated a distance d apart as shown in figure. AB is 20m high and CD is 30m high from the ground. An object of mass m is thrown from the top of AB horizontally with a velocity of 10 m/s towards CD.



Simultaneously another object of mass 2m is thrown from the top of CD at an angle of 60° to the horizontal towards AB with the same magnitude of initial velocity as that of the first object. The two objects move in the same vertical plane, collide in mid-air and stick to each other.

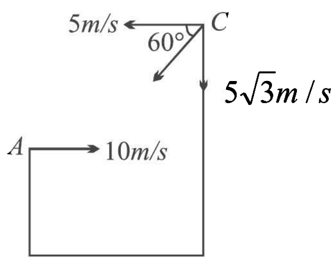
Calculate the distance d between the towers.

Solution Acceleration of A and C both is 9.8 m/s² downwards.



Therefore, relative acceleration between them is zero i.e., the relative motion between them will be straight line. Now assuming A to be at rest, the condition of collision will be that

$\vec{V}_{CA} = \vec{V}_C - \vec{V}_A$ = relative velocity of C w.r.t. A should be along CA.

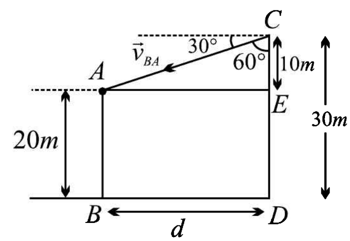


$$\vec{V}_A = 10\hat{i}$$

$$\vec{V}_B = -5\hat{i} - 5\sqrt{3}\,\hat{j}$$

$$\therefore \vec{V}_{BA} = -5\hat{i} - 5\sqrt{3}\,\hat{j} - 10\hat{i}$$

$$\vec{V}_{BA} = -15\hat{i} - 5\sqrt{3}\,\hat{j}$$



$$\therefore \quad \tan 60° = \frac{d}{10}$$

$$\therefore \quad d = 10\sqrt{3} \text{ m}$$

Note

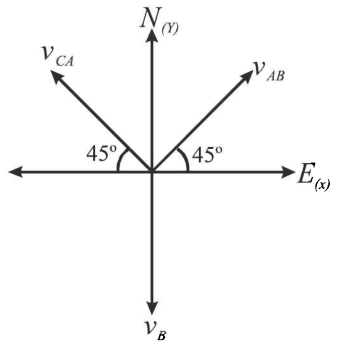
Also check their time of flights.

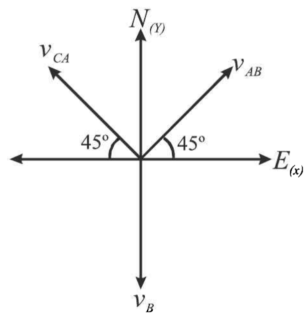
Example 44. The relative velocity of a car '*A*' with respect to car *B* is 30√2 m/s due North-East. The velocity of car '*B*' is 20 m/s due south. The relative velocity of car '*C*' with respect to car '*A*' is 10√2 m/s due North-West. The speed of car '*C*' and the direction (in terms of angle it makes with the east).

(A) $20\sqrt{2}$ m/s, 45° (C) $20\sqrt{2}m/s, 135°$

(B) $10\sqrt{2}\,m/s, 45°$ (D) $10\sqrt{2}\,m/s, 135°$.

Example 45 Given $|\vec{v}_{AB}| = 30\sqrt{2}ms^{-1}$





$|\vec{v}_B| = 20\,ms^{-1}$

$|\vec{v}_{CA}| = 10\sqrt{2}\,ms^{-1}$

Now $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 30\sqrt{2}(\cos 45°\,\hat{i} + \sin 45°\,\hat{j})$

or, $\vec{v}_A - \vec{v}_B = (30\hat{i} + 30\hat{j})ms^{-1}$ ...(i)

and, $\vec{v}_B = (-20\hat{j})ms^{-1}$ ...(ii)

and $\vec{v}_{CA} = \vec{v}_C - \vec{v}_A = 10\sqrt{2}(-\cos 45°\,\hat{i} + \sin 45°\,\hat{j})$

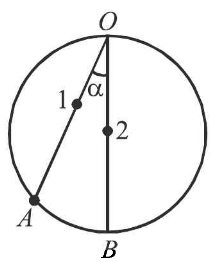
or $\vec{v}_C - \vec{v}_A = (-10\hat{i} + 10\hat{j})ms^{-1}$ ...(iii)

solving equation (i) (ii) and (iii) we'll get $\vec{v}_C = 20\hat{i} + 20\hat{j}$

Hence, (*A*) is the correct answer.

Example 46. Two particles 1 and 2 are allowed to descend on the two frictionless chord *OA* and *OB* of a vertical circle, at the same instant from point *O*. The ratio of the velocities of the particles 1 and 2 respectively, when they reach on the circumference will be (*OB* is the diameter)

(A) sin α (B) tan α (C) cos α (D) none of these.



Solution $OA = d\cos\alpha, \quad a_{OA} = g\cos\alpha$

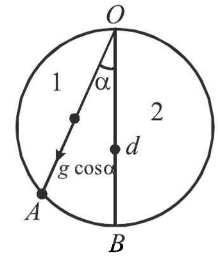
Along OA

$\Rightarrow \quad v_A^2 = 2g\cos\alpha.\cos\alpha$

Along OB

$v_B^2 = 2gd$

$\Rightarrow \quad \frac{v_B}{v_B} = \cos\alpha$



Hence, (C) is correct option.

Example 47. A man running at 6 km/hr on a horizontal road in vertically falling rain observes that the rain hits him at 30° from the vertical. The actual velocity of rain has magnitude

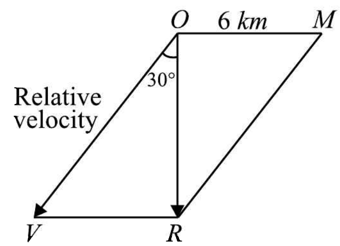
(A) 6 km/hr (B) $6\sqrt{3}\,km/hr$ (C) $2\sqrt{3}\,km/hr$ (D) 2 km/hr

Solution Velocity of rain = Velocity of man + Relative velocity of rain OR gives the actual velocity.

$$\tan 30° = \frac{VR}{OR}$$

$$\frac{1}{\sqrt{3}} = \frac{6}{OR}$$

$$OR = \mathbf{6\sqrt{3}}$$



∴ Hence, the answer is (B).

Example 48. A boat which has a speed of 5 km/hr in still water crosses a river of width 3 km along the shortest possible path in t min. The river flows at the rate of 3 km/hr. The time taken t is

(A) 20 min (B) 25 min (C) 45 min (D) 55 min

Solution $t = \frac{AB}{\sqrt{5^2 - 3^2}} = \frac{3}{4} =$ **45 minutes**

∴ Answer is (C)

Example 49. A boat which has a speed of 5 km/hr in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in kilometres per hour is

(A) 1 (B) 3 (C) 4 (D) $\sqrt{41}$

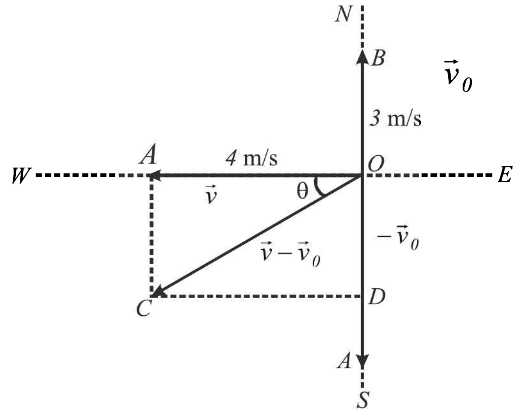
Solution Distance covered in 15 minutes = $5\,km/hr \times \frac{15}{60} hr$ = 1.25 km

Extra distance along river covered = $\sqrt{(1.25)^2 - (1)^2} = 0.75\ km$

Velocity of river = $\frac{0.75}{(15/60)hr} = \frac{0.75 \times 4}{1} =$ **3 km/hr**

∴ Answer is (B)

Example 50. A body moving in a curved path possesses a velocity 3 m/s towards north at any instant of its motion. After 10s, the velocity of the body was found to be 4 m/s towards west. Calculate the average acceleration during this interval.



Solution To solve this problem the vector nature of velocity must be taken into account. In the figure, the initial velocity v_0 and the final velocity v are drawn from a common origin. The vector difference of them is found by the parallelogram method.

The magnitude of difference is

$$|v - v_0| = OC = \sqrt{OA^2 + AC^2}$$

$$= \sqrt{4^2 + 3^2} = 5 m/s$$

The direction is given by

$$\tan\theta = \frac{3}{4} = 0.75 \ , \ \theta = 37°$$

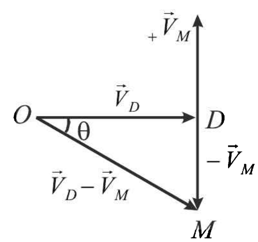
∴ Average acceleration

$$= \frac{|\vec{v} - \vec{v}_0|}{t} = \frac{5}{10}$$

= 0.5 m/s² at 37ŗ South of West.

Example 51. A monkey is climbing a vertical tree with a velocity of 10 m/s while a dog runs towards the tree chasing the monkey with a velocity of 15 m/s. Find the velocity of the dog relative to the monkey.

Solution Velocities are shown in the figure



Velocity of the dog relative to the monkey

= velocity of Dog – velocity of monkey

$$= \vec{V}_D - \vec{V}_M$$

$$= \vec{V}_D + (-\vec{V}_B)$$

This velocity is directed along *OM* and its magnitude is

$$\sqrt{15^2 + 10^2} = \sqrt{225 + 100} = 18 \text{ m/s}$$

This velocity makes an angle θ with the horizontal, where $\tan\theta = \frac{10}{15} = \frac{2}{3}$

or $\theta = \tan^{-1}\left(\frac{2}{3}\right) = 33.7°$

Example 52. A man wants to cross a river 500 m wide. The rowing speed of the man relative to water is 3 km/hr and the river flows at the speed of 2 km/hr. If the man's walking speed on the shore is 5 km/hr, then in which direction should he start rowing in order to reach the directly opposite point on the other bank in the shortest time.

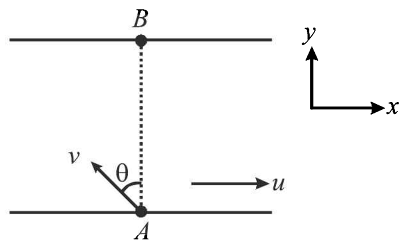
Solution Lets $\vec{v}_m$ = velocity of the man relative to ground.

$\vec{v}$ = velocity of the man relative to water

$\vec{u}$ = velocity of water,

given $|\vec{u}| = 2kmh^{-1}$, $|\vec{v}| = 3kmh^{-1}$

Let him start at an angle θ with the normal



Net velcoity of man $\vec{v}_m = \vec{u} + \vec{v}$

$$\vec{v}_m = (u - v\sin\theta)\hat{i} + v\cos\theta\hat{j}$$

Hence time taken by the man to cross the river is $t_1 = \dfrac{0.5}{v\cos\theta}$

∴ Drift of the man along the river is

$x = (u - v\sin\theta)t_1$

$$x = (u - v\sin\theta)\frac{0.5}{v\cos\theta}$$

Time taken by the man to cover this distance is

$$t_2 = \frac{0.5\left(\dfrac{u\sec\theta}{v} - \tan\theta\right)}{5} = 0.1\left(\frac{u}{v}\sec\theta - \tan\theta\right)$$

Therefore, total time $T = t_1 + t_2$

$$\Rightarrow \quad T = \frac{0.5}{v}\sec\theta + \frac{0.1u}{v}\sec\theta - 0.1\tan\theta$$

Putting the value of u and v, we get

$$T = \frac{0.5}{3}\sec\theta + \frac{0.1\times 2}{3}\sec\theta - 0.1\tan\theta = \frac{0.7}{3}\sec\theta - 0.1\tan\theta$$

$$\Rightarrow \quad \frac{dt}{d\theta} = \frac{0.7}{3}\sec\theta\tan\theta - 0.1\sec^2\theta$$

for T to be minimum, $\dfrac{dT}{d\theta} = 0$

⇒ $\sin\theta = (3/7) \Rightarrow \theta = \sin^{-1}(3/7)$

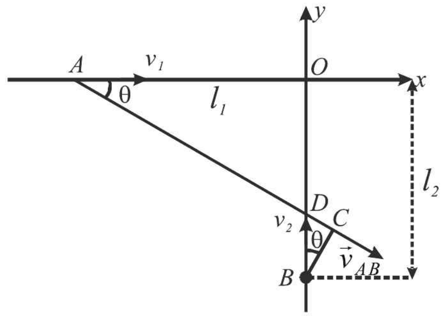
Example 53. Two particles A and B move with constant velocities v_1 and v_2 along two mutually perpendicular straight lines towards the intersection point O. At moment $t = 0$, the particle were located at distance l_1 and l_2 from O, respectively. Find the time, when they are nearest and also the shortest distance between them.

Solution **Method I:**

$$\because \quad \vec{v}_{AB} = \vec{v}_A - \vec{v}_B = v_1\hat{i} - v_2\hat{j}$$

Minimum distance is the length of the perpendicular to $\vec{v}_{AB}$ from B.

If θ is the angle between the x – axis and $\vec{v}_{AB}$, then



$$\tan\theta\left|-\frac{v_2}{v_1}\right| = \frac{v_2}{v_1}$$

In $\Delta AOD, OD = OA\tan\theta = \frac{v_2}{v_1}l_1$

Therefore $BD = l_2 - OD = \frac{v_1 l_2 - v_2 l_1}{v_1}$

In $\quad \Delta BCD,\ \cos\theta = \frac{BC}{BC}$

$$\Rightarrow \qquad BC = BD\cos\theta = \frac{v_1 l_2 - v_2 l_1}{v_1} \times \frac{v_1}{\sqrt{v_1^2 + v_2^2}}$$

$$\Rightarrow \qquad BC = \frac{|v_1 l_2 - v_2 l_1|}{\sqrt{v_1^2 + v_2^2}}$$

The required time $t = \frac{AC}{|\vec{v}_{AB}|} = \frac{AD + DC}{|\vec{v}_{AB}|}$

$$\Rightarrow \quad \frac{l_1\sec\theta + BC\tan\theta}{\sqrt{v_1^2 + v_2^2}} = \frac{\frac{l_1}{v_1}\sqrt{v_1^2 + v_2^2} + \frac{v_1 l_2 - v_2 l_1}{\sqrt{v_1^2 + v_2^2}} \times \frac{v_2}{v_1}}{\sqrt{v_1^2 + v_2^2}} = \frac{l_1 v_1 + l_2 v_2}{v_1^2 + v_2^2}.$$

Method II:

After time 't', the position of the point A and B are $(l_1 - v_1 t)$ and $(l_2 - v_2 t)$, respectively.

The distance L between the points A' and B' are

$$L^2 = (l_1 - v_1 t)^2 + (l_2 - v_2 t)^2 \qquad \text{...(i)}$$

Differentiating with respect to time,

$$2L\frac{dL}{dt} = 2(l_1 - v_1 t)(-v_1) + 2(l_2 - v_2 t)(-v_2)$$

From minimum value of $L, \frac{dL}{dt} = 0$

$$(v_1^2 + v_2^2)t = l_1 v_1 + l_2 v_2 \qquad \text{or } t = \frac{l_1 v_1 + l_2 v_2}{v_1^2 + v_2^2}$$

Putting the value of t in equation (1)

$$L_{\min} = \frac{|l_1 v_2 - l_2 v_1|}{\sqrt{v_1^2 + v_2^2}}.$$

***Example* 54.** Two particles are projected at the same instant from two points A and B on the same horizontal level where $AB = 28$ m, the motion taking place in a vertical plane through AB. The particle from A has an initial velocity of 39 m/s at an angle $\sin^{-1}\left(\frac{5}{13}\right)$ with AB and the particle from B has an initial velocity of 25 m/s at an angle $\sin^{-1}\left(\frac{3}{5}\right)$ with *BA*. Show that the particle would collide in mid-air find when and where the impact occurs.

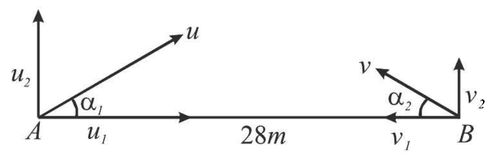
Solution $AB = 28$m.

At *A*, a particle is projected with velocity $u = 39$ m/s. u_1 and u_2 are its horizontal and vertical components respectively. The angle *u* makes with *AB* $\cos\alpha_1$.

Given that $\sin\alpha_1 = \frac{5}{13}$ $\therefore$ $\cos\alpha_1 = \frac{12}{13}$.

Similarly for the particle projected from *B*, with velocity v =25 m/s, v_1 and v_2 are the horizontal and vertical components respectively.

$$\sin\alpha_2 = \frac{3}{5} \quad \therefore \quad \cos\alpha_2 = \frac{4}{5}.$$



Now $u_2 = u\sin\alpha_1 = 39 \times \frac{5}{13} = 15$ m/s

$$v_2 = v\sin\alpha_2 = 25 \times \frac{3}{5} = 15 \text{ m/s}$$

The vertical components of the velocities are the same at the start. Subsequently at the other instant *t* their vertical displacement are equal and have a value

$$h = 15t - 4.9t^2$$

which means that the line joining their positions at the instant *t* continues to be horizontal and the particles come closer to each other.

Their relative velocity in the horizontal direction

$$= 39\cos\alpha_1 + 25\cos\alpha_2$$

$$= 39 \times \frac{12}{13} + 25 \times \frac{4}{5} = 36 + 20 = 56\, m/s$$

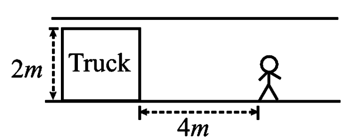
Time of collision $= \frac{AB}{56} = \frac{28}{56} = 0.5s$, after they were projected.

Height at which the collision occurs $= ut - \frac{1}{2}at^2 = 15(0.5) - \frac{1}{2}(9.8)(0.5)^2$ = 6.275 m

The horizontal distance of the position of collision from *A* $= (u_1 \cos\alpha_1)$ (time of collision)

$$= 39 \times \frac{12}{13} \times 0.5s = 18\,\text{m}$$

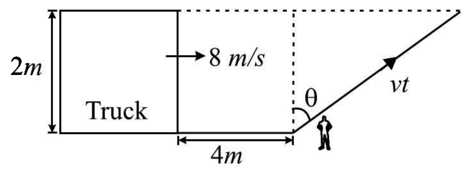
Example 55. A 2m wide truck is moving with a uniform speed of 8 m/s along a straight horizontal road. A pedestrian starts crossing the road at an instant when the truck is 4 m away from him. The minimum constant velocity with which he should run to avoid an accident is



(A) $1.6\sqrt{5}$ m/s (B) $1.2\sqrt{5}$ m/s

(C) $1.2\sqrt{7}$ m/s (D) $1.6\sqrt{7}$ m/s

Solution



$vt = 2 \sec\theta$

Distance covered by truck = $8t = 4 + vt \sin\theta = 4 + 2\tan\theta$

$\Rightarrow \quad 8.\dfrac{2\sec\theta}{v} = 4 + 2\tan\theta$

$\Rightarrow \quad v = \dfrac{8\sec\theta}{2+\tan\theta} = \dfrac{8}{2\cos\theta \times \sin\theta}$

For minimum velocity, $\dfrac{dv}{d\theta} = 0 \qquad \Rightarrow \tan\theta = \dfrac{1}{2}$

$\therefore \quad V_{min} = \dfrac{8\sqrt{1+1/4}}{2+1/2} = 1.6\sqrt{5}$.

Hence (*A*) is correct option.

Example 56. A man crosses the river perpendicular to river flow in time t seconds and travels an equal distance down the stream in *T* seconds. The ratio of man's speed in still water to the speed of river water will be:

(A) $\dfrac{t^2 - T^2}{t^2 + T^2}$ (B) $\dfrac{T^2 - t^2}{T^2 + t^2}$ (C) $\dfrac{t^2 + T^2}{t^2 - T^2}$ (D) $\dfrac{T^2 + t^2}{T^2 - t^2}$

Solution Let velocity of man in still water be *v* and that of water with respect to ground be *u*.

Velocity of man perpendicular to river flow with respect to ground = $\sqrt{v^2 - u^2}$

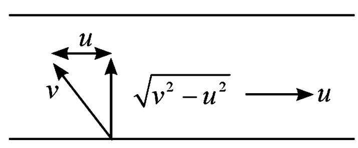
Velocity of man downstream = $v + u$

As given, $\sqrt{v^2 - u^2}\ t = (v + u)\ T$

$\Rightarrow \quad (v^2 - u^2)\ t^2 = (v + u)^2\ T^2$

$\Rightarrow \quad (v - u)\ t^2 = (v + u)\ T^2$

$\therefore \quad \dfrac{v}{u} = \dfrac{t^2 + T^2}{t^2 - T^2}$



$\therefore$ (*C*) is correct option.

Example 57. A train is standing on a platform , a man inside a compartment of a train drops a stone . At the same instant train starts to move with constant acceleration. The path of the particle as seen by the person who drops the stone is:

(A) parabola

(B) straight line for sometime & parabola for the remaining time

(C) straight line
(D) variable path that cannot be defined

[Ans. (*C*)**]**

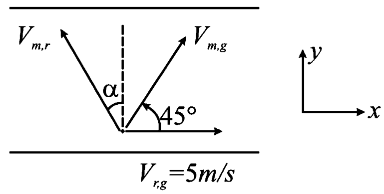
Example 58. A swimmer crosses the river along the line making an angle of 45ţ with the direction of flow. Velocity of the river is 5 m/s. Swimmer takes 6 seconds to cross the river of width 60 m. The velocity of the swimmer with respect to water will be:

(A) 10 m/s (B) 12 m/s (C) $5\sqrt{5}$ m/s (D) $10\sqrt{2}$ m/s

Solution $\vec{V}_{m,g} = \vec{V}_{m,r} + \vec{V}_{r,g}$

As resulting velocity $\vec{V}_{m,g}$ is at 45° with river flow

i.e. $\vec{V}_{r,g} - \vec{V}_{m,r}\sin\alpha = \vec{V}_{m,r}\cos\alpha$ ...(1)



and $\dfrac{60m}{V_{mr}\cos\alpha} = 6$ sec. ...(2)

Solving (1) & (2)

$V_{m,r} = 5\sqrt{5}$ m/s

∴ (*C*)is the correct answer.

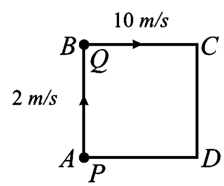
Example 59. A man wearing a hat of extended length 12 cm is running in rain falling vertically downwards with speed 10 m/s. The maximum speed with which man can run, so that rain drops does not fall on his face (the length of his face below the extended part of the hat is 16 cm) will be:

(A) 7.5 m/s (B) 13.33 m/s (C) 10 m/s (D) zero

Solution **[Ans.** (*A*)**]**

Example 60 Two men *P* & *Q* are standing at corners *A* & *B* of square *ABCD* of side 8 m. They start moving along the track with constant speed 2 m/s and 10 m/s respectively. The time when they will meet for the first time, is equal to :

(A) 2 sec (B) 3 sec
(C) 1 sec (D) 6 sec



Solution **[Ans.** $a = 8$ m**]**

They meet when *Q* displace 8 × 3 m

more that *p* ⇒ relative displacement = relative velocity × time.

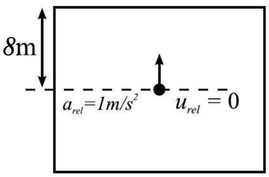
$8 \times 3 = (10 - 2)\, t$

$t = 3$ sec

∴ (*B*) is the correct answer.

Example 61. A coin is released inside a lift at a height of 2 m from the floor of the lift. The height of the lift is 10 m. The lift is moving with an acceleration of 11 m/s² downwards. The time after which the coin will strike with the lift is:

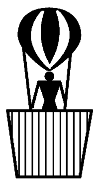
(A) 4 *s* (B) 2 *s* (C) $\dfrac{4}{\sqrt{21}}\, s$ (D) $\dfrac{2}{\sqrt{11}}\, s$



Solution Relative to lift initial velocity and acceleration of coin are 0 m/s and 1 m/s² upwards

$\therefore \quad 8 = \frac{2}{\sqrt{11}} s\ t^2$ or $t = 4$ second

Example 62. A man in a balloon, throws a stone downwards with a speed of 5 m/s with respect to balloon . The balloon is moving upwards with a constant acceleration of 5 m/s². Then velocity of the stone relative to the man after 2 second is:



(A) 10 *m/s* (B) 30 *m/s* (C) 15 *m/s* (D) 35 *m/s*

Solution Relative velocity of stone = 5 *m/s*

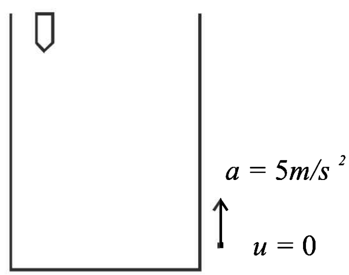
relative acceleration of stone = 10 + 5 = 15 m/s^2

$\therefore \quad v = u + at = 5 + 15 \times 2 = 35\ m/s$

$\therefore$ relative velocity after $t = 2$ second is 35 m/s

$\therefore$ Hence, (*D*) is correct answer.

Example 63. A lift starts with rest and moves in upward direction with constant acceleration $a = 5m/s^2$. After two second a bolt is dropped from height h = 1m. Find the time after which the bolt strikes the lift floor.



Solution After one second the velocity of the lift

$v = u + at = 0 + 5 \times 1$

$v = 5$ m/s

Since the velocity of bolt and left are same

Hence Relative initial velocity of bolt w.r.t. lift

$u_{b\ell} = u_b - u_\ell = v - v = 0$

Relative acceleration of bolt with respect to lift

$a_{b\ell} = a_b - a_\ell = g - (-5) = 10 + 5 = 15$ m/s²

$h = ut + \frac{1}{2}at^2 \qquad 1 = 0 + \frac{1}{2} \times 15 \times t^2$

$t^2 = \frac{2}{15}\ t = \sqrt{\frac{2}{15}}$ sec.

Example 64. A man can swim in still water at a speed of 3km/hr. He wants to cross a river that flows at 2km/hr and reach the point directly opposite to his starting point.

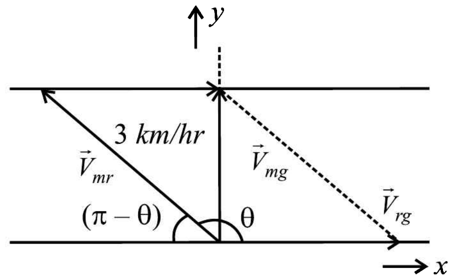
(a) In which direction should he try to swim, (that is, find the angle his body makes with river flow).

(b) How much time will he take to cross the river. If the river is 500 m wide.

Example

(a) $\vec{V}_{rg} = 2\hat{i}$ km/hr Velocity of river w.r.t ground

$\vec{V}_{mr} = 3$ km/hr Velocity of man w.r.t river



$\vec{V}_{mr} = \vec{V}_{mg} - \vec{V}_{rg}$

$$\boxed{\vec{V}_{mg} = \vec{V}_{mr} - \vec{V}_{rg}}$$

$\vec{V}_{mr} = -3\cos(\pi-\theta)\hat{i} + 3\sin(\pi-\theta)\hat{j}$

$\vec{V}_{mr} = 3\cos\theta\,\hat{i} + 3\sin\theta\hat{j}$

$\vec{V}_{mg} = \vec{V}_{mr} + \vec{V}_{rg} \quad = 3\cos\hat{i} + 3\sin\theta\hat{j} + 2\hat{i}$

$\vec{V}_{mg} = (3\cos\theta + 2)\hat{i} + 3\sin\theta\hat{j}$

$$\left\{ \begin{aligned} \sin\theta &= \sqrt{1-\cos^2\theta} \\ &= \sqrt{1-\frac{4}{9}} \\ &= \sqrt{\frac{5}{3}} \end{aligned} \right.$$

Since $\vec{V}_{mg}$ is along y-axis

$\therefore\ 3\cos\theta + 2 = 0$

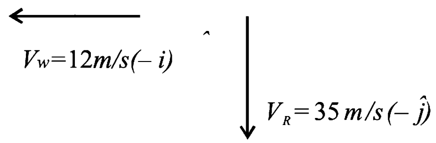
$\cos\theta = -2/3 \qquad \vec{V}_{mg} = 3\sin\theta\hat{j}$

$\vec{V}_{mg} = 3\times\frac{\sqrt{5}}{3}\hat{j} \qquad \vec{V}_{mg} = \sqrt{5}\,\hat{j}$

(b) Time taken to cross the river

$$t = \frac{d}{\vec{v}_{mg}} = \frac{0.5}{\sqrt{5}} = \frac{\sqrt{5}}{10}\,hr.$$

***Example* 65.** Rain is falling vertically with a speed of 35m/s. A woman rides a bicycle with a speed of 12 m/s in east to west direction. What is the direction in which she should hold his umbrella?



Solution The relative velocity of rain with respect to woman.

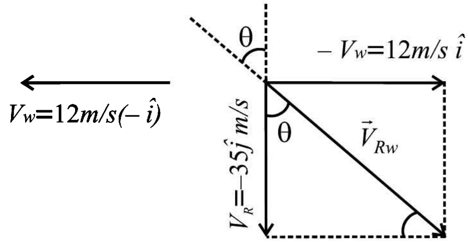
$\vec{V}_{Rw} = \vec{V}_{Rg} - \vec{V}_{wg}$

$\vec{V}_{Rw} = \vec{V}_{Rg} + (-\vec{V}_{wg})$

$$\vec{V}_{Rw} = -35\hat{j}\, m/s + 12\hat{i}\ m/s$$

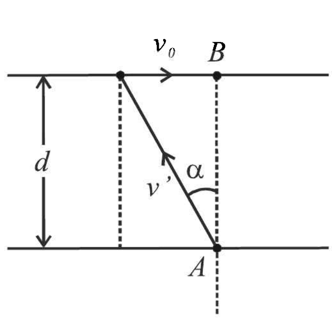
$$\tan\theta = \frac{12}{35}$$

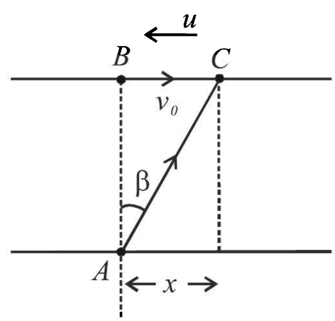
$$\theta = \tan^{-1}\left(\frac{12}{35}\right)$$



Example 66. Two swimmers leave point *A* on one bank of the river to reach point *B* lying right across on the other bank. One of them crosses the river along the straight line *AB* while the other swims at right angles to the stream and then walks the distance that he has been carried away by the stream to get to point *B*. What was the velocity *u* of his walking if both swimmers reached the destination simultaneously? The stream velocity v_0 = 2.0 km/hr and the velocity *v*' of each swimmer with respect to water equals 2.5 km/hr.

Solution Let the I^st swimmer swims in river making an angle α with *AB* in order to reach at opposite point *B*.





$$t_{AB} = \frac{d}{v'\cos\alpha}$$

$$\cos\alpha = \sqrt{1-\sin^2\alpha}$$

$$\cos\alpha = \sqrt{1-\left(\frac{v_0}{v'}\right)^2}$$

$$\therefore t_{AB} = \frac{d}{v'\sqrt{1-\left(\frac{v_0}{v'}\right)^2}} \qquad ...(1)$$

The II^nd swimmer who swims at right angle to river stream reaches at point *C*, then he walks with velocity *u* to reach the point *B*.

$$t_{AB} = \frac{d}{v'} + \frac{x}{u} \qquad ...(2)$$

The resultant velocity along *AC*

$$v = \sqrt{v'^2 + v_0^2}$$

total time to reach at *B*, from the equation (2)

$$\Rightarrow t_{AB} = \frac{d}{v'} + \frac{\sqrt{v_0^2+v'^2}\sin\beta}{v'/d.u} = \frac{d}{v'}\left(1+\frac{\sqrt{v_0^2+v'^2}}{u}.\frac{v_0}{\sqrt{v_0^2+v'^2}}\right)$$

from the equation (1)

$$\frac{d}{v'\left[1-\left(\frac{v_0}{v'}\right)^2\right]^{1/2}} = \frac{d}{v'}\left(1+\frac{v_0}{u}\right) \Rightarrow u = \frac{v_0}{\left(1-\frac{v_0^2}{v'^2}\right)^{-1/2} - 1}$$

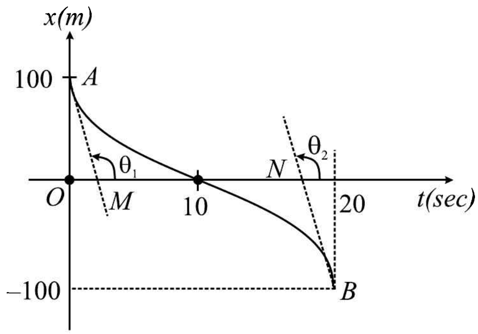
$\Rightarrow u = 3\,km/hr$

Comprehension Questions

Comprehension - 1

Read the following write up and answer the questions that follow.

The graph below gives the displacement of a particle travelling along the x-axis as a function of time. AM is the tangent to the curve at the starting moment and BN is tangent at the end moment ($\theta_1 = \theta_2 = 120°$).



Example 1. The average velocity during the first 20 seconds is

(A) – 10 m/s (B) 10 m/s (C) zero (D) 20 m/s

[Ans. (A)**]**

Example 2. The average acceleration during the first 20 seconds is

(A) – 10 m/s^2 (B) 10 m/s^2 (C) zero (D) 20 m/s^2

[Ans. (C)**]**

Example 3. The direction ($\hat{i}$ or – $\hat{i}$) of acceleration during the first 10 seconds is _____________ .

[Ans. ($\hat{i}$)**]**

Example 4. Time interval during which the motion is retarded.

(A) 0 to 20sec. (B) 10 to 20sec. (C) 0 to 10sec. (D) None of these

[Ans. (C)**]**

Solution (i) $\langle \vec{v} \rangle = \frac{x_f - x_i}{\Delta t} = \frac{-100-100}{20} = -\ 10\text{m/s}$

(ii) $\langle \vec{a} \rangle = \frac{v_f - v_i}{\Delta t} = \frac{\tan\theta_2 - \tan\theta_1}{20} = 0$ since ($\theta_2 = \theta_1$)

(iii) during first 10 sec, speed decreases

∴ acceleration is opposite to the velocity

∴ acceleration is in $\hat{i}$

(iv) during first 10 sec., magnitude of the slope of x-t curve is decreasing and hence speed is decreasing

$\therefore$ motion is retarded.

$t = 0$ to $t = 10$ s

[**Ans.** (i) – 10m/s (ii) 0 (iii) $\hat{i}$ (iv) t = 0 to t = 10 s**]**

Comprehension - 2

Following are three functions which relates the parameters of motion.

$$S(t) = ut + \frac{1}{2}at^2 \qquad v(s) = \sqrt{u^2 + 2as} \qquad v(t) = u + at$$

Where S, u, t, a, v are respectively the displacement (dependent variable), initial velocity (constant), time taken (independent variable), acceleration (constant) and final velocity (dependent variable) of the particle after time t.

Example 1. Find displacement of a particle after 10 seconds starting from rest with an uniform acceleration of $2 m/s^2$.

(A) 10 m (B) 100 m (C) 50 m (D) 200 m

Solution $u = 0$, $a = 2$, $t = 10$

$S = \frac{1}{2} \times 2 \times (10)^2 = 100$ m. **[Ans. (B)]**

Example 2. Find the velocity of the particle after 100 m –

(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 0 m/s

Solution $v = \sqrt{u^2 + 2as}$

$= \sqrt{0^2 + 2 \times 2 \times 100}$

$= 2 \times 10 = 20$ m/sec. **[Ans. (B)]**

Example 3. Find the velocity of the particle after 10 seconds if its acceleration is zero in interval (0 to 10 s) –

(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 0 m/s

Solution Since acceleration is zero, and final velocity is dependent variable on a and u

So, $v = 0$ m/sec. **[Ans. (D)]**

Example 4. Find the displacement of the particle when its velocity becomes 10 m/s if acceleration is $5 m/s^2$ all through–

(A) 50 m (B) 200 m (C) 10 m (D) 100 m

Solution $V^2 = u^2 + 2as$

$(10)^2 = 0 + 2 \times 5 \times S \quad \Rightarrow S = 10$ m. **[Ans. (C)]**

Comprehension - 3

Raindrops are falling with a velocity $10\sqrt{2}$ m/s making an angle of 45^0 with the vertical. The drops appear to be falling vertically to a man running with constant velocity. The velocity of rain drops change such that the rain drops now appear to be falling vertically with $\sqrt{3}$ times the velocity it appeared earlier to the same person running with same velocity.

Example 1. The magnitude of velocity of man with respect to ground is

(A) $10\sqrt{2}$ m/s (B) $10\sqrt{3}$ m/s (C) 20 m/s (D) 10 m/s

[Ans. (D)]

Example 2. After the velocity of rain drops change, the magnitude of velocity of raindrops with respect to ground is

(A) 20 m/s (B) $20\sqrt{3}$ m/s (C) 10 m/s (D) $10\sqrt{3}$ m/s

[Ans. (A)]

Example 3. The angle (in degrees) between the initial and the final velocity vectors of the raindrops with respect to the ground is

(A) 8 (B) 15 (C) 22.5 (D) 37

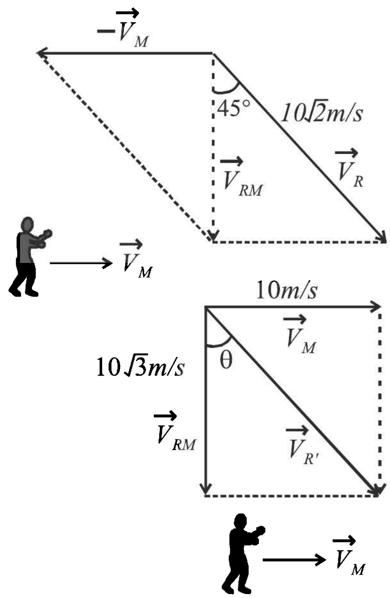
[Ans. (B)]

Solution **In the first case :**

From the figure it is clear that

$\vec{V}_{RM}$ is 10 m/s downwards and

$\vec{V}_M$ is 10 m/s towards right



In the second case :

Velocity of rain as observed by man becomes $\sqrt{3}$ times in magnitude.

∴ New velocity of rain

$$\vec{V}_{R'} = \vec{V}_{R'M} + \vec{V}_M$$

∴ The angle rain makes with vertical is

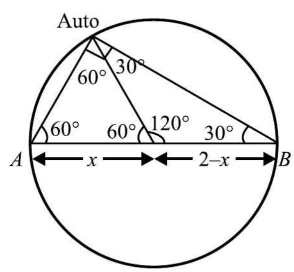
$$\tan\theta = \frac{10}{10\sqrt{3}} \quad \text{or} \quad \theta = 30°$$

∴ Change in angle of rain = 45 – 30 = 15°.

Comprehension - 4

A overhead bridge, a subway and a road start from A and again meet at B. The minimum distance between A and B, which is same as the length of the road AB, is 2 km. The overbridge and the subway form a semicircular arc above and below the road. A laser sensor is fixed (embedded) in the road.

An Autorickshaw takes the overbridge from A and a taxi takes the subways from B.The laser sensor gives a beep when the linear distances between point A and the autorickshaw is same as that between the rickshaw and the laser sensor which also equals the distance of laser source from point A.



Example 1. If the time t, for the laser starts when the autorickshaw just enters the bridge from point A and at t = 240 sec, laser the gives a beep, what is the speed of the autorickshaw ?

(A) 4.36 m/s (B) 1.21 m/s

(C) 8.16 m/s (D) 16.32 m/s

[Ans. (A)]

Solution $x = 2 - x \rightarrow x = 1$ km so sensor is at mid of the Road.

Arc $AB \Rightarrow \dfrac{2\pi R \times \pi}{2\pi 3}$

$$V = \frac{\pi \times R}{3 \times 240} = \frac{3.14 \times 1000}{3 \times 240} = 4.36 \text{ m/s}$$

Example 2. The autorickshaw takes the overhead bridge from A and on reaching B, immediately takes the subway to come back to A, while the taxi starts from B travels to and fro from B to A continuously by road. If the auto and the taxi travel with constant speeds of $\frac{\pi}{2}$ km/hr and 3 km/hr respectively, how frequently do they meet at A ?

(A) every 4 hours

(B) every $\frac{2\pi n}{3}$ hours, n = 3,6,9,....

(C) every $\frac{2}{3}$ hours

(D) they never meet

Solution $T_{\text{Auto}} = \dfrac{2\pi R x 2}{\pi} = 4$ hrs.

$T_{\text{Taxi}} = \dfrac{4}{3}$ hrs.

Common time period is not found so they never meet. **[Ans. (D)]**

Example 3. Due to heavy rains, the flyover and the roads were blocked and all the vehicles had to take the subway. The autorickshaw and the taxi started from A and B respectively with the speeds $\frac{\pi}{2}$ and $\frac{2\pi}{3}$ km/hr respectively. After how much time did they meet ?

(A) $\frac{7}{6}$ hours (B) $\frac{6}{7}$ hours (C) $\frac{2\pi}{3}$ hours (D) $\frac{6\pi}{7}$ hours

Solution $V_R = \dfrac{\pi}{2} + \dfrac{2\pi}{3} = \dfrac{7\pi}{6} \Rightarrow t = \dfrac{\pi R}{\frac{7\pi}{6}} = \dfrac{6}{7}\, H$ **[Ans. (B)]**

Comprehension-5

Direction for questions 16-20 : Read the following data and answer the questions that follows:

Three brothers, Ram, Shyam and Mohan travelled by road. They all left the college at the same time – 12 noon. The description of the motions of the three are detailed below:

Name	**Ram**	**Shyam**	**Mohan**
Phase I	Bus for	Bike for	Foot for
	2 hours @ 10 mph	1 hr. @ 30 mph	3 hrs. @ 3.33 mph
Phase II	Bike for	Foot for	Bus for
	1.5 hrs. @ 40 mph	3 hrs. @ 3.33 mph	3 hrs. @ 10 mph
Phase III	Foot for	Bus for	Bike for
	3 hrs. @ 3.33 mph	4 hrs. @ 10 mph	2 hrs. @ 30 mph

Example 1. When did Ram overtake Shyam?

(A) 3 : 15 p.m. (B) 2 : 22 p.m. (C) 3 : 30 p.m. (D) 2 : 20 p.m.

[Ans. (B)]

Example 2. At what distance from the start does Mohan overtake Shyam?

(A) 40 miles (B) 57 miles (C) 70 miles (D) 80 miles

[Ans. (C)]

Example 3. If Ram travelled by bike instead of foot in the last leg of his journey (for the same distance as he had covered by foot), what is the difference in the time of Ram and Mohan to cover 90 miles?

(A) 6 hrs 50 minutes (B) 10 hrs 40 minutes

(C) 3 hrs 55 minutes (D) 4 hrs 10 minutes

[Ans. (C)]

Example 4. If all of them travelled a distance of 100 miles, who reached first and at what time (assume the last leg time increases to cover 100 miles)?

(A) Mohan at 6 p.m. (B) Ram at 8 p.m.

(C) Shyam at 6 p.m. (D) Mohan at 8 p.m.

[Ans. (D)]

Example 5. In the above question, who reached last and at what time?

(A) Ram at 9:30 p.m. (B) Ram at 10 p.m.

(C) Shyam at 9:30 p.m. (D) Shyam at 10 p.m.

[Ans. (D)]

Comprehension - 6

At time t the position of a body moving such that its position varies with time and is given by $s = t^3 - 6t^2 + 9t$ m.

Example 1. Find the body's acceleration each time the velocity is zero.

Example 2. Find the body's speed each time the acceleration is zero

Example 3. Find the total distance traveled by the body from $t = 0$ to $t = 2$.

[Ans. (1) – 6m/s^2 , 6m/s^2 (2) 3m/s (3) 6m]

Comprehension - 7

Two trains of 1 km and 2 km length are running in opposite directions on parallel tracks. The speeds of 1 km train is 25 m/s and that of 2 km train is 35 m/s. The drivers of the two trains apply breaks when they cross each other such that the trains stop with the guards facing each other. Assume that both trains stop together.

Example 1. Find the time taken for this phenomenon. [**Ans.** 100 sec]

Example 2. Find the distance travelled by both the trains before stopping. [**Ans.** 500 m , 2500 m]

Example 3. Find the magnitude of relative acceleration of the trains. [**Ans.** 0.6 m/s^2]

Solution **(1-3)**

Let retardation of B is a so retardation A is $2a$. Relative acceleration of A w.r.t. B is $-3a$

$$V_{AB}^2 = U_{AB}^2 + 2a_{AB}\ S_{AB}$$

$$0 = 60^2 - 6a \times 3000$$

$a = 1/5\ \text{m/sec}^2$

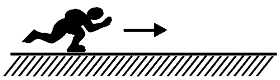
$V_{AB} = U_{AB} + a_{AB}\, t$

$0 = 60 - 3(1/5) \times t$

$t = 100$

Comprehension - 8

If a man has a velocity which is varying with time given as $v = 3t^2$, then :



Example 1. Find out the velocity of the man after 3 sec.

(A) 18 m/s (B) 9 m/s (C) 27 m/s (D) 36 m/s

Solution $v = 3t^2$

$v = 3(3)2 = 2.7$ m/s **[Ans. (C)]**

Example 2. Find out his displacement after 2 seconds of his start :

(A) 10 m (B) 6 m (C) 12 m (D) 8 m

Solution $\int_o^s dS = \int_0^2 3t^2 dt$

$S = \left[t^3\right]_0^2 = 8$ **[Ans. (D)]**

Example 3. Find out his acceleration after 3 seconds :

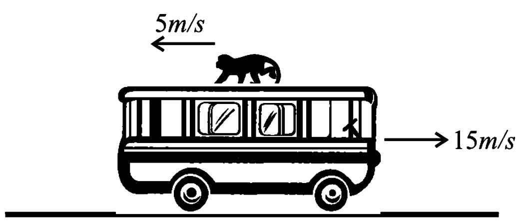
(A) 9 m/s² (B) 18 m/s² (C) 12 m/s² (D) 6 m/s²

Solution $f = 6t$

$f = 6 \times 3 = 18$ m/s² **[Ans. (B)]**

Comprehension-9

A bus is moving rightward with a velocity of 15 m/sec and on the bus a monkey is running oppositely with a velocity of 5 m/sec (with respect to the bus). Nearby a helicopter is rising vertically up with a velocity of 10 m/sec.



Example 1. Find out the direction of the helicopter as seen by the monkey.

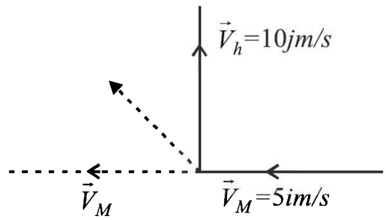
Ans. (↖)

Example 2. Find out the direction of the bus as seen by the helicopter's pilot.

Ans.(↘) .

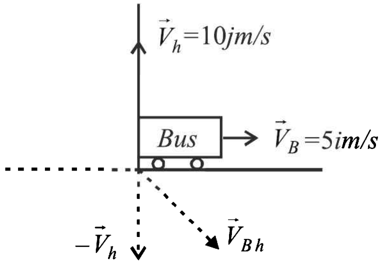
Solution $\vec{V}_{hM} = \vec{V}_h - \vec{V}_M = 10j - 10i = -10i + 10j$

$\therefore$ $\vec{V}_{hM} = 10(-i) + 10j$



$\therefore$ As seen by the monkey helicopter is moving in (↖) direction.

$\vec{V}_{Bh} = \vec{V}_B - \vec{V}_h = 15\,i - 10\,j = 15\,i + 10\,(-j)$



$\therefore$ As seen by helicopter's pilot the bus is moving in (↘) direction.

Comprehension - 10

Rain is falling with a velocity $(-4\hat{i} + 8\hat{j} - 10\hat{k})$. A person is moving with a velocity of $(6\hat{i} + 8\hat{j})$ on the ground.

Example 1. Find the velocity of rain with respect to man and the direction from which the rain appears to be coming.

Solution $\vec{V}_{rm} = \vec{V}_r - \vec{V}_m = (-10\hat{i} - 10\hat{k})$

[**Ans.** $(-10\hat{i} - 10\hat{k})$ rain appears to come 45° with $\hat{i}$]

Example 2. The speed with which the rain drops hit the person is :

(A) 10 m/s (B) $10\sqrt{2}$ m/s (C) $\sqrt{180}$ m/s (D) $\sqrt{360}$ m/s

Solution $V_m = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ m/sec. [**Ans.** (B)]

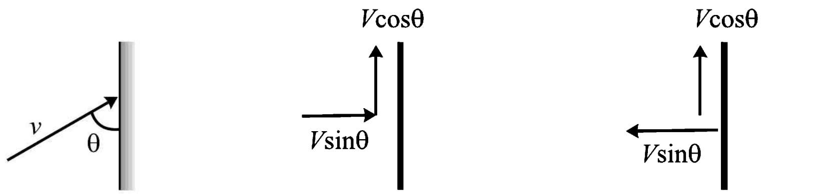
Example 3. The velocity of man w.r.t. rain is :

(A) $-6\hat{i} - 8\hat{j}$ (B) $4\hat{i} - 8\hat{j} + 10\hat{k}$ (C) $-10\hat{i} - 10\hat{k}$ (D) $10\hat{i} + 10\hat{k}$

Solution $\vec{V}_{mr} = -\vec{V}_{rm} = 10\hat{i} + 10\hat{k}$ [**Ans.** (D)]

Comprehension - 11

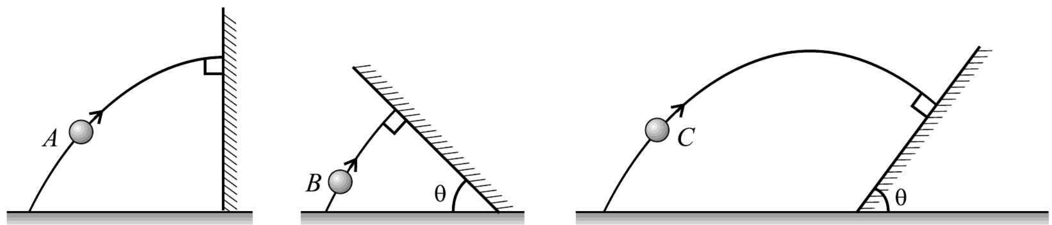
We know how by neglecting the air resistance, the problems of projectile motion can be easily solved and analysed. Now we consider the case of the collision of a ball with a wall. In this case the problem of collision can be simplified by considering the case of elastic collision only. When a ball collides with a wall we can divide its velocity into two components, one perpendicular to the wall and other parallel to the wall. If the collision is elastic then the perpendicular component of velocity of the ball gets reversed with the same magnitude.



Velocity just before collision — Components of velocity just before collision — Components of velocity just after collision

The other parallel component of velocity will remain constant if wall is given smooth.

Now let us take a problem. Three balls 'A' and 'B' & 'C' are projected from ground with same speed at same angle with the horizontal. The balls A, B and C collide with the wall during their flight in air and all three collide perpendicularly with the wall as shown in figure.



Example 1. Which of the following relation about the maximum height H of the three balls from the ground during their motion in air is correct :

(A) $H_A = H_C > H_B$ (B) $H_A > H_B = H_C$ (C) $H_A > H_C > H_B$ (D) $H_A = H_B = H_C$

Solution $H_A = H_C > H_B$

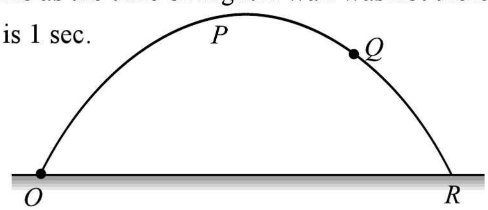
Obviously A just reaches its maximum height and C has crossed its maximum height which is equal to A as u and θ are same. But B is unable to reach its max. height. **[Ans.** (A)**]**

Example 2. If the time taken by the ball A to fall back on ground is 4 seconds and that by ball B is 2 seconds. Then the time taken by the ball C to reach the inclined plane after projection will be :

(A) 6 sec. (B) 4 sec. (C) 3 sec. (D) 5 sec.

Solution Time of flight of A is 4 seconds which is same as the time of flight if wall was not there.

Time taken by C to reach the inclined roof is 1 sec.



$T_{OR} = 4$

$T_{QR} = 1$

∴ $T_{OQ} = T_{OR} - T_{QR} = 3$ seconds.

[Ans. (C)**]**

Example 3. The maximum height attained by ball 'A' from the ground is :

(A) 10 m (B) 15 m

(C) 20 m (D) Insufficient information

Solution from above $= \dfrac{2u\sin\theta}{g} = 4$

∴ $u\sin\theta = 20$ m/s ⇒ vertical component is 20 m/s.

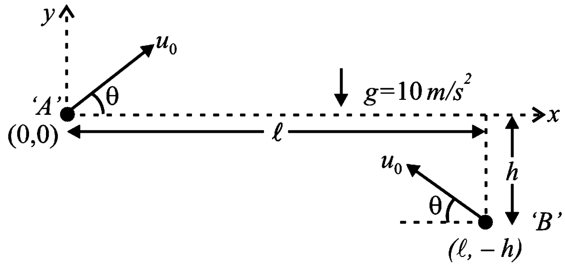
for maximum height

$v^2 = u^2 + 2as \quad \Rightarrow \quad 0^2 = 20^2 - 2 \times 10 \times s$

$s = 20$ m. **[Ans.** (C)**]**

Comprehension - 12

Two particles 'A' and 'B' are projected in the vertical plane with same initial velocity u_0 from part (0, 0) and (ℓ , $-h$) towards each other as shown in figure at $t = 0$.



Example 1. The path of particle 'A' with respect to particle 'B' will be

(A) parabola
(B) straight line parallel to x-axis.
(C) straight line parallel to y-axis
(D) none of these.

[Ans. (B)**]**

Example 2. Minimum distance between particle A and B during motion will be :

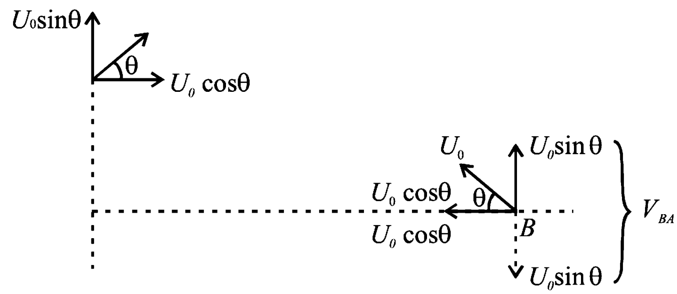
(A) ℓ
(B) h
(C) $\sqrt{\ell^2 + b^2}$
(D) $\ell + h$

Solution **(1 and 2) [Ans.** (B)**]**

The path of a projectile as observed by other projectile is a straight line.

$V_A = u\cos\theta\ \hat{i} + (u\sin\theta - gt)\ \hat{j}$. $V_{AB} = (2u\cos\theta)\ \hat{i}$

$V_B = -u\cos\theta\ \hat{i} + (u\sin\theta - gt)\ \hat{j}$. $a_{BA} = g - g = 0$



The vertical component $u_0 \sin\theta$ will get cancelled. The relative velocity will only be horizontal which is equal to $2u_0 \cos\theta$.

Hence B will travel horizontally towards left w.r.t. A with constant speed $2u_0 \cos\theta$ and minimum distance will be h.

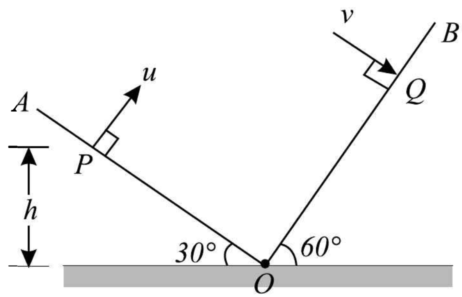
Example 3. The time when separation between A and B is minimum is :

(A) $\dfrac{x}{u_0 \cos\theta}$
(B) $\sqrt{\dfrac{2h}{g}}$
(C) $\dfrac{\ell}{2u_0 \cos\theta}$
(D) $\dfrac{2\ell}{u_0 \cos\theta}$

Solution Time to attain this separation will obviously be $\frac{S_{rel}}{V_{rel}} = \frac{\ell}{2u_0 \cos\theta}$. **[Ans. (C)]**

Comprehension - 13

Two inclined planes *OA* and *OB* having inclinations 30° and 60° with the horizontal respectively intersect each other at *O*, as shown in figure. A particle is projected from point *P* with velocity $u = 10\sqrt{3}\ \frac{m}{s}$ along a direction perpendicular to plane *OA*. If the particle strikes plane *OB* perpendicular at *Q* (Take $g = 10$ m/s²). Then



Example 1. The time of flight is from *P* to *Q* is
(A) 5 Sec. (B) 2 sec (C) 1 sec (D) None of these
[Ans. (B)]

Example 2. The velocity with which the particle strikes the plane *OB* is
(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 40 m/s
[Ans. (A)]

Example 3. The height *h* of point *P* from the ground is
(A) $10\sqrt{3}$ m (B) 10 m (C) 5 m (D) 20 m
[Ans. (C)]

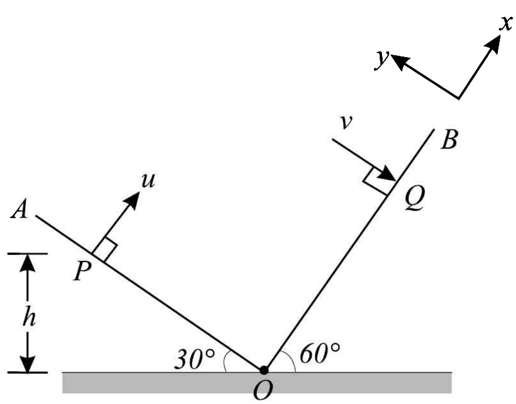
Example 4. The distance *PQ* is
(A) 20 m (B) $10\sqrt{3}$ m (C) 10 m (D) 5 m
[Ans. (A)]

Let us choose the *x* and *y* directions along *OB* and *OA* respectively. Then

$u_x = u = 10\sqrt{3}$ m/s, $u_y = 0$

$a_x = -g \sin 60° = -5\sqrt{3}$ m/s²

and $a_y = -g \cos 60° = -5$ m/s²



Example 1. At point *Q*, *x*-component of velocity is zero. Hence, substituting in

$v_x = u_x + a_x t$

$0 = 10\sqrt{3} - 5\sqrt{3}t$

or $t = \frac{10\sqrt{3}}{5\sqrt{3}} = 2\text{s}$ **Ans.**

Example 2. At point *Q*, $v = v_y = u_y + a_y t$

∴ $v = 0 - (5)(2) = -10$ m/s **Ans.**

Here, negative sign implies that velocity of particle at *Q* is along negative y direction.

Example 3. DistancePO = |displacement of particle along y-direction | = $|s_y|$

Here, $s_y = u_y t + \frac{1}{2} a_y t^2$

$= 0 - \frac{1}{2}(5)(2)^2 = -10$ m

$\therefore$ $PO = 10$ m

Therefore, $h = PO \sin 30° = (10)\left(\frac{1}{2}\right)$

or $h = 5$ m **Ans.**

Example 4. Distance OQ = displacement of particle along x-direction = s_x

Here, $s_x = u_x t + \frac{1}{2} a_x t^2$

$= (10\sqrt{3})(2) - \frac{1}{2}(5\sqrt{3})(2)^2$

$= 10\sqrt{3}$ m

or $OQ = 10\sqrt{3}$ m

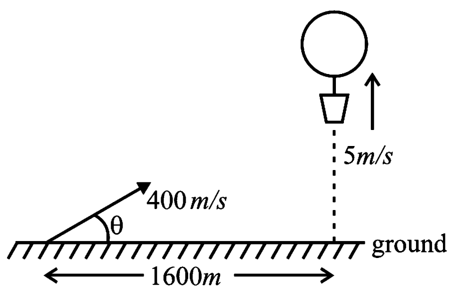
$\therefore$ $PQ = \sqrt{(PO)^2 + (OQ)^2} = \sqrt{(10)^2 + (10\sqrt{3})^2}$

$= \sqrt{100+300} = \sqrt{400}$

$\therefore$ $PQ = 20$ m **[Ans.]**

Comprehension - 14

An observer having a gun observes a remotely controlled balloon. When he first noticed the balloon, it was at an altitude of 800 m and moving vertically upward at a constant velocity of 5m/s. The horizontal displacement of balloon from the observer is 1600 m. Shells fired from the gun have an initial velocity of 400 m/s at a fixed angle θ ($\sin\theta = 3/5$ and $\cos\theta = 4/5$). The observer having gun waits (for some time after observing the balloon) and fires so as to destroy the balloon. Assume $g = 10 m/s^2$. Neglect air resistance.



Example 1. The flight time of the shell before it strikes the balloon is

(A) 2 sec (B) 5 sec. (C) 10 sec (D) 15 sec

Solution The motion in the x-direction is a constant velocity motion. We find the flight time $= \frac{1600\,m}{u_x}$

$$\frac{1600}{400\cos\theta} = 5 \text{ sec.}$$

Flight time = 5 sec.

Example 2. The altitude of the collision above ground level is

(A) 1250m (B) 1325m (C) 1075m (D) 1200m

Solution From the flight time, the initial velocity in the y-direction and the acceleration in the y-direction, we can calculate the altitude of the shell:

$$h = u_y t - \frac{1}{2}gt^2 = \frac{1200}{5} \times 5 - \frac{1}{2} \times 10 \times 25$$

$$= 1200 - 125 = 1075 \text{ m}$$

Example 3. After noticing the balloon, the time for which observer having gun waits before firing the shell is

(A) 50 sec. (B) 55 sec. (C) 60 sec. (D) 45 sec.

Solution After the waiting time plus the flight time, the balloon should reach the same altitude as the shell. Let t_w be the waiting time.

$$\therefore\ t_w + 5\text{sec} = \frac{1075-800}{5} \text{ or } t_w = 50 \text{ sec.}$$

Comprehension - 15

A stone is projected from level ground with speed u and at an angle θ with horizontal. Some how the acceleration due to gravity (g) becomes double (that is 2g) immediately after the stone reaches the maximum height and remains same thereafter. Assume direction of acceleration due to gravity always vertically downwards.

Example 1. The total time of flight of particle is :

(A) $\frac{3}{2}\frac{u\sin\theta}{g}$ (B) $\frac{u\sin\theta}{g}\left(1+\frac{1}{\sqrt{2}}\right)$

(C) $\frac{2u\sin\theta}{g}$ (D) $\frac{u\sin\theta}{g}\left(2+\frac{1}{\sqrt{2}}\right)$

[Ans. (B)]

Example 2. The horizontal range of particle is

(A) $\frac{3}{4}\frac{u^2\sin 2\theta}{g}$ (B) $\frac{u^2\sin 2\theta}{2g}\left(1+\frac{1}{\sqrt{2}}\right)$

(C) $\frac{u^2}{g}\sin 2\theta$ (D) $\frac{u^2\sin 2\theta}{2g}\left(2+\frac{1}{\sqrt{2}}\right)$

[Ans. (B)]

Example 3. The angle ϕ which the velocity vector of stone makes with horizontal just before hitting the ground is given by :

(A) $\tan\phi = 2\tan\theta$ (B) $\tan\phi = 2\cot\theta$ (C) $\tan\phi = \sqrt{2}\tan\theta$ (D) $\tan\phi = \sqrt{2}\cot\theta$

[Ans. (C)]

Solution **1 to 3**

The time taken to reach maximum height and maximum height are

$$t = \frac{u\sin\theta}{g} \quad \text{and} \quad H = \frac{u^2\sin^2\theta}{2g}$$

For remaining half, the time of flight is

$$t' = \sqrt{\frac{2H}{(2g)}} = \sqrt{\frac{u^2\sin^2\theta}{2g^2}} = \frac{t}{\sqrt{2}}$$

∴ Total time of flight is $t + t' = t\left(1+\frac{1}{\sqrt{2}}\right)$

$$T = \frac{u\sin\theta}{g}\left(1+\frac{1}{\sqrt{2}}\right)$$

Also horizontal range is = u cosθ × T = $\frac{u^2\sin 2\theta}{2g}\left(1+\frac{1}{\sqrt{2}}\right)$

Let u_y and v_y be initial and final vertical components of velocity.

∴ $u_y^2 = 2gH$ and $v_y^2 = 4gH$

∴ $v_y = \sqrt{2}\,u_y$

Angle (ϕ) final velocity makes with horizontal is

$$\tan\phi = \frac{v_y}{u_x} = \sqrt{2}\frac{u_y}{u_x} = \sqrt{2}\tan\theta$$

Comprehension - 16

The velocity '*v*' of a particle moving along straight line is given in terms of time *t* as $v = 3(t^2 - t)$ where *t* is in seconds and *v* is in m/s.

Example **1.** The distance travelled by particle from $t = 0$ to $t = 2$ seconds is :

(A) 2 m (B) 3 m (C) 4 m (D) 6 m

[Ans. (B)**]**

Example **2.** The displacement of particle from $t = 0$ to $t = 2$ seconds is

(A) 1 m (B) 2 m (C) 3 m (D) 4 m

[Ans. (B)**]**

Example **3.** The speed is minimum after $t = 0$ second at instant of time

(A) 0.5 sec (B) 1 sec. (C) 2 sec. (D) None of these

[Ans. (B)**]**

Solution **1 to 3**

The velocity of particle changes sign at $t = 1$ sec.

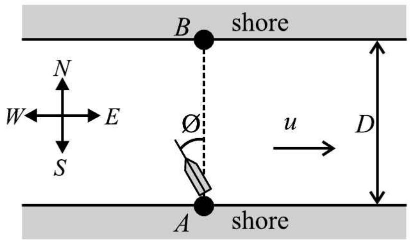
∴ Distance from $t = 0$ to $t = 2$ sec. is = $\int_1^0 v\,dt + \int_2^1 v\,dt$

$$= \left[(t^3 - \frac{3}{2}t^2)\right]_1^0 + \left[(t^3 - \frac{3}{2}t^2)\right]_2^1 = 3 \text{ m}$$

Displacement from $t = 0$ to $t = 2$ sec. is $\int_0^2 v\,dt = \left[(t^3 - \frac{3}{2}t^2)\right]_0^2 = 2$ m.

Comprehension - 17

Two ports, A and B, on a North-South line are separated by a river of width D. The river flows east with speed u. A boat crosses the river starting from port A. The speed of the boat relative to the river is v. Assume $v = 2u$.



Example 1. What is the direction of the velocity of boat relative to river, θ, so that it crosses directly on a line from A to B ?

(A) 30° west of north (B) 30° east of north
(C) 60° west of north (D) 60° east of north

Solution To reach the port B, the x-component of the total velocity must be zero: $v \sin\theta - u = 0$.

So $\sin\theta = 1/2$or $\theta = 30°$

Take positive x-axis along east and positive y-axis along north.

Example 2. Suppose the boat wants to cross the river from A to the other side in the shortest possible time. Then what should be the direction of the velocity of boat relative to river ?

(A) 30° west of north (B) 30° east of north
(C) 60° west of north (D) along north

Solution To cross the river the fastest, we need to maximize the y-component of the total velocity is $v_B \cos\theta$. So $\theta = 0$. The boat should head straight to the North.

Take positive x axis along east and positive y-axis along north.

Example 3. The boat crosses the river from A to the other side in shortest possible time, then how far is the boat from the port B after crossing the river

(A) $D/\sqrt{2}$ (B) $\sqrt{2}D$ (C) $2D$ (D) $D/2$

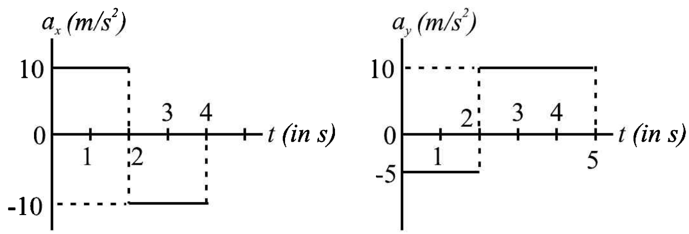
Solution The trip takes time $t = \frac{D}{v}$. The y-component of the total velocity is u. So the boat is at a

distance $u\frac{D}{v} = \frac{D}{2}$ down stream from the port B after crossing.

Take positive x-axis along east and positive y-axis along north.

Comprehension - 18

A particle which is initially (at $t = 0$) at rest at the origin, is subjected to an acceleration with x- and y-components as as shown. After time $t = 5$, the particle has no acceleration.



Example 1. What is the magnitude velocity of the particle at $t = 2$ seconds ?

(A) $10\sqrt{5}$ m/s (B) $5\sqrt{10}$ m/s (C) $5\sqrt{5}$ m/s (D) None of these

Example 2. What is the magnitude of average velocity of the particle between $t = 0$ and $t = 4$ seconds?

(A) $\frac{5}{2}\sqrt{13}$ m/s (B) $\frac{5}{2}\sqrt{17}$ m/s (C) 30 m/s (D) None of these

Example 3. When is the particle at its farthest distance from the y-axis?

(A) 3 sec. (B) 2 sec. (C) 4 sec. (D) 1 sec.

Solution **[Ans.** 1. (A), 2. (B), 3. (C)]

At $t = 2$ sec

$$v_x = u_x + a_x t = 0 + 10 \times 2 = 20 \text{ m/s}$$

$$v_y = u_y + a_y t = 0 - 5 \times 2 = -10 \text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(20)^2 + (-10)^2} = 10\sqrt{5} \text{ m/s}$$

From $t = 0$ to Ist $t = 4$ sec

$$x = \left[\frac{1}{2}(10)(2)^2\right]_{(0\to 2)} + \left[(10\times 2)2 - \frac{1}{2}(10)(2)^2\right]_{(2\to 4)}$$

$$x = 40 \text{ m}$$

$$y = \left[-\frac{1}{2}5(2)^2\right]_{(0\to 2)} - \left[(10(2) - \frac{1}{2}(10)(2)^2\right]_{(2\to 4)}$$

$$y = -10 \text{ m}$$

Hence, average velocity of particle between $t = 0$ to $t = 4$ sec is

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{\sqrt{(40)^2 + (-10)^2}}{4}$$

$$v_{av} = \frac{5}{2}\sqrt{17} \text{ m/s}$$

At $t = 2$ sec, $u = 10 \times 2 = 20$ m/s

After $t = 2$sec

$$v = u + at$$

$0 = 20 - 10t$

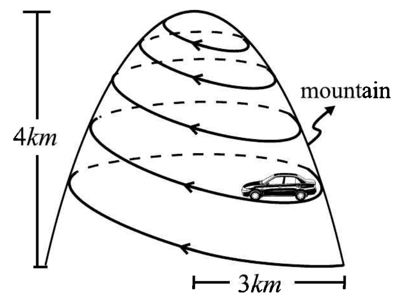
$t = 2$ sec.

Hence, at $t = 4$ sec. the particle is at its farthest distance from the y-axis.

The particle is at farthest distance from y-axis at $t > 4$. Hence the available correct choice is $t = 4$.

Comprehension - 19

Mr. Shyam drives his car at uniform speed from bottom of a mountain to the top in 20 minutes along a helical path as shown.



At the beginning the speedometer of his car shows 8315 km, while on reaching the top it reads 8335 km. (Take upward as positive y-axis and positive x-axis towards right)

Example 1. The total distance covered is :

(A) 10 km (B) 20 km

(C) 25 km (D) can not be determine

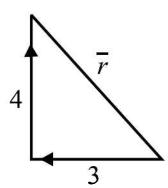
Solution Total distance covered = Final leading – initial leading = 20 km.

Example 2. His displacement vector during the journey is :

(A) $(-3\hat{i} + 4\hat{j})$ km (B) 3 km (C) 5 km (D) none of these

Solution Displacement vector,

$\bar{r} = -3\hat{i} + 4\hat{j}$



Example 3. The average velocity during the journey is :

(A) $(-9\hat{i} + 12\hat{j})$ km/hr (B) $(-2.5\hat{i} + 3.3\hat{j})$ m/s

(C) (25/8) m/s (D) None of these

Solution Average velocity ,

$$<v> = \frac{\bar{r}}{t} = \frac{(-3\hat{i} + 4\hat{j})}{20 \times 60} \times 1000$$

$$= (-2.5\hat{i} + 3.3\hat{j})\ m/s$$

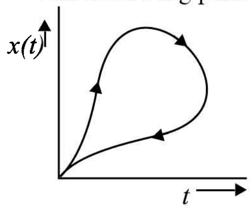
$$= (-9\hat{i} + 12\hat{j})\ \text{km/h}.$$

Comprehension – 20

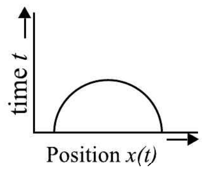
Position time graph, velocity–time graph, speed–time graph are given below:

Example 1. Are the following position time graphs possible ? Give proper reasons.

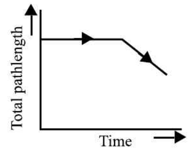
(i)



(ii)



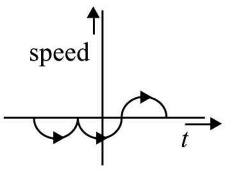
(iii)



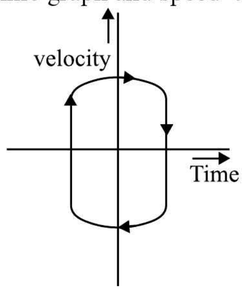
[**Ans.** None]

Example 2. Are the following velocity–time graph and speed–time graph possible ?

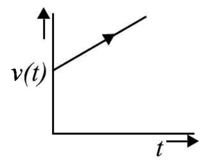
(i)



(ii)



(iii)



[**Ans.** Only graph (iii) is possible]

Match the following

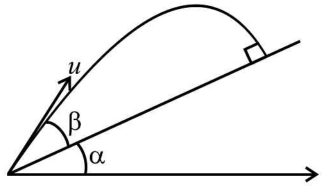
Example 1. A ball is thrown vertically upward in the air by a passenger (relative to himself) from a train that is moving. Correctly match the situation as described in the left column, with the paths given in right column.

(A)	Train moving with constant acceleration on a slope then path of the ball as seen by the passenger.	(P)	Straight line
(B)	Train moving with constant acceleration on a slope then path of the ball as seen by a stationary observer outside.	(Q)	Parabolic
(C)	Train moving with constant acceleration then path of the ball as seen by the passenger.	(R)	Elliptical
(D)	Train moving with constant acceleration then path of the ball as seen by a stationary observer outside.	(S)	Hyperbolic

[**Ans.** (A) Q, (B) Q , (C) Q, (D) Q]

Solution In all cases, angle between velocity and net force (in the frame of observer) is in between 0° and 180° (excluding both values, in that path is straight line).

Example 2. The projectile collides perpendicularly with the inclined plane. (Refer the figure)



(A) Maximum height attained by the projectile from the ground — (P) zero

(B) Maximum height attained by the projectile from Inclined plane — (Q) g

(C) Acceleration of the projectile before striking the inclined plane — (R) $\dfrac{u^2 \sin^2 \beta}{2g\cos\alpha}$

(D) Horizontal component of acceleration of the projectile. — (S) $\dfrac{u^2 \sin^2(\alpha+\beta)}{2g}$

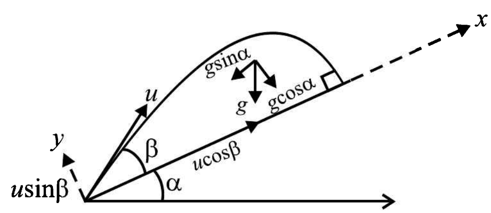
Ans. (A) *S* (B) *R* (C) *Q* (D) *P*

Solution Maximum height from ground of projectile is $\dfrac{u^2 \sin^2(\alpha+\beta)}{2g}$

Maximum height from inclined plane is $\dfrac{U_\perp^2}{2g_\perp}$

$U_\perp = U\sin\beta$ and $g_1 = g\cos\alpha$.

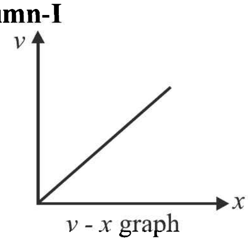
$\therefore$ h from inclined plane $= \dfrac{U^2 \sin^2 \beta}{2g\cos\alpha}$



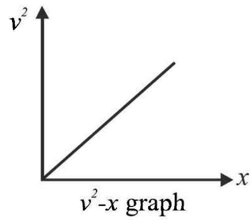
Acceleration of the projectile is always g downward and horizontal component will obviously be zero.

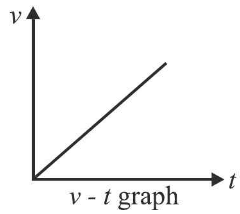
Example 3. Column I gives some graphs for a particle moving along x-axis in positive x–direction. The variables v, x and t represent speed of particle, x–coordinate of particle and time respectively. Column II gives certain resulting interpretation. Match the graphs in Column I with the statements in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

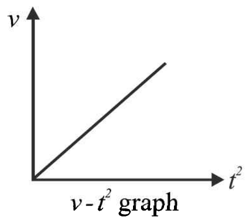
Column-I | **Column-II**

(A)


(p) Acceleration of particle is uniform

(B)  (q) Acceleration of particle is nonuniform

(C)  (r) Acceleration of particle is directly proportional to 't'

(D)  (s) Acceleration of particle is directly proportional to 'x'.

[**Ans.** (A) q, s (B) p (C) p (D) q, r]

Solution From graph (A) $\Rightarrow$ $v = kx$ where k is positive constant

acceleration $= v\dfrac{dv}{dx} = kx \times k = k^2x$

$\therefore$ acceleration is non uniform and directly proportional to x. $\therefore a \to Q, S$

From graph (B) $\Rightarrow$ $v^2 = kx$. Differentiating both sides with respect to x.

$2v\dfrac{dv}{dx} = k$ or $v\dfrac{dv}{dx} = \dfrac{k}{2}$ Hence acceleration is uniform. $\therefore b \to P$

From graph (C) $\Rightarrow$ $v = kt$

acceleration $= \dfrac{dv}{dt} = k$ Hence acceleration is uniform $\Rightarrow c \to P$

From graph (D) $\Rightarrow$ $v = kt^2$

acceleration $= \dfrac{dv}{dt} = 2\text{kt}$ Hence acceleration is non uniform and directly proportional to t.

$\therefore d \to Q, R$

Example 4. Match the following:

(i)	Rate of change of displacement	(A)	Magnitude of average velocity
(ii)	Average speed is always greater than or equal to	(B)	Initial to final position
(iii)	Displacement has the same direction as that of	(C)	Velocity

[**Ans.** (i) C, (ii) A, (iii) B]

Example 5. Consider motion of a particle in one dimension. Initially particle is at origin and has velocity towards positive x - direction. x, v, a and t denote displacement, velocity, acceleration and time respectively. Column II gives subsequent motion of the particle under the conditions in column I. Match the condition in Column I with the resultant motion in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

Column-I	Column-II
(A) $a = -3\,v$	(p) Particle never stops
(B) $v = 6 - 3t$	(q) Particle stops at least once
(C) $x = 3 - 3\cos 2t$	(r) Particle travels finite distance before coming to rest first time.
(D) $x = 3t + 6t^2$	(s) Particle comes back to origin at least once.

Ans. (A) p ; (B) q, r, s (C) q, r, s (D) p

Solution For a = – 3 v

$$\frac{dv}{dt} = -3\,v$$

$$v = v_0\, e^{-3t}$$

The velocity is never zero and is never negative. So it will not come back to origin.

For $v = 6 - 3t$

After $t = 2$s

v is negative. It will return back to origin and continue moving in negative x-direction.

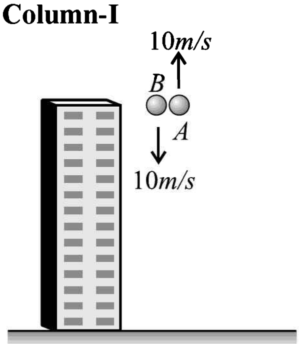
For $x = 3 - 3\cos 2t$

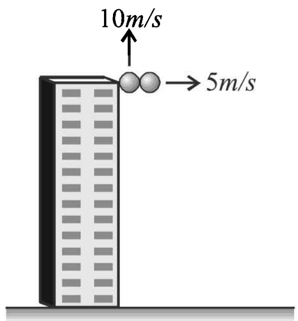
This is an example of SHM

For $x = 3\,t + 6\,t^2$

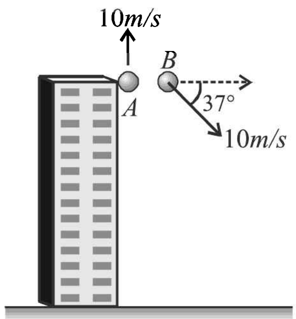
The particle will continue moving in positive x-direction.

Example 6. Both A & B are thrown simultaneously as shown from a very high tower.

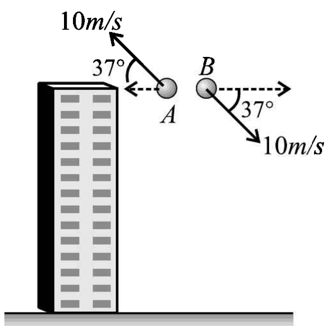
Column-I	Column-II
(A) 	(p) Distance between the two balls after two seconds is $8\sqrt{5}$ m.

(B) 

(q) distance between two balls after 2 seconds is 40 m.

(C) 

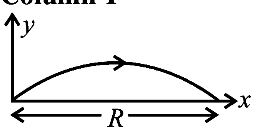
(r) magnitude of relative velocity of B with respect to A is $5\sqrt{5}$ m/s.

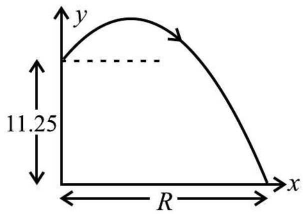
(D) 

(s) Magnitude of relative velocity of B w.r.t A is $5\sqrt{2}$ m/s.

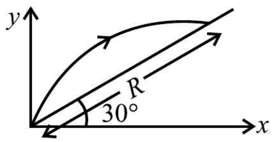
[**Ans.** (A) q, (B) r (C) p (D) q]

***Example* 7.** In the column-1, the path of a projectile (initial velocity 10 m/s and angle of projection with horizontal 60° in all cases) is shown in different cases. Rangle 'R' is to be matched in each case from column-11. Take g = 10 m/s^2. Arrow on the trajectory indicates the direction of motion of projectile.

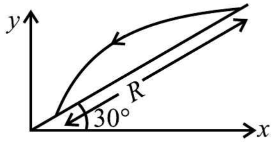
Column-I	Column-II
(A) 	(p) $R = \frac{15\sqrt{3}}{2}$ m

(B) 

(q) $R = \dfrac{40}{3}$ m

(C) 

(r) $R = 5\sqrt{3}$ m

(D) 

(s) $R = \dfrac{20}{3}$ m

Ans. (A) *r* (B) *p* (C) *s* (D) *q*

Solution

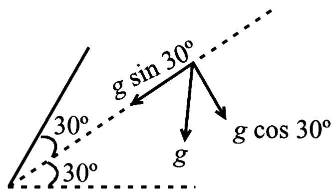
(A) $R = \dfrac{u^2 \sin 2\theta}{g} = \dfrac{100\sqrt{3}}{2(10)} = 5\sqrt{3}m$

(B) $11.25 = -\,10\sin 60^\circ\, t + \dfrac{1}{2}(10)\, t^2 \Rightarrow 5t^2 - 5\sqrt{3}\; t - 11.25 = 0$

$t = \dfrac{5\sqrt{3} \pm \sqrt{25(3) + 4(5)(11.25)}}{10} = \dfrac{5\sqrt{3} \pm \sqrt{3}(10)}{10}$

$= \dfrac{15}{10}\sqrt{3} = \dfrac{3}{2}\sqrt{3}$

(C) $R = 10\cos 60\left(\dfrac{3}{2}\sqrt{3}\right) = 7.5\sqrt{3}$ m



$t = \dfrac{2u\sin 30^\circ}{g\cos 30^\circ} = \dfrac{2(10)\left(\dfrac{1}{2}\right)}{10\left(\dfrac{\sqrt{3}}{2}\right)} = \dfrac{2}{\sqrt{3}}$ sec.

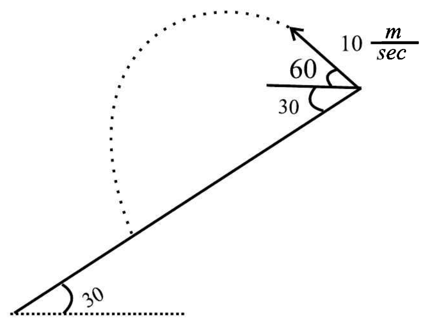
$R = 10\cos 30^\circ\, t - \dfrac{1}{2}$ g sin 30° $\qquad t^2 = \dfrac{10\sqrt{3}}{2}\left(\dfrac{2}{\sqrt{3}}\right) - \dfrac{1}{2}(10)\left(\dfrac{1}{2}\right)\dfrac{4}{3}$

$= 10 - \dfrac{10}{3} = \dfrac{20}{3}$ m

(D) $T = \dfrac{2(10)}{g\cos 30} = \dfrac{2(10)}{10\left(\dfrac{\sqrt{3}}{2}\right)} = \dfrac{4}{\sqrt{3}}$ sec.

$R = \dfrac{1}{2}\, g \sin 30^\circ\, t^2$

$= \dfrac{1}{2}(10)\left(\dfrac{1}{2}\right)\dfrac{16}{3} = \dfrac{40}{3}$ m



Example 8. Two particles *A* and *B* moving in *x*-*y* plane are at origin at $t = 0$ sec. The initial velocity vectors of *A* and *B* are $\vec{u}_A = 8\hat{i}$ m/s and $\vec{u}_B = 8\hat{j}$ m/s. The acceleration of *A* and *B* are constant and are $\vec{a}_A = -2\hat{i}$ m/s² and $\vec{a}_B = -2\hat{j}$ m/s². Column I gives certain statements regarding particle *A* and *B*. Column I gives corresponding results. Match the statements in column I with corresponding results in Column II.

Column I	**Column II**
(A) The time (in seconds) at which velocity of *A* relative to *B* is zero	(p) $16\sqrt{2}$
(B) The distance (in metres) between *A* and *B* when their relative velocity is zero.	(q) $8\sqrt{2}$
(C) The time (in seconds) after $t = 0$ sec, at which *A* and *B* are at same position	(r) 8
(D) The magnitude of relative velocity of *A* and *B* at the instant they are at same position.	(s) 4

Ans. (A) *s* (B) *p* (C) *r* (D) *q*

Solution The initial velocity of *A* relative to *B* is $\vec{u}_{AB} = \vec{u}_A - \vec{u}_B = (8\hat{i} - 8\hat{j})$ m/s $\therefore\ u_{AB} = 8\sqrt{2}$ m/s

Acceleration of *A* relative to *B* is -

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B = (-2\hat{i} + 2\hat{j})$ m/s² $\therefore\ a_{AB} = 2\sqrt{2}$ m/s²

since *B* observes initial velocity and constant acceleration of *A* in opposite directions, Hence *B* observes *A* moving along a straight line.

From frame of *B*

Hence time when $v_{AB} = 0$ is $t = \dfrac{u_{AB}}{a_{AB}} = 4$ sec.

The distance between *A* & *B* when $v_{AB} = 0$ is $S = \dfrac{u_{AB}^2}{2a_{AB}} = 16\sqrt{2}$ m

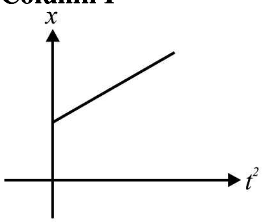
The time when both are at same position is -

$$T = \frac{2u_{AB}}{a_{AB}} = 8 \text{ sec.}$$

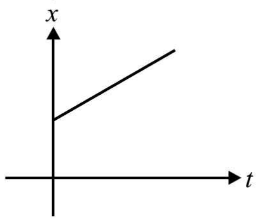
Magnitude of relative velocity when they are at same position in $u_{AB} = 8\sqrt{2}$ m/s.

Example 9. Column 1 gives some graphs for a particle moving along x-axis in positive x–direction. The variables v, x and t represent speed of particle, x–coordinate of particle and time respectively. Column ll gives certain resulting interpretation. Match the graphs in Column 1 with the statements in Column ll and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

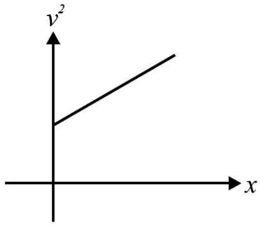
Column I | **Column II**

(A) 

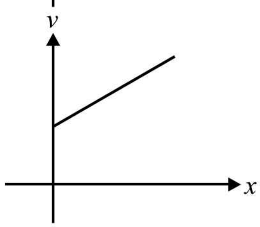
(p) acceleration is uniform and non zero

(B) 

(q) acceleration is non uniform

(C) 

(r) velocity is uniform

(D) 

(s) velocity is non-uniform

[**Ans.** (A) *p*, *s* (B) *r* (C) *p*, *s* (D) *q*, *s*]

Solution

(A) $x = mt^2 + c$ $\quad \therefore v = 2mt \Rightarrow a = 2m$.

Hence velocity is non-uniform and acceleration is uniform and non-zero.

(B) $x = mt$ $\quad \therefore v = m$ and $a = 0$

$\Rightarrow$ velocity is uniform

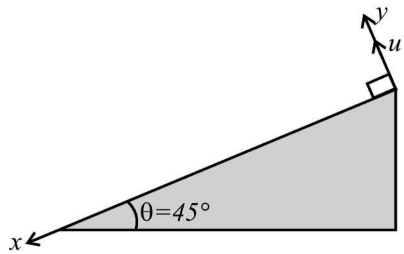
(C) $v^2 = mx + c$ $\quad \therefore a = v\dfrac{dv}{dx} = \dfrac{m}{2}$

$\Rightarrow$ velocity is non-uniform and acceleration is uniform.

(D) $v = mx + \text{c}$ $\quad \therefore a = v\dfrac{dv}{dx} = \text{m}^2x + \text{c}$

$\Rightarrow$ velocity is non-uniform and acceleration is also non-uniform.

***Example* 10.** An inclined plane makes an angle $\theta = 45°$ with horizontal. A stone is projected normally from the inclined plane, with speed u m/s at $t = 0$ sec. x and y axis are drawn from point of projection along and normal to inclined plane as shown. The length of incline is sufficient for stone to land on it and neglect air friction. Match the statements given in column I with the results in column II.



Column-I	Column-II
(A) The instant of time at which velocity of stone is parallel to x-axis	(p) $\frac{2\sqrt{2}u}{g}$
(B) The instant of time at which velocity of stone makes an angle $\theta = 45°$ with positive x-axis.	(q) $\frac{2u}{g}$
(C) The instant of time till which (starting from $t = 0$) component of displacement along x-axis is half the range on inclined plane is	(r) $\frac{\sqrt{2}u}{g}$
(D) Time of flight on inclined plane is	(s) $\frac{u}{\sqrt{2}g}$

[**Ans.** (A) *r* (B) *s* (C) *q* (D) *p*]

Solution

Time of flight, $T = \frac{2u}{g\cos 45°} = \frac{2u}{g\cos 45°}\frac{2\sqrt{2}u}{g} \quad \therefore D \rightarrow p$

Velocity of stone is parallel to x-axis at half the time of flight.

$\therefore \quad A \rightarrow r$

At the instant stone make 45° angle with x-axis its velocity is horizontal.

$\therefore$ The time is $= \frac{u\sin 45°}{g} = \frac{u}{\sqrt{2}g} \qquad \therefore B \rightarrow$ s

The time till its displacement along x-axis is half the range is

$= \frac{1}{\sqrt{2}}T = \frac{2u}{g} \quad \therefore C \rightarrow q$

Example 11. For a particle moving in x-y plane initial velocity of particle is $\vec{u} = u_1\hat{i} + u_2\hat{j}$ and acceleration of particle is always $\vec{a} = a_1\hat{i} + a_2\hat{j}$ where u_1, u_2, a_1, a_2 are constants. Some parameters of motion is given in column-I, match the corresponding path given in column-II.

Column-I	Column-II
(A) If $u_1 \neq 0, u_2 = 0, a_1 \neq 0, a_2 \neq 0$	(p) path of particle must be parabolic
(B) If $u_1 = 0, u_2 \neq 0, a_1 \neq 0, a_2 \neq 0$	(q) path of particle must be straight line
(C) If $u_1 = 0, u_2 = 0, a_1 \neq 0, a_2 \neq 0$	(r) path of particle may be parabolic
(D) If $u_1 \neq 0, u_2 \neq 0, a_1 \neq 0, a_2 \neq 0$	(s) path of particle may be straight line

[**Ans.** (A) *p* (B) *p* (C) *q* (D) *r*, *s*]

Solution If angle between constant acceleration vector $\vec{a}$ and velocity vector $\vec{v}$ is zero or 180° then path is straight line otherwise path must be parabolic.

Example 12. A particle is projected from level ground. Assuming projection point as origin, x-axis along horizontal and y-axis along vertically upwards. If particle moves in x-y plane and its path is given by $y = ax - bx^2$ where a, b are positive constants. Then match the physical quantities given in column-I with the values given in column-II. (g in column II is acceleration due to gravity.)

Column I	Column II
(A) Horizontal component of velocity	(p) $\frac{a}{b}$
(B) Time of flight	(q) $\frac{a^2}{4b}$
(C) Maximum height	(r) $\sqrt{\frac{g}{2b}}$
(D) Horizontal range	(s) $\sqrt{\frac{2a^2}{bg}}$

[**Ans.** (A) *r* (B) *s* (C) *q* (D) *p*]

Solution Equation of path is given as $y = ax - bx^2$

Comparing it with standard equation of projectile;

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\tan\theta = a, \quad \frac{g}{2u^2\cos^2\theta} = b$$

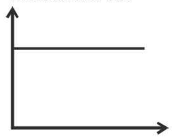
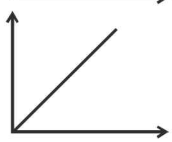
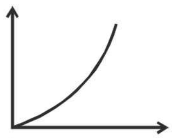
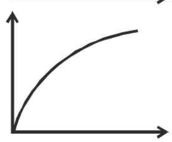
Horizontal component of velocity $= u\cos\theta = \sqrt{\frac{g}{2b}}$

$$\text{Time of flight } T = \frac{2u\sin\theta}{g} = \frac{2(u\cos\theta)\tan\theta}{g} = \frac{2\left(\sqrt{\frac{g}{2b}}\right)a}{g} = \sqrt{\frac{2a^2}{bg}}$$

$$\text{Maximum height } H = \frac{u^2 \sin^2\theta}{2g} = \frac{[u\cos\theta . \tan\theta]^2}{2g} = \frac{\left[\sqrt{\frac{g}{2b}} . a\right]^2}{2g} = \frac{a^2}{4b}$$

$$\text{Horizontal range } R = \frac{u^2 \sin 2\theta}{g} = \frac{2(u\sin\theta)(u\cos\theta)}{g} = \frac{2\left[\sqrt{\frac{g}{2b}} . a\right]\left[\sqrt{\frac{g}{2b}}\right]}{g} = \frac{a}{b}$$

Example 13. A particle is moving along a straight line. Its velocity varies with time as $v = kt$, where k is a positive constant and t is the time. Match the graphs in Column II with the statements in Column I and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

Column-I	Column-II
(A) Acceleration versus time curve	(p) 
(B) Acceleration versus displacement curve	(q) 
(C) Velocity versus time curve	(r) 
(D) Displacement versus velocity curve	(s) 

Solution (A) p (B) p (C) q (D) r

Example 14. A particle is projected horizontally at time $t = 0$ from a given height above the ground level. Then match the physical quantities given in Column - I with the corresponding results given in Column - II. Consider all quantities in Column I from $t = 0$ and before the particle reaches the ground.

Column-I	Column-II
(A) Magnitude of acceleration	(p) remains constant
(B) Magnitude of average velocity from $t = 0$ to any time t	(q) decreases with time t
(C) Angle between acceleration and velocity vector	(r) increases with time t
(D) Distance of particle from its initial position.	(s) depends on initial velocity.

Ans. (A) p (B) r, s (C) q, s (D) r, s

Solution

(A) The magnitude of acceleration remains constant.

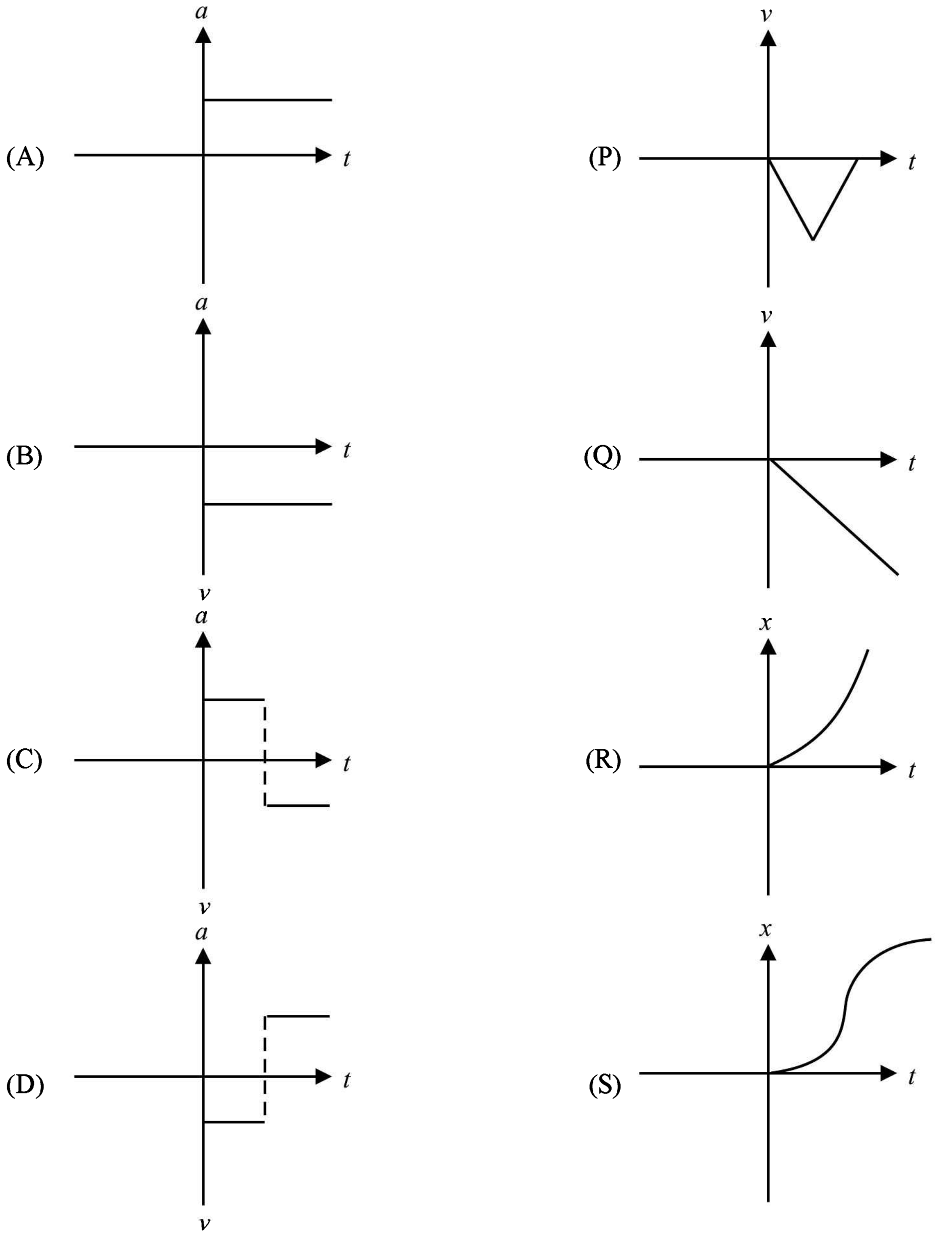
(B) Magnitude of average velocity increases and depends on initial velocity.

(C) Angle between acceleration and velocity vector increases with time.

(D) Distance of particle from its initial position increases with time and depends on initial velocity

***Example* 15.** In each of the situations assume that particle was initially at rest at origin and there after it moved rectilinearly. Some of the graph in left column represent the same motion as represented by graphs in right column match these graphs.

Column-I **Column-II**



[**Ans.** $A-R, B-Q, C-S, D-P$]

Solution

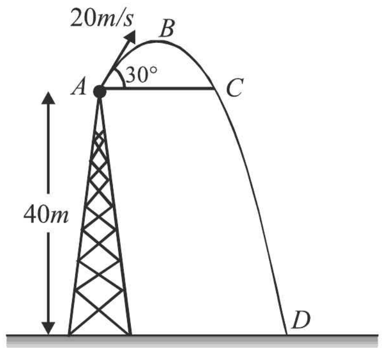
(A) a is +ve constant $\Rightarrow$ velocity will increase linearly with time and position will increase parabolicaly with time.

(B) a is −ve constant $\Rightarrow$ velocity will decrease linearly with time and position will decrease parabolicaly with time.

(C) First acceleration is +ve & then −ve $\Rightarrow$ velocity will first increase and then decrease. Also, position of the particle will increase parabolicaly.

(D) First acceleration is −ve & then +ve $\Rightarrow$ velocity will decrease linearly and then increase.

Example 16. A projectile is fired from top of a 40 m high tower with velocity 20 m/s at an angle of 30° with the horizontal (see figure). $g = 10$ m/s^2.



(A)	Ratio of time taken from A to D with time taken from A to C is equal to	(P) 1
(B)	Ratio of vertical distance travelled from A to D with the maximum height from ground is less than	(Q) 2
(C)	Ratio of final speed at D with the initial speed at A is less than	(R) 3
(D)	Ratio of horizontal displacement from A to D with height of tower is greater than	(S) 4

[**Ans.** (A) Q (B) Q, R, S (C) Q, R, S (D) P]

True/False

Example 1. Negative acceleration and deceleration have same meaning in the context of motion along a straight line.

[**Ans.** False]

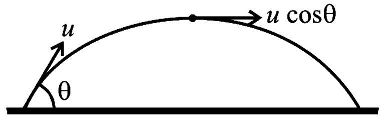
Example 2. A body falling freely from rest from very high building , covers some distance S in t sec. In next t sec, it will cover a distance 3 S.

[**Ans.** True]

Example 3. The speed acquired by a body when falling in vacuum for a given time is dependent on the mass of the falling body.

[**Ans.** False]

Example 4. A projectile fired from the ground follows a parabolic path. The velocity of the projectile is zero at the top most point.



[**Ans.** False]

Solution Velocity at the highest point is $u\cos\theta$.

only the vertical component is zero.

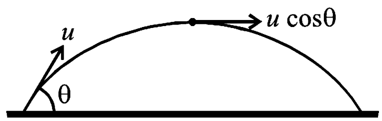
Example 5. In a projectile motion up an inclined plane, the acceleration perpendicular to the trajectory at the farthest point from the incline is g downwards.

[**Ans.** False]

Example 6. For a ball thrown horizontally from the top of a cliff at speed u, the equation of trajectory is: $y = -\frac{gx^2}{2u^2}$. State true/ false. (taking the point of projection as origin)

[**Ans.** True]

Example 7. A projectile fired from the ground follows a parabolic path. The velocity of the projectile is zero at the top most point.



[**Ans.** False]

Solution Velocity at the highest point is $u\cos\theta$.

only the vertical component is zero.

Example 8. Path of a projectile with respect to another projectile is parabolic.

[Ans. False]

Solution Path is a straight line.

Example 9. S_1 : The ranges of two particles projected such that their times of flights are equal are also always equal.

S_2 : The maximum heights attained by two particles projected such that their times of flights are equal are also always equal.

S_3 : For a particle projected at an angle $\theta = \tan^{-1} 4$ will attain maximum height equal to the range.

S_4 : A particle starting from rest from origin with $v = 2 - 3t$ m/s will never return back to the origin again.

(A) $FFTF$ (B) $FTTF$ (C) $TFTT$ (D) $FFFF$

Solution

[**Ans.** (B)]

Example 11. S_1 : A particle having negative acceleration will slow down.

S_2 : For constant acceleration if angle between initial velocity makes an oblique angle with acceleration then path will be parabolic.

S_3 : A particle having its velocity equal to zero at any instant of the motion, then its acceleration is also zero at that instant.

S_4 : If speed of a body is varying, its velocity must be varying and it must have non-zero acceleration

(A) $FTFT$ (B) $FFTT$ (C) $TTFF$ (D) $FTFF$

Solution Negative acceleration necessarily does not mean retardation. Hence S_1 is false

If the initial velocity is not parallel or antiparallel to its uniform acceleration, it shall move in parabolic path. Hence S_2 is True

For a particle projected vertically upwards, the velocity is zero at highest point but the acceleration is non zero. Hence S_3 is false.

Varying speed implies varying velocity. Hence acceleration cannot be zero. Hence S_4 is true

[Ans. (A)]

Assertion Reason Questions

Example 1. **Assertion :** A particle having negative acceleration will slow down.

Reason : Direction of the acceleration is not dependent upon direction of the velocity.

(A) If both assertion and reason are true and reason is the correct explanation of assertion
(B) If both assertion and reason are true but reason is not the correct explanation of assertion.
(C) If assertion is true but reason is false
(D) If assertion is false but reason is true.

[Ans. (D)]

Example 2. **Assertion :** Velocity-time graph for an object in uniform motion along a straight path is a straight line parallel to the time axis.

Reason : In uniform motion of an object velocity increases as the square of time elapsed.

(A) A (B) B (C) C (D) D
(E) E

[Ans. (C)]

Example 3. **STATEMENT-1 :** Three projectiles are moving in different paths in the air. Vertical component of relative velocity between any of the pair does not change with time as long as they are in air. Neglect the effect of air friction.

STATEMENT-2 : Relative acceleration between any of the pair of projectiles is zero.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Solution Acceleration of each of the projectile = $\vec{g}$. Relative acceleration $\vec{a}_r = \vec{g} - \vec{g} = 0$.

[Ans. (A)]

Example 4. STATEMENT-1 : For a particle moving in a straight line, velocity (v in m/s) of the particle in terms of time (t in sec) is given by $v = t^2 - 6t + 8$. Then the speed of the particle is minimum at t = 2 sec.

STATEMENT-2 : For a particle moving in a straight line the velocity v at any time t may be minimum or may be maximum when $\frac{dv}{dt} = 0$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (B)]

Solution The expression for velocity and time can be expressed as $v = (t - 2)(t - 4)$

The speed is therefore zero at t = 2 and t = 4. Hence speed is minimum at t = 2.

But $\frac{dv}{dt} = 2t - 6$ is zero at t = 3 seconds.

Hence statement I is true also we know statement II is true but II is not a correct explanation of I.

Example 5. Assertion : In rectilinear motion when velocity is positive distance travelled increases and when velocity is negative distance travelled decreases.

Reason : Distance is length of the path covered by a particle.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(C) if Assertion is true, but the Reason is false.

(D) if Assertion is false, but the Reason is true.

[Ans. (D)]

Solution Either velocity is positive or negative distance travelled by a body is always positive, hence assertion is wrong.

Example 6. Assertion : If velocity is negative magnitude of position may increase.

Reason : Velocity is rate of change of position w.r.t. time.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(C) if Assertion is true, but the Reason is false.

(D) if Assertion is false, but the Reason is true.

Solution If velocity is –ve, position may be increasing is –ve direction, so magnitude of position is increasing. Also, velocity is rate of change of position and if position is increasing in –ve direction velocity will be –ve.

Example 7. Assertion : If acceleration of a particle is decreasing then it is possible that velocity is increasing with time.

Reason : Acceleration is rate of change of velocity.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(C) if Assertion is true, but the Reason is false.

(D) if Assertion is false, but the Reason is true.

[Ans. (A)]

Solution A decreasing acceleration means the rate of change of velocity is decreasing, it doesn't mean that the velocity is decreasing. As long as acceleration is +ve, velocity will keep on increasing.

Example 8. **Assertion :** A particle is at rest in time interval $[t_1, t_2]$. Then, the displacement of the patticle is zero but position of the particle can be non–zero in this time interval.

Reason : Change in position is called displacement.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

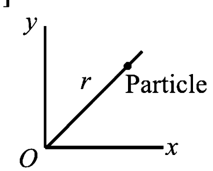
(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(C) if Assertion is true, but the Reason is false.

(D) if Assertion is false, but the Reason is true.

[Ans. (A)]

Solution



so r is not zero but Particle is at rest.

Example 9. **Assertion :** For a projectile up the incline maximum angle of projection can be $\left(\frac{\pi}{4}-\frac{\beta}{2}\right)$ where β is angle made by incline with horizontal.

Reason : Maximum range up the incline is given by $\frac{u^2}{g(1+\sin\beta)}$ where β is angle made by incline with horizontal.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(C) if Assertion is true, but the Reason is false.

(D) if Assertion is false, but the Reason is true.

[Ans. (D)]

Example 10. **Statement 1 :** Magnitude of average velocity is equal to average speed.

Statement 2 : Magnitude of instantaneous velocity is equal to instantaneous speed.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

[Ans. (D)]

Solution Since Distance $\geq$ Dispacement

so Av speed $\geq$ Av. velocity

so statement I^{st} is false. But II^{nd} is true by definition.

Example 11. **Statement 1 :** When velocity of a particle is zero then acceleration of particle is also zero.

Statement 2 : Acceleration is equal to rate of change of velocity.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

[Ans. (D)]

Solution When a body is projected upward then at the maximum height its velocity is zero but acceleration is always g downward statement 2 is true by theory.

Example 12. **Assertion :** If slope of a given curve is zero at a given point then their may exist a maxima at that point.

Reason : At a maxima slope change its sign.

(A) If both Assertion and Reason are true and the Reason is correct explanation of the Assertion.

(B) If both Assertion and Reason are true, but Reason is not correct explanation of the Assertion.

(C) if Assertion is true, but the Reason is false.

(D) if Assertion is false, but the Reason is true.

[Ans. (A)]

Solution $\frac{dy}{dx} = 0 \quad \Rightarrow$ there may be a maxima or minima.

So, Assertion is true.

At maxima sign of slope change, so there is change in sign there must be a zero in between.

Example 13. **STATEMENT–1 :** The equation of distance travelled by a particle moving in a straight line with constant acceleration in n^{th} second is $S_n = u + (2n - 1)\frac{a}{2}$, where letters have usual meaning, is dimensionally incorrect.

STATEMENT–2: For every equation relating physical quantities to be true, it must have dimensional homogenity.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

[Ans. (D)]

Solution The equation of distance travelled in n^{th} second is dimensionally correct because the interval of time 1 second has already been substituted into the equation and its dimension should be taken into account. Therefore statement-1 is false.

Example **14. STATEMENT-1 :** Three projectiles are moving in different paths in the air. Vertical component of relative velocity between any of the pair does not change with time as long as they are in air. Neglect the effect of air friction.

STATEMENT-2 : Relative acceleration between any of the pair of projectiles is zero.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (A)]

Solution Acceleration of each of the projectile = $\vec{g}$. Relative acceleration $\vec{a}_r = \vec{g} - \vec{g} = 0$.

Example **15. STATEMENT-1 :** For a particle moving along straight line with constant acceleration, magnitude of displacement is less than distance covered in same time interval. Then the magnitude of average velocity will be less than initial speed in that time interval.

STATEMENT-2: Average velocity $= \dfrac{\text{Total Displacement}}{\text{Total time taken}}$ and Average speed $= \dfrac{\text{Total distance covered}}{\text{Total time taken}}$

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (D)]

Solution Let the direction of initial velocity 'u' and constant acceleration 'a' of the particle be opposite. Then after the particle turns back and acquire a velocity of magnitude larger than 3u, the magnitude of average velocity $\left|\dfrac{u+v}{2}\right|$ shall be greater than v. Hence statement I is false.

Example **16. STATEMENT-1 :** Two stones are simultaneously projected from level ground from same point with same speeds but different angles with horizontal. Both stones move in same vertical plane. Then the two stones may collide in mid air.

STATEMENT-2 : For two stones projected simultaneously from same point with same speed at different angles with horizontal, their trajectories may intersect at some point.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (D)]

Solution Both the stones cannot meet (collide) because their horizontal component of velocities are different. Hence statement I is false.

Example 17. STATEMENT-1 : Speed of a particle is non-negative. The speed v of a particle moving along a straight line at time t will be least if $\frac{dv}{dt} = 0$ and $\frac{d^2v}{dt^2}$ is non-negative.

STATEMENT-2 : For speed v to be least at time t, $\frac{dv}{dt}$ may be zero.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (D)]

Solution If a body is projected vertically upwards, then its speed is least at the highest point, but $\frac{dv}{dt}$ (acceleration) is not zero, hence statement-1 is false and statement-2 is true.

Example 18. STATEMENT-1 : The magnitude of velocity of two boats relative to river is same. Both boats start simultaneously from same point on one bank may reach opposite bank simultaneously moving along different paths.

STATEMENT-2 : For boats to cross the river in same time. The component of their velocity relative to river in direction normal to flow should be same.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

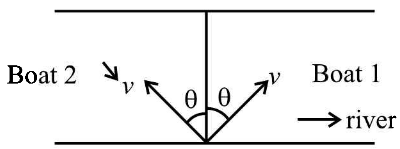
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (A)]

Solution If component of velocities of boat relative to river is same normal to river flow (as shown in figure) both boats reach other bank simultaneously.



Example 19. STATEMENT-1 : A man projects a stone with speed u at some angle. He again projects a stone with same speed such that time of flight now is different. The horizontal ranges in both the cases may be same.

STATEMENT-2 : The horizontal range is same for two projectiles if one is projected at an angle θ with the horizontal and other is projected at an angle 90° – θ with the horizontal.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (C)]

Solution In statement 2, if speed of both projectiles are different, horizontal ranges will be different. Hence statement 2 is false.

Example **20. STATEMENT-1 :** A particle moves in a straight line with constant accleration. The average velocity of this particle cannot be zero in any time interval

STATEMENT-2 : For a particle moving in straight line with constant acceleration, the average velocity in a time interval is $\frac{u+v}{2}$, where u and v are initial and final velocity of the particle of the given time interval.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (D)]

Solution A particle is projected vertically upwards. In duration of time from projection till T reaches back to point of projection, average velocity is zero. Hence statement I is false.

Example **21. STATEMENT-1 :** For a particle moving in a straight line, velocity (v in m/s) of the particle in terms of time (t in sec) is given by $v = t^2 - 6t + 8$. Then the speed of the particle is minimum at t = 2 sec.

STATEMENT-2 : For a particle moving in a straight line the velocity v at any time t may be minimum or may be maximum when $\frac{dv}{dt} = 0$.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (B)]

Solution The expression for velocity and time can be expressed as $v = (t - 2)(t - 4)$

The speed is therefore zero at t = 2. Hence speed is minimum at t = 2.

But $\frac{dv}{dt} = 2t - 6$ is zero at t = 3 seconds.

Hence statement I is true also we know statement II is true but II is not a correct explanation of I.

Example **22. Statement 1 :** Magnitude of average velocity is equal to average speed.

Statement 2 : Magnitude of instantaneous velocity is equal to instantaneous speed.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

[Ans. (D)]

Example **23. Statement 1 :** When velocity of a particle is zero then acceleration of particle is also zero.

Statement 2 : Acceleration is equal to rate of change of velocity.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

[Ans. (D)]

Example **24. STATEMENT-1 :** A stone is projected (not vertically upwards) from level ground . The average velocity of this stone is in horizontal direction in between the two instants of time when velocity of stone makes same angle(in magnitude) with horizontal . (Neglect air friction)

STATEMENT-2 : The average velocity of a projectile (not projected vertically upwards) in between any two instants of time is always in horizontal direction.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (C)]

Solution Between the two instants of time when the projectile makes same angle with horizontal, it is at same height from level ground.

In between the given time interval the displacement of the projectile is horizontal.

Then the average velocity is horizontal. Hence statement-1 is true.

The average velocity of projectile between any two instants of time is not always horizontal.

Hence statement-2 is false.

Example **25. STATEMENT-1 :** For a particle moving in straight line with constant acceleration, the plot of its velocity versus time is a straight line.

STATEMENT-2 : If acceleration is constant for a particle moving along straight line ; the velocity of particle is always directly proportional to time.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (C)]

Solution For a particle moving in straight line with constant acceleration; velocity is given by

$$v = u + at$$

Hence, velocity-time graph will be a straight line. Hence statement-1 is true.

But velocity of particle is not directly proportional to time. Hence statement-2 is false.

Example **26. STATEMENT-1 :** Two stones are simultaneously projected from level ground from same point with same speeds but different angles with horizontal. Both stones move in same vertical plane. Then the two stones may collide in mid air.

STATEMENT-2 : For two stones projected simultaneously from same point with same speed at different angles with horizontal, their trajectories may intersect at some point.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

[Ans. (D)]

Solution Both the stones cannot meet (collide) because their horizontal component of velocities are different. Hence statement I is false.

Example **27. Assertion :** The distance covered can never be less than the magnitude of displacement.

Reason : Distance covered can never decrease where as displacement can.

(A) If both Assertion and Reason are true and the Reason is correct explanation of Assertion.

(B) If both Assertion and Reason are true but Reason is not a correct explanation of Assertion.

(C) If Assertion is true but Reason is false.

(D) If both Assertion and Reason are false.

[Ans. (A)]

EXERCISE

Exercise-1 : Subjective Problems

1. Men are running along a road at 15 km/h behind one another at equal intervals of 20 m. Cyclist are riding in the same direction at 25 km/h at equal intervals of 30 m. At what speed an observer travel along the road in opposite direction so that whenever he meets a runner he also meets a cyclist?
2. Two perpendicular rail tracks have two trains A & B respectively. Train A moves north with a speed of 54 km h^{-1} and train B moves west with a speed of 72 km h^{-1}. Assume that both trains starts from same point. Calculate the
   (a) rate of separation of the two trains
   (b) relative velocity of ground with respect to B
   (c) relative velocity of A with respect to B.
3. A man is swimming in a lake in a direction of 30° East of North with a speed of 5 km/hr and a cyclist is going on a road along the lake shore towards East at a speed of 10 km/hr. In what direction and with what speed would the man appear to swim to the cyclist.
4. A motor boat has 2 throttle position on its engine. The high speed position propels the boat at 10 km hr^{-1} in still water and the low position gives half the higher speed. The boat travels from its dock downstream on a river with the throttle at low position and returns to its dock with throttle at high position. The return trip took 15 % longer time than it did for the downstream trip. Find the velocity of the water current in the river.
5. (I) A man can swim with a speed of 4 km h^{-1} in still water. How long does he take to cross a river 1 km wide if the river flows steadily at 3 km h^{-1} and he makes his strokes normal to the river current ? How far down the river does he go when he reaches the other bank?
   (II) If he keeps himself always at an angle of 120°C with the river flow while swimming.

(A) Find the time he takes to cross the river.

(B) At what point on the opposite bank will he arrive ?

6. A river is flowing from west to east at a speed of 5 m/min. A man on the south bank of the river, capable of swimming at 10 m/min in still water, wants to swim across the river in shortest distance. In what direction should he swim ?

7. An airplane is flying with velocity $50\sqrt{2}$ km/hour in north-east direction. Wind is blowing at 25 km/hr from north to south. What is the resultant displacement of airplane in 2 hours ?

8. When a train has a speed of 10 m s^{-1} eastward, raindrops that are falling vertically with respect to the earth make traces that are inclined 30ŗ to the vertical on the windows of the train.

   (a) What is the horizontal component of a drop's velocity with respect to the earth? With respect to the train?

   (b) What is the velocity of the raindrop with respect to the earth ? With respect to the train?

9. To a man walking at 7 km/h due west, the wind appears to blow from the north-west, but when he walks at 3 km/h due west, the wind appears to blow from the north. What is the actual direction of the wind and what is its velocity ?

10. When a motorist is driving with velocity $6\hat{i} + 8\hat{j}$, the wind appears to come from the direction $\hat{i}$. When he doubles his velocity the wind appears to come from the direction $\hat{i} + \hat{j}$. Then the true velocity of the wind expressed in the form of $a\hat{i} + b\hat{j}$ is ______.

11. 'n' numbers of particles are located at the vertices of a regular polygon of 'n' sides having the edge length 'a'. They all start moving simultaneously with equal constant speed 'v' heading towards each other all the time. How long will the particles take to collide?

12. Two ships are 10 km apart on a line running south to north. The one further north is streaming west at 40 km/hr. The other is streaming north at 40 km/hr. What is their distance of closest approach and how long do they take to reach it?

13. A ship is sailing towards north at a speed of $\sqrt{2}$ m/s. The current is taking it towards East at the rate of 1 m/s and a sailor is climbing a vertical pole on the ship at the rate of 1 m/s. Find the velocity of the sailor in space.

14. A motorboat is observed to travel 10 km hr^{-1} relative to the earth in the direction 37ŗ north of east. If the velocity of the boat due to the wind only is 2 km hr^{-1} westward and that due to the current only is 4 km hr^{-1} southward, what is the magnitude and direction of the velocity of the boat due to its own power ?

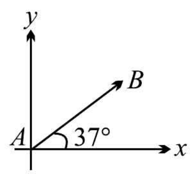
15. A person P sitting on a wooden block (which does not move relative to water) in a flowing river sees two swimmers A and B. A and B both have constant speed v_m relative to water. P observes that A starts from one point of the river bank and appears to move perpendicular to the river flow. P also observes that B starts from some point on the other bank at the same time and moves downstream. The width of the river is 'd' and it flows with velocity v_r. If A and B both reach a point at the same time, than find the initial separation between A and B.

16. A motorboat going down stream overcome a float at a point M. 60 minutes later it turned back and after some time passed the float at a distance of 6 km from the point M. Find the velocity of the stream assuming a constant velocity for the motorboat in still water?

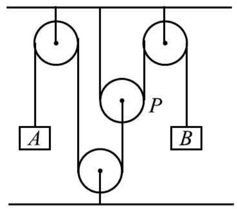
17. 2 swimmers start from point A on one bank of a river to reach point B on the other bank, lying directly opposite to point A. One of them crosses the river along the straight line AB, while the other swims at right angles to the stream and then walks the distance which he has been

carried away by the stream to get to point B. What was the velocity (assumed uniform) of his walking if both the swimmers reached point B simultaneously. Velocity of each swimmer in still water is 2.5 km hr^{-1} and the stream velocity is 2 km hr^{-1}.

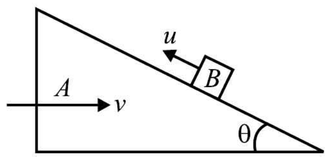
18. An airplane pilot sets a compass course due west and maintains an air speed of 240 km. hr^{-1}. After flying for $\frac{1}{2}$ hr, he finds himself over a town that is 150 km west and 40 km south of his starting point.
    (a) Find the wind velocity, in magnitude and direction.
    (b) If the wind velocity were 120 km. hr^{-1} due south, in what direction should the pilot set his course in order to travel due west ? Take the same air speed of 240 km hr^{-1}.
19. Two straight AOB and COD meet each other right angles. A person walking at a speed of 5 km/hr along AOB is at the crossing O at noon. Another person walking at the same speed along COD reaches the crossing O at 1:30 PM. Find at what time the distance between them is least and what is its value?
20. A stone is dropped from a height h. Simultaneously another stone is thrown up from the ground with such a velocity that it can reach a height of 4h. Find the time when two stones cross each other.
21. A balloon is ascending vertically with an acceleration of $0.2 m/s^2$. Two stones are dropped from it at an interval of 2 sec. Find the distance between them 1.5 sec after the second stone is released.(use $g = 9.8 m/s^2$)
22. A glass wind screen whose inclination with the vertical can be changed is mounted on a car. The car moves horizontally with a speed of 2m/s. At what angle α with the vertical should the wind screen be placed so that the rain drops falling vertically downwards with velocity 6 m/s strike the wind screen perpendicularly?
23. Two particles are moving along two long straight lines, in the same plane, with the same speed = 20 cm/s. The angle between the two lines is 60°, and their intersection point is O. At a certain moment, the two particles are located at distances 3m and 4m from O, and are moving towards O. Find the shortest distance between them subsequently?
24. A man crosses a river in a boat. If he crosses the river in minimum time he takes 10 minutes with a drift 120 m. If he crosses the river taking shortest path, he takes 12.5 minutes. Assuming $v_{b/r} > v_r$, find
    (i) width of the river,
    (ii) velocity of the boat with respect to water,
    (iii) speed of the current.
25. A butterfly is flying with velocity $10\hat{i} + 12\hat{j}$ m/s and wind is blowing along x axis with velocity u. If butterfly starts motion from A and after some time reaches point B, find the value of u.



26. Rain is falling vertically with a speed of 20 ms^{-1} relative to air. A person is running in the rain with a velocity of 5 ms^{-1} and a wind is also blowing with a speed of 15 ms^{-1} (both towards east). Find the angle with the vertical at which the person should hold his umbrella so that he may not get drenched.
27. Find the accelerations of movable pulley P and block B if acceleration of block A = 1 m/s^2↓.

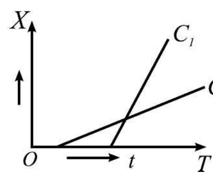


28. The block B moves with a velocity u relative to the wedge A. If the velocity of the wedge is v as shown in figure, what is the value of θ so that the block B moves vertically as seen from ground.

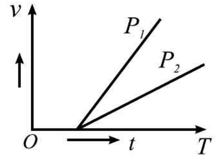


Exercise–2: Objective Problems

1. A stone is thrown upwards with a velocity 50 ms^{-1}. Another stone is simultaneously thrown downwards from the same location with a velocity 50 ms^{-1}. When the first stone is at the highest point, the relative velocity of the second stone w.r.t. the first stone is
   (A) Zero (B) 50 ms^{-1}
   (C) 100 ms^{-1} (D) 150 ms^{-1}
2. A thief is running away on a straight road in a jeep moving with a speed of 9 m s^{-1}. A police man chases him on a motor cycle moving at a speed of 10 m s^{-1}. If the instantaneous separation of the jeep from the motorcycle is 100m, how long will it take for the police man to catch the thief?
   (A) 1s (B) 19s
   (C) 90s (D) 100s
3. Two cars are moving in the same direction with a speed of 30 km h^{-1}. They are separated from each other by 5 km. Third car moving in the opposite direction meets the two cars after an interval of 4 minutes. What is the speed of the third car?
   (A) 30 km h^{-1} (B) 35 km h^{-1}
   (C) 40 km h^{-1} (D) 45 km h^{-1}
4. Shown in the figure are the displacement time graph for two children going home from the school. Which of the following statements about their relative motion is true? Their relative velocity
   (A) First increases and then decreases
   (B) First decreases and then increases
   (C) Is zero
   (D) Is non zero constant.

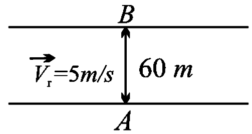


5. A person standing on the escalator takes time t_1 to reach the top of a tower when the escalator is moving. He takes time t_2 to reach the top of the tower when the escalator is standing. How long will he take if he walks up a moving escalator?
   (A) $t_2 - t_1$ (B) $t_1 + t_2$
   (C) $t_1 t_2/(t_1 - t_2)$ (D) $t_1 t_2/(t_1 + t_2)$
6. Shown in the figure are the velocity time graphs of the two particles P_1 and P_2. Which of the following statements about their relative motion is true?
   Their relative velocity:
   (A) Is zero (B) Is non-zero but constant
   (C) Continuously decreases (D) Continuously increases



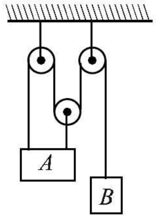
7. Two particles P_1 and P_2 are moving with velocities v_1 and v_2 respectively. Which of the statement about their relative velocity v_{r12} is true?
   (A) $v_{r12} > (v_1 + v_2)$ (B) v_{r12} cannot be greater than $v_1 - v_2$
   (C) v_{r12} cannot be greater than $v_1 + v_2$ (D) $v_{r12} < (v_1 + v_2)$
8. Two identical trains take 3 sec to pass one another when going in the opposite direction but only 2.5 sec if the speed of one is increased by 50 %. The time one would take to pass the other when going in the same direction at their original speed is
   (A) 10 sec (B) 12 sec
   (C) 15 sec (D) 18 sec
9. Two billiard balls are rolling on a flat table. One has velocity components $v_x = 1$m/s, $v_y = \sqrt{3}$ m/s and the other has components $v_x = 2$m/s and $v_y = 2$ m/s. If both the balls start moving from the same point, the angle between their path is
   (A) 60° (B) 45°
   (C) 22.5° (D) 15°
10. A battalion of soldiers is ordered to swim across a river 500 ft wide . At what minimum rate should they swim perpendicular to river flow in order to avoid being washed away by the waterfall 300 ft downstream. The speed of current being 3 m.p.h.
   (A) 6 *m.p.h.* (B) 5 *m.p.h.*
   (C) 4 *m.p.h.* (D) 2 *m.p.h.*
11. A boat, which has a speed of 5 km/hr in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water in km/hr is
   (A) 1 (B) 3
   (C) 4 (D) $\sqrt{41}$
12. A bucket is placed in the open where the rain is falling vertically. If a wind begins to blow at double the velocity of the rain, how will be rate of filling of the bucket change?
   (A) Remain unchanged (B) Doubled
   (C) Halved (D) Become four times
13. A car with a vertical wind shield moves along in a rain storm at the speed of 40 km/hr. The rain drops fall vertically with a terminal speed of 20 m/s. The angle with the vertical at which the rain drop strike the wind shield is
   (A) $\tan^{-1}(5/9)$ (B) $\tan^{-1}(9/5)$
   (C) $\tan^{-1}(3/2)$ (D) $\tan^{-1}(3)$

14. A swimmer swims in still water at a speed = 5 km/hr. He enters a 200 m wide river, having river flow speed = 4 km/hr at point *A* and proceeds to swim at an angle of 127° with the river flow direction. Another point *B* is located directly across *A* on the other side. The swimmer lands on the other bank at a point *C*, from which he walks the distance *CB* with a speed = 3 km/hr. The total time in which he reaches from *A* to *B* is
    (A) 5 minutes (B) 4 minutes
    (C) 3 minutes (D) None
15. A boat having a speed of 5 km/hr. in still water, crosses a river of width 1 km along the shortest possible path in 15 minutes. The speed of the river in Km/hr is
    (A) 1 (B) 3
    (C) 4 (D) $\sqrt{41}$
16. A man is crossing a river flowing with velocity of 5 m/s. He reaches a point directly across at a distance of 60 m in 5 sec. His velocity in still water should be



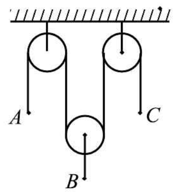
    (A) 12 m/s (B) 13 m/s
    (C) 5 m/s (D) 10 m/s
17. A motor boat is to reach at a point 30° upstream (w.r.t. normal) on other side of a river flowing with velocity 5m/s. Velocity of motorboat w.r.t. water is $5\sqrt{3}$ m/s. The driver should steer the boat at an angle
    (A) 120° w.r.t. stream direction
    (B) 30° w.r.t. normal to the bank
    (C) 30° w.r.t. the line of destination from starting point.
    (D) None of these
18. A flag is mounted on a car moving due North with velocity of 20 km/hr. Strong winds are blowing due East with velocity of 20 km/hr. The flag will point in direction
    (A) East (B) North - East
    (C) South - East (D) South - West
19. Three ships *A*, *B* & *C* are in motion. The motion of *A* as seen by *B* is with speed *v* towards north – east. The motion of *B* as seen by *C* is with speed *v* towards the north – west. Then as seen by *A*, *C* will be moving towards
    (A) North (B) South
    (C) East (D) West
20. Wind is blowing in the north direction at speed of 2 m/s which causes the rain to fall at some angle with the vertical. With what velocity should a cyclist drive so that the rain appears vertical to him :
    (A) 2 m/s south (B) 2 m/s north
    (C) 4 m/s west (D) 4 m/s south
21. When the driver of a car A sees a car B moving towards his car and at a distance 30 m, takes a left turn of 30°. At the same instant the driver of the car B takes a turn to his right at an angle 60°. The two cars collides after two seconds, then the velocity (in m/s) of the car A and B respectively will be : [assume both cars to be moving along same line with constant speed]
    (A) 7.5, $7.5\sqrt{3}$ (B) 7.5, 7.5
    (C) $7.5\sqrt{3}$, 7.5 (D) None

22. At a given instant, A is moving with velocity of 5m/s upwards. What is velocity of B at that time



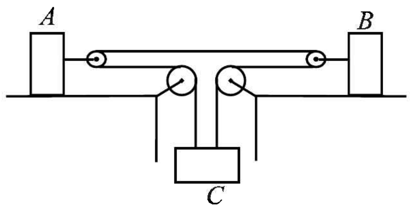
(A) 15 m/s ↓ (B) 15 m/s ↑
(C) 5 m/s ↓ (D) 5 m/s ↑

23. The pulleys in the diagram are all smooth and light. The acceleration of A is a upwards and the acceleration of C is *f* downwards. The acceleration of B is



(A) ½(f – a) up (B) ½(a + f) down
(C) ½(a + f) up (D) ½(a - f) up

24. If acceleration of A is 2 m/s^2 to left and acceleration of B is 1m/s^2 to left, then acceleration of C is



(A) 1 m/s^2 upwards (B) 1 m/s^2 downwards
(C) 2 m/s^2 downwards (D) 2 m/s^2 upwards

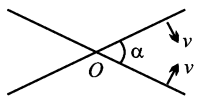
25. Two particles start simultaneously from the same point and move along two straight lines, one with uniform velocity *v* and other with a uniform acceleration a. If α is the angle between the lines of motion of two particles then the least value of relative velocity will be at time given by

(A) (v/a) sin α (B) (v/a) cos α
(C) (v/a) tan α (D) (v/a) cot α

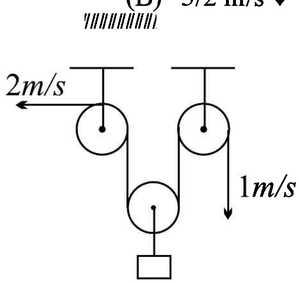
26. A man swimming down stream overcome a float at a point M. After travelling distance D he turned back and passed the float at a distance of D/2 from the point M, then the ratio of speed of swimmer with respect to still water to the speed of the river will be

(A) 2 (B) 3
(C) 4 (D) 2.5

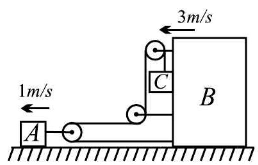
27. A glass wind screen whose inclination with the vertical can be changed is mounted on a car. The car moves horizontally with a speed of 2m/s. At what angle α with the vertical should the wind screen be placed so that the rain drops falling vertically downwards with velocity 6 m/s strike the wind screen perpendicularly.
    (A) $\tan^{-1}(3)$ (B) $\tan^{-1}(1/3)$
    (C) $\cos^{-1}(3)$ (D) $\sin^{-1}(1/3)$
28. Three particles, located initially on the vertices of an equilateral triangle of side L, start moving with a constant tangential acceleration towards each other in a cyclic manner, forming spiral loci that coverage at the centroid of the triangle. The length of one such spiral locus will be
    (A) L/3 (B) $2L/\sqrt{3}$
    (C) $L/\sqrt{2}$ (D) 2L/3
29. Find the speed of the intersection point O of the two wires if the wires starts moving perpendicular to itself with speed v as shown in figure.



    (A) $v\,\text{cosec}(\alpha/2)$ (B) $v\,\text{cosec}(\alpha)$
    (C) $v\cos(\alpha/2)$ (D) $v\sec(\alpha/2)$
30. Find the velocity of the hanging block if the velocities of the free ends of the rope are as indicated in the figure.
    (A) 3/2 m/s ↑ (B) 3/2 m/s ↓ (C) 1/2 m/s ↑ (D) 1/2 m/s ↓



31. The velocities of A and B are marked in the figure. The velocity of block C is (assume that the pulleys are ideal and string inextensible)
    (A) 5 m/s (B) 2 m/s
    (C) 3 m/s (D) 4 m/s

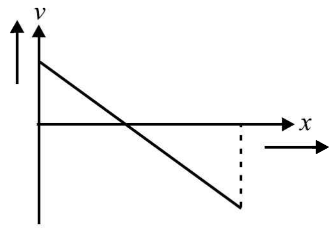


Exercise–3 : Assertion Reason Questions

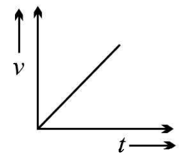
1. **Statement-1 :** Positive acceleration in rectilinear motion of a body does not imply that the body is speeding up.
   Statement-2 : Both the acceleration and velocity are vectors.
   (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
   (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

2. **Statement -1 :** A particle having zero acceleration must have constant speed.
Statement -2 : A particle having constant speed must have zero acceleration.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is false, statement-2 is true.
(D) Statement-1 is true, statement-2 is false.

3. **Statement-1:**
A student performed an experiment by moving a certain block in a straight line. The velocity position graph cannot be as shown.


Statement-2:
When a particle is at its maximum position in rectilinear motion its velocity must be zero.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

4. **Statement-1 :**
If the velocity time graph of a body moving in a straight line is as shown here, the acceleration of the body must be constant.


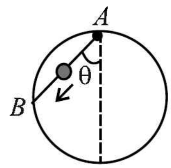
Statement-2 : The rate of change of quantity which is constant is always zero.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is false, statement-2 is true.
(D) Statement-1 is true, statement-2 is false.

5. **Statement-1:** The speed of a projectile is minimum at the highest point.
Statement-2: The acceleration of projectile is constant during the entire motion.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

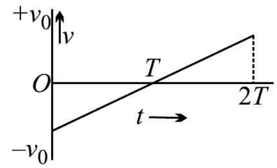
6. **Statement-1:** Two stones are simultaneously projected from level ground from same point with same speeds but different angles with horizontal. Both stones move in same vertical plane. Then the two stones may collide in mid air.
   Statement-2: For two stones projected simultaneously from same point with same speed at different angles with horizontal, their trajectories may intersect at some point.
   (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
   (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
   (C) Statement-1 is true, statement-2 is false.
   (D) Statement-1 is false, statement-2 is true.
7. **Statement-1 :** If seperation between two particles does not change then their relative velocity will be zero.
   Statement-2 : Relative velocity is the rate of change of position of one particle with respect to another.
   (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
   (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
   (C) Statement-1 is true, statement-2 is false.
   (D) Statement-1 is false, statement-2 is true.
8. **Statement-1:** The magnitude of relative velocity of A with respect to B will be always less than V_A.
   Statement-2: The relative velocity of A with respect to B is given by $\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$.
   (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
   (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
   (C) Statement-1 is true, statement-2 is false.
   (D) Statement-1 is false, statement-2 is true.
9. **Statement-1 :** Three projectiles are moving in different paths in the air. Vertical component of relative velocity between any of the pair does not change with time as long as they are in air. Neglect the effect of air friction.
   Statement-2 : Relative acceleration between any of the pair of projectiles is zero.
   (A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
   (B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
   (C) Statement-1 is true, statement-2 is false.
   (D) Statement-1 is false, statement-2 is true.

One or More than One Option Correct

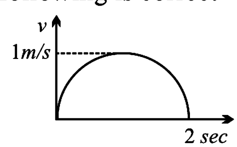
10. A particle moves with constant speed v along a regular hexagon ABCDEF in the same order. Then the magnitude of the average velocity for its motion from A to
    (A) F is v/5 (B) D is v/3
    (C) C is $v\sqrt{3}/2$ (D) B is v
11. A particle moving with a speed v changes direction by an angle θ, without change in speed.
    (A) The change in the magnitude of its velocity is zero.
    (B) The change in the magnitude of its velocity is $2v\sin(\theta/2)$.
    (C) The magnitude of the change in velocity is $2v\sin(\theta/2)$
    (D) The magnitude of the change in its velocity is $v(1 - \cos\theta)$.
12. A particle has initial velocity 10 m/s. It moves due to constant retarding force along the line of velocity which produces a retardation of 5 m/s^2. Then
    (A) the maximum displacement in the direction of initial velocity is 10 m
    (B) the distance travelled in first 3 seconds is 7.5 m
    (C) the distance travelled in first 3 seconds is 12.5 m
    (D) the distance travelled in first 3 seconds is 17.5 m.
13. A bead is free to slide down a smooth wire tightly stretched between points A and B on a vertical circle. If the bead starts from rest at A, the highest point on the circle



    (A) Its velocity v on arriving at B is proportional to $\cos\theta$
    (B) Its velocity v on arriving at B is proportional to $\tan\theta$
    (C) Time to arrive at B is proportional to $\cos\theta$
    (D) Time to arrive at B is independent of θ
14. The figure shows the velocity (v) of a particle plotted against time (t)



    (A) The particle changes its direction of motion at some point
    (B) The acceleration of the particle remains constant
    (C) The displacement of the particle is zero
    (D) The initial and final speeds of the particle are the same
15. Velocity-time graph for a car is semicircle as shown here. Which of the following is correct



    (A) Car must move in circular path.
    (B) Acceleration of car is never zero.
    (C) Mean speed of the particle is $\pi/4$ m/s.
    (D) The car makes a turn once during its motion.
16. A projectile of mass 1 kg is projected with a velocity of $\sqrt{20}$ m/s such that it strikes on the same level as the point of projection at a distance of $\sqrt{3}$ m. Which of the following options are incorrect
    (A) The maximum height reached by the projectile can be 0.25 m.
    (B) The minimum velocity during its motion can be $\sqrt{15}$ m/s

(C) The time taken for the flight can be $\sqrt{\frac{3}{5}}$ sec.

(D) Minimum kinetic energy during its motion can be 6J.

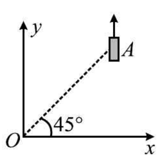
17. Choose the correct alternative (s)
    (A) If the greatest height to which a man can throw a stone is h, then the greatest horizontal distance upto which he can throw the stone is $2h$.
    (B) The angle of projection for a projectile motion whose range R is n times the maximum height is $\tan^{-1}(4/n)$
    (C) The time of flight T and the horizontal range R of a projectile are connected by the equation $gT^2 = 2R\tan\theta$ where θ is the angle of projection.
    (D) A ball is thrown vertically up. Another ball is thrown at an angle θ with the vertical. Both of them remain in air for the same period of time. Then the ratio of heights attained by the two balls 1 : 1.
18. If T is the total time of flight, h is the maximum height & R is the range for horizontal motion, the x & y co-ordinates of projectile motion and time t are related as
    (A) $y = 4h\left(\frac{t}{T}\right)\left(1-\frac{t}{T}\right)$ (B) $y = 4h\left(\frac{X}{R}\right)\left(1-\frac{X}{R}\right)$
    (C) $y = 4h\left(\frac{T}{t}\right)\left(1-\frac{T}{t}\right)$ (D) $y = 4h\left(\frac{R}{X}\right)\left(1-\frac{R}{X}\right)$
19. A particle moves in the xy plane with a constant acceleration 'g' in the negative y-direction. Its equation of motion is $y = ax - bx^2$, where a and b are constants. Which of the following are correct?
    (A) The x-component of its velocity is constant.
    (B) At the origin, the y-component of its velocity is $a\sqrt{\frac{g}{2b}}$.
    (C) At the origin, its velocity makes an angle $\tan^{-1}(A)$ with the x-axis.
    (D) The particle moves exactly like a projectile.
20. A particle is projected from the ground with velocity u at angle θ with horizontal. The horizontal range, maximum height and time of flight are R, H and T respectively. They are given by,
    $R = \frac{u^2 \sin 2\theta}{g}$, $H = \frac{u^2 \sin^2\theta}{2g}$ and $T = \frac{2u\sin\theta}{g}$
    Now keeping u as fixed, θ is varied from 30° to 60°. Then,
    (A) R will first increase then decrease, H will increase and T will decrease
    (B) R will first increase then decrease while H and T both will increase
    (C) R will decrease while H and T will increase
    (D) R will increase while H and T will increase
21. A ball is rolled off along the edge of a horizontal table with velocity 4 m/s. It hits the ground after time 0.4 s. Which of the following are correct?
    (A) The height of the table is 0.8 m
    (B) It hits the ground at an angle of 60° with the vertical
    (C) It covers a horizontal distance 1.6 m from the table
    (D) It hits the ground with vertical velocity 4 m/s

22. An observer moves with a constant speed along the line joining two stationary objects. He will observe that the two objects
    (A) Have the same speed (B) Have the same velocity
    (C) Move in the same direction (D) Move in opposite directions
23. A man on a rectilinearly moving cart, facing the direction of motion, throws a ball straight up with respect to himself
    (A) The ball will always return to him
    (B) The ball will never return to him
    (C) The ball will return to him if the cart moves with constant velocity
    (D) The ball will fall behind him if the cart moves with some positive acceleration
24. A block is thrown with a velocity of 2 ms^{-1} (relative to ground) on a belt, which is moving with velocity 4 ms^{-1} in opposite direction of the initial velocity of block. If the block stops slipping on the belt after 4 sec of the throwing then choose the correct statements (s)
    (A) Displacement with respect to ground is zero after 2.66 and magnitude of displacement with respect to ground is 12 m after 4 sec.
    (B) Magnitude of displacement with respect to ground in 4 sec is 4 m.
    (C) Magnitude of displacement with respect to belt in 4 sec is 12 m.
    (D) Displacement with respect to ground is zero in 8/3 sec.

Exercise-4

JEE/REE Questions of Previous Years

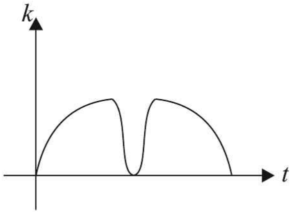
1. An aeroplane is flying vertically upwards with a uniform speed of 500 m/s. When it is at a height of 1000 m above the ground a shot is fired at it with a speed of 700 m/s from a point directly below it. What should be the acceleration of the aeroplane so that it may escape from being hit? **[REE 1994, 2006]**
2. The width of a river is 25 m and in it water is flowing with a velocity of 4 m/min. A boatman is standing on the bank of the river. He wants to sail the boat to a point at the other bank which is directly opposite to him. In what time will he cross the river, if he can sail the boat at 8 m/ min, relative to the water. **[REE 1995, 2006]**
3. On a frictionless horizontal surface, assumed to be the x-y plane a small trolley A is moving along a straight line parallel to the y-axis as shown in the figure with a constant velocity of ($\sqrt{3}$ – 1) m/s. At a particular instant, when the line OA makes an angle of 45ş with the x–axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x –axis when it hits the trolley.

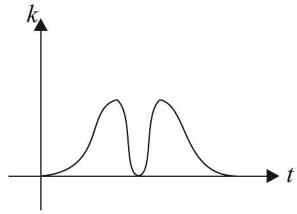


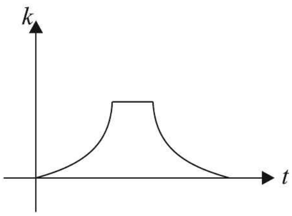
   (a) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity of the ball with the x-axis in this frame.
   (b) Find the speed of the ball with respect to the surface, if $\phi = 4\theta/3$.

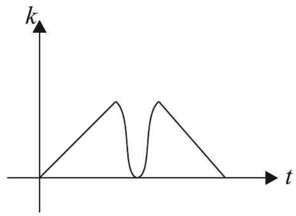
[JEE 2002, 2+3 marks]

4. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describe the variation of its kinetic energy k with time (t) most appropriately? The figures are only illustrative and not to the scale. **[JEE Advanced 2014]**

(A) 

(B) 

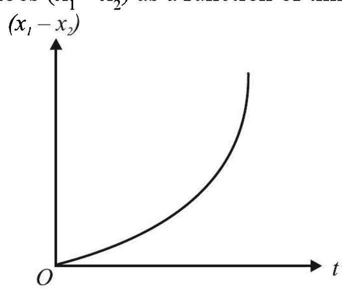
(C) 

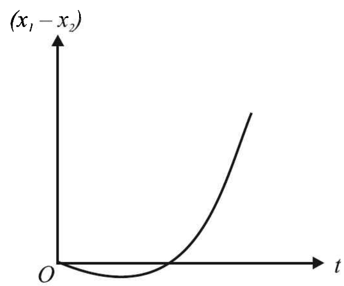
(D) 

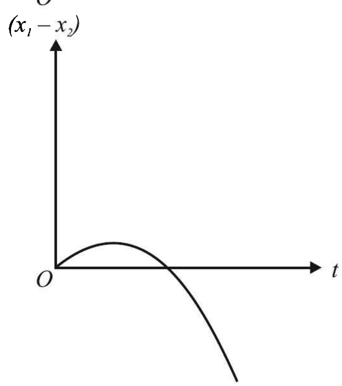
[Ans. (B)**]**

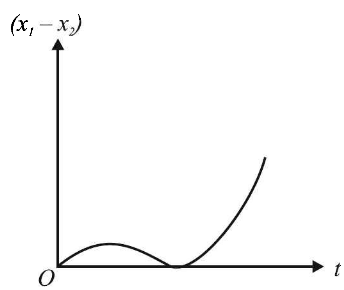
Previous Years' AIEEE Questions

1. A body is at rest at $x = 0$. At t = 0, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time 't' and that of second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time 't'? **[AIEEE 2008]**

(A) 

(B) 

(C) 

(D) 

ANSWER KEY

Exercise - 1

1. 5 km/h
2. (A) 25 m/s or 90 km/hr
   (B) 20 m/s or 72 km/hr due east
   (C) 25 m/s or 90 km/hr at 37°N of E
3. 30ŗ N of W at $5\sqrt{3}$ km/hr.
4. 3 km/hr.
5. (I) 0.75 km (II) (A) $\frac{1}{2\sqrt{3}}$h (B) $\frac{1}{2\sqrt{3}}$km.
6. At an angle 30° west of north.
7. $50\sqrt{5}\,km$
8. (A) 0, 10 m/s West
   (B) $10\sqrt{3}$ m/s, 20 m/s
9. Coming from 5 km/hr, 53° N of E
10. $\left(4\hat{i} + 8\hat{j}\right)$
11. $\dfrac{a}{v\left(1 - \cos\frac{2\pi}{n}\right)}$
12. $\frac{10}{\sqrt{2}}, \frac{1}{8}$ hr
13. 2m/s in a direction making an angle of 60ŗ with E, 45ŗ with N and 60ŗ with the vertical
14. $10\sqrt{2}$ km/hr, 45ŗ N of E
15. $\sqrt{2}$d
16. 3 km/hr
17. 3 km/hr towards B
18. (A) 100 km/hr, 37° W of S (B) 30° N of W
19. 12 : 45 PM
20. $\sqrt{\left(\frac{h}{8g}\right)}$
21. 50m
22. $\tan^{-1}(3)$
23. $50\sqrt{3}$cm
24. 200 m, 20 m/min, 12 m/min

25. 6 m/s
26. $\tan^{-1}(1/2)$
27. $a_P = 1\ m/s^2 \downarrow$, $a_B = 2\ m/s^2 \uparrow$
28. $\cos^{-1}\left(\frac{v}{u}\right)$

Exercise - 2

1.	C	2.	D	3.	D	4.	D
5.	D	6.	D	7.	C	8.	C
9.	D	10.	B	11.	B	12.	A
13.	A	14.	B	15.	B	16.	B
17.	C	18.	C	19.	B	20.	B
21.	C	22.	A	23.	A	24.	A
25.	B	26.	B	27.	A	28.	D
29.	A	30.	A	31.	A		

Exercise - 3

1.	A	2.	D	3.	A	4.	B
5.	B	6.	D	7.	D	8.	D
9.	A	10.	A,C,D	11.	A,C	12.	A,C
13.	A,D	14.	A,B,C,D	15.	C	16.	D
17.	A,B,C,D	18.	A, B	19.	A,B,C,D	20.	B
21.	A,C,D	22.	A,B,C	23.	C,D	24.	B,C,D

Exercise - 4

1. a > 10 m/s2
2. 3.6 minute
3. (A) $\theta = 45$ŗ ; (B) 2 m/s

Previous Years' AIEEE Questions

1. (B)

Chapter

3

Newton's Laws of Motion

FORCE

A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object is called force. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

Effect of Resultant Force

(a) Produces or tries to produce motion in a body at rest.

(b) Stops or tries to stop a moving body.

(c) Changes or tries to change the direction of motion of body.

(d) Produces a change in the shape of the body.

Unit of force: newton and $\frac{kg \cdot m}{s^2}$ (*MKS* System)

dyne and $\frac{g.cm}{s^2}$ (*CGS* System)

1 newton = 10^5 dyne

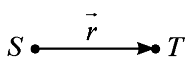
Dimensional Formula of force: $[MLT^{-2}]$

Fundamental Forces

All the forces observed in nature such as muscular force, tension, reaction, friction, elastic, weight, electric, magnetic, nuclear, etc., can be explained in terms of only following four basic interactions:

[A] Gravitational Force

The force of interaction which exists between two particles of masses m_1 and m_2, due to their masses is called gravitational force.



$$\vec{F} = -G\frac{m_1 m_2}{r^3}\vec{r}$$

$\vec{r}$ = position vector of test particle 'T' with respect to source particle 'S'. and G = universal gravitational constant $= 6.67 \times 10^{-11}$ Nm^2/kg^2.

(i) It is the weakest force and is always attractive.

(ii) It is a long range force as it acts between any two particles situated at any distance in the universe.

(iii) It is independent of the nature of medium between the particles.

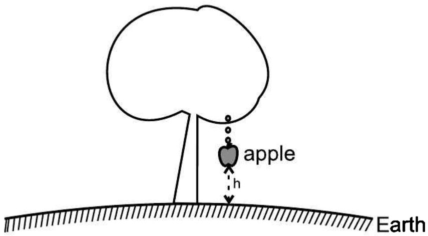


Fig 3.1: Gravitational Force of Earth

An apple is freely falling as shown in figure, When it is at a height h, force between earth and apple is given by

$$F = \frac{GM_e m}{(R_e + h)^2} \quad \text{where } M_e - \text{mass of earth}, \quad R_e - \textit{radius} \text{ of earth}$$

It acts towards earth's centre. Now rearranging above result,

$$F = m \cdot \frac{GM_e}{R_e^2} \cdot \left(\frac{R_e}{R_e + h}\right)^2$$

$$F = mg\left(\frac{R_e}{R_e + h}\right)^2 \qquad \left\{g = \frac{GM_e}{R_e^2}\right\}$$

Here $h << R_e$, so $\dfrac{R_e}{R_e + h} \simeq 1$

$\therefore$ $F = mg$

This is the force exerted by earth on any particle of mass m near the earth surface. The value of $g = 9.81 \text{ m/s}^2 \simeq 10 \text{ m/s}^2 \simeq \pi^2 \text{ m/s}^2 \simeq 32 \text{ ft/s}^2$. It is also called acceleration due to gravity near the surface of earth.

[B] Electromagnetic Force

Force exerted by one particle on the other because of the electric charge on the particles is called electromagnetic force.

Following are the main characteristics of electromagnetic force.

(a) These can be attractive or repulsive.

(b) These are long range forces.

(c) These depend on the nature of medium between the charged particles.

(d) All macroscopic forces (except gravitational) which we experience as push or pull or by contact are electromagnetic, i.e., tension in a rope, the force of friction, normal reaction, muscular force, and force experienced by a deformed spring are electromagnetic forces. These are manifestations of the electromagnetic attractions and repulsions between atoms/ molecules.

[C] Nuclear Force

It is the strongest force. It keeps nucleons (neutrons and protons) together inside the nucleus inspite of large electric repulsion between protons. Radioactivity, fission, and fusion, etc. result because of unbalancing of nuclear forces. It acts within the nucleus that too upto a very small distance.

[D] Weak Force

It acts between any two elementary particles. Under its action a neutron can change into a proton emitting an electron and a particle called antineutrino. The range of weak force is very small, in fact much smaller than the size of a proton or a neutron.

It has been found that for two protons at a distance of 1 fermi:

$F_N : F_{EM} : F_W : F_G :: 1 : 10^{-2} : 10^{-7} : 10^{-38}$

Classification of Forces on the Basis of Contact

(A) Field Force

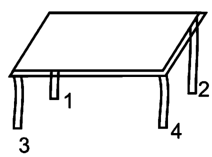
Force which acts on an object at a distance by the interaction of the object with the field produced by other object is called field force. Examples

(a) Gravitation force

(b) Electromagnetic force

(B) Contact Force

Forces which are transmitted between bodies by short range atomic molecular interactions are called contact forces. When two objects come in contact they exert contact forces on each other. Tension, Normal Reaction, Friction, etc. are contact forces. These are forces that act between bodies in contact. Examples



(a) Normal force (N): It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force.

A table is placed on Earth as shown in figure.

Here table presses the earth, so normal force exerted by four legs of table on earth are as shown in figure 3.2.

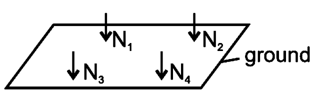
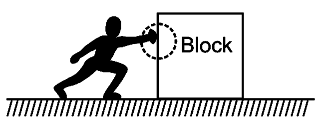
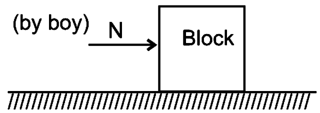


***Fig 3.2:** Force Exerted by a Table*

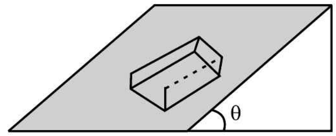
Now a boy pushes a block kept on a frictionless surface.



Here, force exerted by boy on block is electro-magnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is normal force.



A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.



Normal force exerted by block on the surface of inclined plane is shown in figure 3.3.

Force acts perpendicular to the surface

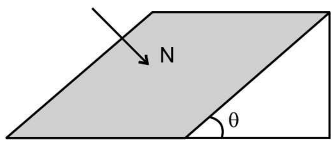
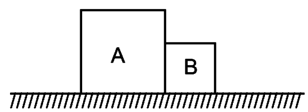


***Fig 3.3:** Force exerted by a block*

Example 1. Two blocks are kept in contact on a smooth surface as shown in figure. Draw normal force exerted by *A* on *B*.

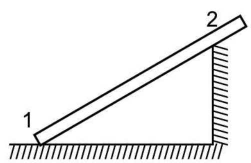


Solution In above problem, block *A* does not push block *B*, so there is no molecular interaction between *A* and *B*. Hence normal force exerted by *A* on *B* is zero.

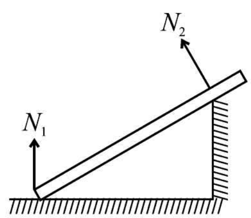
Note

Normal is a dependent force, it comes in role when one surface presses the other.

Example 2. Draw normal forces on the massive rod at point 1 and 2 as shown in figure.



Solution Normal force acts perpendicular to extended surface at point of contact.



N Does not Always Equal mg

In the situation shown in figure **3.4** and in many others, we find that $N = mg$ (the normal force has the same magnitude as the gravitational force.) However, this is not generally true. If an object is on an incline, if there are applied forces with vertical components, or if there is a vertical acceleration of the system, then $N \neq mg$. Always apply Newton's second law to find the relationship between n and mg.

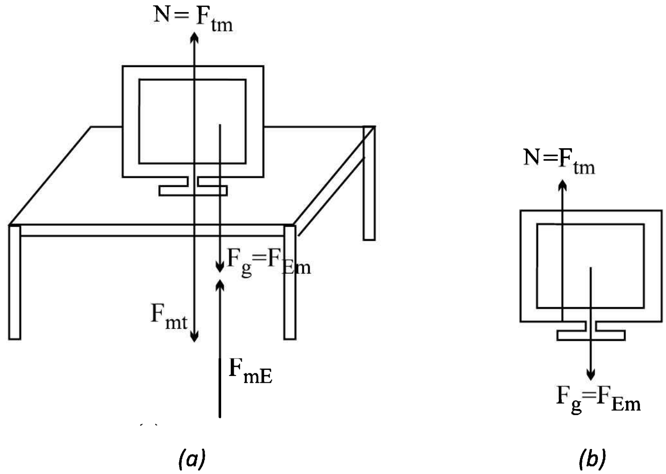
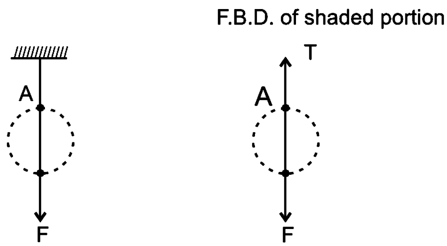


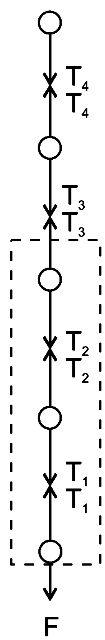
Fig 3.4

(b) Tension: Tension in a string is a electromagnetic force. It arises when a string is pulled. If a massless string is not pulled, tension in it is zero. A string suspended by rigid support is pulled by a force '*F*' as shown in figure 3.5, for calculating the tension at point '*A*' we draw *F.B.D.* of shaded portion of the string; Here string is massless.



(a)

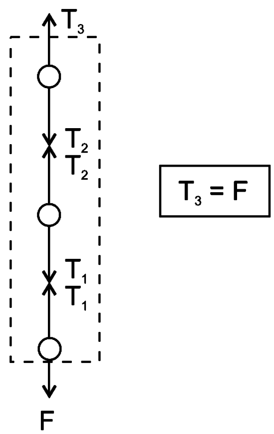
$\Rightarrow \qquad T = F$



String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature as shown in figure. The attraction force between two segments is equal and opposite due to newton's third law.

(b)

For calculating tension at any segment, we consider two or more than two parts as a system.



(c)

Fig 3.5

Here interaction between segments are considered as internal forces, so they are not shown in *F.B.D.*

(c) Frictional force: It is the component of contact force tangential to the surface. It opposes the relative motion (or attempted relative motion) of the two surfaces in contact.

NEWTON'S LAWS OF MOTION

First Law of Motion

Each body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

Newton's first law is really a statement about reference frames in that it defines the types of reference frames in which the laws of Newtonian mechanics hold. From this point of view the first law is expressed as:

If the net force acting on a body is zero, it is possible to find a set of reference frames in which that body has no acceleration.

Newton's first law is sometimes called the law of inertia and the reference frames that it efines are called inertial reference frames.

Newton's law from an 1803 translation from Latin as Newton wrote

"Every body preserves in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon."

Examples of this law:

(a) A bullet fired on a glass window makes a clean hole through it while a stone breaks the whole of it.

(b) A passenger sitting in a bus gets a jerk when the bus starts or stops suddenly.

Newton's First Law and Inertial Frames

Newton's first law of motion, sometimes called the law of inertia, defines a special set of reference frames called inertial frames. This law can be stated as follow:

If an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration. Such a reference frame is called an inertial frame of reference.

Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame.

For our purposes we can consider the Earth as being such a frame. The Earth is not really an inertial frame because of its orbital motion around the Sun and its rotational motion about its own axis, both of which result in centripetal acceleration. However, these accelerations are small compared with g and can often be neglected. For this reason, we assume that the Earth is an inertial frame, as is any other frame attached to it.

Another Statement of Newton's First Law

In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is, with a constant speed in a straight line).

If nothing acts to change the object's motion, then its velocity does not change. From the first law, we conclude that any isolated object (one that does not interact with its environment) is either at rest or moving with constant velocity when viewed from an inertial frame. The tendency of an object to resist any attempt to change its velocity is called **inertia**.

Second Law of Motion

The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts.

Newton's law from an 1803 translation from Latin as Newton wrote

"The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."

Mathematically $\vec{F} = \dfrac{d\vec{p}}{dt}$ or $\vec{F} = m\vec{a}$

where $\vec{p} = m\vec{v}$, $\vec{p}$ = Linear momentum.

In case of two particles having linear momentum $\vec{P}_1$ and $\vec{P}_2$ and moving towards each other under mutual forces, from Newton's second law ;

$$\frac{d}{dt}(\vec{p}_1 + \vec{p}_2) = \vec{F} = 0 \qquad \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$$

$\vec{F}_1 + \vec{F}_2 = 0$ $\quad\quad$ $\vec{F}_2 = -\vec{F}_1$

which is Newton's third law.

Important Points about the Second Law

(a) The Second Law is obviously consistent with the First Law as $F = 0$ Implies $a = 0$.

(b) The Second Law of motion is a vector law. It is actually a combination of three equations, one for each component of the vectors:

$$F_x = \frac{dp_x}{dt} = ma_x; \qquad F_y = \frac{dp_y}{dt} = ma_y; \qquad F_z = \frac{dp_z}{dt} = ma_z$$

This means that if a force is not parallel to the velocity of the body, but makes some angle with it, it changes only the component of velocity along the direction of force. The component of velocity normal to the force remains unchanged.

(b) The Second Law of motion given above is strictly applicable to a single point mass. The force ***F*** in the law stand for the net external force on the particle and **a** stands for the acceleration of the particle. Any internal forces in the system are not to be included in ***F.***

(c) The Second Law of motion is a local relation. What this means is that the force ***F*** at a point in space (location of the particle) at a certain instant of time is related to **a** at the same point at the same instant. That is acceleration here and now is determined by the force here and now not by any history of the motion of the particle.

Third Law of Motion

To every action, there is always an equal and opposite reaction. Newton's law from an 1803 translation from Latin as Newton wrote

"To every action there is always opposed an equal and opposite reaction: to the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

Important Points about the Third Law

(a) The terms 'action' and 'reaction' in the Third Law mean nothing else but 'force'. A simple and clear way of stating the Third Law is as follows: Forces always occur in pairs. Force on a body *A* by *B* is equal and opposite to the force on the body *B* by *A*.

(b) The terms 'action' and 'reaction' in the Third Law may give a wrong impression that action comes before reaction i.e. action is the cause and reaction the effect. There is no such cause-effect relation implied in the Third Law. The force on *A* by *B* and the force on *B* by *A* act at the same instant. Any one of them may be called action and the other reaction.

(c) Action and reaction forces act on different bodies, not on the same body. Thus if we are considering the motion of any one body (*A* or *B*), only one of the two forces is relevant. It is an error to add up the two forces and claim that the net force is zero.

However, if you are considering the system of two bodies as a whole, $\boldsymbol{F}_{AB}$ (force on *A* due to *B*) and $\boldsymbol{F}_{BA}$ (force on *B* due to *A*) are internal forces of the system ($A + B$). They add up to give a null force.

Internal forces in a body or a system of particles thus cancel away in pairs. This is an important fact that enables the Second Law to be applicable to a body or a system of particles.

SYSTEM

Two or more than two objects which interact with each other form a system.

Classification of Forces on the Basis of Boundary of System

(A) **Internal Forces:** Forces acting each with in a system among its constituents.

(B) **External Forces:** Forces exerted on the constituents of a system by the outside surroundings are called as external forces.

(C) **Real Force:** Force which acts on an object due to other object is called as real force. An isolated object (far away from all objects) does not experience any real force.

FREE BODY DIAGRAM

A free body diagram consists of a diagrammatic representations of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.

Steps for F.B.D.

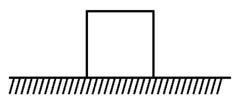
Step 1: Identify the object or system and isolate it from other objects clearly specify its boundary.

Step 2: First draw non-contact external force in the diagram. Generally it is weight.

Step 3: Draw contact forces which acts at the boundary of the object or system. Contact forces are normal, friction, tension and applied force.

In *F.B.D*, internal forces are not drawn, only external are drawn.

***Example* 3.** A block of mass 'm' is kept on the ground as shown in figure.

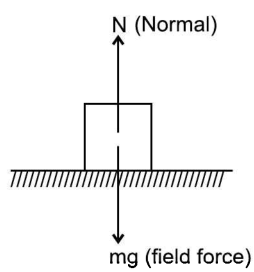


(i) Draw *F.B.D.* of block.

(ii) Are forces acting on block action–reaction pair.

(iii) If answer is no, draw action reaction pair.

Solution



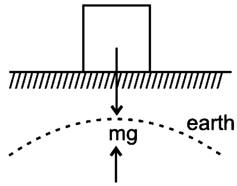
(i) *F.B.D.* of block

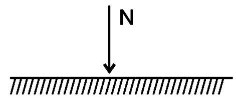
(ii) 'N' and Mg are not action-reaction pair.

Since pair act on different bodies, and they are of same nature.

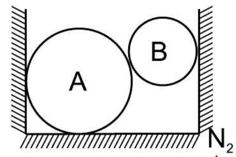
(iii) Pair of 'mg' of block acts on earth in opposite direction.

and pair of 'N' acts on surface as shown in figure.

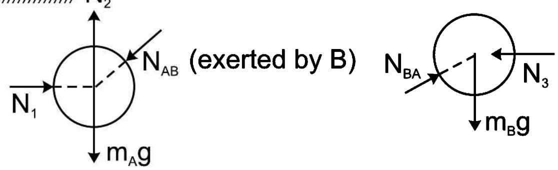




Example 4. Two sphere A and B are placed between two vertical walls as shown in figure. Draw the free body diagrams of both the spheres.



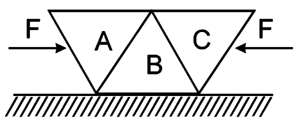
Solution *F.B.D.* of sphere '*A*':
F.B.D. of sphere '*B*':
(exerted by A)



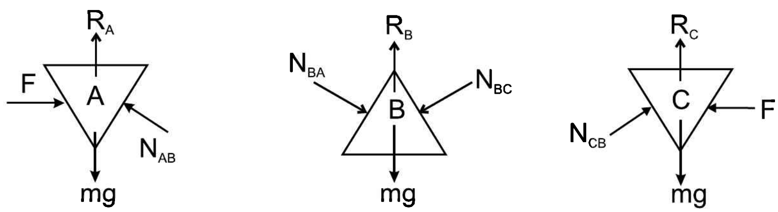
Note

Here N_{AB} and N_{BA} are the action–reaction pair (Newton's third law).

Example 5. Three triangular blocks A, B and C of equal masses 'm' are arranged as shown in figure. Draw *F.B.D.* of blocks A, B and C. Indicate action–reaction pair between A, B and B, C.



Solution



Applications of Newton's Laws

(a) When objects are in equilibrium

To solve problems involving objects in equilibrium:

Step 1. Make a sketch of the problem.

Step 2. Isolate a single object and then draw the **free-body diagram** for the object. Label all external forces acting on it.

Step 3. Choose a convenient coordinate system and resolve all forces into x and y components.

Step 4. Apply the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

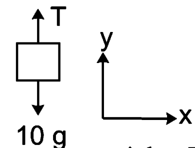
Step 5. Step 4 will give you two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

Step 6. If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps.

Eventually at step 5 you will have enough equations to solve for all unknown quantities.

Example 6. A 'block' of mass 10 kg is suspended with string as shown in figure. Find tension in the string. (g = 10 m/s²)

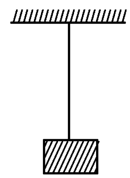
Solution F.B.D. of block



$\Sigma F_y = 0$

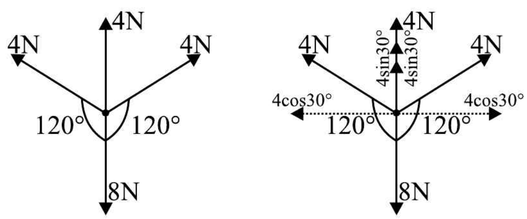
$T - 10\,g = 0$

$\therefore\ T = 100\,N$



Example 7. The diagram shows the forces that are action on a particle. Has the particle an acceleration?

Solution To check whether the particle will have any acceleration or not, let us see net force is zero or not. Resolving the forces in horizontal and vertical directions.



Net force in horizontal direction = 4 cos30° – 4 cos30° = 0

Net force in vertically downward direction = 8 – 4 sin30° – 4 sin30° – 4 = 0

As net force is zero, so the particle will have no acceleration.

Example 8. A body suspended with the help of strings. A body of mass 25 kg is suspended with the help of strings as shown in figure. Find tension in three strings. Strings are light [g = 10 m/s²]

Solution Let the tensions in strings *ab*, *bc* and *bd* are respectively T_1, T_2 and T_3. As the body is hanging in equilibrium, we can use the condition that net force on block is zero. This will give the value of T_3. To know the values of T_1 and T_2 we need to draw *FBD* of knot b also.

For equilibrium of hanging body. $T_3 = 250\,N$ ...(1)

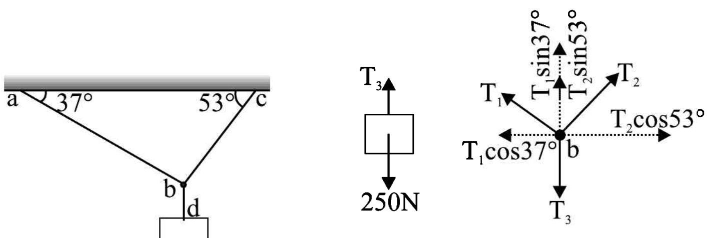
For equilibrium of knot,

$$T_2\cos 53° - T_1\cos 37° = 0 \quad ...(2)$$

and, $$T_3 - T_1\sin 37° - T_2\sin 53° = 0 \quad ...(3)$$

From (i), (ii) and (iii)

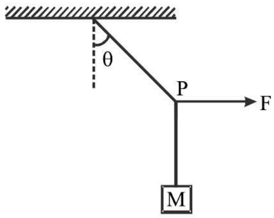
$T_1 = 150\ N;\quad T_2 = 200$ N



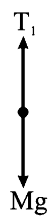
[FBD of hanging body] [FBD of knot b]

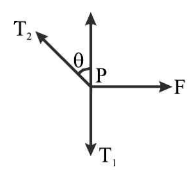
***Example* 9.** A block of mass M is suspended through a light string A horizontal force $F = \sqrt{3}\ mg$ is applied at the middle point of string. Find the angle of the string with the vertical in equilibrium and tension in two points of string

Solution Let middle point of string be P, force acting on point P is shown below. Let tension in the string is given for two parts as shown.



***F.B.D.* of block** ***F.B.D.* of Point *P* of string**





Block is under equilibrium

Consider vertical $T_1 - Mg = 0$

Direction (Y) $\Rightarrow$ $T_1 = Mg$ ...(i)

For horizontal equilibrium (Along X-axis)

$F - T_2\sin\theta = 0 \Rightarrow T_2\sin\theta = F$ ...(ii)

For vertical equilibrium (Along y-axis)

$T_2\cos\theta = T_1$ ...(iii)

dividing (ii) by (iii)

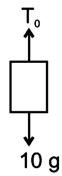
$$\tan\theta = \frac{F}{T_1} = \frac{F}{Mg} = \sqrt{3} \quad\Rightarrow\quad \boldsymbol{\theta = 60°}$$

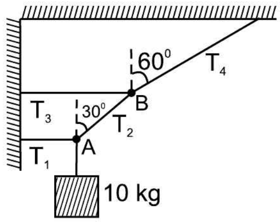
***Example* 10. The system shown in figure is in equilibrium. Find the magnitude of tension in each string; T_1, T_2, T_3 and T_4. ($g = 10$ m/s^{-2})**

Solution ***F.B.D.*** **of block 10 kg**

$T_0 = 10\,g$

$T_0 = 100\,N$





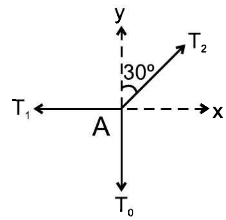
F.B.D. **of point '*A*'**

$\Sigma F_y = O \Rightarrow T_2 \cos 30^\circ = T_0 = 100$ N

$T_2 = \dfrac{200}{\sqrt{3}} N$

$\Sigma F_x = O \Rightarrow T_1 = T_2 \sin 30^\circ$

$= \dfrac{200}{\sqrt{3}} \cdot \dfrac{1}{2} = \dfrac{100}{\sqrt{3}} N.$

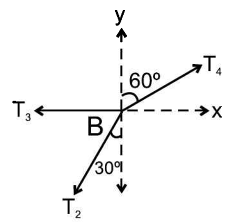


F.B.D. **of point of '*B*'**

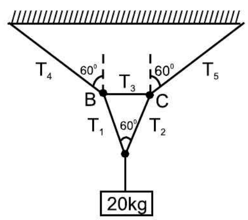
$\Sigma F_y = O \Rightarrow T_4 \cos 60^0 = T_2 \cos 30^0$

and $\Sigma F_x = O \Rightarrow T_3 + T_2 \sin 30^0 = T_4 \sin 60^0$

$\therefore$ $T_3 = \dfrac{200}{\sqrt{3}} N,\ T_4 = 200\,N$



Example 11. The system shown in figure is in equilibrium, find the tension in each string; T_1, T_2, T_3, T_4 and T_5.

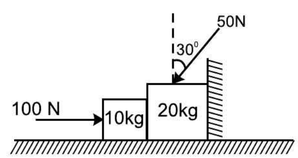


Solution $T_1 = T_2 = \dfrac{200}{\sqrt{3}} N,\ T_4 = T_5 = 200\,N,\ T_3 = \dfrac{200}{\sqrt{3}} N.$

Example 12. Two blocks are kept in contact as shown in figure. Find

(a) forces exerted by surfaces (floor and wall) on blocks.

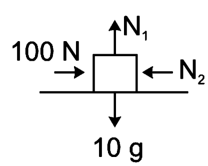
(b) contact force between two blocks.



Solution **F.B.D. of 10 kg block**

$N_1 = 10\,g = 100\,N$ ...(1)

$N_2 = 100\,N$ ...(2)



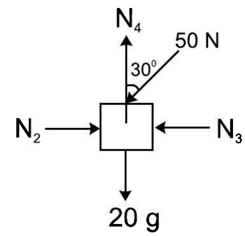
F.B.D. of 20 kg block

$N_2 = 50 \sin 30^\circ + N_3$

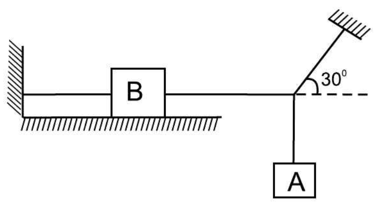
$\therefore \qquad N_3 = 100 - 25 = 75\ N \qquad ...(3)$

and $\qquad N_4 = 50 \cos 30^0 + 20\ g$

$N_4 = 243.30\ N$

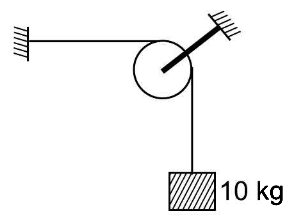


Example 13. The breaking strength of the string connecting wall and block *B* is 175 *N*, find the magnitude of weight of block A for which the system will be stationary. The block *B* weighs 700 *N*. **(g = 10 m/s²)**

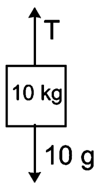


Solution $\frac{175}{\sqrt{3}} N.$

Example 14. Find magnitude of force exerted by string on pulley.

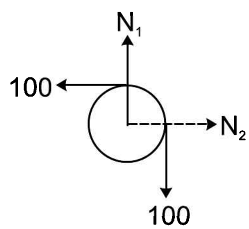


Solution ***F.B.D.* of 10 kg block:**



$T = 10\ g = 100\ N$

F.B.D. of pulley:



Since string is massless, so tension in both sides of string is same.

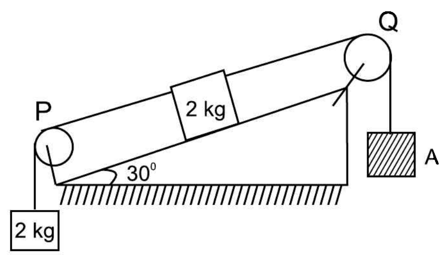
Force exerted by string= $\sqrt{(100)^2 + (100)^2} = 100\sqrt{2}\ N$

Note

Since pulley is in equilibrium position, so net forces on it is zero.

Hence force exerted by hinge on it is $100\sqrt{2}\ N$.

Example 15. In the arrangement shown in figure, what should be the mass of block A so that the system remains at rest. Also find force exerted by string on the pulley *Q*. **(g = 10 m/s²)**



Solution $m = 3$ kg, $30\sqrt{3}\ N$.

(b) Accelerating Objects

To solve problems involving objects that are in accelerated motion:

Problem Solving Steps

Step 1. Make a sketch of the problem.

Step 2. Isolate a single object and then draw the **free-body diagram** for that object. Label all external forces acting on it. Be sure to include all the forces acting on the chosen body, but be equally carefully not include any force exerted by the body on some other body. Some of the forces may be unknown; label them with algebraic symbols.

When we draw the *F.B.D.* w.r.t. inertial frame of reference than we include only real forces such as

(a) weight (b) Tension
(c) Normal reaction (d) friction
(e) spring Force (f) External force
(g) Air resistance force

in our *F.B.D.*

When we draw the *F.B.D.* w.r.t. Non inertial frame of reference then we include real forces as well as Pseudo force in our *F.B.D.*

Step 3. Identify the direction of acceleration and choose *X* and *Y* axis. Choose *X* axis in the direction of acceleration and *Y* axis perpendicular to the direction of acceleration.

Step 4. Resolve the forces along *X* and *Y* direction.

Step 5. Apply Newton's Second Law of motion.

$\Sigma F_x = ma_x$; $\Sigma F_y = ma_y$

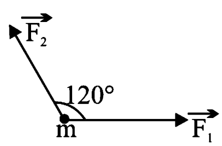
In the direction of equilibrium take acceleration zero.

Step 6. If no. of unknowns in greater than no. of equations then try to co-relate acceleration of different bodies by the help of constraint relation.

Step 7. Solve the written equaztions in Step (5) and Step (6) to find unknown acceleration and forces.

Example 16. Two forces $\vec{F}_1$ *and* $\vec{F}_2$ act on a 2 kg mass. If $F_1 = 10N$ and $F_2 = 5N$, find the acceleration.

Solution Acceleration will be in the direction of net force and will have the magnitude given by



$$\sum \vec{F} = m\vec{a}$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

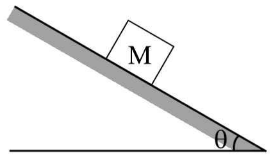
$$\Rightarrow |\vec{F}| = \sqrt{10^2 + 5^2 + 2.10.5\cos 120^\circ} = 5\sqrt{3}\ N$$

$$\Rightarrow |\vec{a}| = 2.5\sqrt{3}\ m/\sec^2$$

If the resultant force is at angle a with $\vec{F}_1$

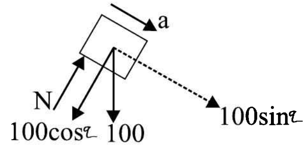
$$\tan\alpha = \frac{5\sin 120^\circ}{10 + 5\cos 120^\circ} \Rightarrow \alpha = 30^\circ$$

Therefore, acceleration is $2.5\sqrt{3}$ m/s² at an angle 30° with the direction of $\vec{F}_1$.



Example 17. Motion of a block on a frictionless incline. A block of mass 10 kg placed on a frictionless, inclined plane of angle θ, as shown in the figure. Determine the acceleration of the block after it is released. What is force exerted by the incline on the block?

Solution When the block is released, it will move down the incline. Let its acceleration be a . As the surface is frictionless, so the contact force will be normal to the plane.



[FBD of block]

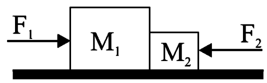
Let it be N.

Here, for the block we can apply equation for motion along the plane and equation for equilibrium perpendicular to the plane.

i.e., $Mg\sin\theta = Ma \Rightarrow a = g\sin\theta$

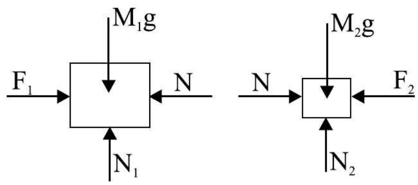
Also, $Mg\cos\theta - N = 0 \Rightarrow N = Mg\cos\theta = 100\cos\theta$

Example 18. Two blocks of masses M_1 and M_2 are placed in contact with each other on a frictionless horizontal surface as shown in figure. A constant force F_1 and F_2 is applied on M_1 and M_2 as shown. Find magnitude of acceleration of the system. Also calculate the contact force between the blocks.



Solution Here accelerations of both blocks will be same as they are rigid and in contact. As the surfaces are frictionless, contact force on any surface will be normal force only. Let the acceleration of each block is a and contact forces are N_1, N_2 and N as shown in free body diagrams of blocks.

Applying, Newton's Second Law for M_1



[FBD of M_2] [FBD of M_1]

$$F_1 - N = M_1 a \quad ...(1)$$

$$M_1 g - N_1 = 0 \quad ...(2)$$

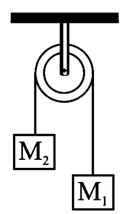
Applying, Newton's second law for M_2

$$N - F_2 = M_2 a \quad ...(3)$$

$$M_2 g - N_2 = 0 \quad ...(4)$$

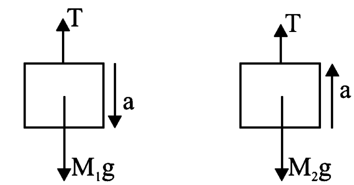
Solving (1) and (3) $a = \dfrac{F_1 - F_2}{M_1 + M_2}$ and $N = \dfrac{M_2 F_1 + M_1 F_2}{M_1 + M_2}$

Example 19. Two blocks of unequal masses M_1 and M_2 are suspended vertically over a frictionless pulley of negligible mass as shown in figure. Find accelerations of each block and tension in the string.



Solution As the string is inextensible, the magnitude of acceleration of two blocks will be same. Pulley in question is mass less and frictionless so tension in strings on two sides of pulley will be same.

Let acceleration of M_1 be 'a' (downward) then acceleration of M_2 will be 'a' (upward). Let the tension in string be T.



[FBD of M_1] [FBD of M_2]

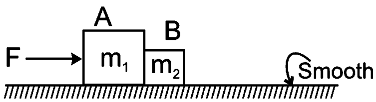
Applying Newton's second law for the blocks,

For M_1, $M_1g - T = M_1a$ ...(i)

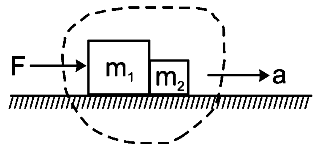
For M_2, $T - M_2g = M_2a$ ...(ii)

Solving equation (i) and (ii), $a = \dfrac{(M_1 - M_2)g}{M_1 + M_2}$ and $T = \dfrac{2M_1M_2}{M_1 + M_2}g$

Example 20. A force F is applied horizontally on mass m_1 as shown in figure. Find the contact force between m_1 and m_2.



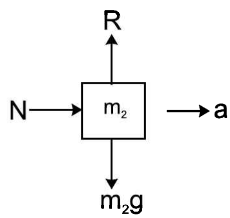
Solution Considering both blocks as a system to find the common acceleration.



Common acceleration $a = \dfrac{F}{(m_1 + m_2)}$ ...(1)

To find the contact force between 'A' and 'B' we draw $F.B.D.$ of mass m_2.

***F.B.D.* of mass m_2**



$$\Sigma F_x = ma_x$$

$$N = m_2 . a$$

$$N = \frac{m_2F}{(m_1 + m_2)}$$

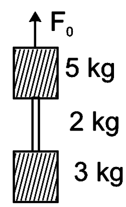
Example 21. The velocity of a particle of mass 2 kg is given by $\vec{v} = at\hat{i} + bt^2\hat{j}$. Find the force acting on the particle.

Solution From second law of motion:

$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v}) = 2.\ \frac{d}{dt}\ (at\hat{i} + bt^2\hat{j})$$

$$\Rightarrow \qquad \vec{F} = 2a\hat{i} + 4b\hat{j}$$

Example 22. A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at 2 m/s^2 by an external force F_0.



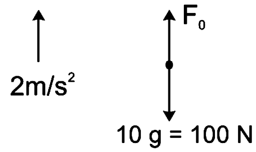
(a) What is F_0 ?

(b) What is the net force on rope ?

(c) What is the tension at middle point of the rope ? (g = 10 m/s^2)

Solution For calculating the value of F_0, consider two blocks with the rope as a system.

F.B.D. of whole system

(a) 

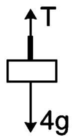
$F_0 - 100 = 10 \times 2$

$F = 120\ N$ ...(1)

(b) According to Newton's second law, net force on rope.

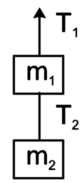
$F = ma = (2)\ (2) = 4\ N$ ...(2)

(c) For calculating tension at the middle point we draw F.B.D. of 3 kg block with half of the rope (mass 1 kg) as shown.



$T - 4g = 4.(2)$; $T = 48\ N$

Example 23. Two blocks with masses $m_1 = 0.2$ kg and $m_2 = 0.3$ kg hang one under other as shown in figure. Find the tensions in the strings (massless) in the following situations : ($g = 10$ m/s²)



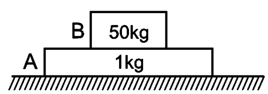
(a) the blocks are at rest
(b) they move upward at 5 m/s
(c) they accelerate upward at 2 m/s²
(d) they accelerate downward at 2 m/s²
(e) if maximum allowable tension is 10 N. What is maximum possible upward acceleration ?

Solution

(a) 5 N, 3 N (b) 5 N, 3 N (c) 6 N, 3.6 N (d) 4 N, 2.4
(e) 10 m/s²

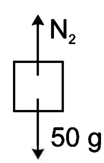
Example 24. A block of mass 50 kg is kept on another block of mass 1 kg as shown in figure. A horizontal force of 10 N is applied on the 1kg block. (All surface are smooth). Find ($g = 10$ m/s²)

(a) Acceleration of block A and B.
(b) Force exerted by B on A.



Solution

(a) F.B.D. of 50 kg



$N_2 = 50\ g = 500\ N$

along horizontal direction, there is no force $a_B = 0$

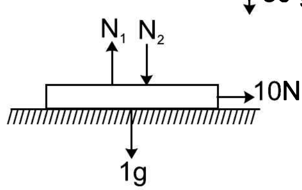
(b) F.B.D. of 1 kg block: along horizontal direction

$10 = 1\ a_A$.

$a_A = 10$ m/s²

along vertical direction

$\therefore N_1 = N_2 + 1g = 500 + 10 = 510\ N$



Example 25. Repeat above example if force is applied on 50 kg block rather on the 1kg block.

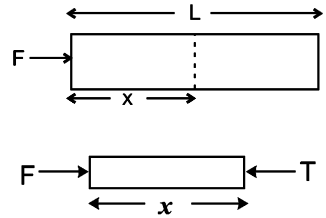
Solution

(a) 2 m/s², 0 (b) 510 N

Example 26. A horizontal force is applied on a uniform rod of length L kept on a frictionless surface. Find the tension in rod at a distance 'x' from the end where force is applied.

Solution Considering rod as a system, we find acceleration of rod

$$a = \frac{F}{m}$$



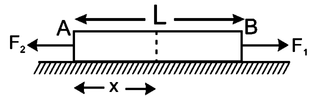
now draw F.B.D. of rod having length 'x' as shown in figure.

Using Newton's second law

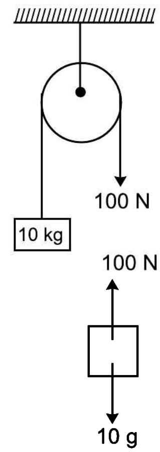
$$F - T = \left(\frac{M}{L}\right) x.a; \qquad T = F - \frac{M}{L}x.\frac{F}{M}; \qquad T = F(1 - \frac{x}{L}).$$

Example 27. Two forces F_1 and F_2 (> F_1) are applied at the free ends of uniform rod kept on a horizontal frictionless surface. Find tension in rod at a distance x from end 'A',

Solution $T = F_2 - \dfrac{(F_2 - F_1)}{L}.x$



Example 28. One end of string which passes through pulley and connected to 10 kg mass at other end is pulled by 100 N force. Find out the acceleration of 10 kg mass. **(g = 9.8 m/s²)**



Solution Since string is pulled by 100 N force. So tension in the string is 100 N.

F.B.D. of 10 kg block

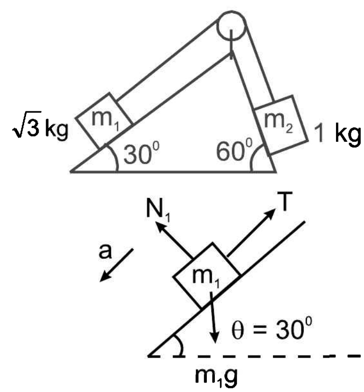
$$100 - 10\,g = 10\,a$$
$$100 - 10 \times 9.8 = 10\,a$$
$$a = 0.2 \text{ m/s}^2.$$

Example 29. Two blocks m_1 and m_2 are placed on a smooth inclined plane as shown in figure. If they are released from rest. Find:

(i) acceleration of mass m_1 and m_2

(ii) tension in the string

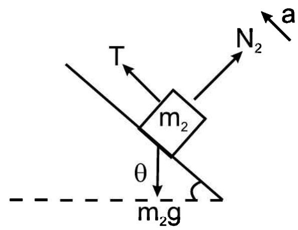
(iii) net force on pulley exerted by string



Solution **F.B.D. of m_1:**

$$m_1 g\sin\theta - T = m_1 a$$

$$\frac{\sqrt{3}}{2}g - T = \sqrt{3}\ a \qquad ...(i)$$



F.B.D. of m_2:

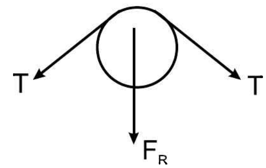
$$T - m_2 g\sin\theta = m_2 a$$

$$T - 1.g = 1.a \quad ...(ii)$$

Adding eq.(i) and (ii) we get $a = 0$

Putting this value in eq.(i) we get

$$T = \frac{\sqrt{3}g}{2},$$

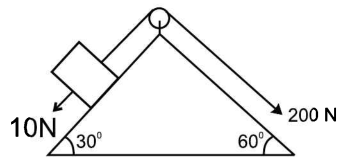


F.B.D. of pulley:

$$F_R = \sqrt{2}\,T$$

$$F_R = \frac{\sqrt{3}}{2}g$$

Example 30. A 10 kg block kept on an inclined plane is pulled by a string applying 200 *N* force. A 10 *N* force is also applied on 10 kg block as shown in figure.



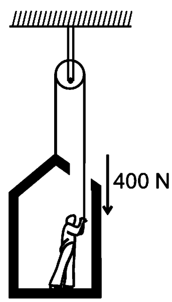
Find: (a) tension in the string.

(b) acceleration of 10 kg block.

(c) net force on pulley exerted by string

Solution (a) 200 *N*, (b) 14 m/s^2, (c) $200\sqrt{2}\,N$

Example 31. A 60 kg painter stands on a 15 kg platform. A rope attached to the platform and passing over an overhead pulley allows the painter to raise himself along with the platform.



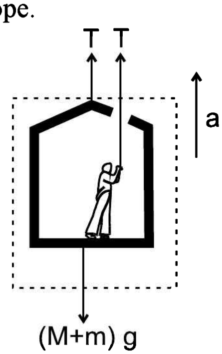
(i) To get started, he pulls the rope down with a force of 400 *N*. Find the acceleration of the platform as well as that of the painter.

(ii) What force must he exert on the rope so as to attain an upward speed of 1 m/s in 1s?

(iii) What force should he apply now to maintain the constant speed of 1 m/s?

Solution The free body diagram of the painter and the platform as a system can be drawn as shown in the figure.

Note that the tension in the string is equal to the force by which he pulls the rope.



(i) Applying Newton's Second Law

$$2T - (M+m)g = (M+m)a$$

or $$a = \frac{2T - (M+m)g}{M+m}$$

Here $M = 60$ kg; $m = 15$ kg; $T = 400\,N$

$g = 10$ m/s^2

$$a = \frac{2(400) - (60+15)(10)}{60+15} = 0.67 \text{ m/s}^2$$

(ii) To attain a speed of 1 m/s in one second, the acceleration a must be 1 m/s^2.

Thus, the applied force is

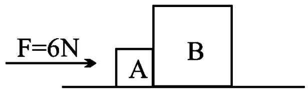
$$F = \frac{1}{2}(M+m)(g+a) = \frac{1}{2}(60+15)(10+1) = 412.5\ N$$

(iii) When the painter and the platform move (upward) together with a constant speed, it is in a state of dynamic equilibrium.

Thus, $2F - (M+m)g = 0$

or $$F = \frac{(M+m)g}{2} = \frac{(60+15)(10)}{2} = 375\ N$$

Example 32. Suppose that blocks A and B have masses of 2 and 6kg, respectively, and are in contact on a smooth horizontal surface. If a horizontal force of $6N$ pushes them, calculate (a) the acceleration of the system and (b) the force that the 2kg block exerts on the other block.



(a)

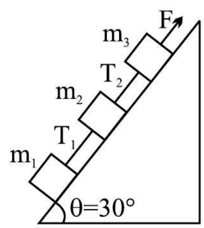
Solution

(a) Considering the blocks to move as unit,
$M = m_0 + m_b = 8$ kg, $F = Ma = 6N$, $a = 0.75$ m/s^2.

(b) If we now consider block B to be our system, the only force acting on it is the force due to block A, F_{ab}. Then since the acceleration is the same as in part (a), we have $F_{ab} = M_b a = 4.5\ N$

Example 33. Three blocks, of masses 2.0, 4.0 and 6.0 kg, arranged in the order lower, middle, and upper, respectively, are connected by strings on a frictionless inclined plane of 60°. A force 120 N is applied upward along the incline to the uppermost block, causing an upward movement of the blocks. The connecting cords are light. What is the acceleration of the blocks?

Solution The situation is depicted in fig. with $F = 120\ N$



$m_1 = 2.0$ kg $m_2 = 4.0$ kg

and $m_3 = 6.0$ kg

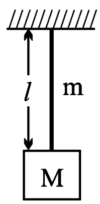
Applying Newton's second law to each block, we have

$$F - T_2 - m_3 g \sin 30° = m_3 a$$
$$T_2 - T_1 - m_2 g \sin 30° = m_2 a$$
$$T_1 - m_1 g \sin 30° = m_1 a$$

Adding these equations,

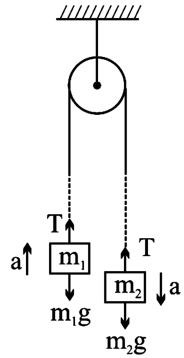
$$F - (m_1 + m_2 + m_3)g \sin 30° = (m_1 + m_2 + m_3)\ a$$
$$a = 5 \text{ m/s}^2$$

Example 34. A heavy block of mass M hangs in equilibrium at the end of a rope of mass m and length ℓ connected to a ceiling. Determine the tension in the rope at a distance x from the ceiling.



Solution $$T(x) = \frac{m}{1}(l-x)g + Mg$$

Example 35. The device in fig. is called an Atwood's machine. In terms of m_1 and m_2 with $m_2 > m_1$. What is the tension in the light cord that connects the two masses? Assume the pulley to be frictionless and massless.



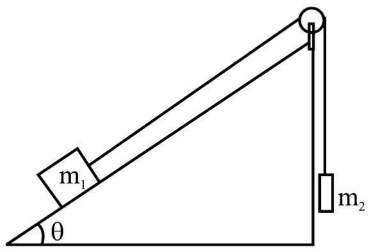
Solution Isolate the forces on each mass and write Newton's second law

$T - m_1 g = m_1 a$ and $m_2 g - T = m_2 a$.

Eliminating T gives $a = (m_2 - m_1)\, g / (m_1 + m_2)$.

From the above equations, $T = 2m_1 m_2 g/(m_1 + m_2)$

Example 36. Two bodies of masses m_1 and m_2 are connected by a light string going over a smooth light pulley at the end of an incline. The mass m_1 lies on the incline and m_2 hangs vertically. Find acceleration of m_1.



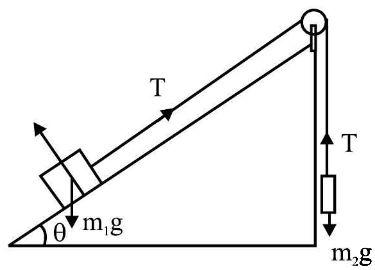
Solution

(a) Figure shows the situation with the forces on m_1 and m_2 shown.

Take the body of mass m_2 as the system. The forces acting on it are:

(i) $m_2 g$ vertically downward (by the earth)

(ii) T vertically upward (by the string)



This gives $m_2 g - T = m_2 a$ ...(i)

Next, consider the body of mass m_1 as the system.

The forces acting on this system are

(i) $m_1 g$ vertically downward (by the earth)

(ii) T along the string up the incline (by the string)

(iii) N normal to the incline (by the incline)

As the string and the pulley are all light and smooth, the tension in the string is uniform everywhere.

Taking components parallel to the incline,

$$T - m_1 g \sin\theta = m_1 a \qquad ...(ii)$$

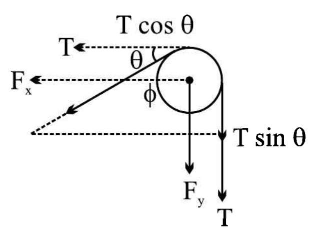
Taking components along the normal to the incline,

$$N = m_1 g \cos\theta$$

Eliminating T from (i) and (ii),

$$a = \frac{m_2 g - m_1 g \sin\theta}{(m_1 + m_2)}$$

(b) Find force acting on the pulley due to rod which attaches it to incline.



The pulley is in equilibrium and ropes passing over pulley must be applying forces on pulley.

Forces applied by ropes and the rod on pulley should cancel. We have to remember that they are vectors.

We construct *FBD* of pulley direction of force due to rod on the pulley is unknown we can not say that it is along the rod only.

Writing Newton's 2nd Law

$F_y - T\sin\theta - T = 0 \Rightarrow F_y = T(1 + \sin\theta)$

$F_x - T\cos\theta = 0 \Rightarrow F_x = T\cos\theta$

$|\vec{F}| = \sqrt{F_x^2 + F_y^2}$; $|\vec{F}| = T\sqrt{2 + 2\sin\theta}$ **Ans.**

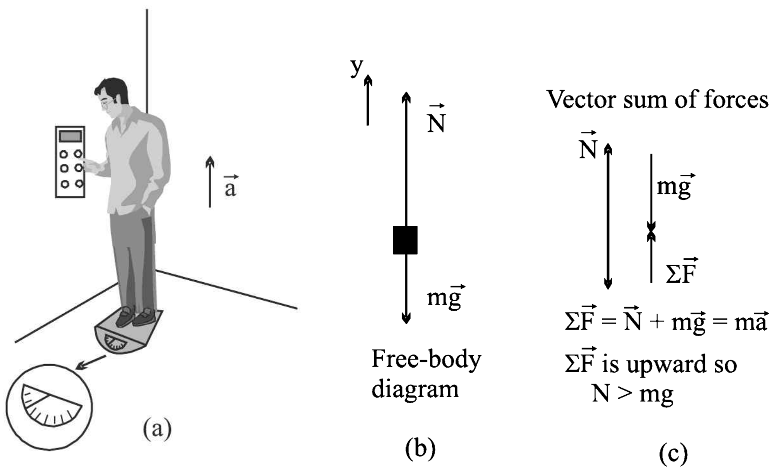
Example 37. Apparent weight in an Elevator

A passenger weighing 600N rides in an elevator. What is the apparent weight of the passenger in each of the following situations? In each case, the magnitude of the elevator's acceleration is 0.500 m/s^2. (a) The passenger is on the first floor and has pushed the button for the 15th floor; the elevator is beginning to move upward. (b) The elevator is slowing down as it nears the 15th floor.

Solution Lets understand this situation by making F.B.D. of the person inside the elevator.

Please note that ma is not being shown in F.B.D. *as ma is not the force it is effect of the net force acting on the person.* Only two actual forces act on the person in elevator normal contact force by the scale's surface and weight. By writing Newton's 2nd law we can find the normal force from the known weight and the acceleration. $W = 600\ N$; magnitude of the acceleration is $a = 0.500$ m/s^2. To find: W′.

We expect the apparent weight $W' = N$ to be greater then the true weight – the floor must push up with a force greater then W to cause an upward acceleration.



$$N - W = ma_y$$

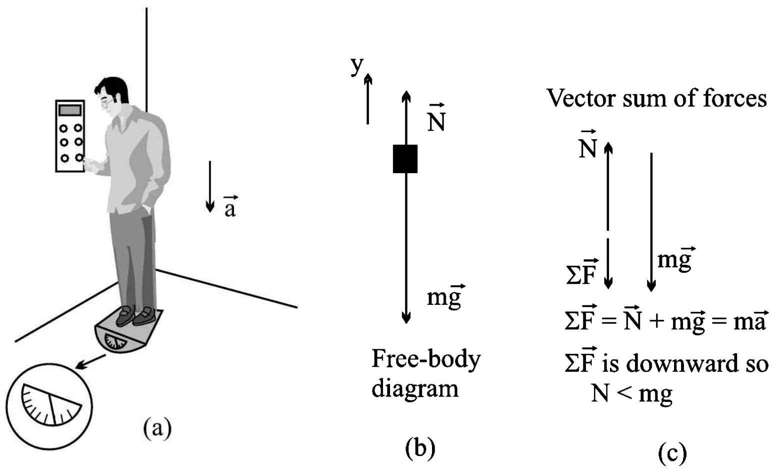
Since $W = mg$, we can substitute $m = W/g$.

$$N = W + ma = W + \frac{W}{g}a_y = W\left(1 + \frac{a_y}{g}\right)$$

$$= 600\,N \times \left(1 + \frac{0.500 m/s^2}{10 m/s^2}\right) = 630\,N$$

By Newton's third law same force will be acting on the scale. Hence scale will measure your weight larger than the actual weight.

(b) When the elevator approaches the 15th floor, it is slowing down while still moving upward; its acceleration is downward ($a_y < 0$) as in figure.



The normal force must be less than the weigtht to have an downward net force.

$$N = W\left(1 - \frac{a}{g}\right) = 600\,N \times \left(1 - \frac{0.500 m/s^2}{10 m/s^2}\right) = 570\,N$$

Example Question: If cable of lift break what will be the reading shown by scale. Explain your result qualitatively.

Example **38.** A man of mass M stands in a basket of mass m as shown in the figure. A rope is attached to the basket and passing over a pulley as shown. The man raises himself and the basket by pulling the rope downward.

(a) With what minimum force should the man pull the rope so as to prevent himself from falling down.

(b) If the man pulls the rope with a force F greater than the minimum force, then determine the acceleration of the (man + basket) system.

(c) Determine the normal reaction between the man and the trolley in part b.

Solution Let the whole system moves upward with an acceleration a. Applying Newton's Second law,

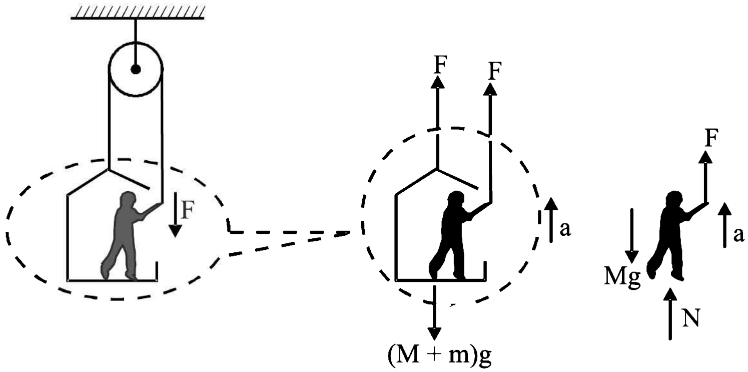
$$2F - (M + m)g = (M + m)a$$

(a) When $F = F_{min}$; $a = 0$, thus

$$F_{min} = \frac{(M + m)g}{2}$$

(b) When $F > F_{min}$, then acceleration of the system is

$$a = \frac{2F}{m+M} - g$$



(A) (B) (C)

(c) Considering the free body diagram of the man, we have from Newton's Second Law,

$$F + N - Mg = Ma$$

or $$F + N - Mg = m\left[\frac{2F}{M+m} - g\right]$$

or $$N = (M - m)\left[\frac{F + (m+M)g}{m+M}\right]$$

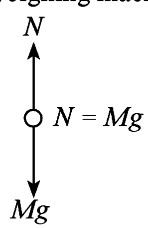
WEIGHING MACHINE

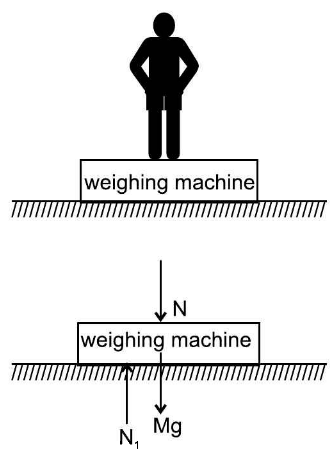
A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

***Example* 39.** (i) A man of mass 60 Kg is standing on a weighing machine placed on ground. Calculate the reading of machine ($g = 10$ m/s^2).

Solution For calculating the reading of weighing machine, we draw F.B.D. of man and machine separately.

F.B.D. of man, F.B.D. of weighing machine



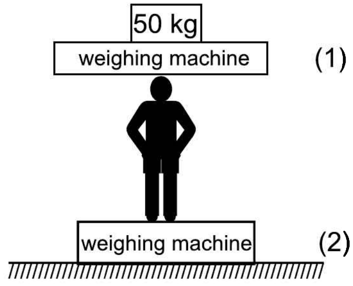


Here force exerted by object on upper surface is N

Reading of weighing machine

$N = Mg = 60 \times 10$

$N = 600\ N.$



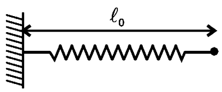
Example 40. A man of mass 60 kg is standing on a weighing machine (2) of mass 5kg placed on ground. Another same weighing machine is placed over man's head. A block of mass 50kg is put on the weighing machine (1). Calculate the readings of weighing machines (1) and (2).

Solution 500 N, 1150 N.

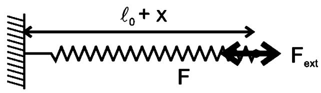
SPRING FORCE

Every spring resists any attempt to change its length; when it is compressed or extended, it exerts force at its ends. The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is the stiffness constant or spring constant (unit Nm^{-1}).

When spring is in its natural length, spring force is zero.



$F = 0$



$F = -kx$

Many springs follow hooke's law for small extension and compression. That is, the extension or compression – the increase or decrease in length from the relaxed length - is proportional to the force applied to the ends of the spring.

Hooke's law for an ideal spring: $F = k\ \Delta L$

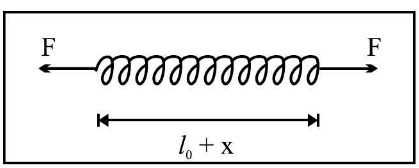
In Eq., F is the magnitude of the force exerted on each end of the spring and ΔL is the modulus of change in length of the spring from its relaxed length. The constant k is called the spring constant for a particular spring. The SI units of a spring constant are N/m.

When we say an ideal spring, we mean a spring that obeys Hooke's law and is also massless.

Since we have assumed spring to be massles we know forces acting on both ends have to be equal and opposite, to have net force on spring to be zero.

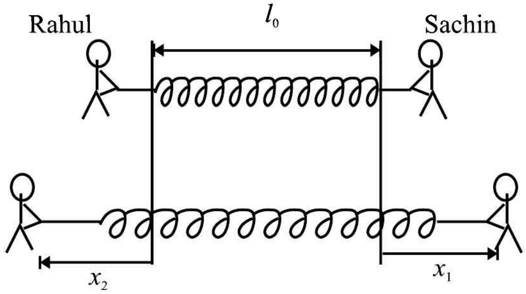
CAUTION

If we look at F.B.D. of the spring we will note that force on spring must act from both ends.



l_0 is natural length of spring

Lets say Rahul and Sachin are pulling a spring from two ends as shown. Rahul moves x_2 and Sachin moves x_1.



The force acting on Rahul and Sachin is $k(x_1 + x_2)$, Not kx_2 on Rahul and kx_1 on Sachin. Force due to spring is kx where x is defined as $|l - l_0|$, where l is present length and l_0 is natural length.

Equivalent Spring Constant

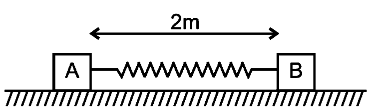
(a) When Springs are connected in Parallel then we can replace them by single spring of spring consant k_e where $k_e = k_1 + k_2$.

For more than two spring $k = k_1 + k_2 + k_3 + \;......$

(b) When Springs are connected in sereies then we can replace them by single spring of spring consant k_e where $1/k_e = 1/k_1 + 1/k_2$. As spring constants are not equal so extensions will not be equal, but total extension y can be written as sum of two extensions $y = y_1 + y_2$

For more than two springs $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \;..........$

Example 41. Two blocks are connected by a spring of natural length 2 m. The force constant of spring is 200 N/m.



Find spring force in following situations:

(a) If block '*A*' and '*B*' both are displaced by 0.5 m in same direction.

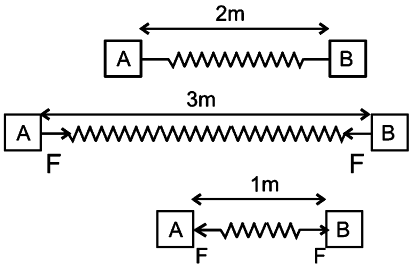
(b) If block '*A*' and '*B*' both are displaced by 0.5 m in opposite direction.

Solution

(a) Since both blocks are displaced by 0.5 m in same direction, so change in length of spring is zero. Hence, spring force is zero.

(b) In this case, change in length of spring is 1 m. So spring force is $F = -Kx = -(200).(1)$

$$F = -200\,N$$



Natural length

when spring is extended

when spring is compressed

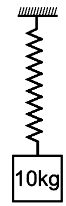
Example 42. Repeat above problem if

(a) *A* is kept at rest and *B* is displaced by 1 m in right direction.

(b) *B* is kept at rest and *A* is displaced by 1m in left direction.

(c) *A* is displaced by 0.75 m in right direction, and B is 0.25 m in left direction.

Solution (a) $F = 200\,N$, (b) $200\,N$, (c) $200\,N$.

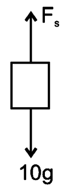


Example 43. Force constant of a spring is 100 N/m. If a 10 kg block attached with the spring is at rest, then find extension in the spring. ($g = 10$ m/s^2)

Solution In this situation, spring is in extended state so spring force acts in upward direction.

Let x be the extension in the spring.

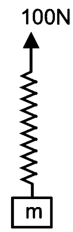
F.B.D. of 10 kg block: $F_s = 10g$



$\Rightarrow \quad Kx = 100$

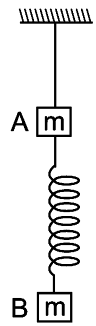
$\Rightarrow \quad (100)x = (100)$

$\Rightarrow \quad x = 1\text{m}$



Example 44. If force constant of spring is 50 N/m. Find mass of the block, if it at rests in the given situation ($g = 10$ m/s^2)

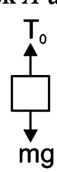
Solution $m = 10\,kg$



Example 45. Two blocks '*A*' and '*B*' of same mass '*m*' attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block '*A*' and '*B*' just after the string is cut.

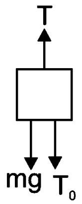
Solution When block *A* and *B* are in equilibrium position

F.B.D of '*B*'



$T_0 = mg \quad \ldots \text{(i)}$

F.B.D of '*A*'

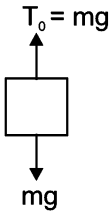


$T = mg + T_0$

$T = 2\,mg$ ...(ii)

when string is cut, tension T becomes zero. But spring does not change its shape just after cutting. So spring force acts on mass B, again draw F.B.D. of blocks *A* and *B* as shown in figure

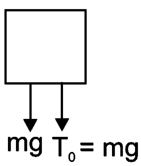
F.B.D. of '*B*'



$T_0 - mg = m.a_B$

$a_B = 0$

F.B.D. of '*A*'

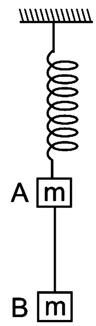


$mg + T_0 = m \,.\, a_A$

$2\,mg = m \,.\, a_A$

$a_A = 2g$

Example 46. Two blocks '*A*' and '*B*' of same mass '*m*' attached with a light string are suspended by a spring as shown in figure. Find the acceleration of block '*A*' and '*B*' just after the string is cut.

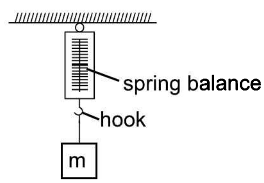


Solution *g*, *g*

Spring Balance

It does not measure the weight. It measures the force exerted by the object at the hook.

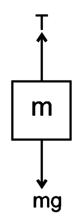
Symbolically, it is represented as shown in figure.



A block of mass '*m*' is suspended at hook.

When spring balance is in equilibrium, we draw the F.B.D. of mass *m* for calculating the reading of balance.

F.B.D. of '*m*'.

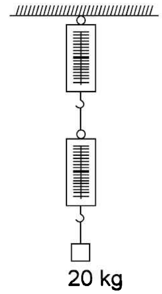


$mg - T = 0$

$T = mg$

Magnitude of *T* gives the reading of spring balance.

Example 47. A block of mass 20 kg is suspended through two light spring balances as shown in figure. Calculate the:

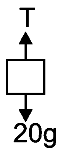


(1) reading of spring balance (1).

(2) reading of spring balance (2).

Solution For calculating the reading, first we draw F.B.D. of 20 kg block.

F.B.D of 20 kg.

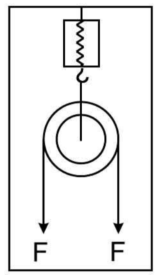


$$mg - T = 0$$

$$T = 20\,g = 200\,N$$

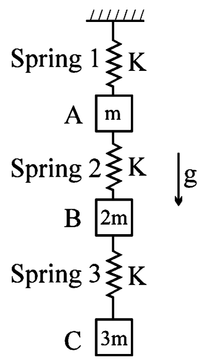
Since both balances are light so, both the scales will read 20 kg.

Example 48. Find the reading of spring balance in the adjoining figure, pulley and strings are ideal.

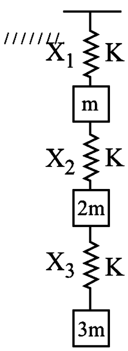


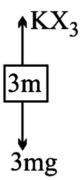
Solution $2F$

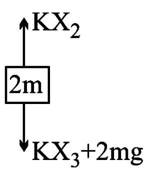
Example 49. The system shown in the figure is in equilibrium. Find the initial acceleration of A, B and C just after the spring-2 is cut.

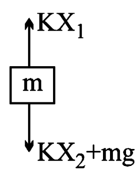


Solution









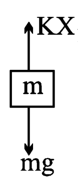
$3mg = KX_3$ ...(1)

$2mg + KX_3 = KX_2$

$\therefore\ 2mg + 3mg = KX_2 \Rightarrow 5\,mg = KX_2$ ...(2)

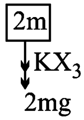
$KX_1 = 6\,mg$ ...(3)

when spring 2 is cut spring force in other two strings remain unchanged.

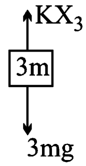


$KX_1 - mg = ma_3 \qquad \Rightarrow a_3 = 5g$

$KX_3 + 2mg = 2ma_2 \Rightarrow a_2 = \frac{5g}{2} \downarrow$



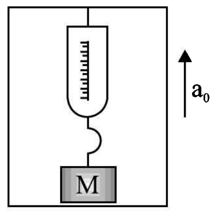
(acceleration of 3 m will be zero)



Concept

It is important to remember that ropes can change tension instantaneously while spring need to move to change tension, so in this example tension in spring is not changing instantaneously

Example 50. A block of mass M is suspended with the help of a spring balance. The spring balance is attached to the celling of an elevator moving with upward acceleration a_0 as shown in figure.



What is reading of spring balance?

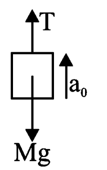
Solution A person outside the elevator will observer the block moving with the elevator upward with an acceleration a_0. Also spring balance will given the reading according to tension in spring. So calculating reading of spring balance means to find tension in the spring of spring balance.

Let tension in spring is T.

Applying Newton's Second law for the block,

$T - Mg = Ma_0 \quad \Rightarrow \quad T = M(g + a_0)$

This will be the reading of spring balance. Note that the reading given by spring balance is different from the weight of block.



[FBD of block]

CONSTRAINED MOTION

Method of virtual work

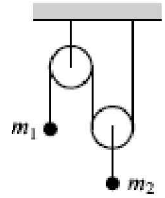
The method of virtual work for finding constraint relation is very useful in complicated situations where visual inspection is difficult and number of strings is more.

Step 1. Constraint forces are those forces whose work on the entire system is zero. To apply this method we should write the tension acting on each block.

Step 2. Displace each of the movable bodies in $+ve$ direction by $S_A.S_B$ etc. Here we need not bother whether these displacements are physically possible or not. Automatically the analysis will tell the relationship between them.

Step 3. Find the work done by tension on each of the bodies. The sum total of all these works should be zero.

Assume that m_1 moves a distance S_1 down and m_2 moves a distance S_2 down. This is not physically possible, but we are dealing with vectors here. If the displacements are in opposite directions, the answer will be negative for them.

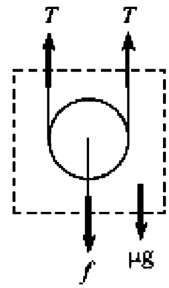


$W_1 = \vec{F}.\vec{S} = Fs \cos\theta$

$= TS_1.\cos 180° = -TS_1$

Since the pulley is massless, the tension in teh string connecting m_2 to the pulley can be found out

Newton's law for pulley



$$f = 2T$$

$$W_2 = 2TS_2.\cos 180° = -2TS_2$$

- $$W_1 + W_2 = 0$$

  This principle that the work done by the string is 0 is called the principle of virtual work.

- Here we are actually using the fact that the work done by the two strings on the total system is 0. But that is as good, becuase sum of two zeroes will also be zero.

  $\Rightarrow \quad -TS_1 - 2\,TS_2 = 0$

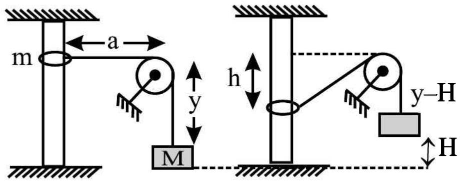
  $\Rightarrow \quad S_1 + 2S_2 = 0$

  $\Rightarrow \quad V_1 + 2V_2 = 0$

  $\Rightarrow \quad a_1 + 2a_2 = 0$

  Principle of virtual work seems to be more complicated, but once we get an understanding of it, it becomes a very easy tool.

Example 51. In the figure shown, the ring starts moving down from rest. What will be the relation between the velocity of the ring and the velocity of the block at any position?



Solution To find the relationship between the velocities of the block and the ring, we will use the concept of virtual work. We have already studied that the total work done by tension on a system is always zero. Assuming small displacements of the bodies when the angle made by the string is θ (displacement is assumed to be small so that the angle made by the string does not change appreciably)

$$-TS_R\cos\theta - TS_B = 0$$

$$S_R\cos\theta + S_B = 0$$

$\Rightarrow \quad v_R\cos\theta + v_B = 0$

String Constraint

When two objects are connected through a string and if the string have the following properties :

(a) The length of the string remains constant i.e. inextensible string.

(b) Always remains tight, does not slacks.

Then the parameters of the motion of the objects along the length of the string and in the direction of extension have a definite relation between then.

Steps for String Constraint

Step 1. Identify all the objects and number of strings in the problem.

Step 2. Assume variable to represent the parameters of motion such as displacement, velocity acceleration etc.

(i) Object which moves along a line can be specified by one variable.

(ii) Object moving in a plane are specified by two variables.

(iii) Objects moving in 3-D requires three variables to represent the motion.

Step 3. Identify a single string and divide it into different linear sections and write in the equation format. $\ell_1+\ell_2+\ell_3+\ell_4+\ell_5+\ell_6=\ell$

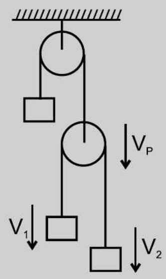
Step 4. Differentiate with respect to time

$$\frac{d\ell_1}{dt}+\frac{d\ell_2}{dt}+\frac{d\ell_3}{dt}+\ldots=0$$

$\frac{d\ell_1}{dt}$ = represents the rate of increment of the portion 1, end points are always in contact with some object so take the velocity of the object along the length of the string $\frac{d\ell_1}{dt}=V_1+V_2$

Take positive sign if it tends to increase the length and negative sign if it tends to decrease the length. Here $+V_1$ represents that upper end is tending to increase the length at rate V_1 and lower end is tending to increase the length at rate V_2

Step 5. Repeat all above steps for different-different strings.

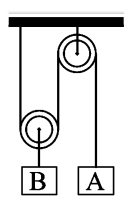


Remember:

$$V_P=\frac{V_1+V_2}{2}$$

$$a_P=\frac{a_1+a_2}{2}$$

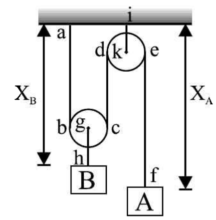
Example 52. Find the relation between accelerations of blocks A and B.



Solution The physical property that we can use is the inextensibility of string.

i.e., $ab + bc + cd + de + ef$ = constant

Let at any moment A and B are at distances X_A and X_B from the support as shown in figure.



Let us taken $gh = \ell_1$ and $ik = \ell_2$ and express the length in equation (i) in terms of X_A, X_B, ℓ_1 and ℓ_2.

we get,

$$X_B - \ell_1 + bc + (X_B - \ell_1 - \ell_2) + de + (X_A - \ell_2) = \text{constant}$$

Here, ℓ_1, ℓ_2, bc and de are constant

$$2X_B + X_A = \text{constant} \qquad ...(1)$$

let at time Δt, X_B charge to $X_B + \Delta X_B$ and X_A changes to $X_A - \Delta X_A$

[B is assumed to move downward]

then, $2(X_B + \Delta X_B) + (X_A - \Delta X_A) = \text{constant}$ ...(2)

from (1) and (2)

$$2\Delta X_B - \Delta X_A = 0$$

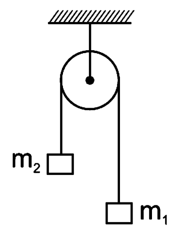
Also, $$\left(\frac{2\Delta X_B}{\Delta t}\right) - \left(\frac{\Delta X_A}{\Delta t}\right) = 0 \Rightarrow 2V_B - V_A = 0$$

Also, $$2\Delta V_B - \Delta V_A = 0 \Rightarrow \frac{2\Delta V_B}{\Delta t} - \frac{\Delta V_A}{\Delta t} = 0 \Rightarrow 2a_B = a_A$$

Hence magnitude of acceleration of A is two times magnitude of acceleration of B.

Here we get the relation between the acceleration by using the inextensibility of string but after some practice such relation can easily be written by observation.

Let us think B moves by a distance x during an interval of time, this will cause movement of pulley g by X. an extra length of $2X$ of string will come to the left of pulley k. This must be coming from right side of pulleys. Hence displacement of A will be $2x$. On the basic of this discussion we can say if the acceleration of block B is a, then the acceleration of A will be $2a$.



Example 53. Two blocks of masses m_1 and m_2 are attached at the ends of an inextensible string which passes over a smooth massless pulley. If $m_1 > m_2$, find:

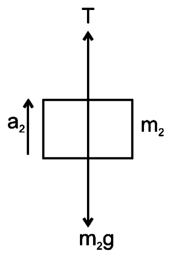
(i) the acceleration of each block

(ii) the tension in the string.

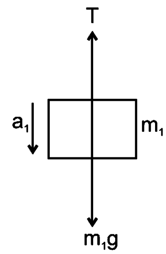
Solution The block m_1 is assumed to be moving downward and the block m_2 is assumed to be moving upward. It is merely an assumption and it does not imply the real direction. If the values of a_1 and a_2 come out to be positive then only the assumed directions are correct; otherwise the body moves in the opposite direction. Since the pulley is smooth and massless, therefore, the tension on each side of the pulley is same.

The free body diagram of each block is shown in the figure.

F.B.D. of m_2 **F.B.D. of m_1**



F.B.D. of m_2



F.B.D. of m_1

Applying Newton's second Law on blocks m_1 and m_2

Block m_1 $\qquad m_1g - T = m_1a$ ...(1)

Block m_2 $\qquad -m_2g + T = m_2a_2$ ...(2)

Number of unknowns: T, a_1 and a_2 (three)

Number of equations: only two

Obviously, we require one more equation to solve the problem. Note that whenever one finds the number of equations less than the number of unknowns, one must think about the constraint relation. Now we are going to explain the mathematical procedure for this.

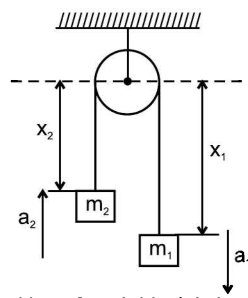
How to determine Constraint Relation ?

(1) Assume the direction of acceleration of each block, e.g. a_1 (downward) and a_2 (upward) in this case.

(2) Locate the position of each block from a fixed point (depending on convenience), e.g. centre of the pulley in this case.

(3) Identify the constraint and write down the equation of constraint in terms of the distance assumed. For example, in the chosen problem, the length of string remains constant is the constraint or restriction.

Thus, $x_1 + x_2 =$ constant



Position of each block is located w.r.t. centre of the pulley

Differentiating both the sides w.r.t. time we get $\dfrac{dx_1}{dt} + \dfrac{dx_2}{dt} = 0$

Each term on the left side represents the velocity of the blocks.

Since we have to find a relation between accelerations,

therefore we differentiate it once again w.r.t. time.

Thus $\dfrac{d^2x_1}{dt^2} + \dfrac{d^2x_2}{dt^2} = 0$

Since, the block m_1 is assumed to be moving downward (x_1 is increasing with time)

$$\therefore \qquad \frac{d^2x_1}{dt^2} = +a_1$$

and block m_2 is assumed to be moving upward (x_2 is decreasing with time)

$\therefore \quad \frac{d^2x_2}{dt^2} = -a_2$

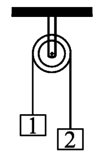
Thus $\quad a_1 - a_2 = 0$

or $a_1 = a_2 = a$ (say) is the required constraint relation.

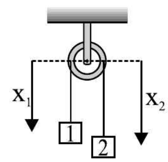
Substituting $a_1 = a_2 = a$ in equations (1) and (2) and solving them, we get

(i) $a = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] g$ (ii) $T = \left[\frac{2m_1m_2}{m_1 + m_2}\right] g$

Example 54. Using constraint method find the relation between accelerations of 1 and 2.



Solution At any instant of time let x_1 and x_2 be the displacements of 1 and 2 from a fixed line (shown dotted).



Then, $\quad x_1 + x_2 = \text{constant}$

or $\quad x_1 + x_2 = l$ (length of string)

Differentiating with respect to time, we have

$v_1 + v_2 = 0 \quad$ or $\quad v_1 = -v_2$

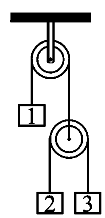
Again differentiating with respect to time, we get

$a_1 + a_2 = 0$

or $\quad a_1 = -a_2$

This is the required relation between a_1 and a_2, i.e., accelerations of 1 and 2 are equal but in opposite directions.

Example 55. Find the constraint relation between the acceleration of block 1, 2 and 3. Their acceleratino are a_1, a_2 and a_3 respectively.



Solution Points 1, 2, 3 and 4 are movable. Let their displacements from a fixed line be x_1, x_2, x_3 and x_4. We have

$x_1 + x_4 = \ell_1$(length of first string) ...(i)

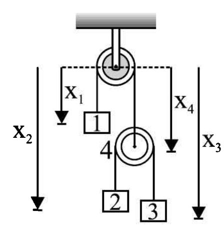
and $(x_2 - x_4) + (x_3 - x_4) = \ell_2$ (length of second string)

or $x_2 + x_3 - 2x_4 = \ell_2$ ..(ii)

On double, differentiating with respect to time, we get

$a_1 + a_4 = 0$ ...(iii)

and $a_2 + a_3 - 2a_4 = 0$ ...(iv)



But since $\quad a_4 = -a_1$ [From equation (iii)]

We have, $\quad \boldsymbol{a_2 + a_3 + 2a_1 = 0}$

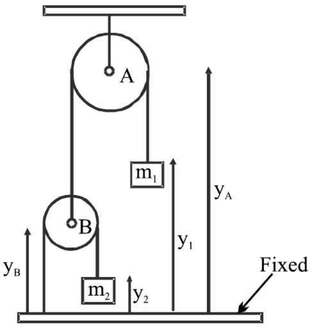
This the the required constraint relation between a_1, a_2 and a_3.

Example 56. In the shown figure masses of the pulleys and strings as well as friction between the string and pulley is negligible. Find the acceleration of the masses m_1 and m_2.

Solution Let the lengths of the strings passes over A is l_1 and of that passes over B is l_2 from the ground level and y_A and y_B be distance of pulleys.

$\because \ (y_A - y_1) + (y_A - y_B) = l_1 \qquad \Rightarrow \quad 2y_A - y_1 - y_B = l_1$

$\because \ y_A$ is constant $\qquad \because \quad y_1 + y_B = 2y_A - l_1 =$ constant



Differentiating twice this equation w. r. t. time we get

$$\frac{d^2y_1}{dt^2} + \frac{d^2y_B}{dt^2} = 0 \ \Rightarrow \ a_1 = a_s \qquad \text{...(i)}$$

Similarly $\quad y_B + y_B - y_2 = l_2 \ \Rightarrow \ 2y_B - y_2 =$ constant

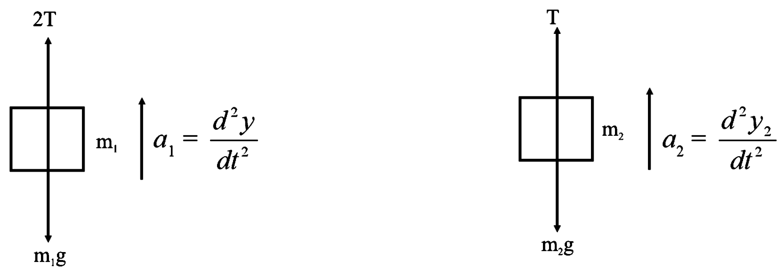
$$\Rightarrow \quad 2\frac{d^2y_B}{dt^2} - \frac{d^2y_2}{dt^2} = 0 \Rightarrow 2a_s = a_2 \qquad \text{...(ii)}$$

Since mass of the pulley is negligible hence net force on it is zero.

$$\Rightarrow \qquad T_1 = 2T$$

Let all the masses moves up,

F.B.D. of the masses



Equation of motion

$2T - m_1g = m_1a_1 \qquad \text{...(iii)}$

$T - m_2g = m_2a_2 \qquad \text{...(iv)}$

Solving equation (i) and (ii), we get

$(2m_2 - m_1)g = m_1a_1 - 2m_2a_2 \qquad \text{...(v)}$

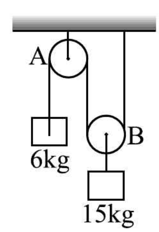
$\because \ \frac{d^2y_2}{dt^2} = 2\frac{d^2y_B}{dt^2}$ Using Eq. (i), (ii) and (v) we get

$(2m_2 - m_1)g = m_1(-a_B) - 2m_2(2a_B)$

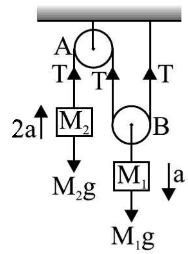
$$\Rightarrow a_B = \frac{(m_1 - 2m_2)}{(m_1 + 4m_2)}g \qquad \Rightarrow \quad a_1 = \frac{(2m_2 - m_1)}{(m_1 + 4m_2)}g$$

$$\Rightarrow a_2 = \frac{2(m_1 - 2m_2)}{(m_1 + 4m_2)}g$$

Example 57. A mass of 15 kg and another of mass 6 kg are attached to a pulley system as shown. *A* is a fixed pulley while *B* is a movable one. Both are considered light and frictionless.



Find the acceleration of 6 kg mass.



Solution Tension is the same throughout the string. It is clear that M_1 will descend downwards while M_2 rises up. If the acceleration of M_1 is a downwards, M_2 will have an acceleration 2a upward.

Now, $\quad M_1g - 2T = M_1a$

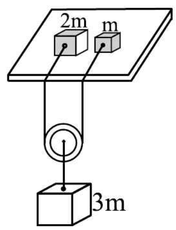
$\quad T - M_2g = M_2.2a$

or, $\quad M_1g - 2M_2g = a(M_1 + 4M_2)$

$\Rightarrow \quad a = \dfrac{M_1 - 2M_2}{M_1 + 4M_2} g = \dfrac{15-12}{15-24} g = \dfrac{3}{39} g$

$\therefore\ a = \dfrac{g}{13}$ $\quad \therefore$ acceleration of 6 kg mass = $2a$ = $\mathbf{\dfrac{2g}{13}}$

Example **58.** Two particles of masses m and $2m$ are placed on a smooth horizontal table. A string, which joins them hangs over the edge supporting a light pulley, which carries a mass 3m. The two parts of the string on the table are parallel and perpendicular to the edge of the table. The parts of the string outside the table are vertical.

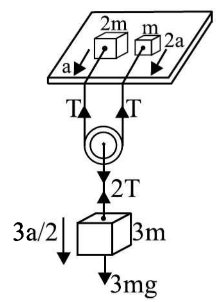


Find the acceleration of the particle of mass 3m.

Solution Let T be the tension in the string; a be the acceleration of the mass 2m; 2a be the acceleration of mass m.

$$T = m.2a$$

The mass 3m will come down with an acceleration $\dfrac{a+2a}{2} = \dfrac{3a}{2}$



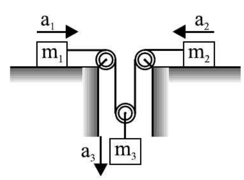
$\therefore\ 3mg - 2T = 3m.\dfrac{3a}{2}$

or $3mg - 4ma = \dfrac{9ma}{2}$

or $\dfrac{17a}{2} = 3g$ $\quad$ or $\quad a = \dfrac{6}{17} g$

the acceleration of 3m mass $\dfrac{3}{2}a = \mathbf{\dfrac{9}{17}g}$

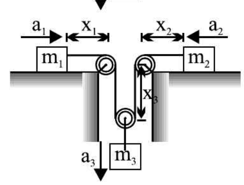
Example **59.** Calculate the relation between acceleration a_1, a_2 and a_3.



Solution Let us consider the respective distances of each block as shown in figure. Since the length of the string is constant, therfore,

$$x_1 + x_2 + 2x_3 = \text{constant}$$

On differentiation twice w.r.t. time, we get



$$\frac{d^2x_1}{dt} + \frac{d^2x_2}{dt^2} + 2\frac{d^2x_3}{dt^2} = 0$$

Since x_1 and x_2 are assumed to be decreasing with time, therefore,

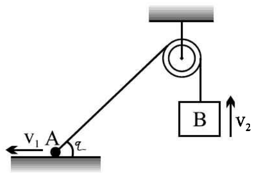
$$\frac{d^2x_1}{dt^2} = -a_1 \quad \text{and} \quad \frac{d^2x_2}{dt^2} = -a_2$$

and x_3 is assumed to be increasing with time, therefore,

$$\frac{d^2x_3}{dt^2} = +a_3$$

Thus,$-a_1 - a_2 + 2a_3 = 0$ or $\boldsymbol{a_1 + a_2 = 2a_3}$

Example 60. Two blocks A and B are joined together and moving as shown in figure. If v_1 and v_2 are the respective speeds of the block A and B block, then determine the relation between the two velocities.

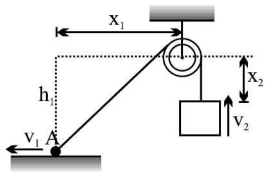


Solution Distances are taken from the centre of the pulley as shown in figure.

Constrain equation:

Length of the string remains constant.

$$\sqrt{x_1^2 + h_1^2} + x_2 = \text{constant}$$



Diferentiating both the sides w.r.t. time, we get

$$\frac{2x_1}{2\sqrt{x_1^2 + h_1^2}}\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$$

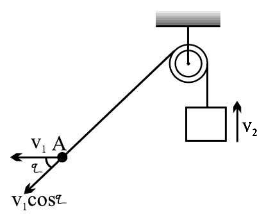
Since the ball moves so as to increase x_1 with time and block moves so as to decrease x_2 with time, therefore

$$\frac{dx_1}{dt} = +v_1 \text{ and } \frac{dx_2}{dt} = -v_2$$

also, $$\frac{x_1}{\sqrt{x_1^2 + h_1^2}} = \cos\theta$$

Thus, $$v_1 \cos\theta - v_2 = 0$$

or $$v_2 = v_1 \cos\theta$$

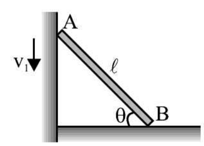


Alternatively, the problem can be solved vaery easily if we look at the problem from a different viewpoint; i.e. velocity of any two points along the string is same.

Obviously, from the figure.

$$\boldsymbol{v_1 \cos\theta = v_2}$$

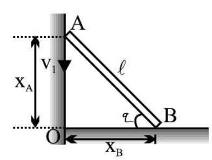
Example 61. In the figure shown, a rod of length ℓ is inclined at an angle θ with the floor against a smooth wall. If the rod A moves instantaneoulsy with v_1, calculate the velocity of end B.



Solution Let us assume the distance of A and B from O as x_A and x_B

From ΔOAB

$$x_A^2 + x_B^2 = R^2$$



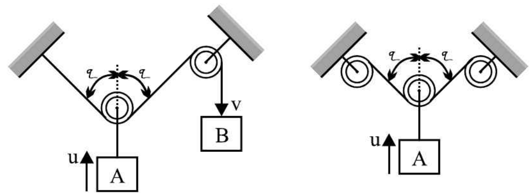
differentiating the equation w.r.t. time

$$\Rightarrow \qquad 2x_A\frac{dx_A}{dt} + 2x_B\frac{dx_B}{dt} = 0$$

from equation $\dfrac{dx_A}{dt} = -v_1 \qquad \Rightarrow \qquad \dfrac{dx_B}{dt} = \dfrac{x_A}{x_B}.v_1 = v_1\tan\theta$

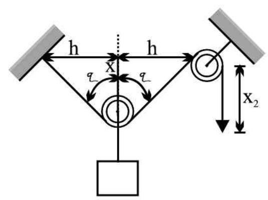
Velocity of end B = $\mathbf{v_1 tan\theta}$

Example 62. In the given figure blocks A and B are moving as shown in figure. Calculate relation between their velocities.



(A) (B)

Solution (A) Length of the string



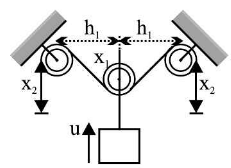
$$2\sqrt{h^2 + x_1^2} + x_2 = L$$

differentiating w.r.t. time

$$\frac{2x_1}{\sqrt{h^2 + x_1^2}}\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$$

$$\Rightarrow \quad 2\cos\theta\,(-u) + v = 0 \quad \Rightarrow \quad u = \frac{v}{2\cos\theta}$$

(B) Length of the string



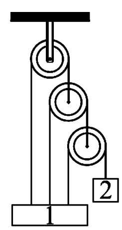
$$2\sqrt{x_1^2 + h_1^2} + 2x_2 = L$$

Differentiating w.r.t. time

$$\frac{2x_1}{\sqrt{x_1^2 + h_1^2}}\frac{dx_1}{dt} + 2.\frac{dx_2}{dt} = 0$$

$$-(\cos\theta)u + v = 0 \qquad \Rightarrow \qquad u = \frac{v}{\cos\theta}$$

Example 63. Using constraint equations find the relation between acceleration of 1 and 2.

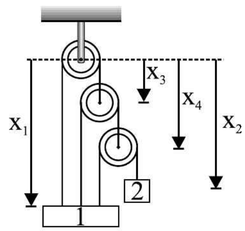


Solution Points 1, 2, 3 and 4 are movable. Let their displacements from a fixed line be x_1, x_2, x_3 and x_4

$$x_1 + x_3 = \ell_1$$

$$(x_1 - x_3) + (x_4 - x_3) = \ell$$

$$(x_1 - x_4) + (x_2 - x_4) = \ell_3$$



On double differentiating with respect to time,
we will get following three constraint relations :

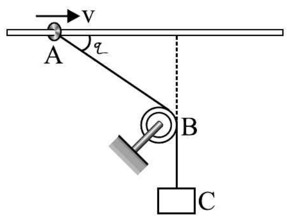
$$a_1 + a_3 = 0 \qquad ...(i)$$

$$a_1 + a_4 - 2a_3 = 0 \qquad ...(ii)$$

$$a_2 + a_2 - 2a_4 = 0 \qquad ...(iii)$$

Solving equation (i), (ii) and (iii), we get

$$\boldsymbol{a_2 = -7a_1}$$

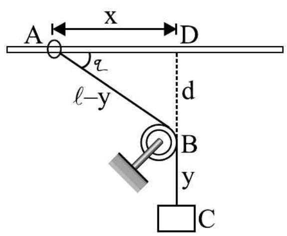


Example 64. A smooth ring A of mass m can slide on a fixed horizontal rod. A string tied to the ring passes over a fixed pulley B and carries a block *C* of mass 2m as shown in figure. At an instant the string between the ring and the pulley makes an angle θ with the rod (a) show that if the ring slides with a speed *v*, the block decends with speed *v*cosθ (b) with what acceleration will the ring start moving if the system is released from rest with θ = 60°.

Solution (a) Let at some instant of time,

$AD = x$ and $BC = y$

and if l = length of string, length *AB* at this instant will be $(\ell - y)$. Distance *DB* (= *d*) is here constant.



In ΔABD, $(\ell - y)^2 = x^2 + d^2$

Differentiating w.r.t. to time, we have

$$2(\ell - y)\left(-\frac{dy}{dt}\right) = 2x\left(\frac{dx}{dt}\right) + 0 \qquad ...(i)$$

or

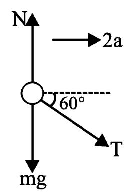
$$\left(\frac{dy}{dx}\right) = \left(\frac{x}{\ell - y}\right)\left(-\frac{dx}{dt}\right) \qquad ...(ii)$$

Here $\frac{dy}{dt}$ = speed of block *C*

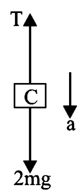
$\left(-\frac{dx}{dt}\right)$ = speed of ring *A* = *v* (given) and $\frac{x}{\ell - y} = \cos\theta$

∴ speed of block *C* = *v*cosθ

(b) Again differentiating Eq. (i) w.r.t. time, we have



FBD of ring



FBD of C

$$(\ell - y)\left(-\frac{d^2y}{dt^2}\right) + \left(\frac{dy}{dt}\right)^2 = x\left(\frac{d^2x}{dt^2}\right) + \left(\frac{dx}{dt}\right)^2 \quad ...(iii)$$

When the system is released from rest at the initial moment.

$$\frac{dy}{dt} = \frac{dx}{dt} = 0$$

Therefore, Eq. (iii) reduces to, $\left(-\frac{d^2y}{dt^2}\right) = \frac{x}{\ell - y}\left(\frac{d^2x}{dt^2}\right)$

or $|a_C| = |a_A|\cos\theta$ or $|a_C| = \frac{|a_A|}{2}$

So, if $a_C = a$, then $a_A = 2a$

Equation of motion for ring is, $T\cos 60° = m(2a)$

or $T = 4ma$ ...(iv)

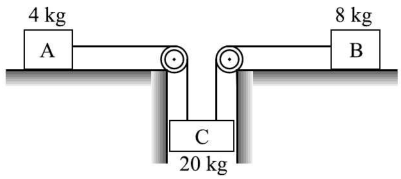
For block C equation of motion is,

$$2mg - T = 2ma \quad \text{...(v)}$$

Solving Eq. (v) and (iv),

acceleration of the ring at the initial moment = $\mathbf{2a = 2g/3}$.

Example 65. Consider the situation shown in figure the block *B* moves on a frictionless surface, while the coefficient of friction between *A* and the surface on which it moves is 0.2. Find the acceleration with which the masses move and also the tension in the strings. (g = 10 m/s²)



Solution Let a be the acceleration with which the masses move and T_1 and T_2 be the tensions in left and right strings. Friction on mass *A* is μmg = 8N. Then equations of motion of masses *A*, *B* and *C* are

For mass *A* $\quad T_1 - 8 = 4a$ ...(i)

For mass B $\quad T_2 = 8a$ ...(ii)

For mass *C* $\quad 200 - T_1 - T_2 = 20a$ ...(iii)

Adding the above three equations, we get $32a = 192$

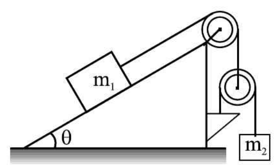
or $\quad \mathbf{a = 6\ m/s^2}$

Form equation (i) and (ii), we have

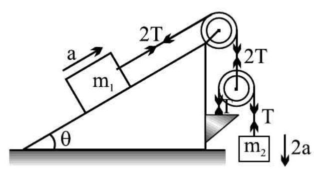
$\quad \mathbf{T_2 = 48\ N}$

and $\quad \mathbf{T_1 = 32\ N}$

Example 66. Find the acceleration of the body of mass m_2 in the arrangement shown in figure. If the mass m_2 is η times great as the mass m_1, and the angle that the inclined plane forms with the horizontal is equal to θ. The mass of the pulleys and threads, as well as the friction, are assumed to be negligible.



Solution Here, by constraint relation we can see that the acceleration of m_2 is two times that of m_1. So, we assume if m_1 is moving up the inclined plane with an acceleration *a*, the acceleration of mass m_2 going down is $2a$. The tension in different strings are shown in figure.



The dynamic equations can be written as

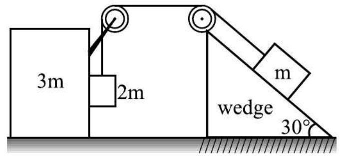
For mass m_1 $\quad 2T - m_1 g\sin\theta = m_1 a$ ...(i)

For mass m_2 $\quad m_2 g - T = m_2(2a)$ ...(ii)

Substituting $m_2 = \eta m_1$ and solving equation (i) and (ii), we get

Acceleration of $\quad m_2 = 2a = \dfrac{2g(2\eta - \sin\theta)}{4\eta + 1}$

Example 67. Neglect friction. Find acceleration of m, $2m$ and $3m$ as shown in the figure. The wedge is fixed



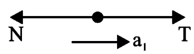
Solution Writing equaiton of motion,

$$T - N = 3ma_1$$

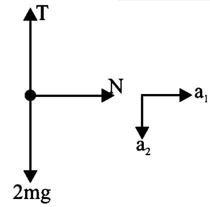
$$N = 2ma_1$$

$$2mg - T = 2ma_2$$

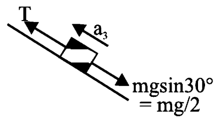
$$T - \frac{mg}{2} = ma_3$$



FBD of 3m



FBD of 2m



FBD of m

Total length $L = x_1 + x_2 + x_3$

differtiation twice w.r.t. time

$$0 = -a_1 + a_2 - a_3$$

From constraint equation,

$$a_2 = a_1 + a_3$$

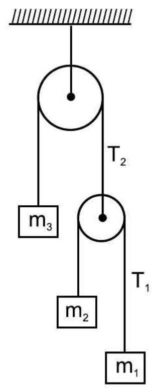
We have five unknowns. Solving the above five equations we get

$$a_1 = \frac{3}{17}g,\ a_2 = \frac{19}{34}g,\ a_3 = \frac{13}{34}g$$

Acceleration of $m = a_3 = \frac{13}{34}g$, acceleration of $2m = \sqrt{a_1^2 + a_2^2} = \frac{\sqrt{397}}{34}g$

and acceleration of $3m = a_1 = \frac{3}{17}g$

Example 68. A system of three masses m_1, m_2 and m_3 are shown in the figure. The pulleys are smooth and massless; the strings are massless and inextensible.

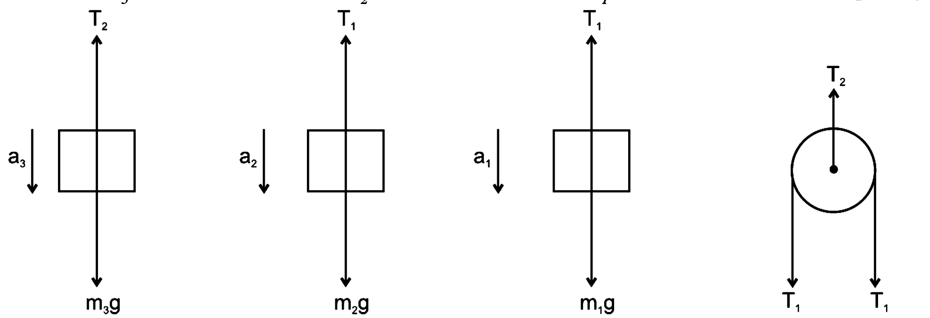


(i) Find the tensions in the strings.

(ii) Find the acceleration of each mass.

Solution All the blocks are assumed to be moving downward and the free body diagram of each block is shown in figure.

F.B.D. m_3 **F.B.D. m_2** **F.B.D. m_1** **F.B.D. of pulley**



Applying Newton's Second Law to

Block m_1: $m_1g - T_1 = m_1a_1$ ...(1)

Block m_2: $m_2g - T_1 = m_2a_2$ ...(2)

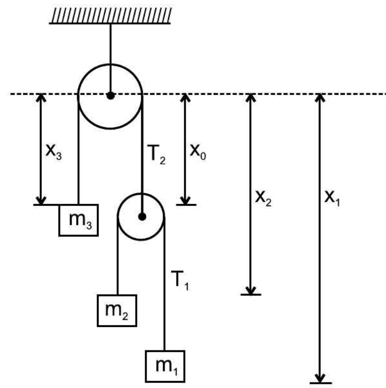
Block m_3: $m_3g - T_2 = m_3a_3$ ...(3)

Pulley: $T_2 = 2T_1$ ...(4)

Number of unknowns a_1, a_2, a_3, T_1 and T_2 (Five)

Number of equations: Four

The constraint relation among accelerations can be obtained as follows



For upper string $x_3 + x_0 = c_1$

For lower string $x_2 - x_0) + (x_1 - x_0) = c_2$

$x_2 + x_1 - 2x_0 = c_2$

Eliminating x_0 from the above two relations,

we get $x_1 + x_1 + 2x_3 = 2c_1 + c_2 =$ constant.

Differentiating twice with respect to time,

we get $\dfrac{d^2x_1}{dt^2} + \dfrac{d^2x_2}{dt^2} + 2\dfrac{d^2x_3}{dt^2} = 0$

or $a_1 + a_2 + 2a_3 = 0$ ...(5)

Solving equations (1) to (5), we get

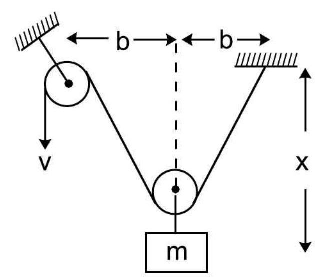
(i) $T_1 = \left[\dfrac{4m_1m_2m_3}{4m_1m_2 + m_3(m_1 + m_2)}\right]g$; $T_2 = 2T_1$

(ii) $a_1 = \left[\dfrac{4m_1m_2 + m_1m_3 - 3m_2m_3}{4m_1m_2 + m_3(m_1 + m_2)}\right]g$

$a_2 = \left[\dfrac{3m_1m_3 - m_2m_3 - 4m_1m_2}{4m_1m_2 + m_3(m_1 + m_2)}\right]g$

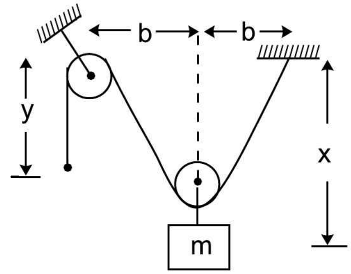
$a_3 = \left[\dfrac{4m_1m_2 - m_3(m_1 + m_2)}{4m_1m_2 + m_3(m_1 + m_2)}\right]g$

Example 69. The figure shows one end of a string being pulled down at constant velocity *v*. Find the velocity of mass '*m*' as a function of '*x*'.



Solution Using constraint equation

$2\sqrt{x^2 + b^2} + y =$ length of string = constant

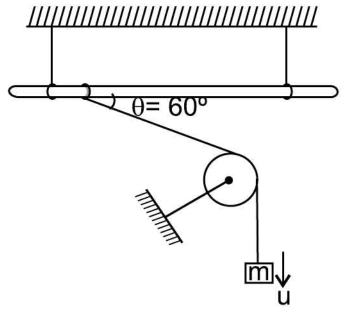


Differentiating w.r.t. time:

$\dfrac{2}{2\sqrt{x^2 + b^2}} \cdot 2x \left(\dfrac{dx}{dt}\right) + \left(\dfrac{dy}{dt}\right) = 0$

$$\left(\frac{dy}{dt}\right) = v \therefore \left(\frac{dx}{dt}\right) = -\frac{v}{2x}\sqrt{x^2+b^2}$$

Example 70. The figure shows mass m moves with velocity u. Find the velocity of ring at that moment. Ring is restricted to move on smooth rod.

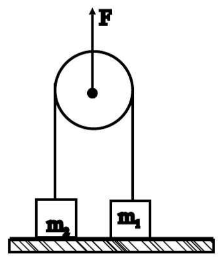


Solution $V_R = \frac{u}{\cos\theta}$,

$V_R = 2u$

Example 71. Two masses m_1 and m_2 are connected by means of a light string, that passes over a light pulley as shown in the figure. If m_1 = 2kg and m_2 = 5kg and a vertical force F is applied on the pulley then find the acceleration of the masses and that of the pulley when

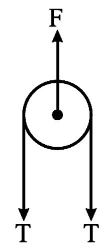
(a) $F = 35\ N$ (b) $F = 70\ N$ (c) $F = 140N$



Solution Since string is massless and friction is absent hence tension in the string is same, every where.

(a) Let acceleration of the pulley be a_p. For

F.B D of the pulley



a_p to be non zero. $F \geq 2T$ ...(1)

also $T \geq m_1 g$

$\Rightarrow \quad T \geq 2g$ ...(2)

From (1) and (2), we get

$F \geq 2 \times (2g) \quad \Rightarrow \quad F \geq 40\ N$

Therefore when $F = 35\ N$

$\boldsymbol{a_p = 0}$ **and hence** $\boldsymbol{a_1 = a_2 = 0}$.

(b) as mass of the pulley is negligible

$F - 2T = 0 \quad \Rightarrow \quad T = F/2 \quad \Rightarrow \quad T = 35N$

to lift m_2 $\quad T \geq m_2 g \quad \Rightarrow \quad T \geq 50\ N$

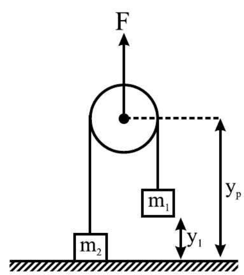
Therefore block m_2 will not move

F.B.D of m_1

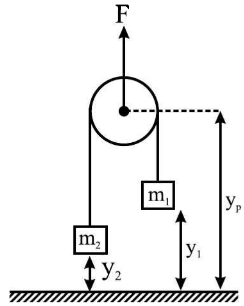
$\Rightarrow \quad T - m_1 g = m_1 a_1$

$\Rightarrow \quad 15 = 2a_1$

$\Rightarrow \quad a_1 = \frac{15}{2} m/s^2$



Constraint equation $y_p + y_p - y_1$ = constant



$\Rightarrow \; 2y_p - y_1 = c$

$\Rightarrow 2\dfrac{d^2y_p}{dt^2} - \dfrac{d^2y_1}{dt^2} = 0 \quad \Rightarrow \quad a_p = \dfrac{a_1}{2} = \dfrac{15}{4} m/s^2$

(c) When $F = 140\ N$

$T = 70N$

F.B.D. of m_1

$\Rightarrow \; T - m_1 g = m_1 a_1$ ...(i)

$\Rightarrow \; 70\ N - 20N = 2 \times a_1$

$\Rightarrow \; a_1 = 25$ m/s²

$T - m_2 g = m_2 a_2$ ...(ii)

$\Rightarrow \; 70\ N - 50N = 5a_2$

$\Rightarrow \; a_2 = 4$ m/s²

Constraint equation

$y_p - y_2 + y_p - y_1 = c \qquad 2y_p - y_2 - y_1 = c$

$\Rightarrow \; 2\dfrac{d^2y_p}{dt^2} - \dfrac{d^2y_1}{dt^2} - \dfrac{d^2y_2}{dt^2} = 0 \; \Rightarrow a_p = \dfrac{a_1 + a_2}{2} = \dfrac{29}{2} m/s^2$

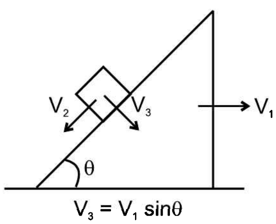
Wedge Constraint

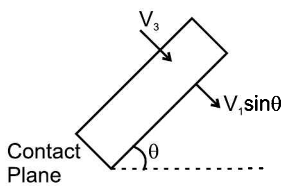
Conditions:

(i) There is a regular contact between two objects.

(ii) Objects are rigid.

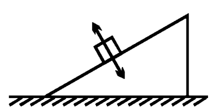
The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.





In other words, Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

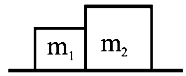
When two or more bodies are connected and their motion are related to maintain connection. ***e.g.*** if we have a block kept on incline plane and we want the block to maintain contact with it. The block can not have velocity and acceleration in direction perpendicular to the incline.



If we have two block kept touching each other on horizontal surface as shown then they must have same velocity and acceleration to maintain the contact

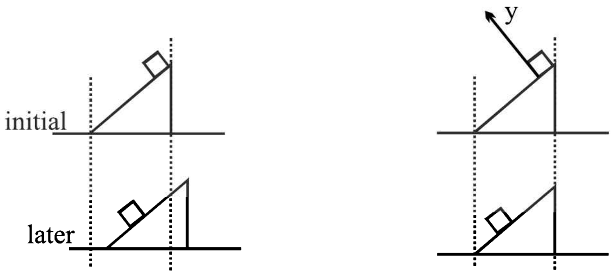
$\vec{v}_1 = \vec{v}_2 \qquad \vec{v}_1 \to \vec{v}_2$

$\vec{a}_1 = \vec{a}_2 \qquad \vec{a}_1 \to \vec{a}_2$



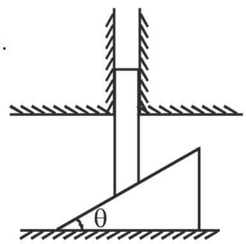
If we keep a block on wedge which can move then again constraint is defined in refrence frame attached to wedge. The block can not have any acceleration '*y*' direction in refrance frame attaced to wedge.

[in ground frame] **[in frame attached to wedge]**



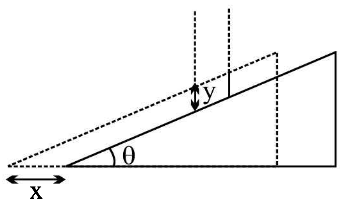
***Example* 72.** Find relation between velocity and acceleration of rod and wedge.

or say in normal direction $V_{A\wedge} = V_{B\wedge}$



Lets imagine what happens when wedge is pushed towards left.

We make a suporimposing diagram on the initial diagram.



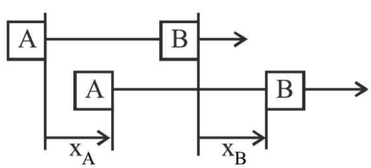
Here there are three constraints involved (a) The wedge can move only horizontally (b) the rod can move only vertically (c) the rod and wedge to have remain in contact thus their motion to be related using geometry.

We can see that when wedge moves *x* along horizontal direction rod rises by *y*.

$\tan\theta = \dfrac{y}{x} \Rightarrow y = x\tan\theta$ hence $v_R = v_w \tan\theta$ differentiating and $a_R = a_w \tan\theta$

Explanatory Notes on Constraint Equation

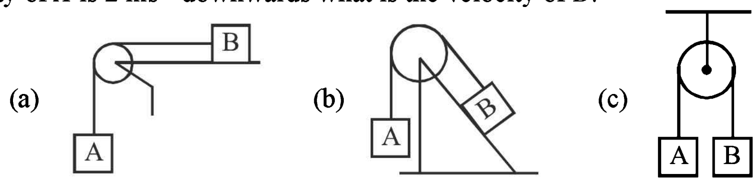
When two bodies are connected by inextensible rope then their motion is interdependent if we want rope to remain taut. If we connect two block as shown in diagram and pull block *B* towards right the block *A* must cover same distance as *B* to keep string tight.



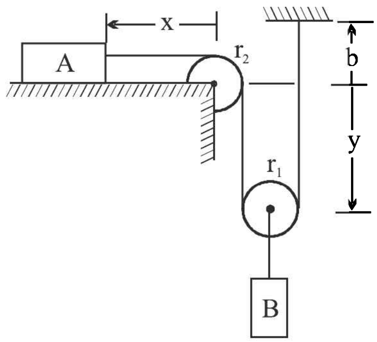
Although if we push B towards left there is no constraint relation as string will slack. If A & B are connected by rod then A will have to move as B is moving in both the above cases. Along the string $V_A = V_B$.

Check Your Skill

If velocity of A is 2 ms^{-1} downwards what is the velocity of B.



Consider first the very simple system of two interconnected particles A and B shown in fig. Although it can be shown by inspection that the horizontal motion of A is twice the vertical motion of B, we will use this example to illustrate the method of analysis which will use for more complex situations where the results cannot be easily reached by inspection.



Clearly, the motion of B is the same as that of the centre of its pulley, so we establish position coordinates x and y measured from a convenient fixed reference. The total length of the cable is

$$\mathrm{L} = x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b$$

With L, r_2, r_1, and b all constant, the first and second time derivatives of the equation give

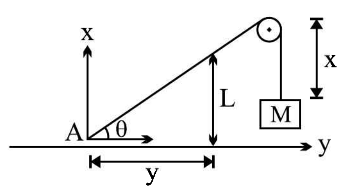
$$0 = \dot{x} + 2\dot{y} \quad \text{or} \quad 0 = \upsilon_A + 2\dot{\upsilon}_B$$

$$0 = \ddot{x} + 2\ddot{y} \quad \text{or} \quad 0 = a_A + 2a_B$$

The velocity and acceleration constraint equations indicate that, for the coordinates selected, the velocity of A must have a sign which is opposite to that of the velocity of B, and similarly for the accelerations.

Example 73. v_R = vel. of ring

v_M = vel. of block



Method 1: Total length $= \sqrt{L^2+y^2} + x = \frac{y}{\sqrt{L^2+y^2}}\frac{dy}{dta} + \frac{dx}{dt}$

$$= -\cos\theta\, v_R + v_M \left[\frac{dy}{dt} = -v_R\right]$$

$$v_M = +v_R \cos\theta$$

Method 2: Wrong method

This point A has vel. equal to v_M along string ring has vel. component along y axis

$$v_R = v_M \cos\theta$$

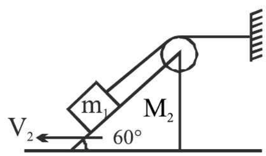
correct method

point A doesn't has along string

$\therefore$ $v_A = v_R$ and its along string

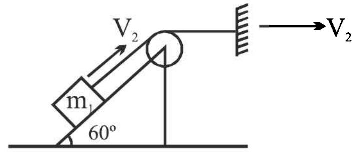
string is $v_R \cos\theta = v_M$

Example 74. Find velocity vector of m_1 if m_2 is pulled with constant velocity $v_2 = 2$ m/s.



Solution This problem involves two constraints. One involves the rope and other is involves m_1 and m_2 remaining in constact with each other.

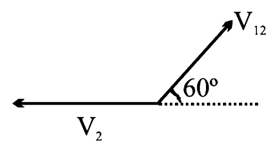
These two constraints can be understood easily if we shift our refrence frame to the wedge.



By doing this we will be able to simplify motion of block m_1 and thus solve constraint of rope easily.

In refrence frame of attached to wedge wall will move horizontally towards right with speed V_2 as shown.

Thus it can be easily deduced that m_1 will move with velocity v_2 upwards. But this is velocity of m_1 on frame attached to m_2 using relation ship of net motion

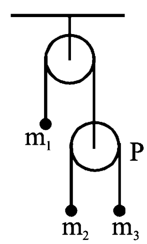


$$\vec{v}_1 = \vec{v}_{12} + \vec{v}_2$$

we can get $\vec{v}_1$ as shown

Solving we get, $|\vec{v}_1| = v_2$

We can check that this velocity vector satisfies condition of m_1 having no component of velocity perpendicular to incline with respect to the wedge.



Example 75. If V_2 = 2m/s upwards;

V_P = 1 m/s upwards

Find the velocity of block 1 and block 3?

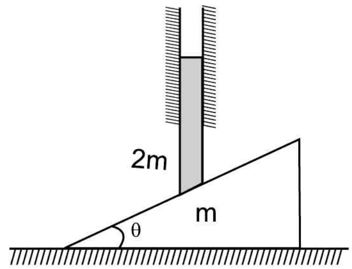
Solution $\Rightarrow$ $V_2 - V_P = V_P - V_3$

$\Rightarrow$ $V_3 = 0$

$\Rightarrow$ $V_1 = -V_P = 1$ m/s $\downarrow$

Example 76. A rod of mass 2m moves vertically downward on the surface of wedge of mass as shown in figure. Find the relation between velocity of rod and that of the wedge at any instant.

Solution Using wedge constraint.



Component of velocity of rod along perpendicular to inclined surface is equal to velocity of wedge along that direction.

$$u \cos\theta = v \sin\theta$$

$$\frac{u}{v} = \tan\theta$$

$$u = v \tan\theta$$

Example 77. In the above solved example, find a relation between acceleration of rod to that of the wedge.

Solution $a_{rod} = a_{wedge} \tan\theta$.

Newton's Law for a System

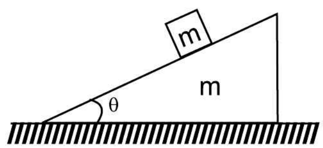
$$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots\ldots$$

$\vec{F}_{ext}$ = Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and

$\vec{a}_1, \vec{a}_2, \vec{a}_3$ are the acceleration of the objects, respectively.

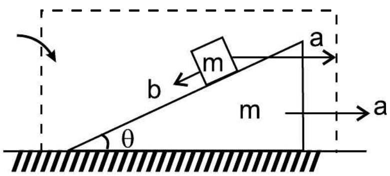
Example 78. The block of mass m slides on a wedge of mass 'm' which is free to move on the horizontal ground. Find the accelerations of wedge and block. (All surfaces are smooth).

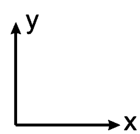


Solution Let $a \Rightarrow$ acceleration of wedge

$b \Rightarrow$ acceleration of block with respect to wedge

Taking block and wedge as a system and applying Newton's law in the horizontal direction





$$F_x = m_1\,\vec{a}_{1x} + m_2\,\vec{a}_{2x} = 0$$

$$0 = ma + m(a - b\cos\theta) \qquad \text{... (i)}$$

here 'a' and 'b' are two unknowns, so for making second equation, we draw F.B.D. of block.

F.B.D of block.

Using Newton's second law along inclined plane

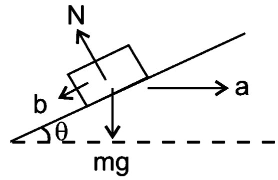
$$mg \sin\theta = m\,(b - a\cos\theta) \qquad \text{... (ii)}$$

Now solving equations (1) and (2) we will get

$$a = \frac{mg\sin\theta\cos\theta}{m(1+\sin^2\theta)} = \frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)}$$

and $$b = \frac{2g\sin\theta}{(1+\sin^2\theta)}$$

So in vector form:

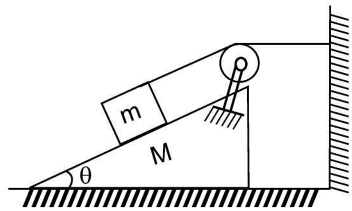


$$\vec{a}_{wedge} = a\hat{i} = \left(\frac{g\sin\theta\cos\theta}{1+\sin^2\theta}\right)\hat{i}$$

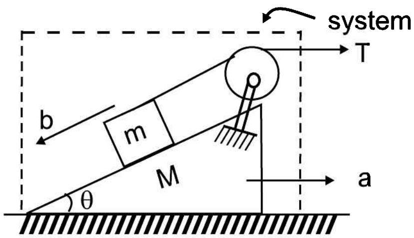
$$\vec{a}_{block} = (a - b\cos\theta)\hat{i} - b\sin\theta\,\hat{j}$$

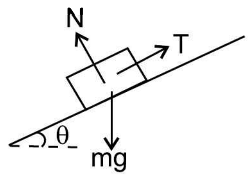
$$\vec{a}_{block} = -\frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)}\hat{i} - \frac{2g\sin^2\theta}{(1+\sin^2\theta)}\hat{j}\ .$$

Example 79. For the arrangement shown in figure when the system is released, find the acceleration of wedge. Pulley and string are ideal and friction is absent.



Solution Considering block and wedge as a system and using Newton's law for the system along x-direction.





$$T = Ma + m\,(a - b\cos\theta) \quad ...(i)$$

F.B.D of m along the inclined plane

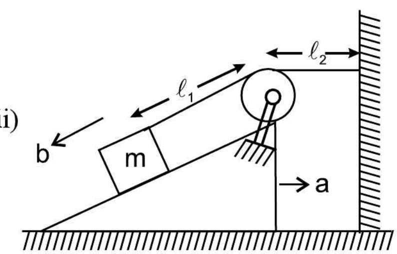
$$mg\sin\theta - T = m\,(b - a\cos\theta) \quad ...(ii)$$

using string constraint equation.

$$\ell_1 + \ell_2 = \text{constant}$$

$$\frac{d^2\ell_1}{dt} + \frac{d^2\ell_2}{dt} = 0$$

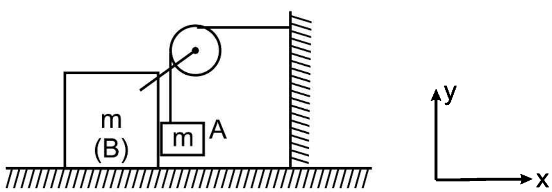
$$b - a = 0 \quad ...(iii)$$



Solving above equations (i),(ii) & (iii) , we get

$$a = \frac{mg\sin\theta}{M + 2m(1-\cos\theta)}$$

Example 80. In the system shown in figure, the block A is released from rest. Find:



(i) the acceleration of both blocks '*A*' and '*B*'.

(ii) Tension in the string.

(iii) Contact force between '*A*' and '*B*'.

Solution (i) $\frac{g}{3}\hat{i} - \frac{g}{3}\hat{j}$, $\frac{g}{3}\hat{i}$ (ii) $\frac{2mg}{3}$ (iii) $\frac{mg}{3}$.

NEWTON'S LAW FOR NON-INTERTIAL FRAME

$$\vec{F}_{Real} + \vec{F}_{Pseudo} = m\vec{a}$$

Net sum of real and pseudo force is taken in the resultant force.

$\vec{a}$ = Acceleration of the particle in the non inertial frame

$$\vec{F}_{Pseudo} = -m\ \vec{a}_{Frame}$$

Pseudo force is always directed opposite to the direction of the acceleration of the frame.

Pseudo force is an imaginary force and there is no action-reaction for it. So it has nothing to do with Newton's Third Law.

Reference Frame

A frame of reference is basically a coordinate system in which motion of object is analyzed. There are two types of reference frames.

(a) **Inertial reference frame:** Frame of reference moving with constant velocity.

(b) **Non-inertial reference frame:** A frame of reference moving with non-zero acceleration.

PESUDO FORCE

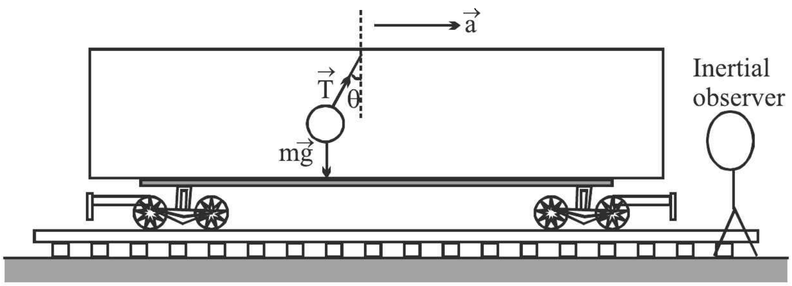
Motion in Accelerated Frames: Till now we have restricted ourselves to apply Newton's laws of motion, only to describe observations that are made in an inertial frame of reference. In this part, we learn how Newton's laws can be applied by an observer in a noninertial reference frame. For example,consider a block kept on smooth surface of a compartment of train.

If the train acclerates, the block accelerates toward the back of the train.We may conclude based on Newton's second law $F = ma$ that a force is acting the block to cause it to accelerate, but the Newton's second law is not applicable from this non-inertial frame. So we can not relate observed accleration with the Force acting on the block.

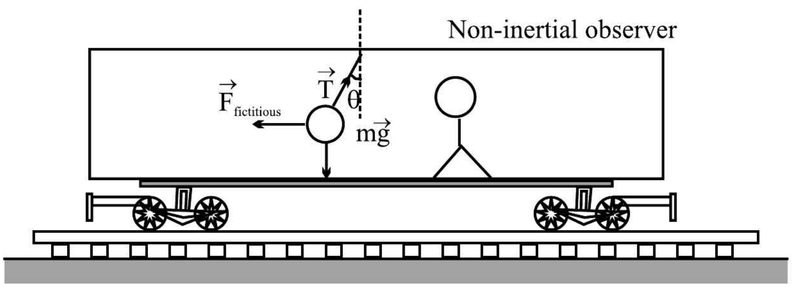
If we still want to use Newton's second law we need to apply a psuedo force, acting in backward direction, ie opposite to the aceleration of noninertial reference frame. This force explains the motion of block towards the back of car. The fictious force is equal to **-ma,** where **a** is the acceleration of the non inertial reference frame. Fictitious force appears to act on an object in the same way as a real force, but real forces are always interactions between two objects. On the other hand there is no second object for a fictitious force.

Example 81. A small ball of mass *m* hangs by a cord from the ceiling of a compartment of a train that is accelerating to the right as shown in following figure. Analyze the situations for two observers *A* & *B*.

Solution The observer A on the ground, is inertial Frame. He sees the compartment is accelerating and knows that the deviation of the cord provides the ball, required horizontal force. The non-inertial observer on the compartment, can not see the car's motion so that he is not aware of its acceleration. Because he does not know of this acceleration, he will say that Newton's second law is not valid as the object has net horizontal force (the horizontal component of tension) but no horizontal acceleration.



(a)



(b)

For the inertial observer, ball has a net force in the horizontal direction and is in equilibrium in the vertical direction. For the noninertial observer, we apply fictitious force towards left and consider it to be in equilibirium.

According to the inertial observer A, the ball experience two forces, T exerted by the cord and the weight.

Apply Newton's second law in in horizontal and vertical direction we get

Inertial observer $$T\sin\theta - mg = 0$$

$$T\cos\theta = ma$$

According to the noninertial observer B riding in the car (Fig. b), the ball is always at rest and so its acceleration is zero. The noninertial observer applies a fictitious force in the horizontal direction of magnitude ma towards left. This fictious force balances the horizontal component of T and thus the net force on the ball is zero.

Apply Newton's second law in horizontal and vertical direction we get

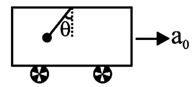
Noninertial observer $$T\sin\theta - mg = 0$$

$$T\cos\theta - ma = 0$$

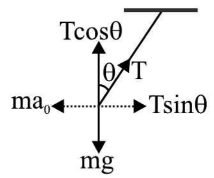
These expressions are equivalent to Equations (1) and (2).

The noninertial observer B obtains the same equations as the inertial observer. The physical explanation of the cord's deflection, however, differs in the two frames of reference.

Example 81. A pendulum is hanging from a ceiling of a car having an acceleration a_0 with respect to the road. Find the angle made by the string with vertical at equilibrium.



Solution The situation is shown in figure. Suppose the mass of bob is m and the string makes an angle θ with vertical, the forces on the bob in the car frame (non-inertial frame) are indicated. The forces are



[FBD of bob w.r.t. car]

(i) Tension in the string

(ii) mg vertically downwards

(iii) ma_0 in the direction opposite to the motion of car (pseudo frame).

Writing the equation of equilibrium

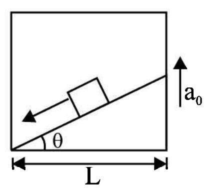
$$T\sin\theta = ma_0$$
$$T\cos\theta = mg$$

$\therefore \quad \tan\theta = \dfrac{a_0}{g}$

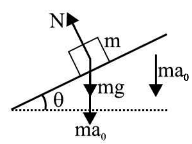
$\therefore$ the string is making an angle $\tan^{-1}\left(\dfrac{a_0}{g}\right)$ with vertical at equilibrium.

Example 83. A block slides down from top of a smooth inclined plane of elevation θ fixed in an elevator going up with an acceleration a_0. The base of incline has length L.

Find the time taken by the block to reach the bottom.



Solution Let us solve the problem in the elevator frame. The free body force diagram is shown. The forces are



(i) N normal to the plane

(ii) mg acting vertically down

(iii) ma_0 (pseudo force).

If a is the acceleration of the body with respect to incline, taking components of forces parallel to the incline

$mg\sin\theta + ma_0\sin\theta = ma \qquad \therefore \qquad a = (g + a_0)\sin\theta$

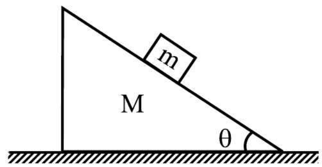
This is the acceleration with respect to elevator.

The distance traveled is $L/\cos\theta$. If t is the time for reaching the bottom of incline

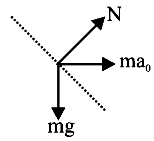
$$\frac{L}{\cos\theta} = 0 + \frac{1}{2}(g + a_0)\sin\theta . t^2$$

$$t = \left[\frac{2L}{(g + a_0)\sin\theta\cos\theta}\right]^{1/2}$$

Example 84. All the surfaces shown in figure are assumed to be frictionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of prism and acceleration of the smaller block with respect to the prism.



Solution Let the acceleration of the prism be a_0 in the backward direction. Consider the motion of the smaller block from the frame of the prism. The forces on the block are



(i) N normal force,

(ii) mg downward (gravity),

(iii) ma_0 forward (psuedo).

The block slides down the plane. Components of the forces parallel to the incline give

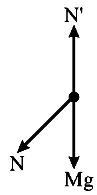
$$ma_0 \cos\theta + mg \sin\theta = ma$$

or $$a = a_0 \cos\theta + g \sin\theta \quad ...(i)$$

Components of the force perpendicular to the incline give

$$N + ma_0 \sin\theta = mg \cos\theta \quad ...(ii)$$

Now consider the motion of the prism from the lab frame. No pseudo force is needed as the frame used is inertial. The forces are



(i) Mg downward,

(ii) N normal to the incline (by the block)

(iii) N' upward (by the horizontal surface).

Horizontal components give,

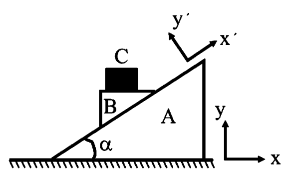
$N \sin\theta = Ma_0$ or $N = Ma_0/\sin\theta$ ...(iii)

Putting in (ii)

$$\frac{Ma_0}{\sin\theta} + ma_0 \sin\theta = mg\cos\theta \text{ or } a_0 = \frac{mg\sin\theta\cos\theta}{M + m\sin^2\theta}$$

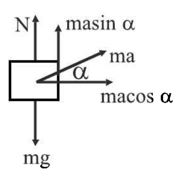
From (i) $$a = \frac{mg\sin\theta\cos^2\theta}{M + m\sin^2\theta} + g\sin\theta = \frac{(M+m)g\sin\theta}{M + m\sin^2\theta}$$

Example 85. In the shown figure the wedge A is fixed to the ground. The prism B of mass M and the block C of mass m is placed as shown. Find the acceleration of the block C w.r.t. B when the system is set free. Neglect any friciton.



Solution Let a be the acceleration of B towards, then

F.B.D. of C relative to B.



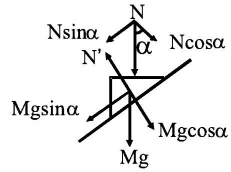
$\Rightarrow$ $mg - N - ma\sin\alpha = 0$, as C is always in contact with B.

$\Rightarrow$ $N = mg - ma\sin\alpha$ ...(i)

and $ma\cos\alpha = ma'$, where a' = acceleration of C relative to B

$a' = a\cos\alpha$ ...(ii)

F.B.D of *B*



Along the incline

$\Rightarrow \quad Mg\sin\alpha + N\sin\alpha = Ma$

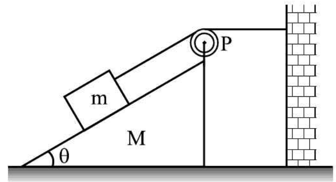
Putting the value of N from (i), we get

$\Rightarrow \quad Mg\sin\alpha + mg\sin\alpha - ma\sin^2\alpha = Ma$ ...(iii)

$$\Rightarrow a = \frac{(M+m)g\sin\alpha}{M+m\sin^2\alpha}$$

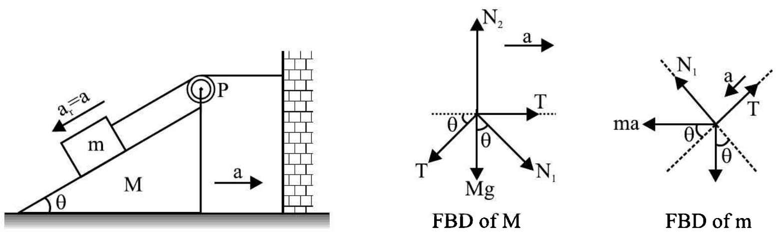
from equation (ii), $a' = \dfrac{(M+m)g\sin\alpha\cos\alpha}{M+m\sin^2\alpha}$

Example 86. In the figure, a bar of mass m is on the smooth inclined face of the wedge of mass M, the inclination to the horizontal being θ. The wedge is resting on a smooth horizontal plane. Assuming the pulley P to be smooth and the string is light and inextensible. Find the acceleration of M.



Assume that M and m are always in contact.

Solution Here, it can be easily shown that if wedge moves toward right by a distance x, the small bar will travel equal distance x on the inclined plane of wedge. Thus, acceleration of wedge towards right (w.r.t. ground) a is equal to the acceleration of bar along incline on wedge a_r (w.r.t. wedge). As the free body diagram of bar is drawn with respect to wedge, a pseudo force has been shown in its FBD.



Let T = tension in the string

N_1 = normal reaction between bar and wedge

N_2 = normal reaction between wedge and ground

Motion equation for M

Along the horizontal $\quad T + N_1\sin\theta - T\sin\theta = Ma$ ...(1)

There is no need of writing equation in vertical direction, as no motion of it is in vertical direction.

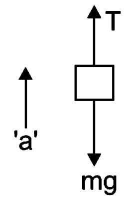
Motion equation for m

Along the plane $\quad ma\cos\theta + mg\sin\theta - T = ma$ ...(2)

Normal to plane $N_1 + ma\sin\theta = mg\cos\theta$ ...(3)

Substituting the value of T and N_1 from equation (2) and (3) in equation (1), we get

$$a = \frac{mg\sin\theta}{M + 2m(1-\cos\theta)}$$

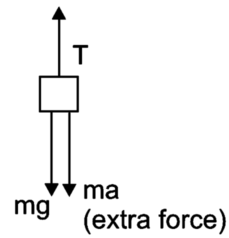


***Example* 87.** A lift having a simple pendulum attached with its ceiling is moving upward with constant acceleration 'a'. What will be the tension in the string of pendulum with respect to a boy inside the lift and a boy standing on earth, mass of bob of simple pendulum is m.

Solution **F.B.D** . of bob (with respect to ground)

$$T - mg = ma$$

$$T = mg + ma \quad ...(i)$$



With respect to boy inside the lift, the acceleration of bob is zero.

So he will write above equation in this manner.

$$T - mg = m(0).$$

$$\therefore \quad T = mg$$

He will tell the value of tension in string is mg. But this is 'wrong'. To correct his result, he makes a free body diagram in this manner, and uses Newton's second law.

$$T = mg + ma \quad ...(ii)$$

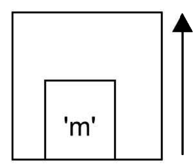
By using this extra force, equations (i) and (ii) give the same result. This extra force is called pseudo force. This pseudo force is used when a problem is solved with a accelerating frame (Non-inertial)

NOTE

Magnitude of Pseudo force = mass of system × acceleration of frame of reference.

Direction of Force

Opposite to the direction of acceleration of frame of reference, (not in the direction of motion of frame of reference)

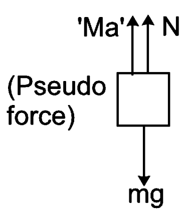


***Example* 88.** A box is moving upward with retardation 'a' < g, find the direction and magnitude of " pseudo force" acting on block of mass 'm' placed inside the box. Also calculate normal force exerted by surface on block

Solution **Pseudo force** acts opposite to the direction of acceleration of reference frame.

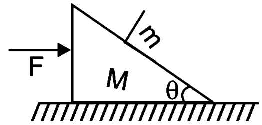
pseudo force = ma in upward direction

F.B.D of 'm' w.r.t. box (non–inertial)



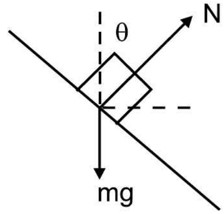
$$N + ma = mg$$
$$N = mg - ma$$

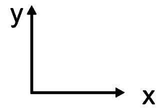
Example 89. All surfaces are smooth in the adjoining figure. Find F such that block remains stationary with respect to wedge.



Solution Acceleration of (block + wedge) is $a = \frac{F}{(M+m)}$

Let us solve the problem by using both frames.





From inertial frame of reference (Ground)

F.B.D. of block w.r.t. ground (Apply real forces):

with respect to ground block is moving with an acceleration 'a'.

$\therefore \quad \sum F_y = 0 \quad \Rightarrow \quad N\cos\theta = mg \quad$ ...(i)

and $\quad \sum F_x = ma \quad \Rightarrow \quad N\sin\theta = ma \quad$ ...(ii)

From Eqs. (i) and (ii)

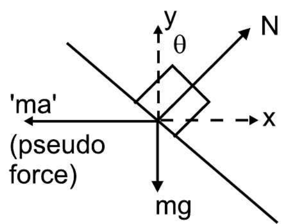
$$a = g\tan\theta$$

$$\therefore \quad F = (M+m)\,a$$

$$= (M+m)\,g\tan\theta$$

From non–inertial frame of reference (Wedge):

F.B.D. of block w.r.t . wedge (real forces + pseudo force)



w.r.t. wedge, block is stationary

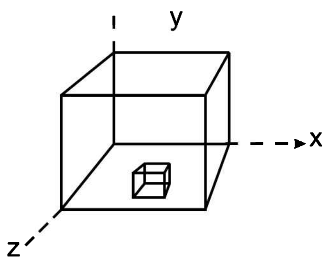
$\therefore \quad \Sigma F_y = 0 \quad \Rightarrow \quad N\cos\theta = mg \quad$ ...(iii)

$\quad \Sigma F_x = 0 \quad \Rightarrow \quad N\sin\theta = ma \quad$ ...(iv)

From Eqs. (iii) and (iv) , we will get the same result

i.e. $\quad F = (M+m)\,g\tan\theta$.

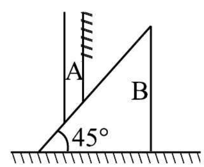
Example 90. A block of mass 2 kg is kept at rest on a big box moving with velocity $2\hat{i}$ and having acceleration $-3\hat{i}+4\hat{j}$ m/s². Find the value of 'Pseudo force' acting on block with respect to box.



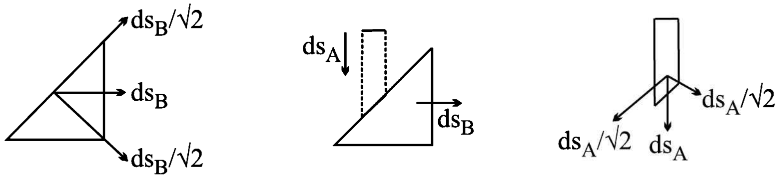
Solution $\vec{F} = -m\vec{a}_{frame} = -2(-3\hat{i}+4\hat{j})$

$F = 6\hat{i} - 8\hat{j}$.

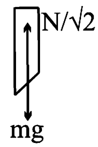
Example 91. Find the acceleration of rod A and wedge B in the arrangement shown in fig. if the mass of rod equal that of the wedge and the friction between all contact surfaces is negligible. Take angle of wedge as 45°.



Solution



Perpendicular to the plane of contact displacement must be same.



$$\frac{ds_B}{\sqrt{2}} = \frac{ds_A}{\sqrt{2}}$$

$$ds_B = ds_A$$

Differentiating, $a_B = a_A$

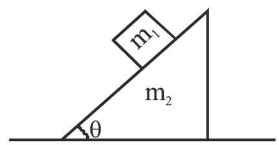
$$mg - N/\sqrt{2} = ma \qquad ...(1)$$

$$N/\sqrt{2} = ma \qquad ...(2)$$

$$mg = 2ma$$

$$\Rightarrow \quad a = g/2]$$

Example 92. There is no friction at any contact. Wedge is free to move Find force acting on wedge due to block. Also find acceleration of wedge.

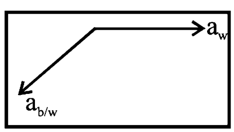


Solution Students may want to directly reach to conclusion that answer is $m_1 g \cos\theta$. Explain that it is being solved in refrence frame of wedge which may beacclerating. Horizontal component of normal contact force applied by block on wedge will accelerate the wedge. Thus refrence frame attached to wedge is non-inertial refrence frame.

Acceration vector of block in ground frame is sum of acceleration of wedge and acceleration of block w.r.t. wedge $(\vec{a}_{b/w})$

$$\vec{a}_b = \vec{a}_{b/w} + \vec{a}_w$$

$\vec{a}_b \equiv$



Consider F.B.D of wedge. Take horizontal component of normal contact force and apply Newton's 2nd Law

$$N \sin\theta = m_2 a_w$$

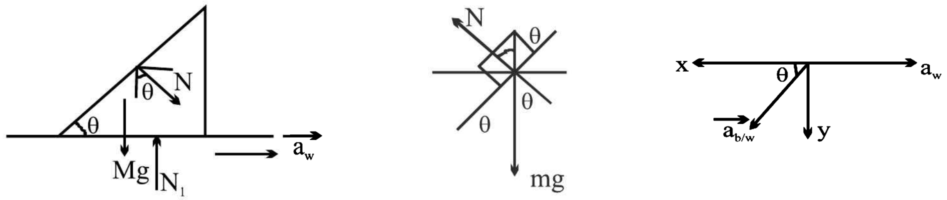
Consider F.B.D. of block and acceleration vector of block, Take horizontal and vertical component of forces and acceleration and apply Newton's second law.

$$a_x = a_{b/w} \cos\theta - a_w$$

$$a_y = a_{b/w} \sin\theta$$

$$N \sin\theta = m_1 (a_{b/w} \cos\theta - a_w)$$

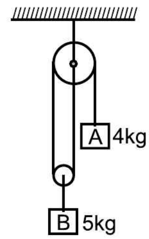
$$m_1 g - N \cos\theta = m_1 (a_{b/w} \sin\theta)$$



Solving we get

$$N = \frac{m_1 m_2 g\cos\theta}{(m_2 + m_1\sin^2\theta)},\ a_w = \frac{m_1 g\cos\theta\sin\theta}{(m_2 + m_1\sin^2\theta)},\ a_{b/w} = \frac{(m_1 + m_2)g\sin\theta}{(m_1\sin^2\theta + m_2)}$$

Example 93. The acceleration of the blocks (*A*) and (*B*) respectively in situation shown in the figure is: (pulleys and strings are massless)



(A) $\frac{2g}{7}$ downward, $\frac{g}{7}$ upward (B) $\frac{2g}{3}$ downward, $\frac{g}{3}$ upward

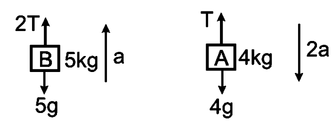
(C) $\frac{10}{13}$ g downward, $\frac{5g}{13}$ upward (D) none of these

[Ans. (A)**]**

Solution The F.B.D. of block *A* and *B* are from constraint, the acceleration of *A* & *B* are '$2a$' and 'a' respectively. Applying Newton's second law to blocks *A* and *B*, we get

$$4g - T = 4(2a) \quad ...(i)$$

$$2T - 5g = 5(a) \quad ...(ii)$$



solving we get acceleration of *A* and *B* as $\frac{2g}{7}$ downward, $\frac{g}{7}$ upward respectively.

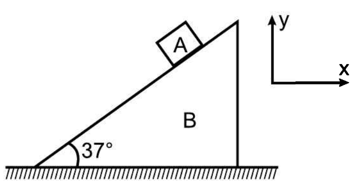
Example 94. A particle has initial velocity, $\vec{v} = 3\hat{i} + 4\hat{j}$ and a constant force $\vec{F} = 4\hat{i} - 3\hat{j}$ acts on the particle. The path of the particle is:

(A) straight line (B) parabolic

(C) circular (D) elliptical

[Ans. (B)**]**

Example 95. In the figure shown the acceleration of A is, $\vec{a}_A = 15\hat{i} + 15\hat{j}$ then the acceleration of *B* is: (A remains in contact with B)

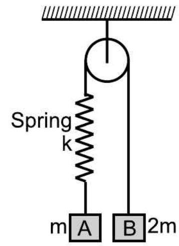


(A) $6\hat{i}$ (B) $-15\hat{i}$

(C) $-10\hat{i}$ (D) $-5\hat{i}$

[Ans. (D)**]**

Example 96. Two blocks A and B of masses m & $2m$ respectively are held at rest such that the spring is in natural length. Find out the accelerations of both the blocks just after release.

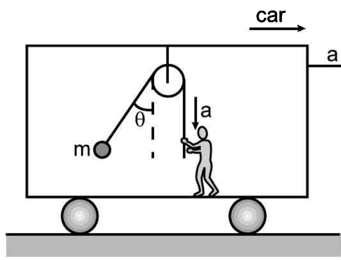


(A) $g\downarrow, g\downarrow$ (B) $\frac{g}{3}\downarrow, \frac{g}{3}\uparrow$

(C) 0, 0 (D) $g\downarrow$, 0

[Ans. (A)**]**

Example 97. A bob is hanging over a pulley inside a car through a string. The second end of the string is in the hand of a person standing in the car. The car is moving with constant acceleration 'a' directed horizontally as shown in figure. Other end of the string is pulled with constant acceleration 'a' vertically. The tension in the string is equal to

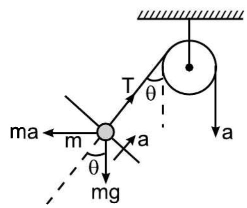


(A) $m\sqrt{g^2+a^2}$ (B) $m\sqrt{g^2+a^2} - ma$

(C) $m\sqrt{g^2+a^2} + ma$ (D) $m(g+a)$

[Ans. (C)**]**

Solution (Force diagram in the frame of the car)



Applying Newton's law perpendicular to string

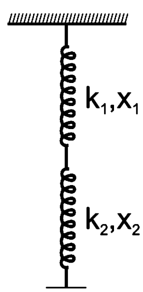
$$mg \sin\theta = ma \cos\theta$$

$$\tan\theta = \frac{a}{g}$$

Applying Newton's law along string

$\Rightarrow$ $T - m\sqrt{g^2+a^2} = ma$ $\boldsymbol{T = m\sqrt{g^2+a^2} + ma}$

Example 98. Two springs are in a series combination and are attached to a block of mass 'm' which is in equilibrium. The spring constants and the extensions in the springs are as shown in the figure. Then the force exerted by the spring on the block is:



(A) $\frac{k_1 k_2}{k_1+k_2}(x_1+x_2)$ (B) $k_1x_1 + k_2x_2$

(C) k_1x_1 (D) None of these

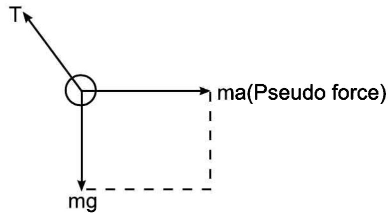
[Ans. (C)**]**

Solution Tension in both springs are same i.e. $k_1x_2 = k_2x_2$ = force exerted by lower spring on the block.

Example 99. Inside a horizontally moving box, an experimenter finds that when an object is placed on a smooth horizontal table and is released, it moves with an acceleration of 10 m/s^2. In this box if 1 kg body is suspended with a light string, the tension in the string in equilibrium position. (w.r.t. experimenter) will be. (Take g = 10 m/s^2)

(A) 10 m/s². (B) $10\sqrt{2}$ m/s² (C) 20 m/s². (D) zero

[Ans. (B)]

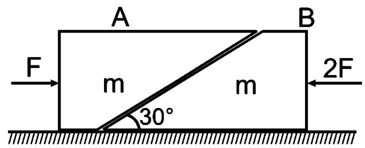


Solution Acceleration of box = 10 m/s²

Inside the box forces acting on bob are shown in the figure

$$T = \sqrt{(mg)^2 + (ma)^2} = 10\sqrt{2}\,N$$

Example 100. Two blocks 'A' and 'B' each of mass 'm' are placed on a smooth horizontal surface. Two horizontal force F and $2F$ are applied on the 2blocks 'A' and 'B' respectively as shown in figure. The block A does not slide on block B. Then the normal reaction acting between the two blocks is:



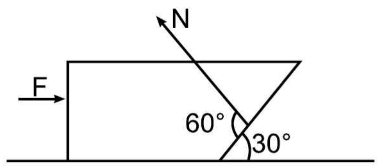
(A) F (B) $F/2$ (C) $\dfrac{F}{\sqrt{3}}$ (D) $3F$

[Ans. (D)]

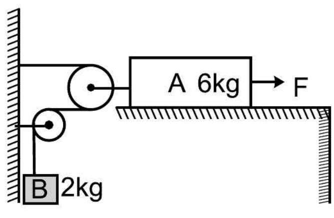
Solution Acceleration of two mass system is $a = \dfrac{F}{2m}$ leftward

FBD of block A

$$N\cos 60° - F = ma = \frac{mF}{2m} \text{ solving } N = 3F$$



Example 101. The system starts from rest and A attains a velocity of 5 m/s after it has moved 5 m towards right. Assuming the arrangement to be frictionless every where and pulley & strings to be light, the value of the force F applied on A is:



(A) 50 N (B) 75 N (C) 100 N (D) 96 N

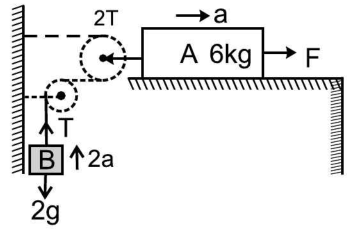
[Ans. (B)]

Solution $a = \dfrac{v^2}{2s} = \dfrac{25}{10} = 2.5$ m/s²

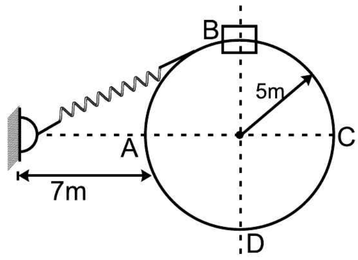
For 6 kg: $F - 2T = 6a$

For 2 kg: $T - 2g = 2(2a)$

From (1) & (2) $F = 75\,N$



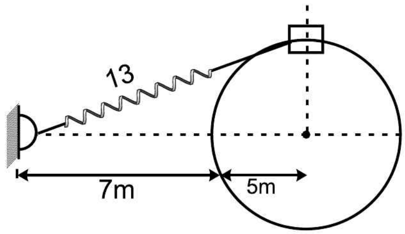
Example 102. A collar 'B' of mass 2 kg is constrained to move along a horizontal smooth and fixed circular track of radius 5 m. The spring lying in the plane of the circular track and having spring constant 200 N/m is undeformed when the collar is at 'A'. If the collar starts from rest at 'B', the normal reaction exerted by the track on the collar when it passes through 'A' is:



(A) 360 N (B) 720 N
(C) 1440 N (D) 2880 N

[Ans. (C)]

Solution Initial extension will be equal to 6 m.



$\therefore$ Initial energy $= \frac{1}{2}(200)(6)^2 = 3600\,J$.

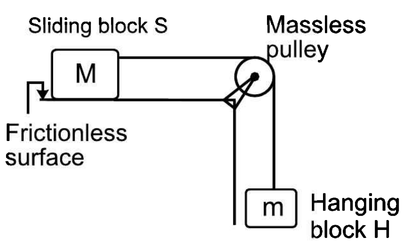
Reaching A: $\frac{1}{2}mv^2 = 3600\,J$

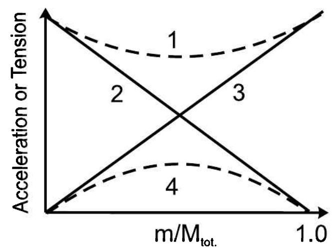
$\Rightarrow$ $mv^2 = 7200\,J$

From F.B.D. at A: $N = \frac{mv^2}{R} = \frac{7200}{5} = 1440\,N$

Analyze the situation and answer Q.103 and 104

Two containers of sand S and H are arranged like the blocks as shown. The containers alone have negligible mass; the sand in them has a total mass M_{tot}; the sand in the hanging container H has mass m.





To measure the magnitude a of the acceleration of the system; a large number of experiments are carried out where m varies from experiment to experiment but M_{tot} does not; that is sand is shifted between the containers before each trial.

Example 103. Which of the curves in graph gives the acceleration magnitude as a function of the ratio m/M_{tot} (the vertical axis is for acceleration) ?

(A) 1 (B) 2 (C) 3 (D) 4

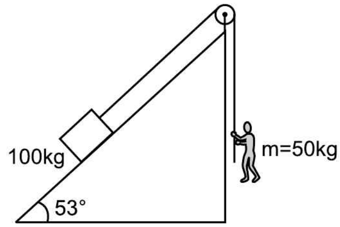
[Ans. (C)]

Example 104. Which of them gives the tension in the connecting cord (the vertical axis is for tension) :

(A) 1 (B) 2 (C) 3 (D) 4

[Ans. (D)]

Example 105. In the arrangement shown, by what acceleration the boy must go up so that 100 kg block remains stationary on the wedge. The wedge is fixed and friction is absent everywhere. Take $g = 10$ m/s^2.



(A) 2 m/s^2 (B) 4 m/s^2
(C) 6 m/s^2 (D) 8 m/s^2

[Ans. (C)]

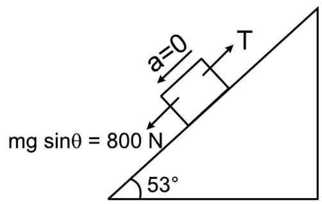
Solution For block to be stationary $T = 800\,N$

If man moves up by acceleration 'a'

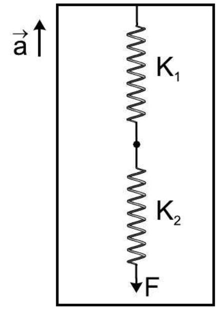
$$T - mg = ma$$

$$800 - 500 = 50\,a$$

$\therefore \quad a = 6$m/s^2.



Example 106. When F force is applied to the combination of two springs (shown in the figure), the elongation in upper spring will be (the whole system is inside a lift which is moving upwards with an acceleration $\vec{a}$). The upper spring is ideal while the lower spring has mass M.

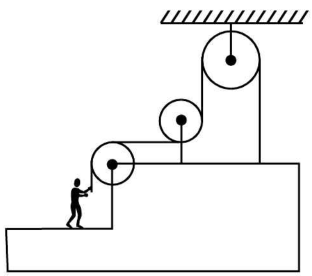


(A) $\dfrac{(M(g+a))}{K_1 + K_2}$ (B) $\dfrac{(F + M(g+a))(K_1 + K_2)}{K_1 K_2}$

(C) $\dfrac{(F + M(g+a))}{K_1 + K_2}$ (D) None of these

[Ans. (B)]

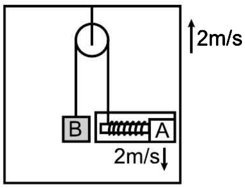
Example 107. A system is shown in the figure. A man standing on the block is pulling the rope. Velocity of the point of string in contact with the hand of the man is 2 m/s downwards. The velocity of the block will be (assume that the block does not rotate):



(A) 3 m/s (B) 2 m/s (C) 1/2 m/s (D) 1 m/s

[Ans. (B)**]**

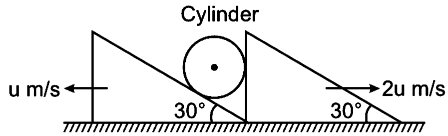
Example 108. In the figure shown the velocity of lift is 2 m/s while string is winding on the motor shaft with velocity 2 m/s and block A is moving downwards with a velocity of 2 m/s, then find out the velocity of block B.



(A) 2 m/s↑ (B) 2 m/s↓
(C) 4 m/s↑ (D) none of these

[Ans. (D)]

Example 109. System is shown in the figure. Assume that cylinder remains in contact with the two wedges. The velocity of cylinder is

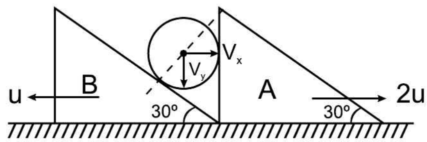


(A) $\sqrt{19-4\sqrt{3}}\,\frac{u}{2}$ m/s (B) $\frac{\sqrt{13}u}{2}$ m/s (C) $\sqrt{3}u$ m/s (D) $\sqrt{7}u$ m/s

[Ans. (D)]

Solution **Method - I:** As cylinder will remain in contact with wedge A

$$V_x = 2u$$



As it also remain in contact with wedge B

$u \sin 30° = V_y \cos 30° - V_x \sin 30°$

$$V_y = V_x \frac{\sin 30°}{\cos 30°} + \frac{U \sin 30°}{\cos 30°}$$

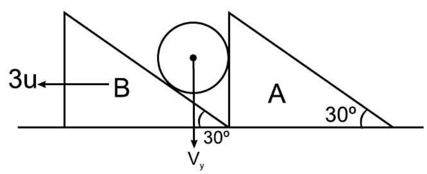
$$V_y = V_x \tan 30° + u \tan 30°$$

$$V_y = 3u \tan 30° = \sqrt{3}\,u$$

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}\,u \textbf{ Ans.}$$

Method II: In the frame of A

$$3u \sin 30° = V_y \cos 30°$$

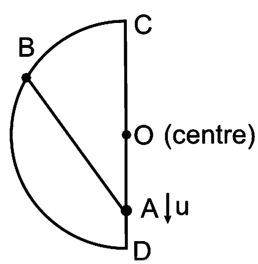


$\Rightarrow$ $V_y = 3u \tan 30° = \sqrt{3}\,u$

and $V_x = 2u$

$\Rightarrow$ $V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}u$ **Ans.**

Example 110. Two beads A and B move along a semicircular wire frame as shown in figure. The beads are connected by an inelastic string which always remains tight. At an instant the speed of A is u, $\angle BAC = 45°$ and $\angle BOC = 75°$, where O is the centre of the semicircular arc.

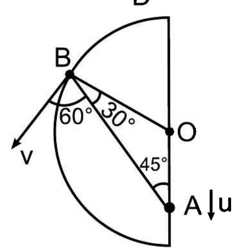


The speed of bead B at that instant is:

(A) $\sqrt{2}u$ (B) u

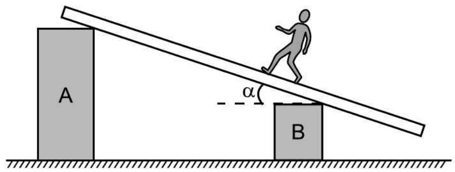
(C) $\frac{u}{2\sqrt{2}}$ (D) $\sqrt{\frac{2}{3}}u$

[Ans. (A)]



Solution **(A)** $u \cos 45° = v \cos 60°$ or $v = \sqrt{2}u$

Ex.111 A plank is held at an angle α to the horizontal (Fig.) on two fixed supports *A* and *B*. The plank can slide against the supports (without friction) because of its weight Mg. Acceleration and direction in which a man of mass m should move so that the plank does not move

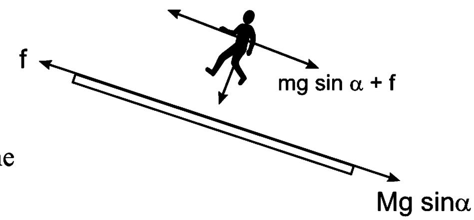


(A) $g\sin\alpha\left(1+\dfrac{m}{M}\right)$ down the incline (B) $g\sin\alpha\left(1+\dfrac{M}{m}\right)$ down the incline

(C) $g\sin\alpha\left(1+\dfrac{m}{M}\right)$ up the incline (D) $g\sin\alpha\left(1+\dfrac{M}{m}\right)$ up the incline

[Ans. (B)]

Solution F.B.D. of man and plank are



For plank be at rest, applying

Newtons second law to plank along the incline

$Mg\sin\alpha = f$ ...(1)

and applying Newton's second law to man along the incline.

$mg\sin\alpha + f = ma$ ...(2)

$a = g\sin\alpha\left(1+\dfrac{M}{m}\right)$ down the incline

Assertion Reason Questions

1. **Assertion :** The pressing force between two blocks is an example of electromagnetic interaction force.
   Reason : At microscopic level, all bodies are made of charged constituents (Nuclei and electron.) so any mechanical contact causes mutual forces between there charges.
   (A) If both Assertion and Reason are true and the Reason is correct explanation of Assertion.
   (B) If both Assertion and Reason are true but Reason is not a correct explanation of Assertion
   (C) If Assertion is true but Reason is false.
   (D) If both Assertion and Reason are false.
   [Ans. (A)]
2. **STATEMENT-1 :** Inertia is the property by virtue of which the body is unable to change by itself the state of rest only.
   STATEMENT-2 : The bodies do not change their state unless acted upon by a resultant force.
   (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
   (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
   (C) Statement-1 is True, Statement-2 is False
   (D) Statement-1 is False, Statement-2 is True
   [Ans. (D)]

3. **STATEMENT-1 :** Blocks A is moving on horizontal surface towards right under action of force F. All surfaces are smooth. At the instant shown the force exerted by block A on block B is equal to net force on block B.

STATEMENT-2 : From Newton's third law, the force exerted by block A on B is equal in magnitude to force exerted by block B on A.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

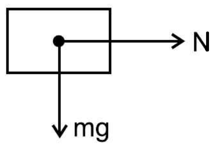
(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Solution The FBD of block A is

The force exerted by B on A is N (normal reaction). The forces acting on A are N (horizontal) and mg (weight downwards).

Hence Statement-I is false.



4. **STATEMENT-1 :** A man standing in a lift which is moving upward, will feel his weight to be greater than when the lift was at rest.

STATEMENT-2 : If the acceleration of the lift is 'a' upward, then the man of mass m shall feel his weight to be equal to normal reaction (N) exerted by the lift given by $N = m(g+a)$ (where g is acceleration due to gravity).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Solution If the lift is retarding while it moves upward, the man shall feel lesser weight as compared to when lift was at rest. Hence statement1 is false and statement 2 is true.

5. **STATEMENT–1 :** According to the Newton's third law of motion, the magnitude of the action and reaction force in an action reaction pair is same only in an inertial frame of reference.

STATEMENT–2 : Newton's laws of motion are applicable in every inertial reference frame.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

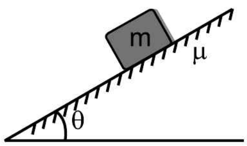
(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

Solution Newton's third law of motion is valid in all reference frames. Hence statement-1 is incorrect.

Comprehension– 1

A block of mass m is placed on a rough fixed inclined plane. The coefficient of friction between the block and the plane is μ and the inclination of the plane is θ. Initially $\theta = 0°$ and the block will remain stationary on the plane. Now the inclination θ is gradually increased. The block presses the inclined plane with a force $mg\cos\theta$. So welding strength between the block and inclined is $\mu\, mg\cos\theta$, and the pulling forces is $mg\sin\theta$. As soon as the pulling force is greater than the welding strength, the welding breaks and the block starts sliding, the angle θ for which the block starts sliding is called angle of repose (λ). During the contact, two contact forces are acting between the block and the inclined plane, the pressing reaction (Normal reaction) and the shear reaction (frictional force). The net contact force will be resultant of both.



Example 1. If the entire system, were accelerated upward with acceleration 'a', the angle of repose, would:

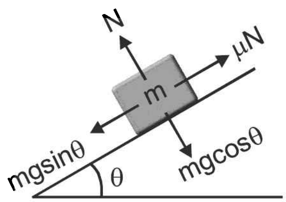
(A) increase (B) decrease

(C) remain same (D) increase of $a > g$

Solution

Initially when the system is not accelerating

For the block to just start sliding its acceleration would be zero.



Equation along the incline. (for just sliding)

$$mg\sin\theta = \mu N \qquad ...(1)$$

Equation perpendicular the incline

$$N = mg\cos\theta \qquad ...(2)$$

Dividing equation (1) by equation (2)

$$\tan\theta = \mu$$

$\therefore \quad \theta = \tan^{-1}(\mu)$ = angle of repose.

When the system is accelerating upwards with a acceleration 'a'.

Equation along the incline.

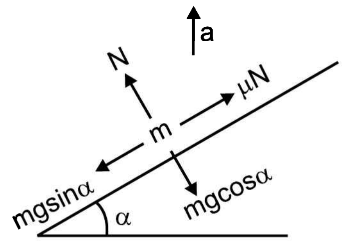
$$\mu N - mg\sin\alpha = ma\sin\alpha \qquad ...(3)$$

Equation perpendicular the incline

$$N - mg\cos\alpha = mg\, a\cos\alpha \qquad ...(4)$$

Diving equation (3) by (4) and eliminating N we get

$$\mu = \frac{m(g+a)\sin\alpha}{m(g+a)\cos\alpha}$$



$\tan\alpha = \mu$.

$\therefore\ \alpha = \tan^{-1}(\mu)$ = angle of repose.

The value of θ and α are same.

Hence the angle of repose will remain same.

(C) is the correct answer.

Example 2. For what value of θ will the block slide on the inclined plane :

(A) $\theta > \tan^{-1}\mu$ (B) $\theta < \tan^{-1}\mu$

(C) $\theta > \cot^{-1}\mu$ (D) $\theta < \cot^{-1}\mu$

Solution Block will start sliding if $mg\sin\theta > \mu N$

$mg\sin\theta > \mu mg\cos\theta$

$\tan\theta > \mu$

$\Rightarrow\ \theta > \tan^{-1}(\mu)$.

(A) is the correct answer.

Example 3. If $\mu = 3/4$ then what will be frictional force (shear force) acting between the block and inclined plane when $\theta = 30^0$:

(A) $\frac{3\sqrt{3}}{8}mg$ (B) $\frac{mg}{2}$ (C) $\frac{\sqrt{3}}{2}mg$ (D) zero

Solution For $\theta = 30°$ and $\mu = \frac{3}{4}$

The pulling force downward (or down the incline) is $mg\sin\theta = mg\sin(30°) = \frac{mg}{2}$

But the resisting force is friction.

Maximum friction force that can act is $\mu\, mg\cos\theta$

$= \frac{3}{4}mg\cos 30° = \frac{3\sqrt{3}}{8}mg.$

The frictional force is greater than the pulling force as $\frac{3\sqrt{3}}{8}mg > \frac{mg}{2}$ $\therefore$ block does not slide.

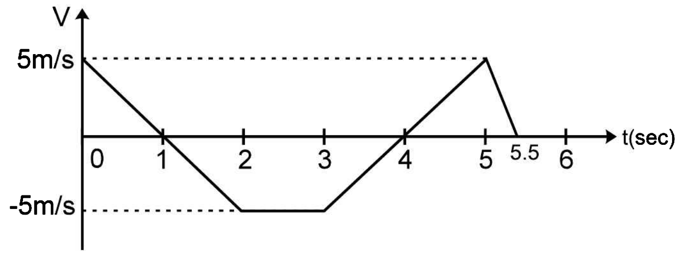
So the friction force acting on the block will be static friction and is equal to the pulling force.

$$f = \frac{mg}{2}.$$

(B is the correct answer.)

Comprehension– 2

An elevator is moving in vertical direction such that its velocity varies with time as shown in figure. Their upward direction is taken as positive. A man of mass 60 kg is standing in the elevator on a weighing machine. When the lift is accelerated up, the reading of the weighing machine increases and when the lift is accelerating down, the reading decreases.



Example 6. The man will not be in contact with weighing machine at time :

(A) $t = 1s$ (B) $t = 5.1s$ (C) $t = 2.4s$ (D) $t = 4.2s$

[Ans. (B)]

Example 7. What will be the maximum reading of the weighing machine.

(A) 90 *kg* (B) 100 *kg* (C) 6 *kg* (D) None

[Ans. (A)]

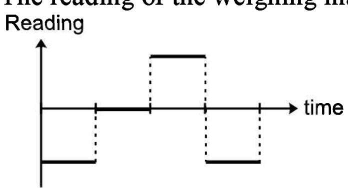
Example 8. If the total mass of (elevator + man) system is 150 *kg*, the maximum tension in the cable supporting the lift will be:

(A) 1500 *N* (B) 3000 *N* (C) 2250 *N* (D) None

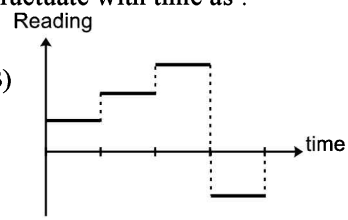
[Ans. (C)]

Example 9. The reading of the weighing machine fluctuate with time as :

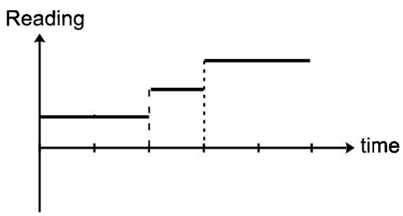
(A)



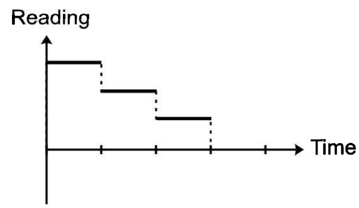
(B)



(C)



(D)



[Ans. (C)]

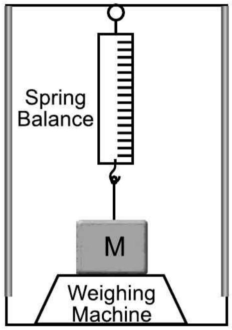
Example 10. Motion of the lift is accelerating for the time interval :

(A) $t \in (0, 1)$ (B*) $t \in (1, 2)$ (C) $t \in (2, 3)$ (D*) $t \in (3, 5)$

[Ans. (A), (B) and (C)]

Comprehension– 3

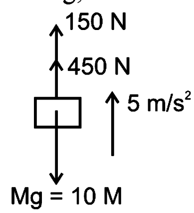
Figure shows a weighing machine kept in a lift. Lift is moving upwards with acceleration of 5 m/s^2. A block is kept on the weighing machine. Upper surface of block is attached with a spring balance. Reading shown by weighing machine and spring balance is 15 kg and 45 kg respectively. Answer the following questions. Assume that the weighing machine can measure weight by having negligible deformation due to block, while the spring balance requires larger expansion :(take $g = 10$ m/s^2)



Example 4. Mass of the object in kg is and the normal force acting on the block due to weighing machine are :

(A) 60 kg, 450 N (B) 40 kg, 150 N (C) 80 kg, 400 N (D) 10 kg, zero

Solution FBD of Block in ground frame :



applying $N.L.$ $150 + 450 - 10\text{ M} = 5M$

$\Rightarrow \quad 15M = 600 \Rightarrow M = \dfrac{600}{15}$

$\Rightarrow$ M = **40 Kg Ans.**

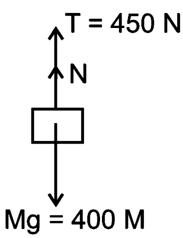
Normal on block is the reading of weighing machine i.e. 150 N.

[Ans. (B)]

Example 5. If lift is stopped and equilibrium is reached. Reading of weighing machine and spring balance will be :

(A) 40 kg, zero (B) 10 kg, 20 kg (C) 20 kg, 10 kg (D) zero, 40 kg

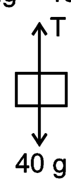
Solution If lift is stopped and equilibrium is reached then



$450 + N = 400$

$\Rightarrow \quad N = -50$

So block will lose the contact with weighing machine thus reading of weighing machine will be zero.



$T = 40$ g

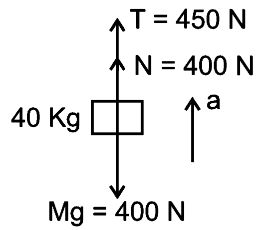
So reading of spring balance will be 40 Kg.

[Ans. (D)]

Example 6. Find the acceleration of the lift such that the weighing machine shows its true weight.

(A) $\dfrac{45}{4}$ m/s^2 (B) $\dfrac{85}{4}$ m/s^2 (C) $\dfrac{22}{4}$ m/s^2 (D) $\dfrac{60}{4}$ m/s^2

[Ans. (A)]



Solution

$$a = \frac{850 - 400}{40}$$

$$\Rightarrow \quad a = \frac{450}{40} = \frac{45}{4} \text{ m/s}^2 \textbf{ Ans.}$$

[Ans. (A)]

Comprehension– 4

Following are three equations of motion

$$S(t) = ut + \frac{1}{2}at^2 \qquad v(s) = \sqrt{u^2 + 2as} \qquad v(t) = u + at$$

Where S, u, t, a, v are respectively the displacement (dependent variable), initial velocity (constant), time taken (independent variable), acceleration (constant) and final velocity (dependent variable) of the particle after time t.

Example 1. Find displacement of a particle after 10 seconds starting from rest with an uniform acceleration of 2m/s^2.

(A) 10 *m* (B) 100 *m* (C) 50 *m* (D) 200 *m*

Solution $S = ut + at^2$

$$S = 0 + \frac{1}{2} \text{x} 2(10)^2 = 100 \, m$$

[Ans. (B)]

Example 2. Find the velocity of the particle after 100 m –

(*A*) 10 m/s (B*) 20 m/s (C) 30 m/s (D) 0 m/s

Solution

$v = u + at$

$v = 0 + 2 \text{ x } 10$

$= 20$ m/s.

[Ans. (B)]

Example 3. Find the velocity of the particle after 10 seconds if its acceleration is zero in interval (0 to 10 s) –

(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 0 m/s

Solution $v = u$

$v = o$ m/s

[Ans. (D)]

Example 4. Find the displacement of the particle when its velocity becomes 10 m/s if acceleration is 5 m/s^2 all through–

(A) 50 *m* (B) 200 *m* (C) 10 *m* (D) 100 *m*

Solution $v^2 = u^2 + 2ar$

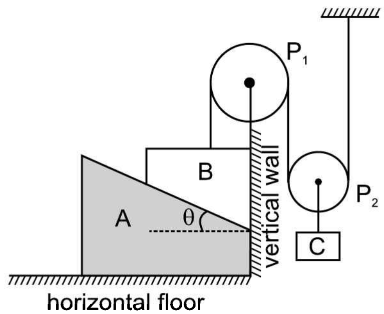
$(10)^2 = 0 + 2 \times 5 \times 5$

$5 = 10$ m

[Ans. (C)]

Comprehension– 5

In the figure shown P_1 and P_2 are massless pulleys. P_1 is fixed and P_2 can move. Masses of A, B and C are $\frac{9m}{64}$, 2m and m respectively . All contacts are smooth and the string is massless. $\theta = \tan^{-1}\left(\frac{3}{4}\right)$. (Take $g = 10$ m/s²)



Example 1. The ratio of magnitude of accelerations of blocks A and C

(A) $\frac{3}{2}$ (B) $\frac{4}{3}$ (C) 2 (D) $\frac{8}{3}$

[Ans. (D)]

Example 2. The acceleration of block C is

(A) 1 m/s² (B) 3 m/s² (C) 4 m/s² (D) 8 m/s²

[Ans. (B)]

Example 3. The tension in string connecting pulley P_2 and block C is (Take m = 1 kg)

(A) 3 N (B) 4.5 N (C) 6.5 N (D) 13 N

[Ans. (D)]

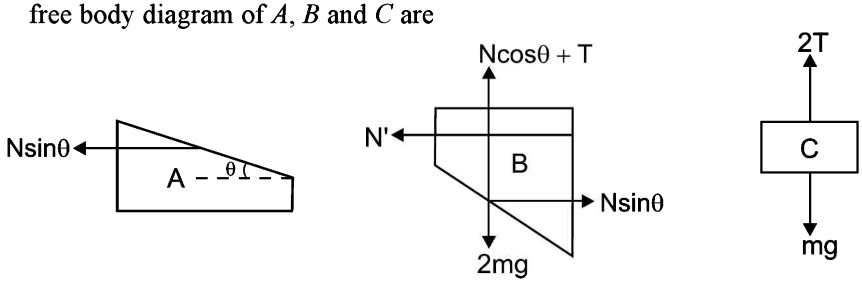
Solution Let the acceleration of B downwards be $a_B = a$

From constraint ; acceleration of A and C are

$$a_A = a\cot\theta = \frac{4a}{3} \text{ towards left}$$

$$a_C = \frac{a}{2} \text{ upwards}$$

free body diagram of A, B and C are



$$N\sin\theta = \frac{9m}{64}(a\cot\theta) \qquad ...(1)$$

$$2mg - T - N\cos\theta = 2ma \qquad ...(2)$$

$$2T - mg = m\frac{a}{2} \qquad ...(3)$$

solving we get

$$a_c = \frac{a}{2} = 3\text{m/s}^2 \text{ and } 2T = 13\ N$$

Comprehension– 6

There are four fundamental interaction force in nature, that are representative of all types of internal forces.

These are gravitational force, electro-magnetic force, strong nuclear force, and weak nuclear force. Gravitational force is the force of mutual attraction, that every massive body apply on other massive body. It's magnitude is very small and is neglected between two small massive bodies, but it is considerable between two big bodies like planets.

Electromagnetic force is the force between charged particle. It is stronger than gravitational force. All mechanical contact forces like tensile force, compressive force, friction force, Vander Walls force etc. are electromagnetic force if we see from microscopic level.

The force that bonds the protons within the nucleus in spite of strong repulsion among themselves, is nuclear force. When separation between the elements is very small, this force become for away stronger than any other force.

Example 1. Match the column

(p)	force between the earth and the falling stone	(i)	Gravitational force
(q)	The pressing force between one block and another block	(ii)	Electromagnetic force
(r)	The stretching force developed in a spring.	(iii)	Strong nuclear force
(s)	The force between the proton and the neutron in a nuclear	(iv)	Weak nuclear force

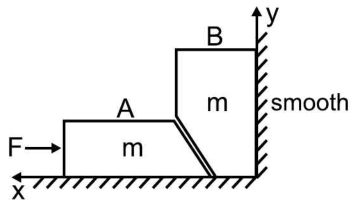
(A) (p-1), (q-ii), (r-iii), (s-iii) (B) (p-ii), (q-i), (r-iv), (s-iii)
(C) (p-iii), (q-iv), (r-i), (s-ii) (D) None of these

Example 2. For very small distance nuclear distance the order of fundamental forces is :
(A) Gravitational force > electromagnetic force > strong nuclear force
(B) Electromagnetic force > gravitational force > strong nuclear force
(C) Strong nuclear force > electromagnetic force > gravitational force
(D) None of these

Example 3. Which of the following interaction force can be repulsive :
(A) gravitational force (B) Electromagnetic force
(C) Strong nuclear force (D) None of these

Comprehension – 7

Two smooth blocks are placed at a smooth corner as shown. Both the blocks are having mass m. We apply a force F on the small block m. Block A presses the block B in the normal direction, due to which pressing force on vertical wall will increase, and pressing force on the horizontal wall decrease, as we increase F. ($\theta = 37°$ with horizontal). As soon as the pressing force on the horizontal wall by block B become zero, it will loose the contact with the ground. If the value of F is further increased, the block B will accelerate in upward direction and simultaneously the block A will move toward right.



Example 1. What will be normal reaction on block B due to the walls when $F = \frac{mg}{4}$.

(A) $\frac{4mg}{25}\hat{i} + \frac{25mg}{22}\hat{j}$ (B) $\frac{2mg}{25}\hat{i} + \frac{22mg}{25}\hat{j}$

(C) $\frac{4mg}{25}\hat{i} + \frac{22mg}{25}\hat{j}$ (D) None of these

Example 2. What is minimum value of F, to lift block B from ground :

(A) $\frac{25}{12}mg$ (B) $\frac{12}{25}mg$ (C) mg (D) None

Example 3. If the force F is greater than that minimum value, acceleration of block A is a rightwards, and normal reaction between block A and B is N then correct relation will be :

(A) $F + N\cos\theta = ma$ (B) $F - N\cos\theta = ma$

(C) $F + 2N\cos\theta = ma$ (D) None of these

Example 4. In the previous question, with how much force will block A presses the ground.

(A) $mg + N\cos\theta$ (B) $mg + N\sin\theta$ (C) $mg - N\cos\theta$ (D) $mg + N\sin\theta$

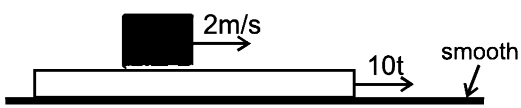
Example 5. In the previous question, the acceleration of B in upward direction (= b), is given by

(A) $N\cos\theta + mg = mb$ (B) $N\sin\theta + mg = mb$

(C) $N\cos\theta - mg = mb$ (D) None of these

Comprehension– 8

A small block of mass 1 kg starts moving with constant velocity 2 m/s on a smooth long plank of mass 10 kg which is also pulled by a horizontal force $F = 10t\,N$ where t is in seconds and F is in newtons. (the initial velocity of the plank is zero).



Example 1. Displacement of 1 kg block with respect to plank at the instant when both have same velocity is

(A) $4\frac{4}{3}m$ (B) $4\,m$ (C) $\frac{8}{3}m$ (D) $2\,m$

Solution

$$a_p = \frac{10t}{10} = t$$

$$\therefore \quad \frac{dv}{dt} = t \quad \Rightarrow \quad \int_0^v dv = \int_0^t t\,dt \quad \Rightarrow \quad v = \frac{t^2}{2}$$

Putting $v = 2$ we have $t = 2$ sec.

Now $\frac{dx}{dt} = \frac{t^2}{2} \therefore x_p = \left[\frac{t^3}{6}\right]_0^2 = \frac{4}{3}$

$x_B = 2 \times 2 = 4$ m

Hence relative displacement $= 4 - \frac{4}{3} = \frac{8}{3}\ m$

[Ans. (C)**]**

Example 2. The time ($t \neq 0$) at which displacement of block and plank with respect to ground is same will be :

(A) 12 *s* (B) $2\sqrt{3}\ s$ (C) $3\sqrt{3}\ s$ (D) $\sqrt{3}/2\ s$

Solution From above

$2t = \frac{t^3}{6} \Rightarrow t^2 = 12 \Rightarrow t = 2\sqrt{3}$ sec.

[Ans. (C)**]**

Example 3. Relative velocity of plank with respect to block when acceleration of plank is 4 m/s² will be

(A) Zero (B) 10 m/s (C) 6 m/s (D) 8 m/s

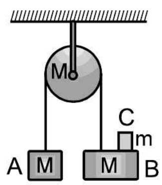
Solution $a = t = 4$ $\therefore$ after 4 seconds V_B = 2 m/s

$V_p = \frac{4^2}{2} = 8$ m/s $\therefore V_{rel} = 8 - 2 = 6$ m/s.

[Ans. (C)**]**

Comprehension– 9

For the following system shown assume that pulley is frictionless, string is massless (*m* remains on *M*) :



Example 1. The acceleration of the block *A* is :

(A) $\frac{mg}{2M+m}$ (B) $\frac{2mg}{2M+m}$ (C) $\frac{mg}{M+2m}$ (D) $\frac{Mg}{M+2m}$

[Ans. (C)**]**

Example 2. Normal reaction on *m* is (force on *C* due to *B*).

(A) $\frac{M\,mg}{2M+m}$ (B) $\frac{2M\,mg}{2M+m}$ (C) $\frac{M\,mg}{M+2m}$ (D) $\frac{2M\,mg}{M+m}$

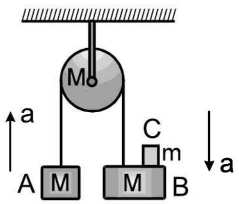
Example 3. The force on the ceiling is

(A) $\frac{(M+m)\,mg}{2M+m}$ (B) $\frac{(6M+5m)\,mg}{M+m}$

(C) $\frac{(M+m)\,mg}{M+m}$ (D) $\frac{(6M+5m)\,Mg}{2M+m}$

[Ans. (D)**]**

Solution



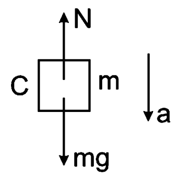
By newtons law on system of (*A*, *B*, *C*)

Solution 1. $(M + m - M)\, g = (2M + m)\, a$

$$\therefore \quad a = \frac{mg}{2M + m}$$

[Ans. (A)**]**

Solution 2. Free body diagram '*C*' block



$$mg - N = ma$$

$$\therefore \quad N = m\left(g - \frac{gm}{2M + m}\right)$$

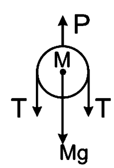
$$N = \frac{2M\,mg}{2M + m}$$

[Ans. (B)**]**

Solution 3. $T - mg = M\dfrac{mg}{2M + m}$ for *A* block

$$\therefore \quad T = Mg + \frac{Mmg}{2M + m}$$

for pulley



$$P = 2T + Mg$$

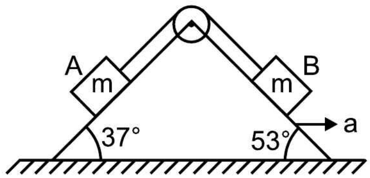
$$= 2Mg + \frac{2Mmg}{2M + m} + Mg = \frac{6M + 3m + 2m}{2M + m}Mg$$

$$P = \left(\frac{6M + 5m}{2M + m}\right)Mg$$

[Ans. (D)**]**

Comprehension – 10

Two blocks *A* and *B* of equal masses *m* kg each are connected by a light thread, which passes over a massless pulley as shown. Both the blocks lie on wedge of mass *m* kg. Assume friction to be absent everywhere and both the blocks to be always in contact with the wedge. The wedge lying over smooth horizontal surface is pulled towards right with constant acceleration *a* (m/s^2). (*g* is acceleration due to gravity).



Example 1. Normal reaction (in N) acting on block B is

(A) $\frac{m}{5}(3g+4a)$ (B) $\frac{m}{5}(3g-4a)$ (C) $\frac{m}{5}(4g+3a)$ (D) $\frac{m}{5}(4g-3a)$

[Ans. (A)]

Example 2. Normal reaction (in N) acting on block A.

(A) $\frac{m}{5}(3g+4a)$ (B) $\frac{m}{5}(3g-4a)$ (C) $\frac{m}{5}(4g+3a)$ (D) $\frac{m}{5}(4g-3a)$

[Ans. (D)]

Example 3. The maximum value of acceleration a (in m/s^2) for which normal reactions acting on the block *A* and block *B* are nonzero.

(A) $\frac{3}{4}g$ (B) $\frac{4}{3}g$ (C) $\frac{3}{5}g$ (D) $\frac{5}{3}g$

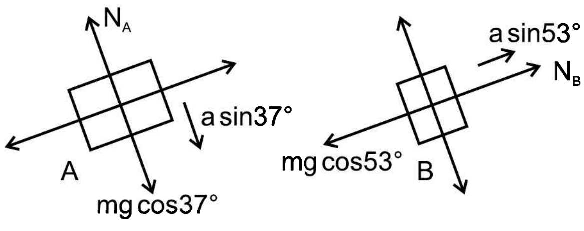
[Ans. (B)]

Solution **14 to 16. (Moderate)**

Applying Newton's second law to block *A* and *B* along normal to inclined surface

$$N_B - mg\cos 53° = ma\sin 53°$$
$$mg\cos 37° - N_A = ma\sin 37°$$

Solving $N_A = \frac{m}{5}(4g-3a)$ and $N_B = \frac{m}{5}(3g+4a)$ The *FBD* of *A* and *B* are



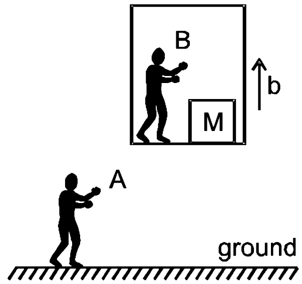
For N_A to be non zero

$$4g - 3a \geq 0$$

or $a \leq \frac{4g}{3}$

Comprehension – 11

A block of mass *M* is kept in elevator (lift) which starts moving upward with constant acceleration '*b*' as shown in figure. Initially elevator at rest. The block is observed by two observers *A* and *B* for a time interval $t = 0$ to $t = T$. Observer *B* is at rest with respect to elevator and observer *A* is standing on the ground.



Example 1. The observer A finds that the work done by gravity on the block is -

(A) $\frac{1}{2}Mg^2T^2$ (B) $-\frac{1}{2}Mg^2T^2$ (C) $\frac{1}{2}MgbT^2$ (D) $-\frac{1}{2}MgbT^2$

Solution Displacement of block in the time interval T is $\frac{1}{2}bT^2$

$$W_{\text{gravity}} = (-mg)\left(+\frac{1}{2}bT^2\right) = -\frac{1}{2}mgbT^2$$

[Ans. (D)**]**

Example 2. The observer A finds that work done by normal reaction acting on the block is -

(A) $\frac{1}{2}M(g+b)^2T^2$ (B) $-\frac{1}{2}M(g+b)^2T^2$

(C) $\frac{1}{2}M(g+b)\,bT^2$ (D) $-\frac{1}{2}M(g+b)\,bT^2$

Solution The normal reaction acting on the block is $M(g+b)$ in upward direction. The upward displacement of the block is $\frac{1}{2}bT^2$. Hence the work done by normal reaction is $M(g+b) \times \frac{1}{2}bT^2$

[Ans. (C)**]**

Example 3. According to observer B

(A) The work done by gravity on the block is zero

(B) The work done by normal reaction on the block is zero

(C) The work done by pseudo force on the block is zero

(D) All the above are correct

Solution According to observer B, the displacement of the block is zero, therefore

Work done by gravity is zero

Work done by normal reaction is zero

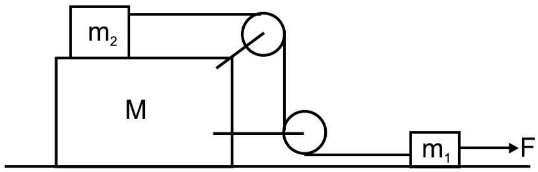
Work done by pseudo force is zero.

[Ans. (D)**]**

Match the following

Example 1. Match the following :

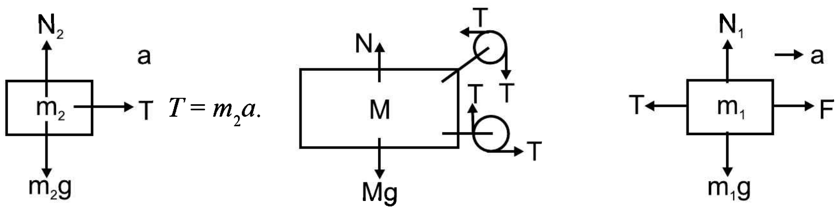
Three blocks of masses m_1, m_2 and M are arranged as shown in figure. All the surfaces are frictionless and string is inextensible. Pulleys are light. A constant force F is applied on block of mass m_1. Pulleys and string are light. Part of the string connecting both pulleys is vertical and part of the strings connecting pulleys with masses m_1 and m_2 are horizontal.



(A)	Acceleration of mass m_1	(P)	$\frac{F}{m_1}$
(B)	Acceleration of mass m_2	(Q)	$\frac{F}{m_1+m_2}$
(C)	Acceleration of mass M	(R)	zero
(D)	Tension in the string	(S)	$\frac{m_2F}{m_1+m_2}$

Solution (A) Q (B) Q (C) R (D) S

FBD's



$$F - T = m_1a$$

$$F = (m_1 + m_2)a$$

$\because$ $T = m_2a$

$\Rightarrow$ $a = \frac{F}{m_1+m_2}$ $\quad\because T = \frac{m_2F}{m_1+m_2}$. $\quad F_x = 0,\ a_M = 0$

Example 2. Column- I gives four different situation. In final statement of each situation two vector quantities are compared. The result of comparison is given in column-II. Match the statement in column-I with the correct comparison(s) in column-II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

	Column I		Column II
(A)	Stone is projected from ground at an angle θ with horizontal ($\theta \neq 90°$). Neglect the effect of air friction. Then between the two instants when it is at same height (above ground), its average velocity and horizontal component of velocity are	(p)	same in magnitude
(B)	For four particles A, B, C & D, the velocities of one with respect to other are given as $\vec{V}_{DC}$ is 20 m/s towards north, $\vec{V}_{BC}$ is 20 m/s towards east and $\vec{V}_{BA}$ is 20 m/s towards south. Then $\vec{V}_{BC}$ and $\vec{V}_{AD}$ are	(q)	different in magnitude

(C) Two blocks of masses 4 and 8 kg are placed on ground as shown

. Then the net force exerted by earth on block of mass 8 kg and normal reaction exerted by 8 kg block on earth are (note that earth includes ground) — (r) same in direction

(D) For a particle undergoing rectilinear motion with uniform acceleration, the magnitude of displacement is half the distance covered in some time interval. The magnitude of final velocity is less than magnitude of initial velocity for this time interval. Then the initial velocity and average velocity for this time interval are — (s) opposite in direction

[Ans. (A) *p, r* (B) *p, r* (C) *q, s* (D) *q, r*]

Solution (A) Let the horizontal component of velocity be u_x. Then between the two instants (time interval T) the projectile is at same height, the net displacement (u_xT) is horizontal

$$\therefore \text{ average velocity} = \frac{u_xT}{T} = u_x \Rightarrow (A)\ p, r$$

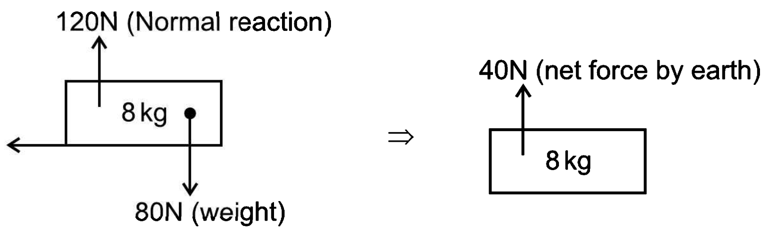
(*B*) Let $\hat{i}$ *and* $\hat{j}$ be unit vectors in direction of east and north respectively.

$\therefore \quad \vec{V}_{DC} = 20\hat{j}$, $\vec{V}_{BC} = 20\hat{i}$ and $\vec{V}_{BA} = -20\hat{j}$

$\therefore \quad -\vec{V}_{AD} = \vec{V}_{DC} + \vec{V}_{CB} + \vec{V}_{BA} = 20\hat{j} - 20\hat{i} - 20\hat{j} = -20\hat{i}$

$\therefore \quad \vec{V}_{AD} = 20\hat{i}$ Hence $\vec{V}_{AD} = \vec{V}_{BC}$ $\Rightarrow (B)\quad p, r$

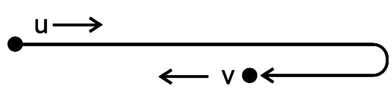
(C) Net force exerted by earth on block of mass 8 kg is shown in *FBD* and normal reaction exerted by 8 kg block on earth is 120 N downwards.



Hence both forces in the statement are different in magnitude and opposite in direction.

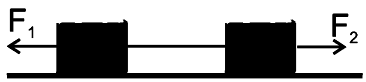
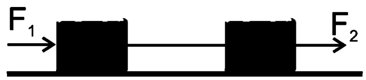
$\Rightarrow$ (C) *q, s*

(*D*) For magnitude of displacement to be less than distance, the particle should turn back. Since the magnitude of final velocity (v) is less than magnitude of initial velocity (u), the nature of motion is as shown.



$\therefore$ Average velocity is in direction of initial velocity and magnitude of average velocity $= \dfrac{u-v}{2}$ is less than u because $v < u$. $\Rightarrow$ (C) q, r

Example 3. Column-I gives four different situations involving two blocks of mass m_1 and m_2 placed in different ways on a smooth horizontal surface as shown. In each of the situations horizontal forces F_1 and F_2 are applied on blocks of mass m_1 and m_2 respectively and also $m_2 F_1 < m_1 F_2$. Match the statements in column I with corresponding results in column-II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

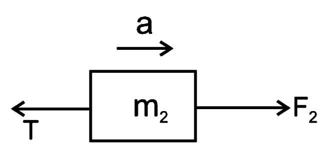
Column I	Column II
(A)  Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is	(p) $\dfrac{m_1 m_2}{m_1+m_2}\left(\dfrac{F_1}{m_1}-\dfrac{F_2}{m_2}\right)$
(B)  Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is	(q) $\dfrac{m_1 m_2}{m_1+m_2}\left(\dfrac{F_1}{m_1}+\dfrac{F_2}{m_2}\right)$
(C)  The magnitude of normal reaction between the blocks is	(r) $\dfrac{m_1 m_2}{m_1+m_2}\left(\dfrac{F_2}{m_2}-\dfrac{F_1}{m_1}\right)$
(D)  The magnitude of normal reaction between the blocks is	(s) $m_1 m_2\left(\dfrac{F_1+F_2}{m_1+m_2}\right)$

[**Ans.** (A) *q* (B) *r* (C) *q* (D) *r*]

Solution Let a be acceleration of two block system towards right

$$\therefore \quad a = \frac{F_2 - F_1}{m_1 + m_2}$$

The *F.B.D.* of m_2 is



$$\therefore \quad F_2 - T = m_2 a$$

Solving $T = \dfrac{m_1 m_2}{m_1+m_2}\left(\dfrac{F_2}{m_2}+\dfrac{F_1}{m_1}\right)$

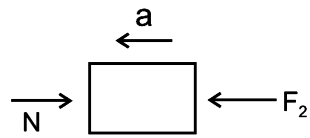
(B) Replace F_1 by $-F_1$ is result of *A*

$$\therefore \quad T = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right)$$

(C) Let a be acceleration of two block system towards left

$$\therefore \quad a = \frac{F_2 - F_1}{m_1 + m_2}$$

The *FBD* of m_2 is



$$\therefore \quad F_2 - N_2 = m_2 a$$

Solving, $N = \dfrac{m_1 m_2}{m_1 + m_2}\left(\dfrac{F_1}{m_1} + \dfrac{F_2}{m_2}\right)$

(D) Replace F_1 by $-F_1$ in result of C

$$N = \frac{m_1 m_2}{m_1 + m_2}\left(\frac{F_2}{m_2} - \frac{F_1}{m_1}\right)$$

True False

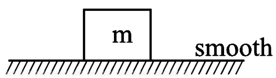
Example 1. The earth is a noninertial frame by definition of noninertial frame.

Solution (True)

EXERCISE

Exercise-1

Draw free body diagram of block of mass m.

1.


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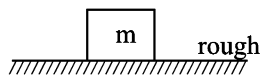

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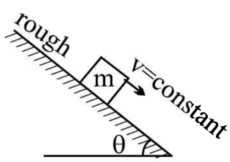

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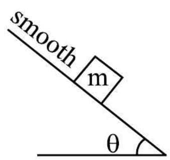

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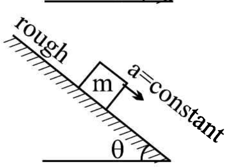

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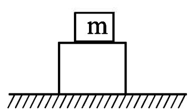

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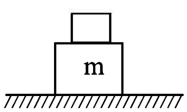

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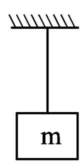

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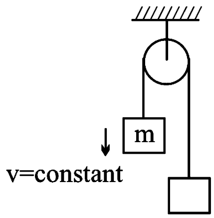

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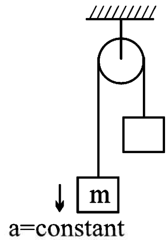

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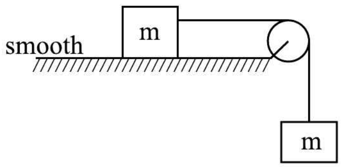

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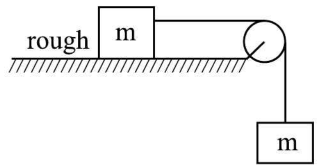

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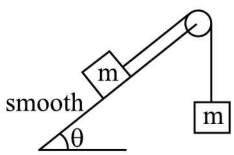

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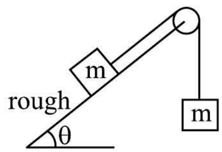

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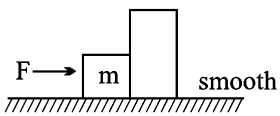

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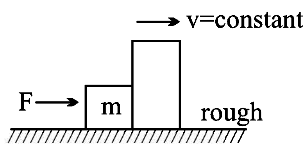

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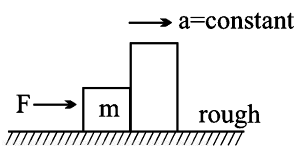

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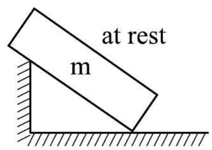

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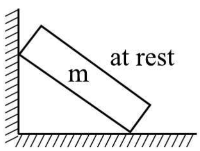

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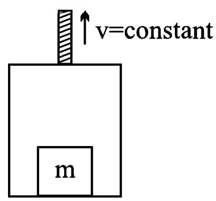
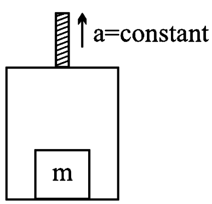


21.



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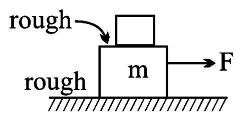

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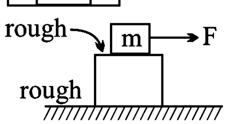

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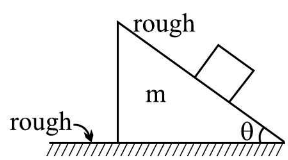

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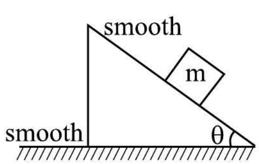

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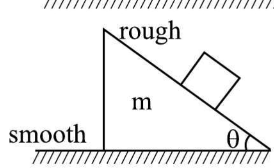

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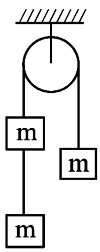

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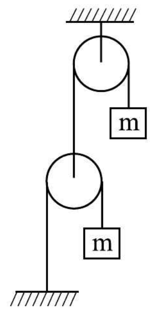

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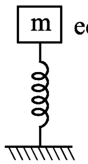
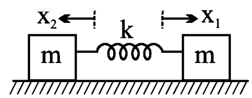


equilibrium

30.



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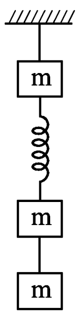

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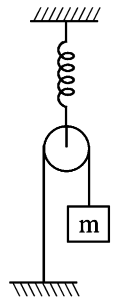

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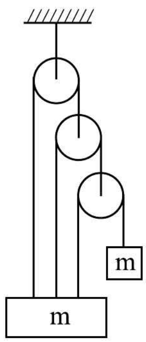

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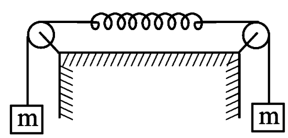

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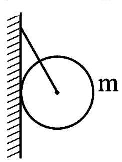
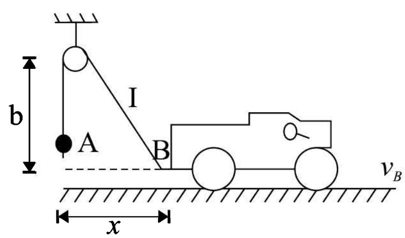


Question on the Constrained Motion

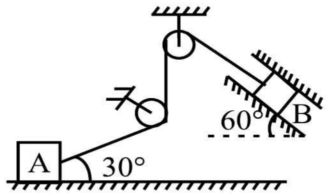
36. In the figure shown determine the velocity v of a bob A as a function of x if the velocity of a jeep v_B is constant. When $x = 0$, ends A and B are coincident at C.



(A) $\dfrac{xv_B}{\sqrt{(h^2+x^2)}}$ (B) $\dfrac{xv_B}{(h^2+x^2)}$

(C) $\dfrac{(h^2+x^2)}{xv_B}$ (D) None

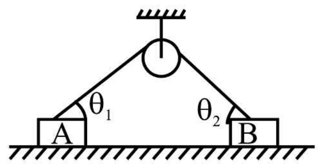
37. In the figure shown A remains in contact with the ground. If velocity of A is 10 m/s, then velocity of B in the figure shown is-



(A) $5\sqrt{3}$ (B) $\dfrac{20}{\sqrt{3}}$

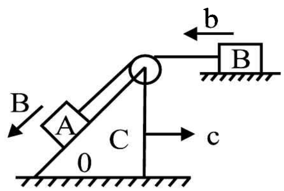
(C) 3 (D) None

38. In the figure shown the block B is moved with constant speed v towards right. Then



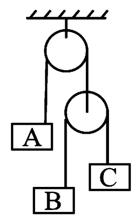
(A) A will move with increasing speed
(B) A will move with decreasing speed
(C) In the position shown the speed of A is $v_1 \cos\theta_2 \sec\theta_1$
(D) In the position shown the speed of A is $v_1 \sec\theta_2 \cos\theta_1$,

39. In the figure shown the relation between the accelerations of A, B & C is (here a = acceleration of A relative to C)



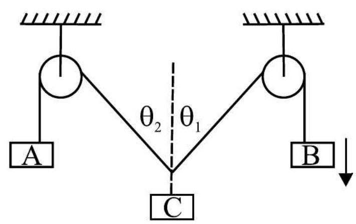
(A) $a+b+c=0$ (B) $a=b+c$
(C) $a-b=c$ (D) None of these

40. In the figure shown A has velocity 10 m/s downward and B has velocity 20 m/s upward. Velocity of C is



(A) 15 (B) 5
(C) 0 (D) None

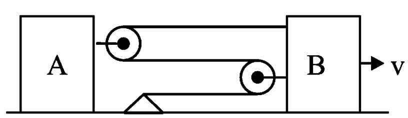
41. In the figure shown ABC can move only vertically. In the figure velocity of B is a u downdard. At that instant the velocity of C is -



(A) $u\cos\theta_1$ (B) $u\sec\theta_1$
(C) $u\sin\theta_1$ (D) none of these

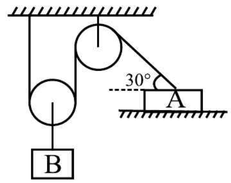
42. In the previous question the velocity of A is -
(A) $u\sec\theta_1\cos\theta_2$ (B) $u\cos\theta_1\cos\theta_2$
(C) $u\sec\theta_1\sec\theta_2$ (D) None of these

43. The block B moves to the right with a velocity v. Velocity of A will be [c is fixed]



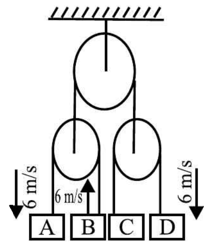
(A) v (B) $(2/3)\,v$
(C) $(3/2)\,v$ (D) $2v$

44. The figure shown the block B moves down with a velocity 10 m/s. The velocity of A in the position shown is -



(A) 12.5 m/s (B) 25 m/s
(C) 6.25 m/s (D) None of these

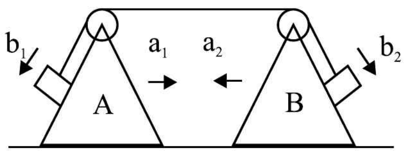
45. In the figure shown the velocity of different blocks is shown. The velocity of C is -



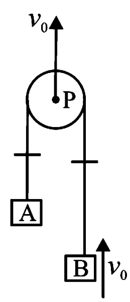
(A) 6 m/s (B) 4 m/s

(C) 0 m/s (D) None

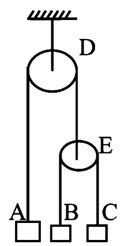
46. Let a_1 and a_2 are the accelerations of A & B. Let b_1 & b_2 the accelerations of C and D relative to the wedges A and B respectively, choose the right relation. (directions of a_1, a_2, b_1 and b_2 are shown in figure)



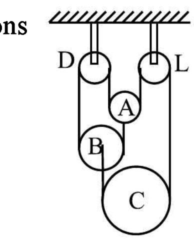
47. Figure shows a pulley over which a string passes and connected to two masses A and B. Pulley moves up with a velocity v_P and mass B is also going up at a velocity v_B. Find the velocity of mass A if -
(A) $v_P = 5$m/s and $v_B = 10$ m/s
(B) $v_P = 5$m/s and $v_B = -20$ m/s



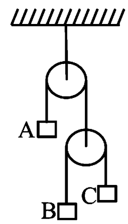
48. In the arrangement shown in figure when the system is released from rest, downward accelerations of blocks B and C relative to A are found to be 5 ms^{-2} and 3 ms^{-2} respectively. Calculate accelerations of blocks B and C, relative to the ground.



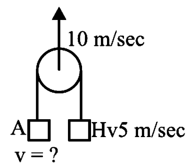
49. A pulley system is set up as in figure. Find the relation between accelerations of pulleys A, B and C.



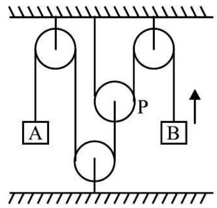
50. The three blocks shows move with constant velocities. Find the velocity of each block, knowing that velocity of A with respect to C is 300 mm/sec upward and that velocity of B with respect to A is 200 mm/sec downward.



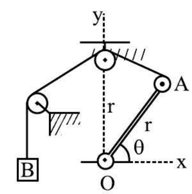
51. The pulley moves up with a velocity of 10 m/sec. Two blocks are tied by a string which passes over a pulley. The velocity V will be _________.
Given: $v_B = 5$ m/s↓.



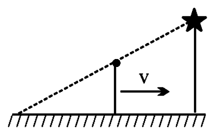
52. Find the acceleration of movable pulley P and block B if acceleration of block $A = 1$ m/s^2↓:
(A) $a_P = 1$m/s^2 ↓, $a_B = 1/2$m/s^2 ↓,
(B) $a_P = 1$m/s^2 ↓, $a_B = 2$m/s^2 ↑,
(C) $a_P = 1$m/s^2 ↑, $a_B = 2$m/s^2 ↓,
(D) $a_P = 1$m/s^2 ↓, $a_B = 2$m/s^2 ↑,



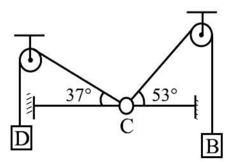
53. The particle A is mounted on a light rod pivoted at O and therefore is constrained to move in a circular arc of radius r. Determine the velocity of A in terms of the downward velocity v_B of the counterweight for any angle θ.



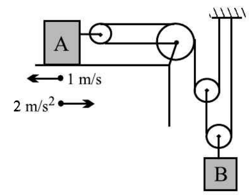
54. A man of height h is walking towards a lamppost of height H in a straight line with a constant speed v. With what speed is his head's shadow moving?



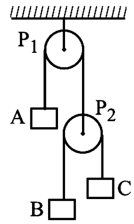
55. A bead C can move freely on a horizontal rod. The bead is connected by blocks B and D by a string as shown in the figure. If the velocity of B is v. Find the velocity of block D.



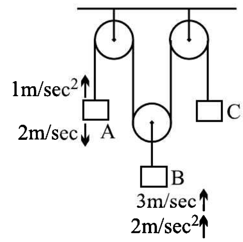
56. In the given figure find the velocity and acceleration of B, if velocity and acceleration of A are as shown.



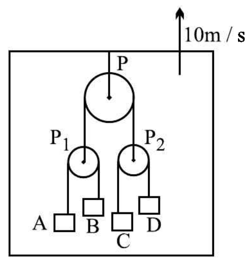
57. The three blocks shown move with constant velocities. Find the velocity of block *A* and *B*. Given V_{P2}=10m/s↓, V_c=2m/s↑.



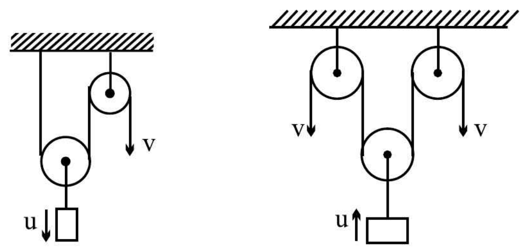
58. For the system shown, calculate velocity and acceleration of *C*.The velocity and acclerations of *A* and *B* with respect to ground are marked.



59. A lift goes up with velocity 10 m/s. *A* pulley *P* is fixed to the ceiling of the lift. To this pulley other two pulleys P_1 and P_2 are attached. P_1 moves up with velocity 30m/s. A moves up with velocity 10m/s. *D* is moving downwards with velocity 10m/s at same instant of time. The velocity of *B* is _______ and that of *C* is ________ at that instant. Assume that all velocities are relative to the ground.



60. In the figure shown, the strings are inextensible. Determine the value of *u* in terms of *v*.

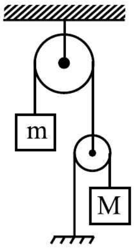


(i) u = _______________. (ii) u = _____________

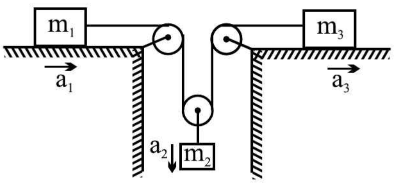
61. If the strings is inextensible, determine the velocity u of each block in terms of *v* and θ.

(i) Fig. (A) *u* = _________. (ii) Fig. (B) *u* = _________.

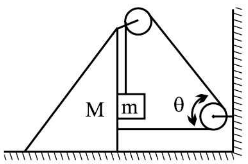
62 In the pulley system shown, find the relationship between acceleration of m and that of M.



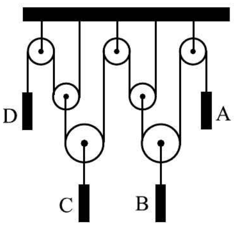
63 In the pulley system shown here, find the relationship between acceleration of m_1, m_2 and m_3.



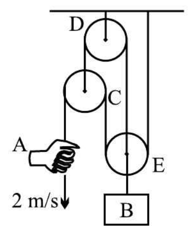
64 M and m are connected as shown in figure. If v and u denote the horizontal velocity of M and vertical velocity component of m respectively then find the ratio of u/v.



65 Determine the relationship that governs the velocity of four cylinder velocities as positive down.

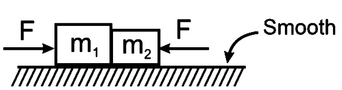


66 Determine the speed with which block B rises in figure if the end of the cord at A is pulled down with a speed of 2 m/s.



Exercise–2: Subjective Problem

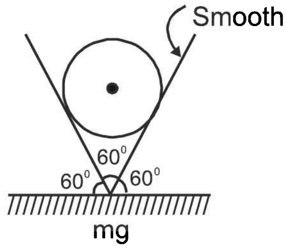
1. Two blocks of masses m_1 and m_2 are placed on ground as shown in figure. Two forces of magnitude F act on m_1 and m_2 in opposite directions.



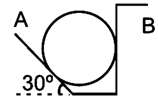
   (a) Draw F.B.D. of masses m_1 and m_2.
   (b) Calculate the contact force between m_1 and m_2.

(c) What will be the value of action-reaction pair between m_1 and m_2.
(d) Calculate force exerted by surface on mass m_1 and m_2

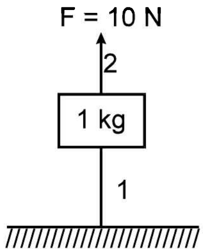
2. A cylinder of weight w is resting on a V-groove as shown in figure.
(a) Draw its free body diagram.
(b) Calculate normal reactions between the cylinder and two inclined walls.



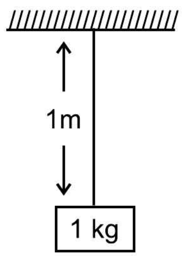
3. The 50 kg homogeneous smooth sphere rests on the 30° incline A and bears against the smooth vertical wall B. Calculate the contact forces at A and B.



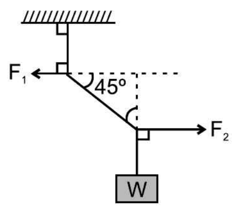
4. A string is connected between surface and a block of mass 1 kg which is pulled by another string by applying force $F = 10\ N$ as shown in figure. ($g = 10\ m/s^2$)
(a) Calculate tension is string (1).
(b) Calculate tension in string (2).



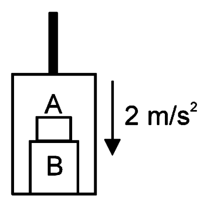
5. A block of mass 1 kg is suspended by a string of mass 1 kg, length 1m as shown in figure. ($g = 10\ m/s^2$) Calculate:
(a) the tension in string at its lowest point.
(b) the tension in string at its mid–point.
(c) force exerted by support on string.



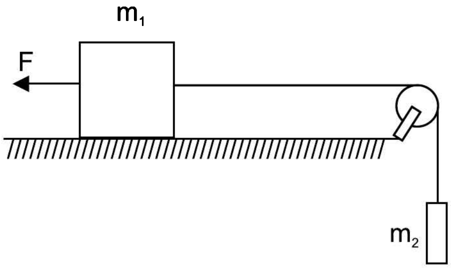
6. In the figure the tension in the diagonal string is 60 N.
(a) Find the magnitude of the horizontal force $\vec{F_1}$ and $\vec{F_2}$ that must be applied to hold the system in the position shown.
(b) What is the weight of the suspended block ?



7. The elevator shown in figure is descending with an acceleration of 2 m/s^2. The mass of the block A is 0.5 kg. What force is exerted by the block A on the block B ? Solvethe problem taking (a) ground as the frame (b) lift as the frame.



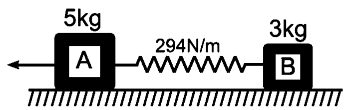
8. A constant force $F = m_2g/2$ is applied on the block of mass m_1 as shown in figure. The string and the pulley are light and the surface of the table is smooth. Find the acceleration of m_1.



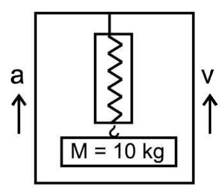
9. A chain consisting of five links each with mass 100gm is lifted vertically with constant acceleration of 2m/s². as shown. Find
   (a) the forces acting between adjacent links
   (b) the force F exerted on the top link by the agent lifting the chain
   (c) the net force on each link.

10. A block of mass 1 kg connected with a spring of force constant 100 N/m is suspended to the ceiling of lift moving upward with constant velocity 2 m/s. Calculate the extension produced in spring.

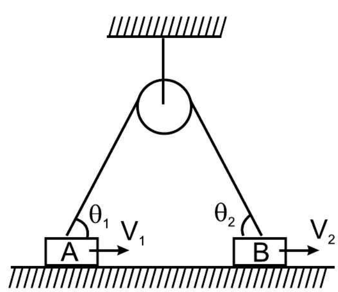
11. Two blocks A (5 kg) and B (3 kg) resting on a smooth horizontal plane are connected by a spring of stiffness 294 N/m. A horizontal force of $F = 3 \times 9.8\ N$ acts on A as shown. At the instant B has an acceleration of 4.9 m/s². Find the acceleration of block A?



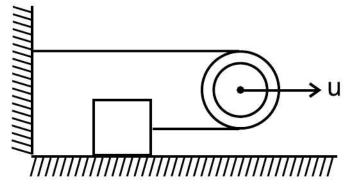
12. What will be the reading of spring balance in the figure shown in following situations. ($g = 10$ m/s²)
   (i) $a = 0$, $v = 0$
   (ii) $a = 0$, $v = 2$ m/s
   (iii) $a = 0$, $v = -2$m/s
   (iv) $a = 2$m/s², $v = 0$
   (v) $a = -2$ m/s² $v = 0$
   (vi) $a = 2$ m/s², $v = 2$ m/s
   (vii) $a = 2$ m/s², $v = -2$ m/s
   (vii) $a = -2$ m/s² $v = -2$ m/s



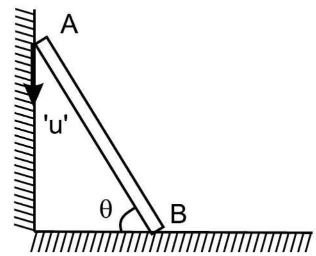
13. In the figure shown, blocks A and B move with velocities v_1 and v_2 along horizontal direction. Find the ratio of $\frac{v_1}{v_2}$.



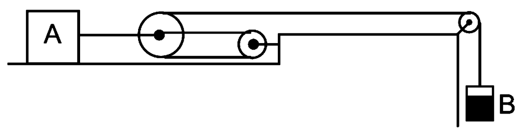
14. In the figure shown, the pulley is moving with velocity u. Calculate the velocity of the block attached with string.



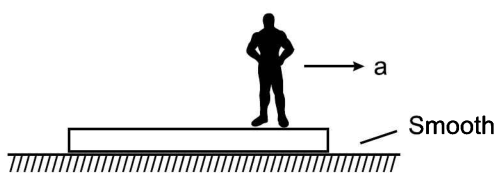
15. The velocity of end 'A' of rigid rod placed between two smooth vertical walls moves with velocity 'u' along vertical direction. Find out the velocity of end 'B' of that rod, rod always remains in contact with the vertical walls.



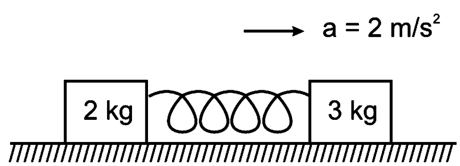
16. If block A has a velocity of 0.6 m/s of the right, determine the velocity of cylinder B



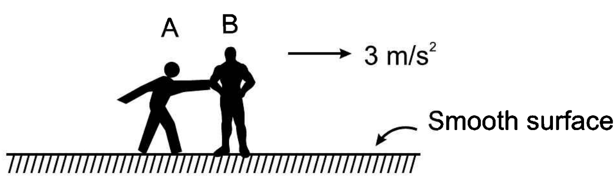
17. A man of mass m standing on a platform of mass '$2m$' jumps horizontally with an acceleration 'a'. Find the acceleration of platform.



18. Two blocks of masses 2 kg and 3 kg connected with a spring are moving on a smooth horizontal surface. Acceleration of mass 3kg is $2 m/s^2$ along right direction. What will be the acceleration of mass 2 kg ?

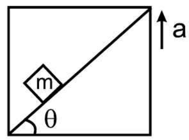


19. Man 'A' of mass 60 kg pushes the other man 'B' of mass 75 kg due to which man 'B' starts moving with acceleration $3 m/s^2$. Calculate the acceleration of man 'A' at that instant.

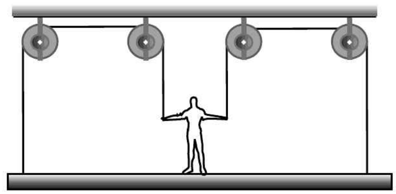


20. An object of mass 2 kg moving with velocity $10\hat{i}$ m/s is seen in a frame moving with velocity $10\hat{i}$ m/s. What will be the value of 'pseudo force' acting on object in this frame.

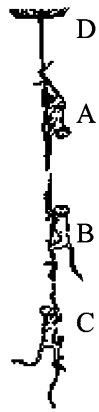
21 In the adjoining figure, a wedge is fixed to an elevator moving upwards with an acceleration 'a'. A block of mass 'm' is placed over the wedge. Find the acceleration of the block with respect to wedge. Neglect friction.



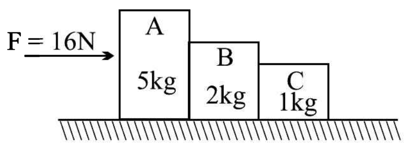
22 A painter of mass M stand on a platform of mass m and pulls himself up by two ropes which hang over pulley as shown. He pulls each rope with the force F and moves upward with uniform acceleration 'a'. Find 'a' (neglecting the fact that no one could do this for long time).



23 Three monkeys A, B and C with masses of 10 , 15 & 8 Kg respectively are climbing up & down the rope suspended from D. At the instant represented , A is descending the rope with an acceleration of 2 m/s² & C is pulling himself up with an acceleration of 1.5 m/s² . Monkeys B is climbing up with a constant speed of 0.8 m/s . Treat the rope and monkeys as a complete system and calculate the tension T in the rope at D. (g = 10 m/s^{-2})



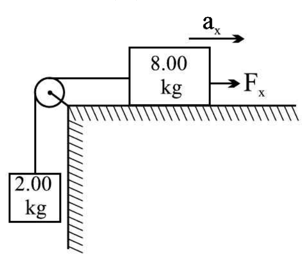
24. Figure shows three blocks in contact and kept on a smooth horizontal surface. What is ratio of force exerted by block A on B to that of B on C.



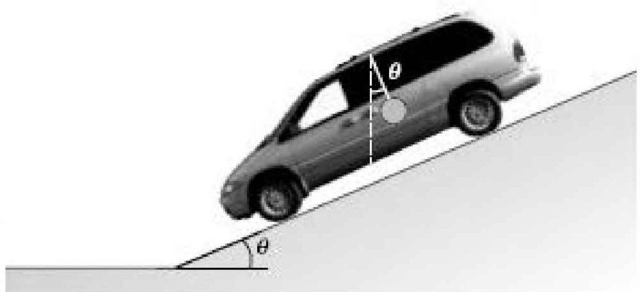
25. A force F applied to an object of mass m_1 produces an acceleration of 3.00 m/s². The same force applied to a second object of mass m_2 produces an acceleration of 1.00 m/s². (a) What is the value of the ratio m_1 / m_2 ? (B) If m_1 and m_2 are combined, find their acceleration under the action of the force F.

26. Two forces, $\boldsymbol{F}_1 = (-6\mathbf{i} - 4\mathbf{j})\ N$ and $\boldsymbol{F}_2 = (-3\mathbf{i} + 7\mathbf{j})\ N$, act on a particle of mass 2.00 kg that is initially at rest at coordinates (– 2.00 m, + 4.00 m). (a) What are the components of the particle's velocity at t = 10.0 s? (B) In what direction is the particle moving at t = 10.0s? (C) What displacement does the particle undergo during the first 10.0s? (D) what are the coordinates of the particle at t = 10.0 s?

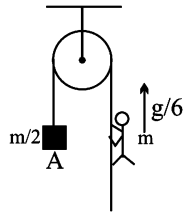
27. In the system shown in figure, a horizontal force of magnitude F_x acts on the 8.00 kg object. The horizontal surface is frictionless. (a) For what values of F_x does the 2.00 kg object accelerate upward? (B) For what values of F_x is the tension in the cord zero? (C) Plot the acceleration of the 8.00 kg object versus F_x. Include values of F_x from – 100 N to + 100 N.



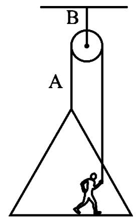
28. A van accelerates down a hill (Fig.), going from rest to 30.0 m/s in 6.00 s. During the acceleration, a toy (m = 0.100 kg) hangs by a string from the van's ceiling. The acceleration is such that the string remains perpendicular to the ceiling. Determine (a) the angle θ and (B) the tension in the string.



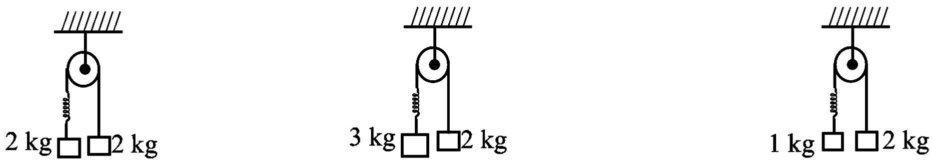
29. Block A of mass m/2 is connected to one end of light rope which passes over a pulley as shown in the Fig. Man of mass m climbs the other end of rope with a relative acceleration of g/6 with respect to rope. Find acceleration of block A and tension in the rope.



30. To point the side of a building, painter normally hoists himself up by pulling on the rope A as in figure. The painter and platform together weigh 200N. The rope B can withstand 300N. Find
(a) the maximum acceleration of the painter.
(b) tension in rope A
(i) when painter is at rest
(ii) when painter moves up with an acceleration 2 m/s^2.

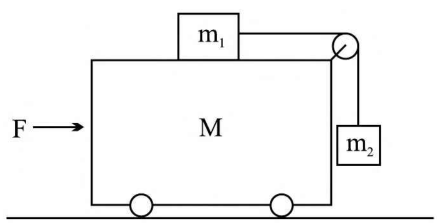


31 Same spring is attached with 2 kg, 3 kg and 1 kg blocks in three different cases as shown in figure. If x_1, x_2 and x_3 be the constan extensions in the spring in these three cases then find the ratio of their extensions.

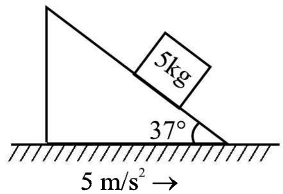


(a) (b) (c)

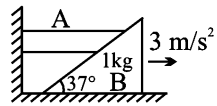
32 What horizontal force must be applied to the cart shown in figure in order that the blocks remain stationary relative to the cart? Assume all surfaces, wheels, and pulley are frictionless.



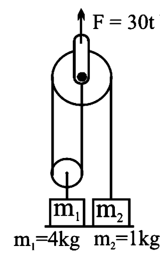
33 Inclined plane is moved towards right with an acceleration of 5 ms^{-2} as shown in figure. Find force in newton which block of mass 5 kg exerts on the incline plane. (All surfaces are smooth)



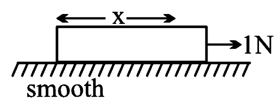
34. Find force in newton which mass A exerts on mass B if B is moving towards right with 3 ms^{-2}. Also find mass of A. (All surfaces are smooth)



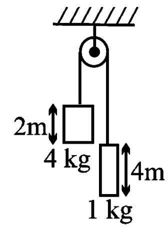
35. Force F is applied on upper pulley. If $F = 30t$ where t is time in second. Find the time when m_1 loses contact with floor.



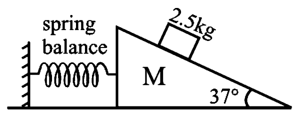
36. A rope of length L has its mass per unit length λ varies according to the function $\lambda(x) = e^{x/L}$. The rope is pulled by a constant force of $1N$ on a smooth horizontal surface. Find the tension in the rope at $x = L/2$.



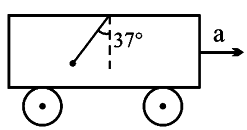
37. In figure shown, both blocks are released from rest. Find the time to cross each other?



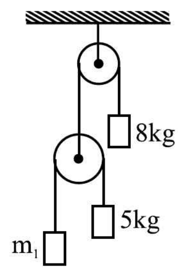
38. Find the reading of spring balance as shown in figure. Assume that mass M is in equilibrium.



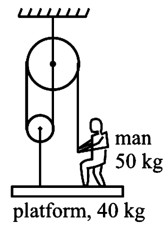
39. At what acceleration of the trolley will the string makes an angle of 37° with vertical if a small mass is attached to bottom of string.



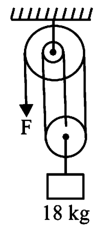
40. At what value of m_1 will 8 kg mass be at rest.



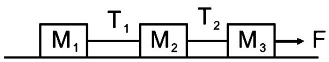
41. What force must man exert on rope to keep platform in equilibrium?



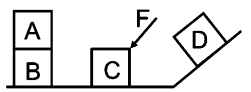
42. In the figure at the free end of the light string, a force F is applied to keep the suspended mass of 18 kg at rest. Assuming pulley is light then the force exerted by the ceiling on the system.



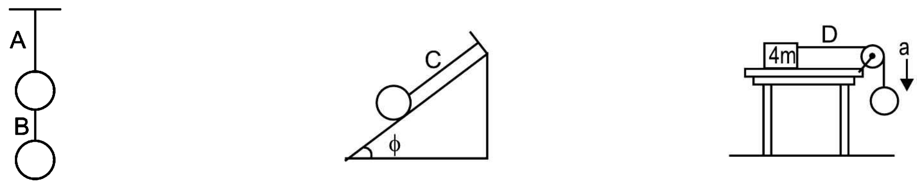
43. Three blocks are connected as shown in the figure, on a horizontal frictionless table and pulled to the right with a force at 60 N. If $M_1 = 10$ kg, $M_2 = 20$ kg and $M_3 = 30$ kg then the value of T_2.



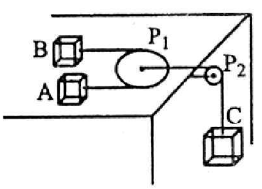
44. Four identical 2 kg blocks are arranged as shown. All surfaces are rough. A force of 10 N is applied to block C parallel to incline as shown. Rank the NORMAL on the BOTTOM surface of each block ?



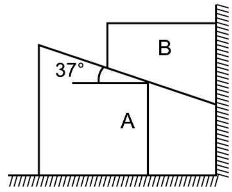
45. Four identical balls of mass m are arranged as shown. The mass of the block in the last diagram is 4 m and having acceleration a, the angle of the incline is ϕ is = 30° and all the surfaces are frictionless. Rank the TENSIONS in the labeled ropes ? Consider wedge and table to be fixed.



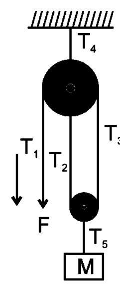
46. Two masses A and B, lie on a frictionless table. They are attached to either end of a light rope which passes around a horizontal movable pulley of negligible mass. Find the acceleration of each mass M_A = kg, M_B = 2kg, M_C = 4kg. The pulley P_2 is vertical.



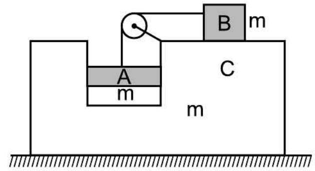
47. The masses of blocks A and B are same and equal to m. Friction is absent everywhere. Find the magnitude of normal force with which block B presses on the wall and accelerations of the blocks A and B.



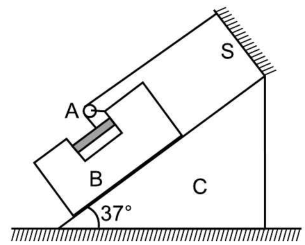
48. A mass M is held in place by an applied force F and a pulley system as shown in figure. The pulleys are massless and frictionless.
   (a) Draw a free body diagram for each pulley
   (b) Find the tension in each section of rope T_1, T_2, T_3, T_4 and T_5.
   (c) Find the magnitude of F.



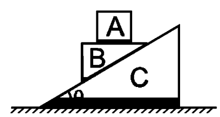
49. A block 'C' of mass m rests on a smooth table. Two blocks A and B each of mass m, are attached to the end of a light inextensible string passing over a smooth pulley fixed to C as shown in the figure. B rests on C and A can move in a frictionless vertical shaft. Find the acceleration of C.



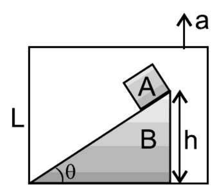
50. In the figure shown C is a fixed wedge. A block B is kept on the inclined surface of the wedge C. Another block A is inserted in a slot in the block B as shown in figure. A light inextensible string passes over a light pulley which is fixed to the block B through a light rod. One end of the string is fixed and other end of the string is fixed to A.S is a fixed support on the wedge. All the surfaces are smooth. Masses of A and B are same. Find the magnitude of acceleration of A and B. (sin 37° = 3/5)



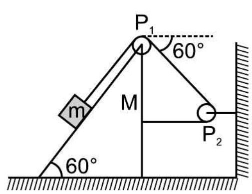
51. In the figure shown all blocks are of equal mass 'm'. All surfaces are smooth. Find the acceleration of all the blocks.



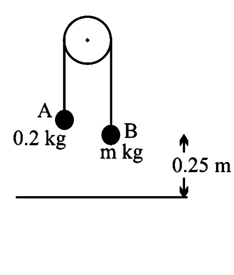
52. A lift L is moving upwards with a constant acceleration $a = g$. A small block A of mass 'm' is kept on a wedge B of the same mass 'm'. The height of the vertical face of the wedge is 'h'. A is released from the top most point of the wedge. Find the time taken by A to reach the bottom of B. All surfaces are smooth and B is also free to move.



53. In the arrangement shown in the Fig., the block of mass m = 2 kg lies on the wedge of mass M = 8 kg. Find the initial acceleration of the wedge if the surfaces are smooth

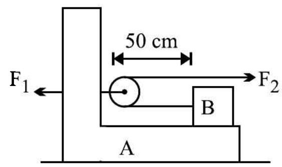


54. The diagram shows particles A and B, of masses 0.2 kg and m kg respectively, connected by a light inextensible string which passes over a fixed smooth peg. The system is released from rest, with B at a height of 0.25 m above the floor. B descends, hitting the floor 0.5 s later. All resistances to motion may be ignored.

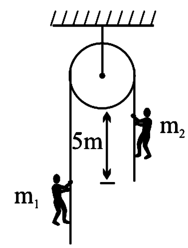


(a) Find the acceleration of B as it descends.
(b) Find the tension in the string while B is descending and find also the value of m.
(c) When B hits the floor it comes to rest immediately, and the string becomes slack. Find the length of time for which B remains at rest on the ground before being jerked into motion again.

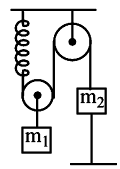
55. A 1kg block 'B' rests as shown on a bracket 'A' of same mass. Constant forces $F_1 = 20N$ and $F_2 = 8N$ start to act at time $t = 0$ when the distance of block B from pulley is 50 cm. Time when block B reaches the pulley is ______.



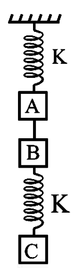
56. Two men of masses m_1 and m_2 hold on the opposite ends of a rope passing over a frictionless pulley. The mass m_1 climbs up the rope with an acceleration of 1.2 m/s² relative to the rope. The man m_2 climbs up the rope with an acceleration of 2.0 m/s² relative to the rope. Find the tension in the rope if m_1 = 40 kg and m_2 = 60 kg. Also find the time after which they will be at same horizontal level if they start from rest and are initially separated by 5m.



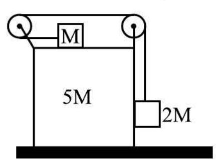
57. In figure shown, pulleys are ideal $m_1 > 2\ m_2$. Initially the system is in equilibrium and string connecting m_2 to rigid support below is cut. Find the initial acceleration of m_2?



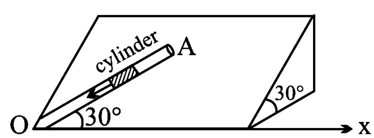
58. The system shown adjacent is in equilibrium. Find the acceleration of the blocks A, B & C all of equal masses m at the instant when (Assume springs to be ideal)
(a) The spring between ceiling and A is cut.
(b) The string (inextensible) between A and B is cut.
(c) The spring between B and C is cut.
Also find the tension in the string when the system is at rest and in the above 3 cases.



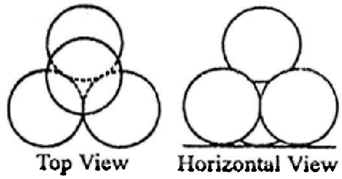
59. In the system shown, find the initial acceleration of the wedge of mass 5M. The pulleys are ideal and the cords are inextensible. (there is no friction anywhere).



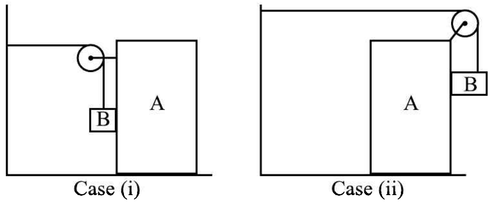
60. An inclined plane makes an angle 30° with the horizontal. A groove OA = 5 m cut in the plane makes an angle 30° with OX. A short smooth cylinder is free to slide down the influence of gravity. Find the time taken by the cylinder to reach from A to O. (g = 10 m/s²)



61. An ornament for a courtyard at a world's fair is to be made up of four identical, frictionless metal sphere, each weighing $2\sqrt{6}$ Newton. The spheres are to be arranged as shown, with three resting on a horizontal surface and touching each other; the fourth is to rest freely on the other three. The bottom three are kept from separating by spot welds at the points of contact with each other. Allowing for a factor of safety of 3, how much tension must the sot welds with stand.



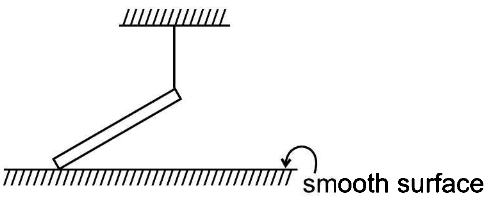
62. A 20 kg block B is suspended from a cord attached to a 40 kg cart A. Find the ratio of the acceleration of the block in cases (i) and (ii) shown in figure immediately after the system is released from rest. (neglect friction)

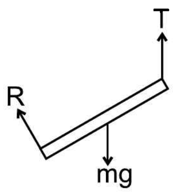


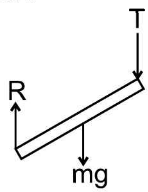
Exercise–3: Objective Problems

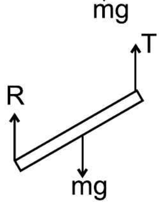
1 Action and reaction
(A) Act on two different objects
(B) Have equal magnitude
(C) Have opposite directions
(D) Have resultant zero.

2 Which figure represents the correct F.B.D. of rod of mass m as shown in figure :



(A) 

(B) 

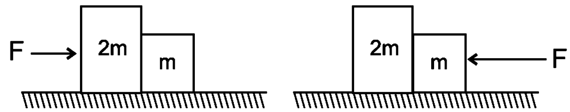
(C) 

(D) None of these

3 In which of the following cases the net force is not zero ?
(A) A kite skillfully held stationary in the sky
(B) A ball freely falling from a height

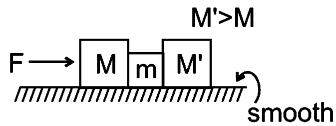
(C) An aeroplane rising upwards at an angle of 45° with the horizontal with a constant speed
(D) A cork floating on the surface of water

4. Two blocks are in contact on a frictionless table. One has mass *m* and the other 2*m*. A force *F* is applied on 2m as shown in the figure. Now the same force *F* is applied from the right on *m*. In the two cases respectively, the ratio force of contact between the two blocks will be



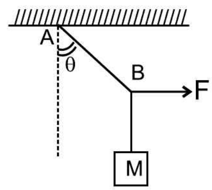
(A) same (B) 1: 2 (C) 2: 1 (D) 1: 3

5. A constant force *F* is applied in horizontal direction as shown. Contact force between *M* and m is *N* and between *m* and *M'* is *N'* then



(A) *N* or *N'* equal (B) $N > N'$
(C) N' > N (D) cannot be determined

6. A mass *M* is suspended by a rope from a rigid support at A as shown in figure. Another rope is tied at the end *B*, and it is pulled horizontally with a force *F*. If the rope *AB* makes an angle θ with the vertical, then the tension in the string *AB* is

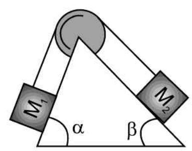


(A) $F \sin\theta$ (B) $F/\sin\theta$
(C) $F \cos\theta$ (D) $F/\cos\theta$

7. Two persons are holding a rope of negligible weight tightly at its ends so that it is horizontal. A 15 kg weight is attached to the rope at the mid point which now no longer remains horizontal. The minimum tension required to completely straighten the rope is

(A) 15 kg (B) $\frac{15}{2}$ kg
(C) 5 kg (D) Infinitely large

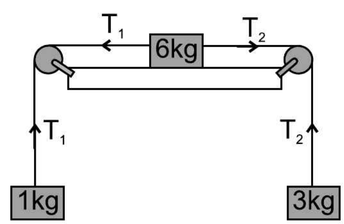
8. Two masses M_1 and M_2 are attached to the ends of a string which passes over a pulley attached to the top of a double inclined plane of angles of inclination α and β. If $M_2 > M_1$, the acceleration a of the system is given by



(A) $\dfrac{M_2 g(\sin\beta)}{M_1 + M_2}$ (B) $\dfrac{M_1 g(\sin\alpha)}{M_1 + M_2}$

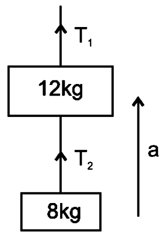
(C) $\left(\dfrac{M_2 \sin\beta - M_1 g \sin\alpha}{M_1 + M_2}\right) g$ (D) Zero

9. Three masses of 1 kg, 6 kg and 3 kg are connected to each other with threads and are placed on table as shown in figure, What is the acceleration with which the system is moving? Take $g = 10\text{m s}^{-2}$.



(A) Zero (B) 1 m s^{-2}
(C) 2 m s^{-2} (D) 3 m s^{-2}

10. A body of mass 8 kg is hanging from another body of mass 12 kg. The combination is being pulled by a string with an acceleration of 2.2 m s^{-2}. The tension T_1 and T_2 will be respectively: (use $g = 9.8 m/s^2$)

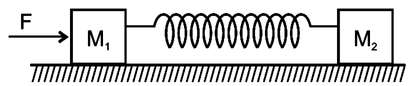


(A) 200 *N*, 80 *N* (B) 220 *N*, 90 *N*
(C) 240 *N*, 96 *N* (D) 260 *N*, 96 *N*

11. A fireman wants to slide down a rope. The rope can bear a tension of $\frac{3}{4}$ the of the weight of the man. With what minimum acceleration should the fireman slide down :
(A) $\frac{g}{3}$ (B) $\frac{g}{6}$ (C) $\frac{g}{4}$ (D) $\frac{g}{2}$

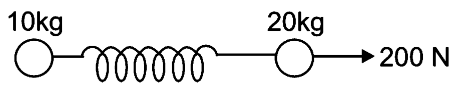
12. A particle of small mass m is joined to a very heavy body by a light string passing over a light pulley. Both bodies are free to move. The total downward force on the pulley is
(A) mg (B) 2 mg
(C) 4 mg (D) >> mg

13. Two blocks of masses M_1 and M_2 are connected to each other through a light spring as shown in figure. If we push mass M_1 with force F and cause acceleration a_1 in mass M_1, what will be the acceleration in M_2?



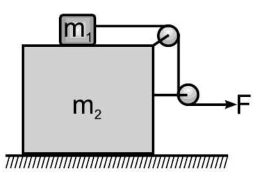
(A) F/M_2 (B) $F/(M_1+M_2)$
(C) a_1 (D) $(F-M_1a_1)/M_2$

14. Two masses of 10 kg and 20 kg respectively are connected by a massless spring as shown in figure. A force of 200 *N* acts on the 20 kg mass at the instant when the 10 kg mass has an acceleration of 12 ms^{-2}, the acceleration of the 20 kg mass is:



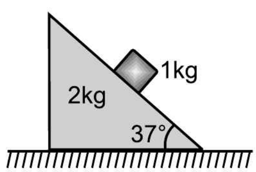
(A) 2 ms–2 (B) 4ms^{-2}
(C) 10ms^{-2} (D) 20ms^{-2}

15. In the arrangement shown in the Figure all surfaces are frictionless, the masses of the block are $m_1 = 20$ kg and $m_2 = 30$ kg. The accelerations of masses m_1 and m_2 will be if $F = 180$ *N*.



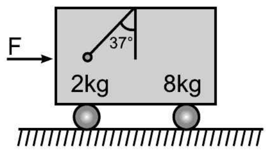
(A) $a_{m_1} = 9 \text{ m/s}^2, a_{m_2} = 0$ (B) $a_{m_1} = 9 \text{ m/s}^2, a_{m_2} = 9 \text{ m/s}^2$
(C) $a_{m_1} = 0, a_{m_2} = 9 \text{ m/s}^2$ (D) None of these

16. Figure shows a wedge of mass 2kg resting on a frictionless floor. A block of mass 1 kg is kept on the wedge and the wedge is given an acceleration of 5 m/sec^2 towards right. Then:



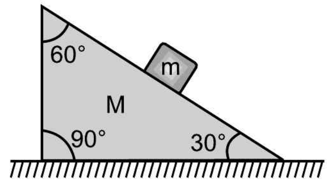
(A) Block will remain stationary w.r.t. wedge
(B) The block will have an acceleration of 1 m/sec^2 w.r.t. the wedge
(C) Normal reaction on the block is 11 *N*
(D) Net force acting on the wedge is 2 *N*

17. A trolley of mass 8 kg is standing on a frictionless surface inside which an object of mass 2 kg is suspended. A constant force F starts acting on the trolley as a result of which the string stood at an angle of 37^0 from the vertical. Then:



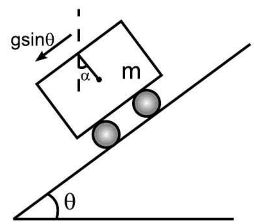
(A) acceleration of the trolley is 40/3 m/sec^2.
(B) force applied in 60 N
(C) force applied is 75 N
(D) tension in the string is 25 N

18. A triangular block of mass M rests on a smooth surface as shown in figure. A cubical block of mass m rests on the inclined surface. If all surfaces are frictionless, the force that must be applied to M so as to keep m stationary relative to M is



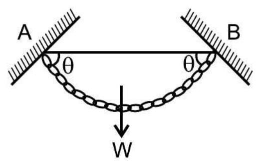
(A) Mg tan 30 (B) mg tan 30
(C) $(M+m)g$ tan 30 (D) $(M+m)g$ cos 30

19. A trolley is accelerating down an incline of angle θ with acceleration $g$sinθ. Which of the following is correct. (α is the angle made by the string with vertical).



(A) α = θ
(B) α = 0^0
(C) Tension in the string, $T = mg$
(D) Tension in the string, $T = mg$ sec θ

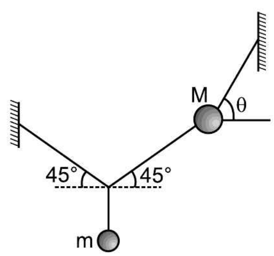
20. A flexible chain of weight W hangs between two fixed points A and B at the same level. The inclination of the chain with the horizontal at the two points of support is θ. What is the tension of the chain at the endpoint.



(A) $\frac{W}{2}$ cosec θ (B) $\frac{W}{2}$ sec θ

(C) W cos θ (D) $\frac{W}{3}$ sin θ

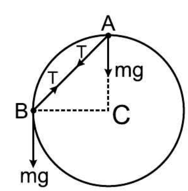
21. Two masses m and M are attached with strings as shown. For the system to be in equilibrium we have



(A) $\tan\theta = 1 + \frac{2M}{m}$ (B) $\tan\theta = 1 + \frac{2m}{M}$

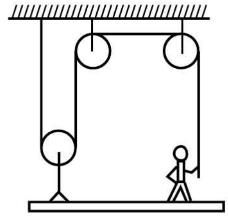
(C) $\tan\theta = 1 + \frac{M}{2m}$ (D) $\tan\theta = 1 + \frac{m}{2M}$

22. Objects A and B each of mass m are connected by light inextensible cord. They are constrained to move on a frictionless ring in a vertical plane as shown in figure. The objects are released from rest at the positions shown. The tension in the cord just after release will be



(A) $mg\sqrt{2}$ (B) $\frac{mg}{\sqrt{2}}$ (C) $\frac{mg}{2}$ (D) $\frac{mg}{4}$

23. A 50 kg person stands on a 25 kg platform. He pulls on the rope which is attached to the platform via the frictionless pulleys as shown in the fig. The platform moves upwards at a steady rate if the force with which the person pulls the rope is:

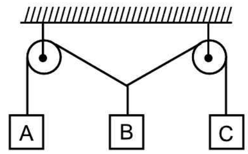


(A) 500 N (B) 250 N
(C) 25 N (D) 50 N

24. A balloon of gross weight w newton is falling vertically downward with a constant acceleration a(<g). The magnitude of the air resistance is:

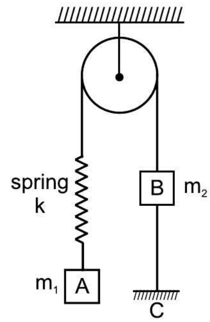
(A) w (B) $w\left(1+\frac{a}{g}\right)$ (C) $w\left(1-\frac{a}{g}\right)$ (D) $w\frac{a}{g}$

25. Three blocks A, B and C are suspended as shown in the figure. Mass of each blocks A and C is m. If system is in equilibrium and mass of B is M, then:



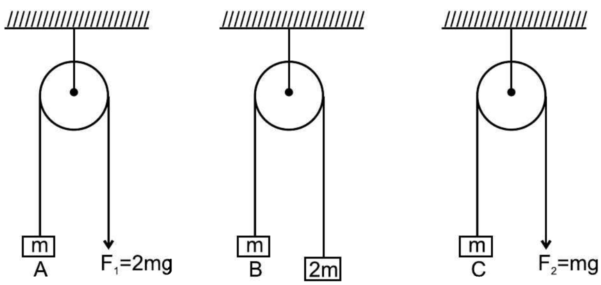
(A) $M = 2m$ (B) $M < 2m$
(C) $M > 2m$ (D) $M = m$

26. In the system shown in the figure $m_1 > m_2$. System is held at rest by thread BC. Just after the thread BC is burnt:



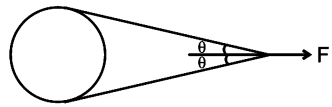
(A) Acceleration of m_2 will be upwards
(B) Magnitude of acceleration of both blocks will be equal to $\left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$
(C) Acceleration of m_1 will be equal to zero
(D) Magnitude of acceleration of two blocks will be nonzero and unequal.

27. In the figure, the blocks A, B and C of mass m each have acceleration a_1, a_2 and a_3 respectively. F_1 and F_2 are external forces of magnitudes 2 mg and mg respectively.



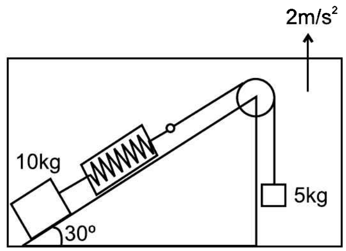
(A) $a_1 = a_2 = a_3$ (B) $a_1 > a_2 > a_3$
(C) $a_1 = a_2$, $a_2 > a_3$ (D) $a_1 > a_2$, $a_2 = a_3$

28. A string is wrapped round a log of wood and it is pulled with a force F as shown in the figure.



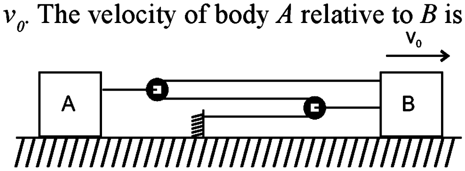
(A) tension T in the string increases with increase in θ
(B) tension T in the string decreases with increase in θ
(C) tension $T > F$ if $\theta > \pi/3$
(D) tension $T > F$ if $\theta > \pi/4$

29. In the figure the reading of the spring balance will be: [$g = 10$ m/s^2]



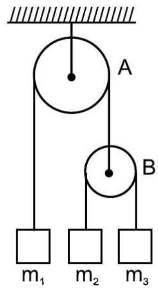
(A) 6 kg$f$
(B) 5 kg$f$
(C) 60 N
(D) 60 kg$f$

30. Block B moves to the right with a constant velocity v_0. The velocity of body A relative to B is



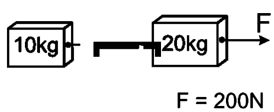
(A) $\frac{v_0}{2}$, towards left (B) $\frac{v_0}{2}$, towards right
(C) $\frac{3v_0}{2}$, towards left (D) $\frac{3v_0}{2}$, towards right

31. In the arrangement shown in figure, pulleys are massless and frictionless and threads are inextensible. Block of mass m_1 will remain at rest if:



(A) $\frac{1}{m_1} = \frac{1}{m_2} + \frac{1}{m_3}$ (B) $\frac{4}{m_1} = \frac{1}{m_2} + \frac{1}{m_3}$
(C) $m_1 = m_2 + m_3$ (D) $\frac{1}{m_3} = \frac{2}{m_2} + \frac{3}{m_1}$

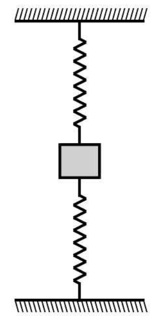
32. Two blocks of masses 10 kg and 20 kg are connected by a light spring as shown. A force of 200 N acts on the 20 kg mass as shown. At a certain instant the acceleration of 10 kg mass is 12 ms^{-2}.



(A) At that instant the 20 kg mass has an acceleration of 12 ms^{-2}.
(B) At that instant the 20 kg mass has an acceleration of 4 ms^{-2}.
(C) The stretching force in the spring is 120 N.
(D) The collective system moves with a common acceleration of 30 ms^{-2} when the extension in the connecting spring is the maximum.

33. A block tied between two springs is in equilibrium. If upper spring is cut then the acceleration of the block just after cut is 6 m/s^2 downwards. Now, if instead of upper spring, lower spring

is being cut then the magnitude of acceleration of the block just after the cut will be: (Take $g = 10$ m/s^2)



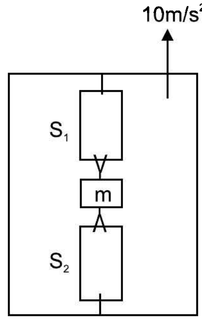
(A) 16 m/s^2

(B) 4 m/s^2

(C) Cannot be determined

(D) None of these

34. Reading shown in two spring balances S_1 and S_2 is 90 kg and 30 kg respectively and lift is accelerating upwards with acceleration 10 m/s^2. The mass is stationary with respect to lift. Then the mass of the block will be:

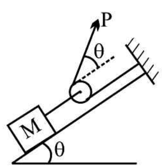


(A) 60 kg
(B) 30 kg
(C) 120 kg
(D) None of these

35. Five persons A, B, C, D & E are pulling a cart of mass 100 kg on a smooth surface and cart is moving with acceleration 3 m/s^2 in east direction. When person 'A' stops pulling, it moves with acceleration 1m/s^2 in the west direction. When person 'B' stops pulling, it moves with acceleration 24 m/s^2 in the north direction. The magnitude of acceleration of the cart when only A & B pull the cart keeping their directions same as the old directions, is

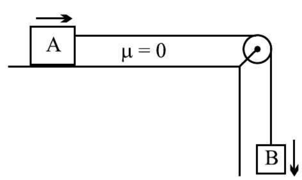
(A) 26 m/s^2 (B) $3\sqrt{71}$ m/s^2
(C) 25 m/s^2 (D) 30 m/s^2

36. What should be the minimum force P to be applied to the string so that block of mass m just begins to move up the frictionless plane.



(A) $Mg \tan \theta/2$ (B) $Mg \cot \theta/2$
(C) $\dfrac{Mg\cos\theta}{1+\sin\theta}$ (D) None

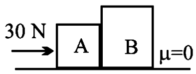
37. Both the blocks shown here are of mass m and are moving with constant velocity in direction shown in a resistive medium which exerts equal constant force on both blocks in direction opposite to the velocity. The tension in the string connecting both of them will be: (Neglect friction)

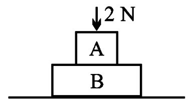


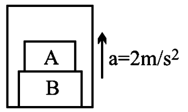
(A) mg (B) $mg/2$
(C) $mg/3$ (D) $mg/4$

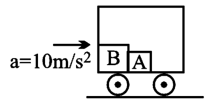
38. In which of the following cases is the contact force between A and B maximum

($m_A = m_B = 1$ kg)

(A)


(B)


(C)


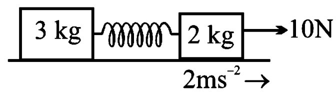
(D)


39. A rope of mass 5 kg is moving vertically in vertical position with an upwards force of 100 N acting at the upper end and a downwards force of 70 N acting at the lower end. The tension at midpoint of the rope is

(A) 100 N (B) 85 N

(C) 75 N (D) 105 N

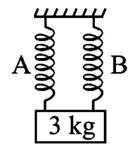
40. Find the acceleration of 3 kg mass when acceleration of 2 kg mass is 2 ms^{-2} as shown in figure.



(A) 3 ms^{-2} (B) 2 ms^{-2}

(C) 0.5 ms^{-2} (D) zero

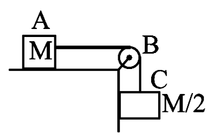
41. Block of 1 kg is initially in equilibrium and is hanging by two identical springs A and B as shown in figures. If spring A is cut from lower point at t=0 then, find acceleration of block in ms^{-2} at t = 0.



(A) 5 (B) 10

(C) 15 (D) 0

42. A block of mass M on a horizontal smooth surface is pulled by a load of mass M/2 by means of a rope AB and string BC as shown in the figure. The length & mass of the rope AB are L $M/2$ and respectively. As the block is pulled from $AB = L$ to $AB = 0$ its acceleration changes from



(A) $\frac{3g}{4}$ to g (B) $\frac{g}{4}$ to $\frac{g}{2}$

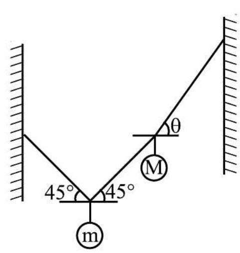
(C) $\frac{g}{4}$ to g (D) $\frac{3g}{2}$ to 2g

43. A particle of mass m, initially at rest, is acted on by a force $F = F_0\left\{1-\left(\frac{2t-T}{T}\right)^2\right\}$ during the interval $0 \le t \le T$. The velocity of the particle at the end of the interval is :

(A) $\frac{5F_0T}{6m}$ (B) $\frac{4F_0T}{3m}$

(C) $\frac{2F_0T}{3m}$ (D) $\frac{3F_0T}{2m}$

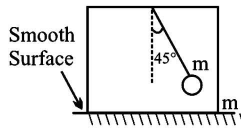
44 Two masses m and M are attached to the strings as shown in the figure. If the system is in equilibrium, then



(A) $\tan\theta = 1 + \frac{2M}{m}$ (B) $\tan\theta = 1 + \frac{2m}{M}$

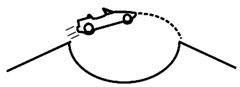
(C) $\cot\theta = 1 + \frac{2M}{m}$ (D) $\cot\theta = 1 + \frac{2m}{M}$

45 A ball connected with string is released at an angle 45° with the vertical as shown in figure. Then the acceleration of the box at this instant will be : [Mass of the box is equal to mass of ball]



(A) $g/4$ (B) $g/3$
(C) $g/2$ (D) none of these

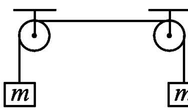
46 A stunt man jumps his car over a crater as shown (neglect air resistance)



(A) During the whole flight the driver experiences weightlessness
(B) During the whole flight the driver never experiences weightlessness
(C) During the whole flight the driver experiences weightlessness only at the highest point
(D) The apparent weight increases during upward journey

47 A ball of mass m is thrown vertically upwards. Assume the force of air resistance has magnitude proportional to the velocity, and direction opposite to the velocity's. At the highest point, the ball's acceleration is
(A) 0 (B) less than g
(C) g (D) greater than g

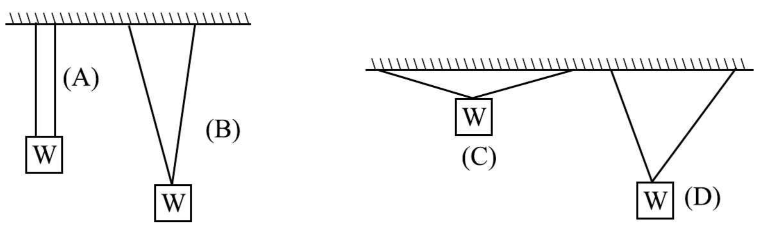
48 Two identical mass m are connected to a massless string which is hung over two frictionless pulleys as shown in figure. If everything is at rest, what is the tension in the cord?



(A) less than mg
(B) exactly mg
(C) more than mg but less than $2mg$
(D) exactly $2mg$

49 A flexible chain of weight W hangs between two fixed points A & B which are at he same horizontal level. The inclination of the chain with the horizontal at both the points of support is θ. What is the tension of the chain at the mid point?
(A) $\frac{W}{2}\operatorname{cosec}\theta$ (B) $\frac{W}{2}\tan\theta$ (C) $\frac{W}{2}\cot\theta$ (D) none

50 A weight can be hung in any of the following four ways by string of same type. In which case is the string most likely to break?
(A) A (B) B
(C) C (D) D



Question No. 51 to 52 (2 questions)

A frictionless pulley is attached to one arm of a balance and a string passed around it carries two masses m_1 and m_2. The pulley is provided with a clamp due to which m_1 and m_2 do not move w.r.t. each other.

51. On removing the clamp, m_1 and m_2 start moving. How much change in counter mass has to be made to restore balance?

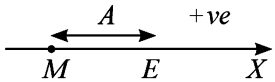
(A) $\dfrac{(m_1+m_2)^2}{m_1-m_2}$ (B) $\dfrac{(m_1-m_2)^2}{m_1+m_2}$ (C) $2m_1 - m_2$ (D) $m_1 - m_2$

52. On removing the clamp, if the counter mass restores balance, then acceleration of centre of mass of the masses m_1 and m_2 will have acceleration of magnitude

(A) zero (B) $\left(\dfrac{m_1-m_2}{m_1+m_2}\right)g$ (C) $\left(\dfrac{m_1-m_2}{m_1+m_2}\right)^2 g$ (D) g

Question No. 53 to 55 (3 questions)

A particle of mass m is constrained to move on x-axis. A force F acts on the particle. F always points toward the position labeled E. For example, when the particle is to the left of E, F points to the right. The magnitude of F is a constant F except at point E where it is zero.

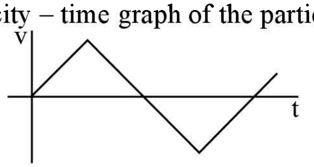


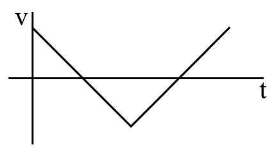
The system is horizontal. F is the net force acting on the particle. The particle is displaced a distance A towards left from the equilibrium position E and released from rest at $t = 0$.

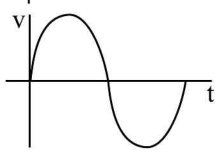
53 What is the period of the motion?

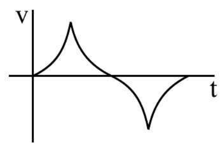
(A) $4\left(\sqrt{\dfrac{2Am}{F}}\right)$ (B) $2\left(\sqrt{\dfrac{2Am}{F}}\right)$ (C) $\left(\sqrt{\dfrac{2Am}{F}}\right)$ (D) None

54 Velocity – time graph of the particle is

(A)


(B)


(C)


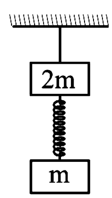
(D)


55 Find minimum time it will take to reach from $x = -\dfrac{A}{2}$ to 0.

(A) $\dfrac{3}{2}\sqrt{\dfrac{mA}{F}}(\sqrt{2}-1)$ (B) $\sqrt{\dfrac{mA}{F}}(\sqrt{2}-1)$

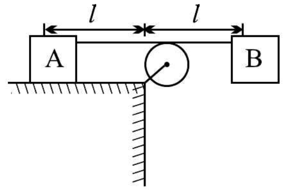
(C) $2\sqrt{\dfrac{mA}{F}}(\sqrt{2}-1)$ (D) None

56 Two blocks are connected by a spring. The combination is suspended, at rest, from a string attatched to the ceiling, as shown in the figure. The string breaks suddenly. Immediately after the string breaks, what is the initial downward acceleration of the upper block of mass 2m ?



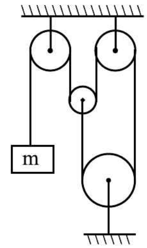
(A) 0 (B) $3g/2$ (C) g (D) 2g

57 Two blocks A and B each of same mass are attached by a thin inextensible string through an ideal pulley. Initially block B is held in position as shown in figure. Now the block B is released. Block A willl slide to right and hit the pulley in time t_A. Block B will swing and hit the surface in time t_B. Assume the surface as frictionless.
[**Hint:** Tension T in the string acting on both blocks is same in magnitude. Acceleration needed for horizontal motion is from T.]



(A) $t_A = t_B$ (B) $t_A < t_B$
(C) $t_A > t_B$
(D) Data are not sufficient to get relationship between t_A and t_B.

58 If the string & all the pulleys are ideal, acceleration of mass m is

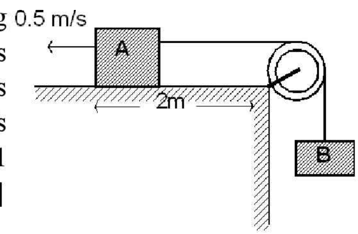


(A) $\frac{g}{2}$
(B) 0
(C) g
(D) Dependent on m

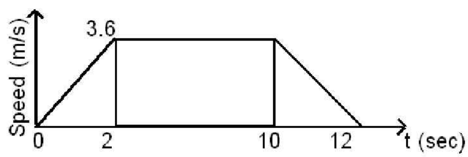
Exercise–4

(JEE/REE Questions of Previous Years)

1 A mass A (0.5 kg) is placed on a smooth table with a string attached to it. The string goes over a frictions pulley and is connected to another mass B (0.2 kg). At $t = 0$ the mass A is at a distance 2m from the end moving with a speed of 0.5 m/s towards the left, what will be its position and speed at $t = 1$ sec? **[IIT, 1975]**

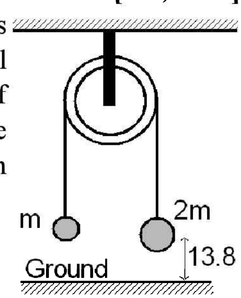


2 A lift is going up. The total mass of the lift and the passengers is 150 kg. The variation in the speed of the lift is given in the graph.

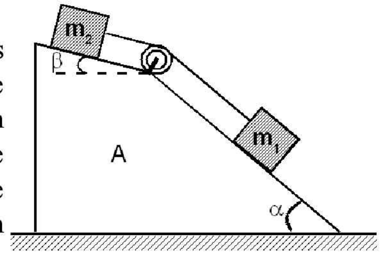


(A) What will be the tension in the rope pulling the lift at t equal to
(i) 1 sec (ii) 6 sec and (iii) 11 sec ?
(B) What is the height through which the lift takes the passengers ?
(C) What will be the average velocity and average acceleration during the course of entire motion? **[IIT, 1976]**

3 Two masses m and $2m$ are connected by a massless string which passes over a pulley as shown in fig. The masses are held initially with equal length of the string on either side of the pulley. Find the velocity of masses at the instant the lighter mass moves up a distance of 6.54 m. The string is suddenly cut at that instant. Calculate the time taken by each mass to reach the ground. **[IIT, 1977]**



4 Two cubes of masses m_1 and m_2 lie on two frictionless slopes of block A which rests on a horizontal table. The cubes are connected by a string which passes over a pulley as shown in the diagram. To what horizontal acceleration f should the whole system (i.e. block and cubes) be subjected so that the cubes do not slide down the planes? What is the tension in the string in this situation?



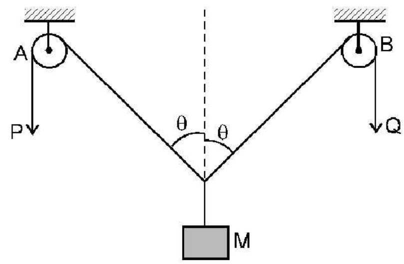
5 A ship of mass 3×10^7 kg initially at rest is pulled by a force of 5×10^4 N through a distance of 3m. Assume that the resistance due to water is negligible, the speed of the ship is

(A) 1.5 m/s (B) 60 m/s (C) 0.1 m/s (D) 5 m/s

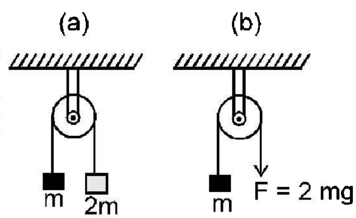
[IIT, 1980]

6 In the arrangement shown in fig. the ends P and Q of an unstretchable string move downwards with uniform speed U. Pulleys A and B are fixed. Mass M moves upwards with a speed.

(A) $2U \cos \theta$
(B) $U \cos \theta$
(C) $2U/\cos \theta$
(D) $U/\cos \theta$

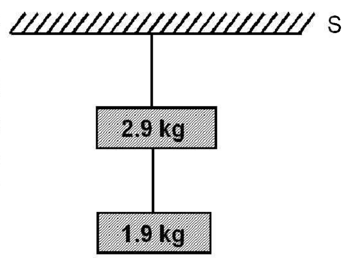


7 The pulley arrangements of fig. (a) and (B) are indentical. The mass of the rope is negligible. In (a), the mass m is lifted up by attaching a mass 2m to the other end of the rope. In (b), m is lifted up by pulling the other end of the rope with a constant downward force $F = 2$ mg. Find the acceleration of m is the same in both cases:



8 Two blocks of mass 2.9 kg and 1.9 kg are suspended from a rigid support S by two inextensible wires each of length 1 m. The upper wire has negligible mass and the lower wire has a uniform mass of 0.2 kg/m. The whole system of blocks, wires and support have an upward acceleration of 0.2 m/s^2. The acceleration due to gravity is 9.8 m/s^2. **[IIT, 1989]**

(i) Find the tension at the midpoint of the lower wire.
(ii) Find the tension at the midpoint of the upper wire.

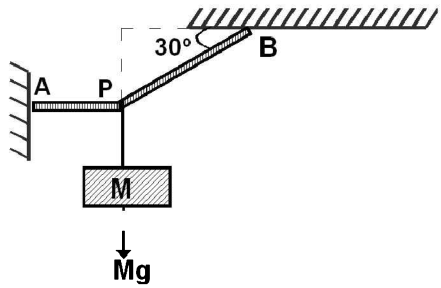


9 Essential characteristic of equilibrium is

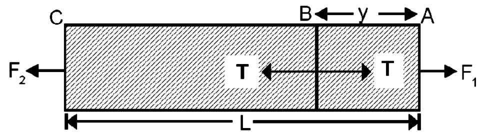
(A) Momentum equal zero
(B) Acceleration equals zero
(C) *K.E.* equals zero
(D) Velocity equals zero

[REE, 1989]

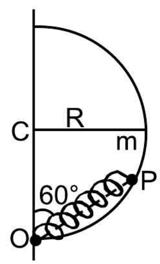
10 A mass is hung with a light inextensible string in figure. Find the tension of horizontal string *AP*. **[IIT, 1990]**



11 What is the tension in a rod of length L and mass M at a distance y from F_1 when the rod is acted on two unequal forces F_1 and F_2 ($<F_1$) as shown in the figure. **[IIT, 1993]**



12 A smooth semicircular wire-track of radius R is fixed in a vertical plane shown in fig. One end of a massless spring of natural length (3R/4) is attached to the lower point O of the wire track. A small ring of mass m, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant $K = mg/R$. Consider the instant when the ring is released, and (i) draw free body diagram of the ring, (ii) Determine the tangential acceleration of the ring and the normal reaction. **[JEE, 1996, 5 marks]**



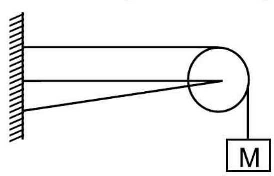
13 A spring of force constant K is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force constant of -

(A) $2/3\,K$ (B) $3/2\,K$

(C) K (D) $2K$ **[JEE, 1999]**

14 Two blocks of masses $m_1 = 3$ kg and $m_2 = \frac{1}{\sqrt{3}}$ kg are connected by a light inextensible string which passes over a smooth peg. The blocks rest on the inclined smooth planes of a wedge and the peg is fixed to the top of the wedge. The planes of the wedge supporting m_1 and m_2 are inclined at 30° and 60°, respectively, with the horizontal. Calculate the acceleration of the masses and the tension in the string. **[REE, 1999]**

15 A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as shown in the figure. The force on the pulley by the clamp is given by-

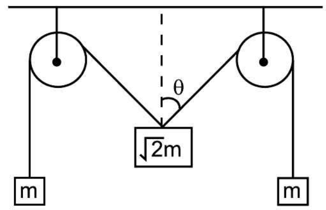


(A) $\sqrt{2}\ Mg$ (B) $\sqrt{2}\ mg$

(C) $\left(\sqrt{(M+m)^2+m^2}\right)g$ (D) $\left(\sqrt{(M+m)^2+M^2}\right)g$

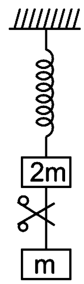
[JEE, 2001]

16 The pulleys and strings shown in the figure are smooth and of negligible mass for the system to remain in equilibrium, the angle θ should be **[JEE, 2002]**



(A) 0°
(B) 30°
(C) 45°
(D) 60°

17 System shown in figure is in equilibrium and at rest. The spring and string are massless Now the string is cut. The acceleration of mass 2m and m just after the string is cut will be: **[JEE, 2006]**

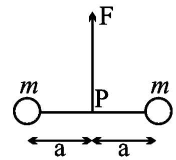


(A) $g/2$ upwards , g downwards
(B) g upwards, $g/2$ downwards
(C) g upwards , $2g$ downwards
(D) $2g$ upwards , g downwards

18 Two particles of mass m each are tied at the ends of a light string of length 2a. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance '*a*' from the center *P* (as shown in the figure). Now, the mid-point of the string is pulled vertically upwards with a small but constant force *F*. As a result, the particles move towards each other on the surfaces. The magnitude of acceleration, when the separation between them becomes 2*x*, is **[JEE, 2007]**

(A) $\frac{F}{2m}\frac{a}{\sqrt{a^2-x^2}}$ (B) $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$

(C) $\frac{F}{2m}\frac{x}{a}$ (D) $\frac{F}{2m}\frac{\sqrt{a^2-x^2}}{x}$



19 A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table because

STATEMENT-2

For every action there is an equal and opposite reaction

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement 1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True **[JEE, 2007]**

20 A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is **[JEE, 2009]**

(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$

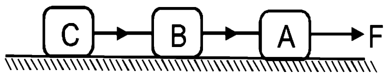
(C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

Previous Years' AIEEE Questions

1. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is : **[AIEEE-2002]**

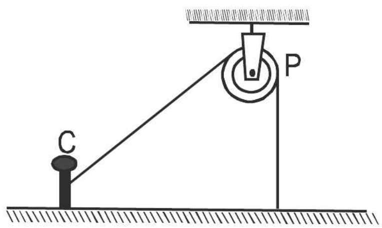
(A) 8 : 1 (B) 9 : 7 (C) 4 : 3 (D) 5 : 3

2. Three identical blocks of masses $m = 2$ kg are drawn by a force $F = 10.2\ N$ on a frictionless surface, then what is the tension (in N) in the string between the blocks B and C ? **[AIEEE-2002]**



(A) 9.2 (B) 3.4 (C) 4 (5) 9.8

3. One end of massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 360 N. With what value of minimum safe acceleration (in ms^{-2}) can a man of 60 kg climb down the rope? **[AIEEE-2002]**



(A) 16 (B) 6 (C) 4 (D) 8

4. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, When the lift is stationary. If the lift moves downward with an acceleration of 5 m/s^2, the reading of the spring balance will be : **[AIEEE-2003]**

(A) 24 N (B) 74 N (C) 15 N (D) 49 N

5. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is : **[AIEEE-2003]**

(A) $\frac{Pm}{M+m}$ (B) $\frac{Pm}{M-m}$ (C) P (D) $\frac{PM}{M+m}$

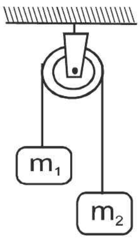
6. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then the true statement about the scale reading is : **[AIEEE-2003]**

(A) Both the scale read M kg each

(B) The scale of the lower one reads M kg and of the upper one zero

(C) The reading of the two scales can be anything but the sum of the reading will be M kg
(D) Both the scales read $M/2$ kg

7. Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when system is free to move ? ($g = 9.8$ m/s^2) **[AIEEE-2004]**



(A) 0.2 m/s^2 (B) 9.8 m/s^2 (C) 5 m/s^2 (D) 4.8 m/s^2

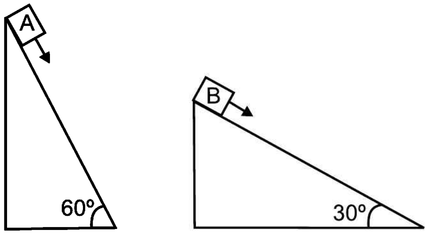
8. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. The a is equal to **[AIEEE-2005]**
(A) g (B) $g \tan \alpha$ (C) $g/\tan\alpha$ (D) g cosec α

9. A ball of mass 0.2 kg is thrown vertically upwards by applying a constant force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2m height further, find the magnitude of the force. Consider g = 10 m/s^2. **[AIEEE-2006]**
(A) 20 N (B) 22 N (C) 4 N (D) 16 N

10. A block of mass m is connected to another block of mass M by a string (massless). The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest. Then a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m **[AIEEE-2007]**

(A) $\dfrac{mF}{m}$ (B) $\dfrac{(M+m)F}{m}$ (C) $\dfrac{mF}{(m+M)}$ (D) $\dfrac{MF}{(m+M)}$

11. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B? **[AIEEE-2010]**



(A) 4.9 ms^{-2} in horizontal direction (B) 9.8 ms^{-2} in vertical direction
(C) Zero (D) 4.9 ms^{-2} in vertical direction

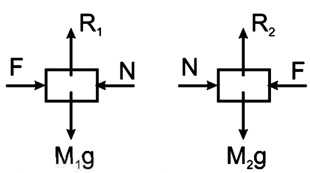
ANSWER KEY

Exercise - 1

36. *A*
37. *A*
38. *C*
39. *B*
40. *C*
41. *B*
42. *A*
43. *C*
44. *B*
45. *B*
46. $a_1 + a_2 = b_1 + b_2$
47. $V_A = 0, V_A = 30 \uparrow m/s$
48. $a_A = 2\ m/s^2 \uparrow, a_B = 3\ m/s^2 \downarrow, a_C = 1\ m/s^2 \downarrow$
49. $V_A + V_B + 2V_C = 0$
50. $V_A = 125\ mm/s \uparrow, V_B = 75\ mm/s \downarrow, V_C = 175\ mm/s \downarrow$
51. *25 m/s*
52. *(C)*
53. $v_A = \dfrac{v_B}{\cos(\pi/4 - \theta/2)}$
54. $\dfrac{H}{H-h}v$
55. $\dfrac{4}{3}$ *V*
56. $v_B = 0.5\ m/s \uparrow\ \&\ a_B = 1\ m/s^2 \downarrow$
57. $V_A = 10 \uparrow\ V_B = 22 \downarrow$
58. $V_c = 4m/s \downarrow, a_C = 5m/s^2 \downarrow$
59. $50m/s \uparrow, 10m/s \downarrow$
60. *(i)* $u = -v/2$, *(ii)* $u = v$
61. *(i)* $\dfrac{v}{2\cos\theta}$, *(ii)* $\dfrac{v}{\cos\theta}$
62. $2a + a_M = 0$
63. $2a_2 + a_3 - a_1 = 0$
64. $1 + \cos\theta$
65. $4v_A + 8v_B + 4v_C + v_D = 0$
66. *0.5 m/s*

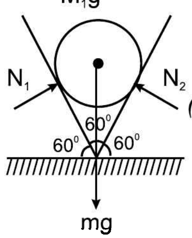
Exercise - 2

1 *(a)*



(b) $N = F$ *(c)* *F* *(d)* m_1g, m_2g.

2 *(a)*



(b) equal magnitude w .

3 $N_A = \dfrac{1000}{\sqrt{3}}\ N, N_B = \dfrac{500}{\sqrt{3}}\ N$

4. *(i) zero, (ii) 10 N*

5 *(a) 10 N, (b) 15 N, (c) 20 N.*

6. *(a)* $|\vec{F}_1| = |\vec{F}_2| = \dfrac{60}{\sqrt{2}}\ N$ *(b)* $W = \dfrac{60}{\sqrt{2}}\ N$

7 4N.

8. $\frac{m_2 g}{2(m_1 + m_2)}$

9 (a) 4.8 N, 3.6 N, 2.4N, 1.2 N(b) F = 6 N (c) 0.2 N

10 0.1 m

11. The acceleration of A is 3 × 0.98 m/s^2

12 (i) 100 N, (ii) 100 N, (iii) 100 N, (iv) 120 N, (v) 80 N, (vi) 120N, (vii) 120N, (viii) 80N.

13 $\frac{\cos\theta_2}{\cos\theta_1}$

14. 2u

15. u tan

16. 1.8 m/s

17. a / 2, towards left.

18. 3 m/s^2, towards left.

19. $\frac{15}{4}$ m/s^2, opposite direction.

20. F = 0

21. (g + a) sin θ

22. $a = \frac{4F}{M+m} - g.$

23. For the system $T - 33\,g = m_A a_A + m_B a_B + m_C a_C = 10\,(-2) + 15\,(0) + 8\,(3/2)$

. $\Rightarrow T = 33\,g - 8 = 322\,N$

24. 3: 1

25. (a) $\frac{m_1}{m_2} = \frac{1}{3}$ (b) a = 3/4 m/s^2]

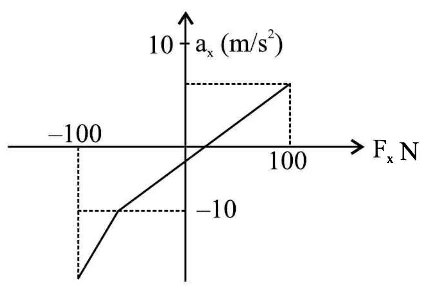
26. (a) 45 m/s along negative x-axis and 15 m/s along positive y-axis

(b) $\theta = \tan^{-1}\left(-\frac{1}{3}\right)$ from positive x-axis

(c) $(-225\hat{i} + 75\hat{j})m$

(d) $(-227\hat{i} + 79\hat{j})m$

27. (a) $F_x > 19.6\,N$ (b) $F_x \leq -78.4\,N$ (c)



28. (a) 30°, (b) $\frac{\sqrt{3}}{2}$

29. $a = \frac{4g}{9}, T = \frac{13mg}{18}$

30. (a) 5m/s^2, (b)(i)100 N, (ii) 120 N

31. $x_2 > x_1 > x_3\, x_1 : x_2 : x_3 :$ 15: 18: 10

32. $(M + m_1 + m_2)\left(\frac{m_2}{m_1} g\right)$

33. 55

34. 5N, 16/31 kg

35. 2 sec

36. $\dfrac{1}{\sqrt{e}+1}$

37. *1 sec*

38. *12 N*

39. *7.5 ms*$^{-2}$

40. *10/3 kg*

41. *300 N*

42. *240 N*

43. *30 N*

44. $N_D < N_A < N_C < N_B$

45. $T_C < T_D < T_B < T_A$

46. $\dfrac{4g}{5}, \dfrac{3g}{5}, \dfrac{3g}{5}$

47. $a = \dfrac{12g}{25}; b = \dfrac{9g}{25}; N_{BW} = \dfrac{12mg}{25}$.

48. (a) $T_1 = T_2 = T_3 = \dfrac{mg}{2}$, $T_5 = Mg$ and $T_4 = \dfrac{3mg}{2}$ (b) $F = \dfrac{mg}{2}$

49. $\dfrac{g}{5}$

50. $a_B = \dfrac{4}{3}$ m/s^2, $a_A = \dfrac{4}{3}\sqrt{2}$ m/s^2.

51. $a_C = \dfrac{2g\sin\theta\cos\theta}{1+3\sin^2\theta}; a_A = \dfrac{4g\sin^2\theta}{1+3\sin^2\theta}; a_B = \dfrac{2g\sin\theta}{\sqrt{1+3\sin^2\theta}}$

52. $t = \sqrt{\dfrac{h(1+\sin^2\theta)}{2g\sin^2\theta}}$

53. $a = \dfrac{30\sqrt{3}}{23}$ m/s^2.

54. (a) 2 ms^{-2}, (b) 2.4 N, 0.3 (c) 0.2 s

55. *0.5 sec*

56. *556.8 N, 1.47 sec*

57. $\left(\dfrac{m_1 - 2m_2}{2m_2}\right)g$

58. (a) $a_A = \dfrac{3g\downarrow}{2} = a_B$; $a_C = 0$; $T = mg/2$;.

(b) $a_A = 2g$, $a_B = 2g\uparrow$, $a_c = 0$, $T = 0$; (c) $a_A = a_B = g/2\uparrow$, $a_c = g\downarrow$, $T = \dfrac{3mg}{2}$; $T = 2mg$

59. *2g/23*

60. *2 sec*

61. *2 N*

62. $\dfrac{3}{2\sqrt{2}}$

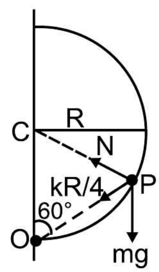
Exercise - 3

1.	(A)	2.	(C)	3.	(B)	4.	(B)
5.	(B)	6.	(B)	7.	(D)	8.	(C)
9.	(C)	10.	(C)	11.	(C)	12.	(C)
13.	(D)	14.	(B)	15.	(A)	16.	(C)
17.	(C,D)	18.	(C)	19.	(A)	20.	(A)
21.	(A)	22.	(B)	23.	(B)	24.	(C)
25.	(B)	26.	(A,C)	27.	(B)	28.	(A,C)
29.	(A,C)	30.	(B)	31.	(B)	32.	(B,C)

33. *(B)*	34. *(B)*	35. *(C)*	36. *(A)*
37. *(B)*	38. *(A)*	39. *(B)*	40. *(B)*
41. *(A)*	42. *(B)*	43. *(C)*	44. *(A)*
45. *(B)*	46. *(A)*	47. *(C)*	48. *(B)*
49. *(C)*	50. *(C)*	51. *(B)*	52. *(C)*
53. *(A)*	54. *(A)*	55. *(B)*	56. *(B)*
57. *(B)*	58. *(C)*		

Exercise - 4

1. *1.1 m; 2.3 m/s*
2. *(a) (i) 17400 N (ii) 14700 N (iii) 12000 N*
   . *(B) 36 m*
   . *(C) Average velocity = 3 m/s; av. acceleration = 0*
3. *2.78s ; 0.665 s*
4. $f = \dfrac{(m_1 \sin\alpha + m_2 \sin\beta)g}{(m_1 \cos\alpha + m_2 \cos\beta)}, T = \dfrac{m_1 m_2 g \sin(\alpha - \beta)}{(m_1 \cos\alpha + m_2 \cos\beta)}$
5. *C*
6. *D*
7. *(a) g/3 (B) g*
8. *20 N, 50 N*
9. *B*
10. $\sqrt{3}Mg$
11. $F_1\left(1-\dfrac{y}{L}\right)+F_2\left(\dfrac{y}{L}\right)$
12. $\dfrac{5\sqrt{3}}{8}g, \ \dfrac{3mg}{8}$



13. *(B)*
14. $\dfrac{10\sqrt{3}}{(3\sqrt{3}+1)}\text{m/s}^2, \dfrac{15(\sqrt{3}+1)}{(3\sqrt{3}+1)}N$
15. *(D)*
16. *(C)*
17. *(A)*
18. *B*
19. *B*
20. *B*

Previous Years' AIEEE Questions

1. *(B)*
2. *(B)*
3. *(C)*
4. *(A)*
5. *(D)*
6. *(A)*

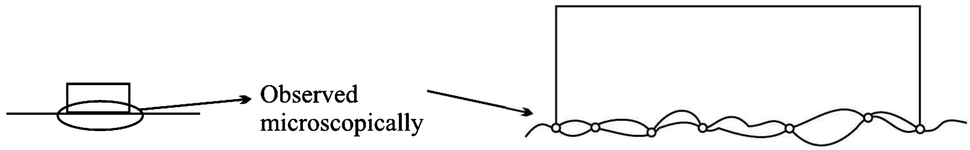
7. *(A)*
8. *(B)*
9. *(B)*
10. *(C)*
11. *(D)*

Chapter 4

Friction

INTRODUCTION

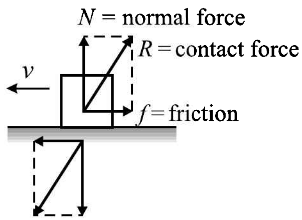
The actual shape of any surface when observed closely is like this



If normal contact force between these bodies is not zero, they will be pressing each other. The pressing leads to extreme pressure at contact points. This pressure leads to locking (cold welding) at the contact points.

Now if they are moving relative to each other or are trying to move relative to each other the elevations and depressions or the joints will inhibit the motion.

When two bodies are kept in contact, electromagnetic forces act between the charged particles (molecules) at the surfaces of the bodies. Thus, each body exerts a contact force of the other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and therefore the contact forces obey Newton's third law.



The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it (in the figure above). In the perpendicular component is called the normal contact force or normal force (generally written as N) and the parallel component is called friction (generally written as f).

Therefore, if R is contact force then

$$R = \sqrt{f^2 + N^2}$$

Reasons for Friction

1. Inter-locking of extended parts of one object into the extended parts of the other object.
2. Bonding between the molecules of the two surfaces or objects in contact.

Friction Force is of Two Types

a. Kinetic
b. Static

Kinetic Friction Force

Kinetic friction exists between two contact surfaces only when there is relative motion between the two contact surfaces. It stops acting when relative motion between two surfaces ceases.

In other words if two contact surface are relatively moving then the depressions and elevations continuously try to inhibit the relative motion. Value of this friction remains fixed. It acts when there is relative motion between two surfaces in contact.

Direction of Kinetic Friction on an Object

It is opposite to the velocity of the object with respect to the other object in contact considered.

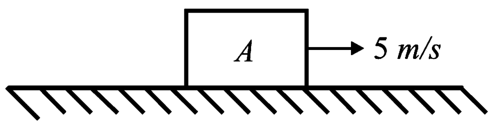
Note that its direction is not opposite to the force applied, it is opposite to the motion of the body considered which is in contact with the other surface.

How to decide the direction of kinetic friction force.

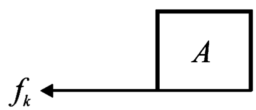
- It is opposite to the relative velocity of the object with respect to the other object in contact considered.
- Note that its direction is not opposite to the force applied, it is opposite to the relative motion of the body considered which is in contact with the other surface.

Examples :

1.

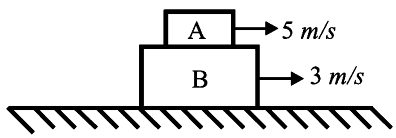


Direction of kinetic friction on the block.

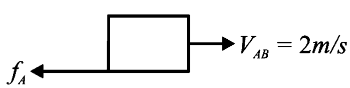


Direction of kinetic friction on the ground.

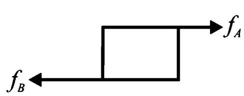
2.



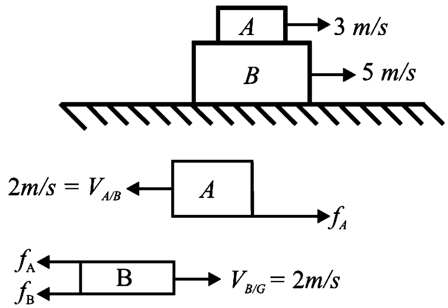
For the direction of Kinetic friction on the block ‘A’, decide the direction of relative velocity of A w.r.t. B.



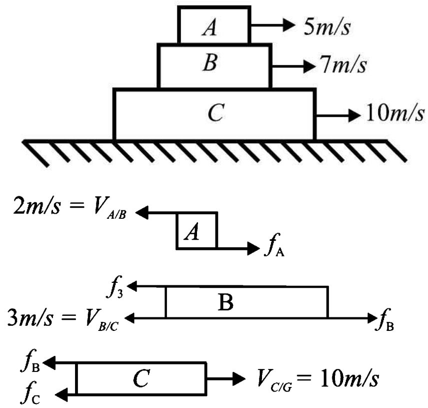
Direction of Kinetic friction on the block ‘B’



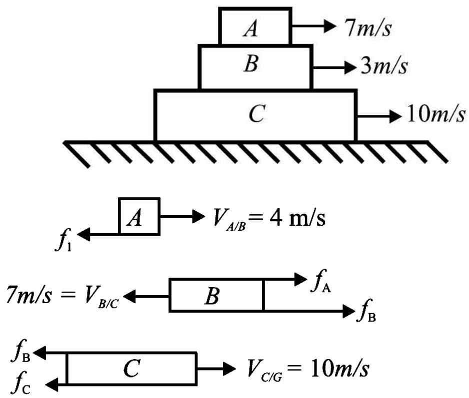
3.



4.



5.

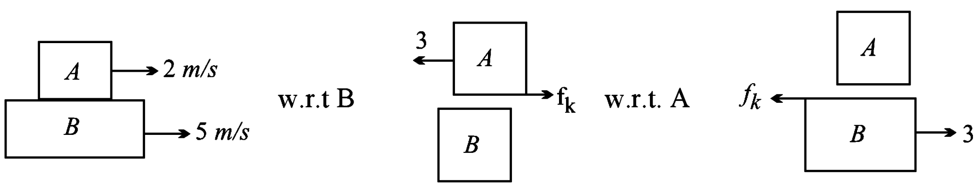


- Magnitude of Kinetic friction is always constant $f_k = \mu_k N$

  Where, μ_k = co-efficient of kinetic friction between the contact surface.

  N = Normal reaction between the contact surface.

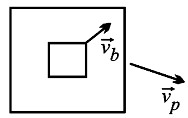
  (a) Direction of kinetic friction acts always oppositively to the relative velocity.



CONCEPT

Kinetic friction oppose relative velocity and vectorial idea of friction.

Example 1. Find unit vector in direction of friction force acting on block

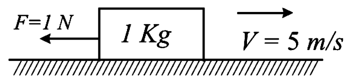


$$\vec{v}_P = 7\hat{i} - 2\hat{j}\ ,\ \vec{v}_B = 3\hat{i} + \hat{j}$$

Solution $\vec{v}_{B/P} = 3\hat{i} + \hat{j} - (7\hat{i} - 2\hat{j})$

on B b/c of p $\hat{f}_k = -\hat{v}_{B/P} = -\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$

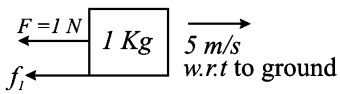
Example 2. Find the direction of kinetic friction force



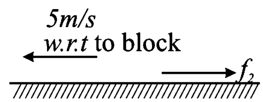
(a) on the block, exerted by the ground.

(b) on the ground, exerted by the block.

Solution (a)



(b)



where f_1 and f_2 are the friction forces on the block and ground respectively.

Example 3. The correct relation between magnitude of f_1 and f_2 is

(A) $f_1 > f_2$ (B) $f_2 > f_1$

(C) $f_1 = f_2$ (D) not possible to decide due to insufficient data.

Solution By Newton's third law the above friction forces are action-reaction pair and equal but opposite to each other in direction. Hence (C).

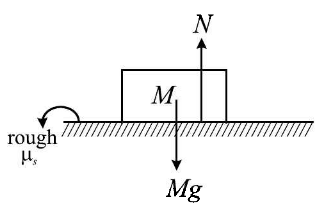
Also note that the direction of kinetic friction has nothing to do with applied force F.

Static Friction

It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surface. If two contact surfaces are relatively at rest then to break away the interlocking some minimum force is required that force is known as static friction force.

For example, consider a bed inside a room ; when we gently push the bed with a finger, the bed does not move. This means that the bed has a tendency to move in the direction of applied force but does not move as there exists static friction force acting in the opposite direction of the applied force.

Example 4. What is value of static friction force on the block?



Solution In horizontal direction as acceleration is zero.

Therefore $\Sigma F = 0$. $\therefore f = 0$

Direction of Static Friction Force

The static friction force on an object is opposite to its impending motion relative to the surface.

The direction and magnitude both are self adjusting such that relative motion is opposed.

Following steps should be followed in determining the direction of static friction force on an object.

(i) Draw the free body diagram with respect to the other object on which it is kept.

(ii) Include pseudo force also if contact surface is accelerating.

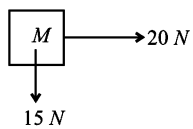
(iii) Decide the resultant force and the component parallel to the surface of this resultant force.

(iv) The direction of static friction is opposite to the above component of resultant force.

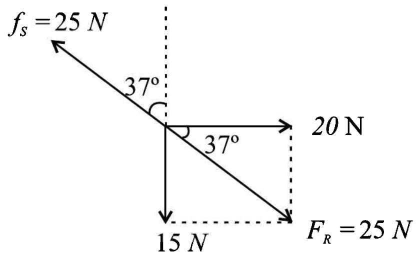
NOTE

Here once again the static friction is involved when there is no relative motion between two surfaces.

Example 5. In the following figure an object of mass M is kept on a rough table as seen from above. Forces are applied on it as shown. Find the direction of static friction if the object does not move.



Solution In the above problem we first draw the free body diagram to find the resultant force.

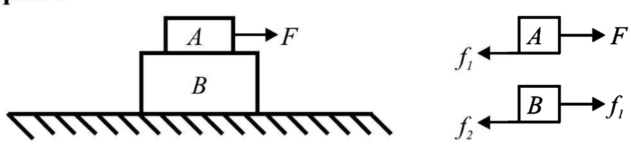


As the object does not move, this is not a case of limiting friction. The direction of static friction is opposite to the direction of the resultant force F_R as shown in figure by f_s. Its magnitude is equal to 25 N.

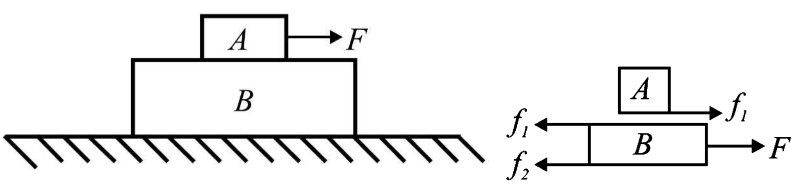
When only one force acting on the object, then direction of static friction opposes the primary cause of its motion.

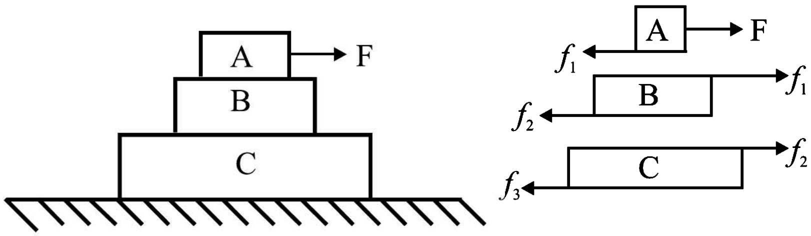
Examples :

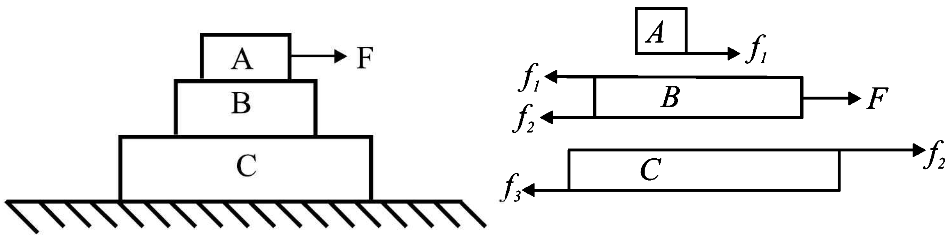
1.

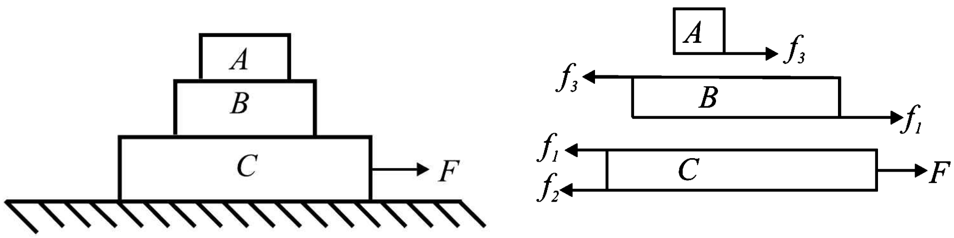


2.

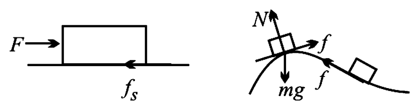


3. 

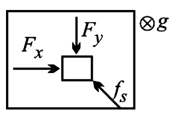
4. 

5. 

Example 6. It acts always tangentially to the contact surface.



Example 7. This example is to show that friction acts against the tendency of relative motion.



$f_s = \sqrt{F_x^2 + F_y^2}$

Important Points about Static Friction Force:

1. Static friction always opposes the primary cause of its motion.
2. It is variable and self adjusting force.
3. Magnitue of static friction (fs) lies between o and $\mu_s N$.

$$0 \le fs \le \mu_s N$$

- Minimum value of static friction = 0
- Maximum value of static friction = $\mu_s N$

Which is known as limitig friction force, it acts on the object when object just begins to slide.

Where, μ_s = co-efficient of static friction force

N = Normal reaction between the contact surface.

MAGNITUDE OF KINETIC AND STATIC FRICTION

Kinetic friction: The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k N$$

where N is the normal force. The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact. If the surfaces are smooth μ_k will be small, if the surfaces are rough μ_k will be large. It also depends on the materials of the two bodies in contact.

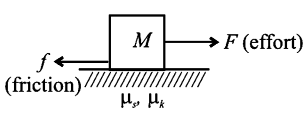
Static friction: The magnitude of static friction is equal and opposite to the external force exerted, till the object at which force is exerted is at rest. This means it is a variable and self adjusting force. However it has a maximum value called limiting friction.

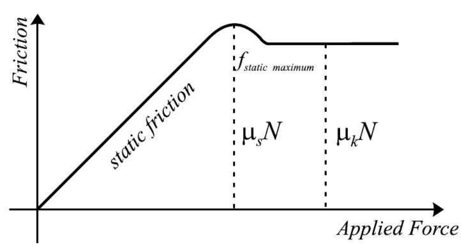
$$f_{max} = \mu_s N$$

The actual force of static friction may be smaller than $\mu_s N$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest.

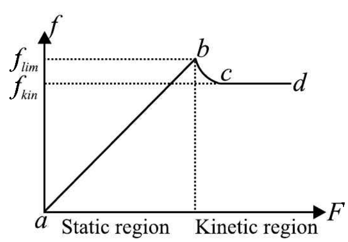
$$0 \leq f_s \leq f_{smax}$$

Here μ_s and μ_k are proportionality constants. μ_s is called coefficient of static friction and μ_k is called coefficient of kinetic friction. They are dimensionless quantities independent of shape and area of contact . It is a property of the two contact surfaces. $\mu_s > \mu_k$ for a given pair of surfaces. If not mentioned then $\mu_s = \mu_k$ can be taken. Value of μ can be from 0 to ∞.





When there is no relative motion between the contact surfaces, frictional force is called static frictional force. It is a self-adjusting force, it adjusts its value according to requirement (of no relative motion). In the taken example static frictional force is equal to applied force. Hence one can say that the portion of graph *ab* will have a slope of 45°. ($f_s \leq \mu_s N$)



Limiting Frictional Force: This frictional force acts when body is about to move. This is the maximum frictional force that can exist at the contact surface. We calculate its value using laws of friction.

Laws of friction : (i) The magnitude of limiting frictional force is proportional to the normal force at the contact surface.

$$f_{\text{lim}} \propto N \Rightarrow f_{\text{lim}} = \mu_s N$$

Here μ_s is a constant the value of which depends on nature of surfaces in contact and is called as 'Coefficient of static friction'. Typical values of μ ranges from 0.05 to 1.5. (ii) The magnitude of limiting frictional force is independent of area of contact between the surfaces.

Table 4.1 gives a rough estimate of the values of coefficient of static friction between certain pairs of materials. The actual value depends on the degree of smoothness and other environmental factors. For example, wood may be prepared at various degrees of smoothness and the friction coefficient will vary.

***Table 4.1:** Values of Coefficient of Static Friction of Materials*

Material	μ_s	Material	μ_s
Steel and steel	0.58	Copper and copper	1.60
Steel and brass	0.35	Teflon and teflon	0.04
Glass and glass	1.00	Rubber tyre on dry concrete road	1.0
Wood and wood	0.35		
Wood and metal	0.40	Rubber tyre on wet concrete road	0.7

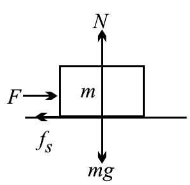
IMPORTANT POINTS

1. Value of μ_k is always less than μ_s ($\mu_k < \mu_s$) from experimental observation.
2. If only coefficient of friction (μ) is given by a problem then $\mu_s - \mu_k = m$ (assumption for)
3. Value of μ_s and μ_k is independent of surface area it depends only on surface properties of contact surface.
4. μ_k is independent of relative speed.
5. μ_s & μ_k are properties of a given pair of surfaces i.e. for wood to wood combination μ_1, then for wood to iron μ_2 and so on.

Magnitude : Maximum strength of the joints formed is directly proportional to the normal contact force because higher the normal contact force higher is the joint strength i.e. $f_{s\,\max} \propto N$

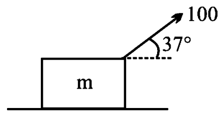
It also depends on the roughness of contact surface $f_{s\,\max}$ (also called f_{limiting}) = $\mu_s N$

Important : Magnitude of static friction is self adjusting such that relative motion do not start. (but still it has maximum value.)

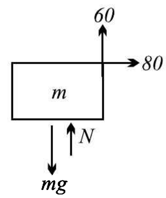


$m = 5\ kg\ /\ \mu_s = 0.1\ \ /\ N = mg$ $\qquad f_{s\,\max} = (0.1)\,N = 5$

1. $F = 1 / f_s \neq 5 / f_s = 1$
2. $F = 3 / f_s \neq 5 / f_s = 3$
3. $f = 6$ / / $f_s = 5$

Example 8. 

$m = 20\ kg$, $\mu_s = 0.5$, find friction on block.



Solution

$N + 60 - mg = 0$

$N = 140$

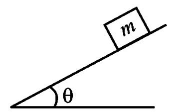
$f_{s\ max} = \mu_s N$ [Here $N \neq mg$]

$f_{s\ max} = 70$, hence answer is 70

CONCEPT

$$f_{max} \neq \mu_s mg \text{ but } \mu_s N$$

Example 9. Find θ at which slipping will start. μ_s is coefficient of static friction. (Angle of repose)



Solution

$N - mg\cos\theta = 0$

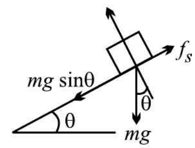
$f_{s\ max} = \mu_s mg\cos\theta$

when slipping starts $f_s = f_{s\ max}$

Thus $mg\sin\theta = \mu\, mg\cos\theta$

$\tan\theta = \mu_s$.

$\tan^{-1}\mu_s$ is called angle of repose.



Example 10. What is friction for θ/2

Solution $f = mg\sin\theta$ [Static friction acts]

CONCEPT

Whenever word contact force is used it means the vector sum of friction & normal contact force and the angle of contact force with normal is known as angle of friction.

Rolling friction : When objects such as a wheel (disc or ring), sphere or a cylinder rolls over a surface, the force of friction that comes play is called rolling friction.

- Rolling friction is directly proportional to the normal reaction (N) and inversely proportional to the radius (r) of the rolling cylinder or wheel.

$$F_{rolling} = \mu_r \frac{N}{r}$$

  μ_r is called coefficient of rolling friction. It would have the dimensions of length and would be measured in metre.

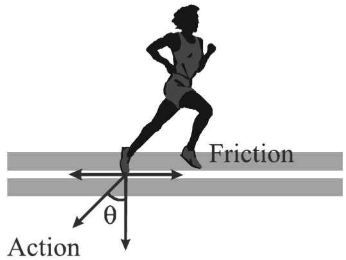
- Rolling friction is often quite small as compared to the sliding friction. That is why heavy loads are transported by placing them on cart with wheels.

- In rolling the surfaces at contact do not rub each other.
- The velocity of point of contact with respect to the surface remains zero all the times although the centre of the wheel moves.

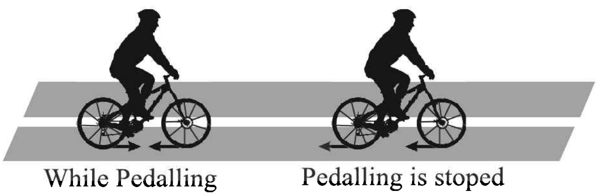
Friction is a Cause of Motion

It is a general misconception that friction always opposes the motion. No doubt friction opposes the motion of a moving body but in many cases it is also the cause of motion. For example:

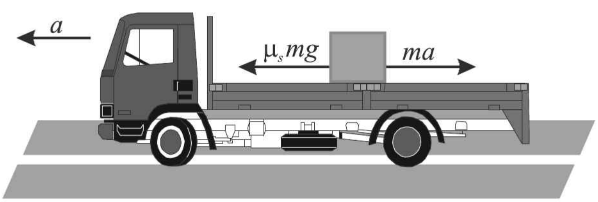
1. While moving, a person or vehicle pushes the ground backwards (action) and the rough surface of ground reacts and exerts a forward force due to friction which causes the motion. If there had been no friction there will be slipping and no motion.



2. During cycling, the rear wheel moves by the force communicated to it by pedalling while front wheel moves by itself. So, when pedalling a bicycle, the force exerted by rear wheel on ground makes force of friction act on it in the forward direction (like walking). Front wheel moving by itself experience force of friction in backward direction (like rolling of a ball). [However, if pedalling is stopped both wheels move by themselves and so experience force of friction in backward direction].



3. If a body is placed in a vehicle which is accelerating, the force of friction is the cause of motion of the body along with the vehicle (i.e., the body will remain at rest in the accelerating vehicle until $ma < \mu_s mg$). If there had been no friction between body and vehicle, the body will not move along with the vehicle.

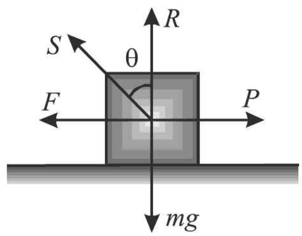


From these examples it is clear that without friction motion cannot be started, stopped or transferred from one body to the other.

Angle of Friction

Angle of friction may be defined as the angle which the resultant of limiting friction and normal reaction makes with the normal reaction.

By definition angle θ is called the angle of friction.



$\tan\theta = \frac{F_{s(max)}}{N}$ $\quad\therefore\ \tan\theta = \mu_s$ $\quad$ [As we know $\frac{F_{s(max)}}{N} = \mu_s$]

or $\theta = \tan^{-1}(\mu_s)$

Hence coefficient of static friction is equal to tangent of the angle of friction.

Resultant Force Exerted by Surface on Block

In the above figure resultant force $S = \sqrt{F^2 + N^2}$

$$S = \sqrt{(\mu mg)^2 + (mg)^2} \qquad S = mg\sqrt{\mu^2 + 1}$$

when there is no friction ($\mu = 0$) S will be minimum i.e., $S = mg$

Hence the range of S can be given by,

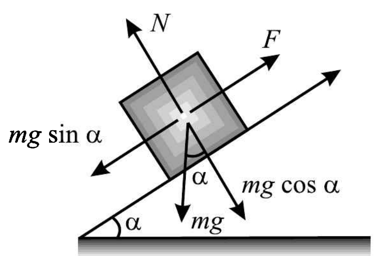
$$mg \le S \le mg\sqrt{\mu^2 + 1}$$

Angle of Respose

Angle of repose is defined as the angle of the inclined plane with horizontal such that a body placed on it is just begins to slide.

By definition, α is called the angle of repose.

In limiting condition $F = mg\sin\alpha$ and $N = mg\cos\alpha$



So $\frac{F}{N} = \tan\alpha$

$\therefore\ \frac{F}{N} = \mu_s = \tan\theta = \tan\alpha$ [As we know $\frac{F}{N} = \mu_s = \tan\theta$]

Thus the coefficient of limiting friction is equal to the tangent of angle of repose.

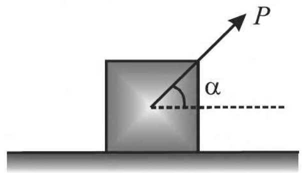
As well as $\alpha = \theta$ i.e., angle of repose = angle of friction.

CALCULATION OF REQUIRED FORCE IN DIFFERENT SITUATION

If W = weight of the body, θ = angle of friction, $\mu = \tan\theta$ = coefficient of friction.

Then we can calculate required force for different situation in the following manner.

1. Minimum pulling force P at an angle α from the horizontal:



By resolving P in horizontal and vertical direction (as shown in fig 4.2)

For the condition of equilibrium

$F = P\cos\alpha$ and $R = W - P\sin\alpha$

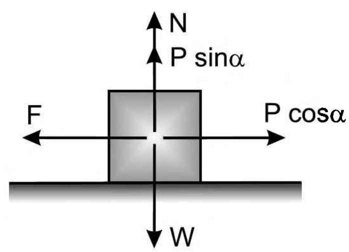


Fig 4.2

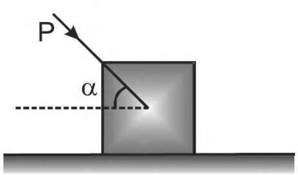
By substituting these value in $F = \mu N$

$$P\cos\alpha = \mu(W - P\sin\alpha)$$

$$\Rightarrow \quad P\cos\alpha = \frac{\sin\theta}{\cos\theta}(W - P\sin\alpha) \qquad [\text{As } \mu = \tan\theta]$$

$$\Rightarrow \quad P = \frac{W\sin\theta}{\cos(\alpha - \theta)}$$

2. Minimum pushing force P at an angle α from the horizontal:



By Resolving P in horizontal and vertical direction (as shown in the fig 4.3).

For the condition of equilibrium

$F = P\cos\alpha$ and $N = W + P\sin\alpha$

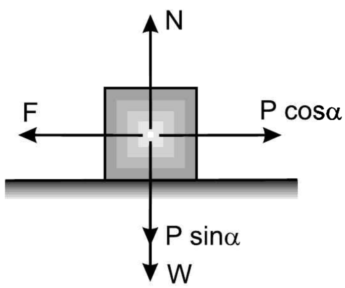


Fig 4.3

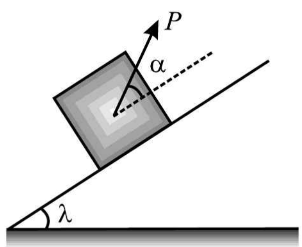
By substituting these value in $F = \mu N$

$\Rightarrow$ $$P \cos \alpha = \mu\,(W + P\sin \alpha)$$

$\Rightarrow$ $$P\cos\alpha = \frac{\sin\theta}{\cos\theta}(W + P\sin\alpha) \quad [\text{As } \mu = \tan\theta]$$

$\Rightarrow$ $$P = \frac{W\sin\theta}{\cos(\alpha+\theta)}$$

3. Minimum pulling force P to move the body up on an inclined plane:



By resolving P in the direction of the plane and perpendicular to the plane (as shown in the fig 4.4)

For the condition of equilibrium

$$N + P\sin\alpha = W\cos\lambda$$

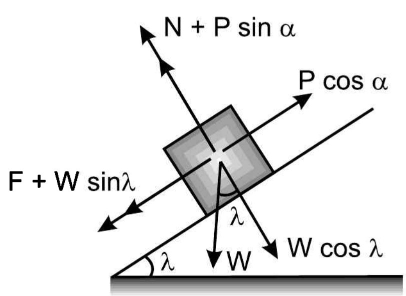


Fig 4.4

$\therefore$ $$N = W\cos\lambda - P\sin\alpha$$

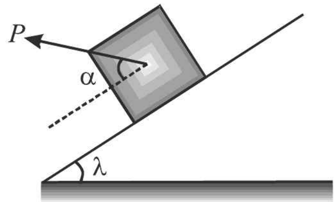
and $F + W \sin\lambda = P\cos\alpha$

$\therefore \qquad F = P\cos\alpha - W\sin\lambda$

By substituting these values in $F = \mu N$ and solving we get

$$P = \frac{W\sin(\theta+\lambda)}{\cos(\alpha-\theta)}$$

4. Minimum force to move a body in downward direction along the surface of inclined plane:



By resolving P in the direction of the plane and perpendicular to the plane (as shown in the fig 4.5).

For the condition of equilibrium

$$N + P\sin\alpha = W\cos\lambda$$

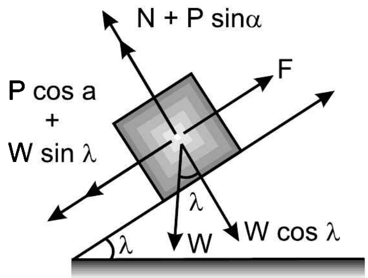


Fig 4.5

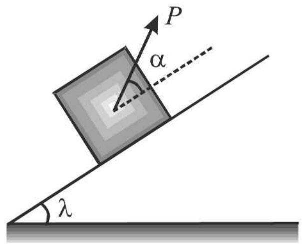
$\therefore \qquad N = W\cos\lambda - P\sin\alpha$

and $\qquad F = P\cos\alpha + W\sin\lambda$

By substituting these values in $F = \mu R$ and solving we get

$$P = \frac{W\sin(\theta-\lambda)}{\cos(\alpha-\theta)}$$

5. Minimum force to avoid sliding of a body down on an inclined plane:



By Resolving P in the direction of the plane and perpendicular to the plane (as shown in the figure).

For the condition of equilibrium

$$N + P \sin \alpha = W \cos \lambda$$

$$\therefore \quad N = W \cos \lambda - P \sin \alpha$$

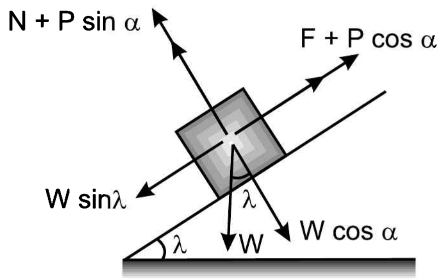


Fig 4.6

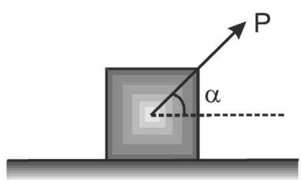
and $\quad P \cos \alpha + F = W \sin \lambda$

$$\therefore \quad F = W \sin \lambda - P \cos \alpha$$

By substituting these values in $F = \mu N$ and solving we get

$$P = W\left[\frac{\sin(\lambda - \theta)}{\cos(\theta + \alpha)}\right]$$

6. Minimum force for motion along horizontal surface and its direction:



Let the force P be applied at an angle α with the horizontal.

By resolving P in horizontal and vertical direction (as shown in fig 4.7).

For vertical equilibrium

$$N + P \sin \alpha = \text{mg}$$

$$\therefore \quad N = mg - P \sin \alpha \qquad \text{...(i)}$$

and for horizontal motion

$$P \cos \alpha \geq F$$

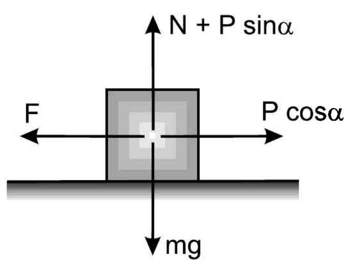


Fig 4.7

i.e., $\quad P \cos \alpha \geq \mu F \qquad \text{...(ii)}$

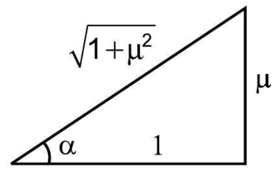
Substituting value of N from (i) in (ii)

$$P \cos \alpha \geq \mu(mg - P \sin \alpha)$$

$$P \geq \frac{\mu mg}{\cos\alpha + \mu \sin\alpha} \quad \text{...(iii)}$$

For the force P to be minimum $(\cos \alpha + \mu \sin \alpha)$ must be maximum i.e.,

$$\frac{d}{d\alpha}[\cos\alpha + \mu \sin\alpha] = 0$$



$\Rightarrow \quad -\sin\alpha + \mu\cos\alpha = 0$

$\therefore \quad \tan\alpha = \mu$

or $\quad \alpha = \tan^{-1}(\mu) =$ *angle* of friction

i.e., For minimum value of P its angle from the horizontal should be equal to angle of friction

As $\tan\alpha = \mu$ so from the figure, $\sin\alpha = \dfrac{\mu}{\sqrt{1+\mu^2}}$

and $\cos\alpha = \dfrac{1}{\sqrt{1+\mu^2}}$

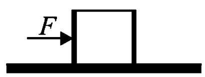
By subsituting these value in equation (iii)

$$P \geq \frac{\mu mg}{\dfrac{1}{\sqrt{1+\mu^2}} + \dfrac{\mu^2}{\sqrt{1+\mu^2}}} \geq \frac{\mu mg}{\sqrt{1+\mu^2}}$$

$$\therefore \quad P_{min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

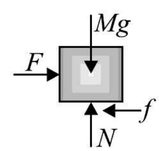
Example 11. A block of mass 5 kg is resting on a rough surface as shown in the figure. It is acted upon by a force of F towards right. Find frictional force acting on block when (a) $F = 5N$, (b) 25 N, (c) 50 N.

($\mu_s = 0.6, \mu_k = 0.5$) ($g = 10$ m/s²)



Solution Maximum value of frictional force that the surface can offer is

$$f_{max} = f_{lim} = \mu_s N = 0.6 \times 5 \times 10 = 30\ N$$



Therefore, if $F \leq f_{max}$ body will be at rest and $f = F$

of $F > f_{max}$ body will more and $f = f_k$

(a) $F = 5N < F_{max}$

So body will not move hence static frictional force will act and,

$f_s = F = \mathbf{5\ N}$

(b) $F = 25\ N < F_{max}$

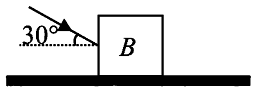
$f_s = \mathbf{25}\ \boldsymbol{N}$

(c) $F = 50\ N > F_{max}$

So body will move and kinetic frictional force will act, it value will be

$$f_k = \mu_k N = 0.5 \times 5 \times 10 = \mathbf{25\ N}$$

Example 12. A block *B* slides with a constant speed on a rough horizontal floor acted upon by a force which is 1.5 times the weight of the block. The line of action *F* makes 30° with the ground. Find the coefficient of friction between the block and the ground.



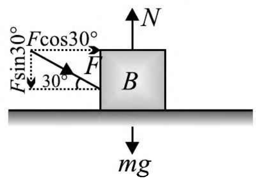
Solution Let *m* be the mass of the block. The weight of the block, then is mg. It is given that $F = 1.5$ mg. *F* can be resolved into two components $F\cos 30°$ parallel to the horizontal floor and $F\sin 30°$ perpendicular to it.

Normal reaction $N = mg + F\sin 30° = mg + \left(1.5mg \times \frac{1}{2}\right) = mg + 0.75mg = 1.75\ mg$

Hence the friction force

$$f = \mu N = \mu \times 1.75,\ mg = 1.75\ \mu\ mg$$

The body moves with constant speed. This means the force $F\cos 30°$ is just able to overcome the frictional force *f*

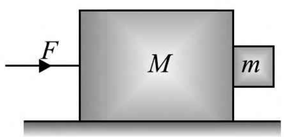


i.e., $f = \mu R = F\cos 30°$

or, $1.75\mu mg = (1.5mg)\frac{\sqrt{3}}{2}$

or, $\mu = \frac{(1.5)\sqrt{3}}{2 \times 1.75} = \mathbf{0.742}$

Example 13. In the adjoining figure, the coefficient of friction between wedge (of mass *M*) and block (of mass *m*) is μ. Find the magnitude of horizontal force *F* required to keep the block stationary with respect to wedge.



Solution Such problems can be solved with or without using the concept of pseudo force.

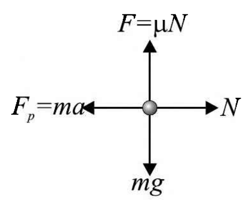
a = acceleration of (wedge + block) in horizontal direction

Non inertial frame of reference (Wedge)

F. B. D. of *m* with respect to wedge

(real + one pseudo force)

with respect to wedge block is stationary.



$\therefore \sum F_x = 0 = \sum F_y$

$\therefore mg = \mu N$ and $N = ma$

$\therefore\ a = \frac{g}{\mu}$ *and* $F = (M+m)a\ =\ (\boldsymbol{M}+\boldsymbol{m})\frac{\boldsymbol{g}}{\boldsymbol{\mu}}$

Example 14. Two blocks A and B of masses 5 kg and 3 kg respectively rest on a smooth horizontal surface with block B resting over A. The coefficient of friction between A and B is 0.5. The maximum horizontal force that can be applied to A so that there will be motion of A and B, without separation is (g = 10 m/s²)

Solution The block A and B move together with acceleration, $a = \frac{F}{m_1 + m_2}$

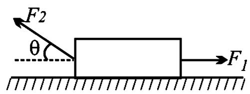
$$a = \frac{F}{8}$$

For mass B, $3a = \mu 3g$

$$a = \mu g = \frac{F}{8}$$

Maximum force required $F = 0.5 \times 8g = \mathbf{40\ N}$

Example 15. A block of mass $m = 3\ kg$ is experiencing two forces acting on it as shown.

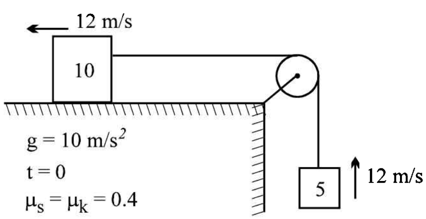


(a) If $F_2 = 20\ N$ and $\theta = 60°$, determine the minimum and maximum values of F_1 so that the block remains at rest. (Take : $\mu_s = 1/\sqrt{3}$ and $\mu_k = 1/3$, $g = 10$ m/s²)

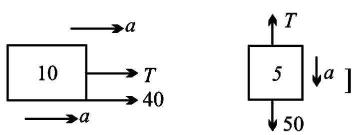
(b) Calculate the magnitude and direction of frictional force on the block if $F_1 = 12\ N$.

[**Ans.** (a) $10\sqrt{3}\ N$, $20 - 10\sqrt{3}\ N$, $2N$, left]

Example 16. Blocks of given velocities as shown at $t = 0$, find velocity and position of 10 kg block at $t = 1$ and $t = 4$.



Solution Making *F.B.D.*



(a) $40 + T = 10a$; $50 - T = 5a$; $a = 6$ m/s²

$u = 12$; $a = -6$

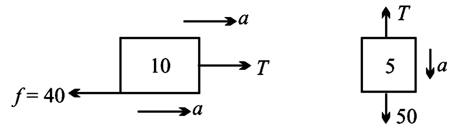
$v = 12 - 6 \times 1 = 6$ m/s; $s = 12 \times 1 - 3 \times 1 = 9$ m

(b) Since velocity has changed the direction during motion, friction would also have changed thus direction and acceleration will change.

$u = 12$; $a = -6$ (till velocity becomes zero)

$v = 0 \quad \Rightarrow \quad t = 2$ sec; $s = 2 \times 2 - 3 \times 4 = 12$ m

now *FBD*



$50 - T = 5a$

$T - 40 = 10a$

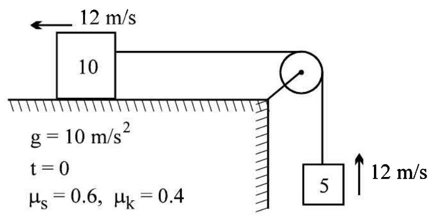
$a = 2/3$ m/s^2

$u = 0, a = 2/3, t = 2, v = 4/3$

$s = \frac{1}{2} \times \frac{2}{3} \times 4 = \frac{4}{3}$; total displacement $12 - \frac{4}{3} = 10\frac{2}{3}$]

CONCEPT

Friction oppose relative velocity not relative acceleration.



Example 17. Find velocity at $t = 4$ sec.

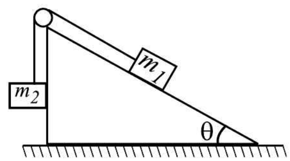
Solution It will come to rest at $t = 2$ sec.

CONCEPT

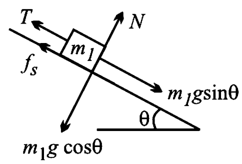
When $\mu_s \neq \mu_k$ then to find relative motion can start or not we must check with μ_s. Once relative motion is started we deal with μ_k to find friction.

Example 18. Find range of m_2 for which m_1 remains at rest.

(given $\theta > \tan^{-1}\mu_s$)



Solution If m_2 is small m_1 will tend to go down the incline for the smallest possible value of m_2, m_1 will experience maximum fricion upwards.



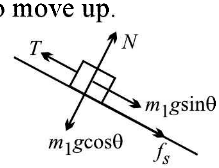
$$T = m_2 g$$

$$T + \mu_s m_1 g \cos\theta - m_1 g \sin\theta = 0$$

$$m_2 = m_1(\sin\theta - \mu_s \cos\theta)$$

Now if m_2 is increased friction on m_1 will decrease as tension is increases.

But it will keep on increasing m_2 then friction will become directed down the incline and for maximum value of m_1 it will be directed downwards and equal to $f_{s\ max}$ because m_1 will tend to move up.



$$T = m_2 g$$

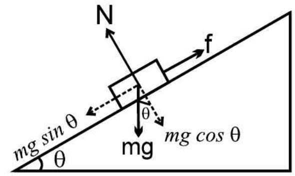
$$m_2 g = m_1 g \sin\theta + \mu_s m_1 g \cos\theta$$

$$m_2 = m_1(\sin\theta + \mu_s \cos\theta)$$

Question: What will change if $\theta < \tan^{-1}\mu_s$

Example 19. The coefficient of static friction between a block of mass m and an incline of angle θ is 0.3. (a) What can be the maximum angle θ of the incline with the horizontal so that the block does not slip on the plane ? (b) If the incline makes an angle $\theta/2$ with the horizontal, find the friction force on the block.

Solution The situation is shown in free body diagram.



(a) The forces acting on the block are

(i) mg, exerted downward by the earth,

(ii) N, normal contact force by the incline, and

(iii) f, friction force f parallel to the incline up the plane, by the incline.

As the block is at rest, these forces should add up to zero. Also, since θ is the maximum angle to prevent slipping, this is a case of limiting equilibrium therefore $f = \mu_s N$

Taking components perpendicular to the incline,

$$N - mg\cos\theta = 0$$

or $$N = mg\cos\theta \quad \text{... (i)}$$

Taking components parallel to the incline,

$$f - mg\sin\theta = 0$$

or $$f = mg\sin\theta$$

or $$\mu_s N = mg\sin\theta. \quad \text{... (ii)}$$

(∵ f can have a maximum value of $\mu_s N$ when θ is increased)

Dividing (ii) by (i) $\mu_s = \tan\theta$

or $\theta = \tan^{-1}\mu_s\theta = \tan^{-1}(0.3)$.

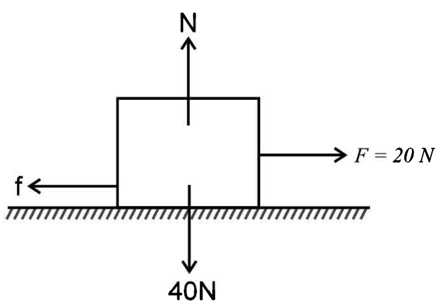
(b) If the angle of incline is θ/2, the equilibrium is not limiting, and hence the force of static friction f is less than $\mu_s N$. To know the value of f, we proceed as in part (a) and get the equations.

$N = mg\cos(\theta/2)$ and $f = mg\sin(\theta/2)$.

Thus, the force of friction is mg sin (θ/2).

***Example* 20.** A horizontal force of 20N is applied to a block of mass 4kg resting on a rough horizontal table. If the block does not move on the table, how much frictional force the table is applying on the block ? What can be said about the coefficient of static friction between the block and the table ? Take $g = 10\ \text{m/s}^2$.

Solution The situation is shown in free body diagram.



The forces on the block are :

(a) 40 N, downward by the Earth,

(b) N, normal force upward by the table,

(c) $F = 20\ N$, applied force,

(d) f, friction force towards left by the table.

As the block is at rest, these forces should add up to zero.

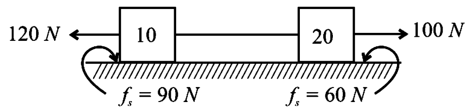
Balancing the forces in horizontal and vertical directions as $a_x = 0$ and $a_y = 0$.

$$f = 20\ N \text{ and } N = 40\ N.$$

Thus, the table exerts a friction (static) force of 20 N on the block in the direction opposite to the applied force. Since there is no relative motion exists hence friction is static.

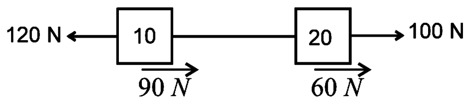
$$f \le \mu_s N, \text{ or, } \mu_s \ge f/N \text{ or, } \mu_s \ge 0.5$$

Example 21. Find the tension in the string in situation as shwon in the figure below. Forces 120 N and 100 N start acting when the system is at rest.



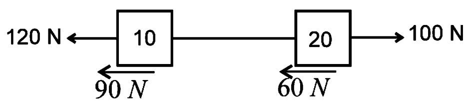
Solution

(i) Let us assume that system moves towards left then as it is clear from *FBD*, net force in horizontal direction is towards right. Therefore the assumption is not valid.



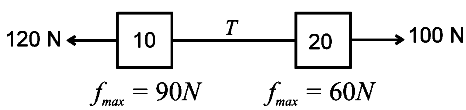
Above assumption is not possible as net force on system comes towards right. Hence system is not moving towards left.

(ii) Similarly let us assume that system moves towards right.

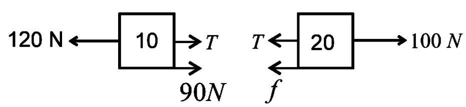


Above assumption is also not possible as net force on the system is towards left in this situation. Hence assumption is again not valid.

Therefore, it can be concluded that the system is stationary.



Assuming that the 10 kg block reaches limiting friction first then using *FBD*'s.



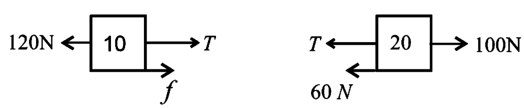
$$120 = T + 90 \quad \Rightarrow \quad T = 30\ N$$

Also $\quad T + f = 100$

$\therefore \quad 30 + f = 100$

$\Rightarrow$ $f = 70\ N$ which is not possible as the limiting value is 60 N for this surface of block.

$\therefore$ Our assumption is wrong and now taking the 20 kg surface to be limiting we have



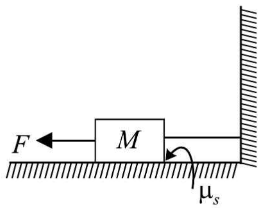
$$T + 60 = 100\,N \Rightarrow T = 40\,N$$

Also $$f + T = 120\,N \Rightarrow f = 80\,N$$

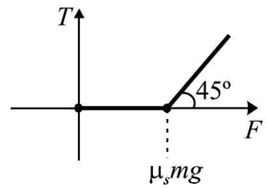
This is acceptable as static friction at this surface should be less than 90 *N*.

Hence the tension in the string is T = 40 *N*.

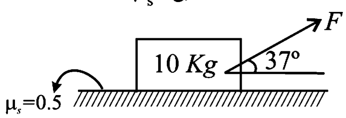
Example 22. In the following figure force *F* is gradually increased from zero. Draw the graph between applied force *F* and tension *T* in the string. The coefficient of static friction between the block and the ground is μ_s.



Solution As the external force *F* is gradually increased from zero it is compensated by the friction and the string bears no tension. When limiting friction is achieved by increasing force *F* to a value till $\mu_s mg$, the further increase in *F* is transferred to the string.

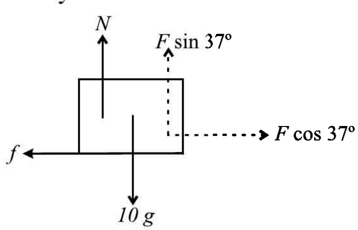


Example 23.



Force *F* is gradually increased from zero. Determine whether the block will first slide or lift up?

Solution



There are minimum magnitude of forces required both in horizontal and vertical direction either to slide on lift up the block. The block will first slide on lift up will depend upon which minimum magnitude of force is lesser.

For vertical direction to start lifting up

$$F \sin 37° + N - Mg \geq 0.$$

N becomes zero just lifting condition.

$$F_{\text{lift}} \geq \frac{10g}{3/5} \therefore F_{\text{lift}} \geq \frac{500}{3} N$$

For horizontal direction to start sliding $\qquad F \cos 37 \geq \mu_s N$

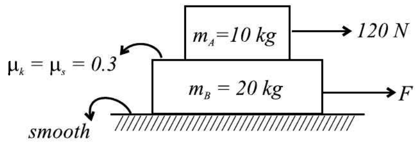
$F \cos 37° > 0.5\,[\,10g - F \sin 37°]$ $\qquad (\because N = 10\,g - F \sin 37°)$

Hence $$F_{\text{slide}} > \frac{50}{\cos 37° + 0.5 \sin 37°}$$

$$F_{\text{slide}} > \frac{500}{11} N \quad \Rightarrow F_{\text{lift}} > \frac{500}{3} N. \qquad \Rightarrow F_{\text{slide}} < F_{\text{lift}}$$

Therefore the block will begin to slide before lifting.

Example 24. In the figure given below force F applied horizontally on lower block, is gradually increased from zero. Discuss the direction and nature of friction force and the accelerations of the block for different values of F (Take $g = 10$ m/s²).



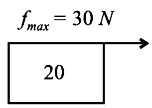
Solution In the above situation we see that the maximum possible value of friction between the blocks is $\mu_s m_A g = 0.3 \times 10 \times 10 = 30\,N$.

Case (i) When $F = O$.

Considering that there is no slipping between the blocks the acceleration of system will be

$$a = \frac{120}{20+10} = 4\ m/s^2$$

But the maximum acceleration of B can be obtained by the following force diagram.

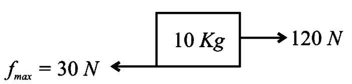


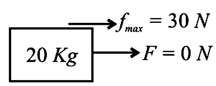
$$a_B = \frac{30}{20} = 1.5\ m/s^2$$

($\because$ only friction force by block A is responsible for producing acceleration in block B)

Because 4 > 1.5 m/s^2 we can conclude that the blocks do not move together.

Now drawing the $F.B.D.$ of each block, for finding out individual accelerations.



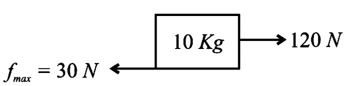


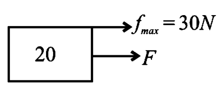
$$a_A = \frac{120-30}{10} = 9\ m/s^2 \text{ towards right}$$

$$a_B = \frac{30}{20} = 1.5\ m/s^2 \text{ towards right.}$$

Case (ii) F is increased from zero till the two blocks just start moving together.

As the two blocks move together the friction is static in nature and its value is limiting. FBD in this case will be





$$a_A = \frac{120-30}{10} = 9\ m/s^2$$

$$a_B = \frac{F+30}{20} = a_A \quad \Rightarrow \quad \frac{F+30}{20} = 9 \quad \therefore\ F = 150\ N$$

Hence, when $0 < F < 150\ N$ the blocks do not move together and the friction is kinetic. As F increases acceleration of block B increases from 1.5 m/s^2.

At $F = 150\ N$ limiting static friction start acting and the two blocks start moving together.

Case (iii) When F is increased above 150 N

In this scenario the static friction adjusts itself so as to keep the blocks moving together. The value of static friction starts reducing but the direction still remains same. This happens continuously till the value of friction becomes zero. In this case the FBD is as follows

$$a_A = a_B = \frac{120 - f}{10} = \frac{F + f}{20}$$

$\therefore$ when friction force f gets reduced to zero the above accelerations become

$$a_A = \frac{120}{10} = 12\ m/s^2$$

$$a_B = \frac{F}{20} = a_A = 12\ m/s^2$$

$$\therefore \qquad F = 240\ N$$

Hence, when $150 \le F \le 240\ N$ the static friction force continuously decreases from maximum to zero at $F = 240\ N$. The accelerations of the blocks increase from 9 m/s^2 to 12 m/s^2 during the change of force F.

Case (iv) When F is increased again from 240 N the direction of friction force on the block reverses but it is still static. F can be increased till this reversed static friction reaches its limiting value. *FBD* at this juncture will be

The blocks move together therefore.

$$a_A = \frac{120 + 30}{10} = 15\ m/s^2$$

$$a_B = \frac{F - 30}{20} = a_A = 15\ m/s^2 \ \therefore\ \frac{F - 30}{20} = 15\ m/s^2$$

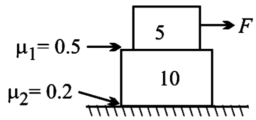
Hence $F = 330\ N$.

Case (v) When F is increased beyond 330 N. In this case the limiting friction is achieved and slipping takes place between the blocks (kinetic friction is involved).

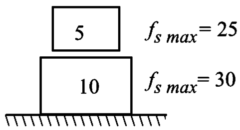
$$\therefore \qquad a_A = 15\ m/s^2 \text{ which is constant}$$

$$a_B = \frac{F - 30}{20}\ m/s^2 \text{ where } F > 330\ N.$$

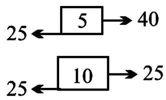
Example 25. Find acceleration of blocks $F = 40\ N$



1. First of all find values of limiting friction at all contact surfaces. ($f_{s\ max}$)

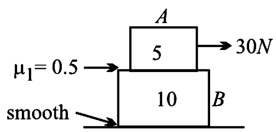


2. Maximum force upper surface of 10 kg can experience is 25 N so it will not make relative to ground.
3. Hence only 5 kg will move

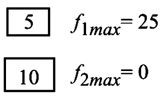


$a_A = 3,\ a_B = 0$

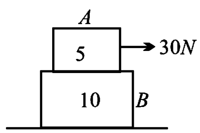
Example 26.



Solution

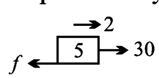
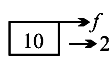


1. 10 kg block must move because some force on upper surface will act on it.
2. B can either move with same velocity and acceleration as A or it can move relative to A
3. Always assume it moves with A and solve



$$a = \frac{30}{15} = 2\ ms^{-2}$$

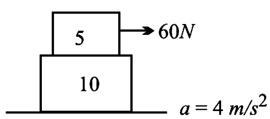
4. Now check if this acceleration is possible by verifying $f \leq f_l$

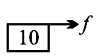
i.e. make *FBD* of  or 

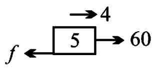
$30 - f = 5 \times 2$ $\qquad f = 10 \times 2 = 20$

$f = 20 < 25$

Example 27.



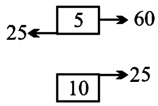
Assuming same acceleration or 



$60 - f = 5 \times 4$ $\quad\quad$ $f = 10 \times 4 = 40$

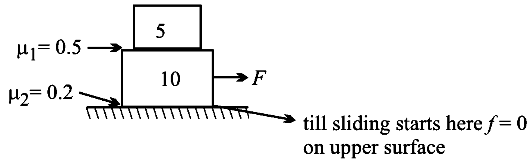
$f = 40 > 25$

hence our assumption is wrong.]



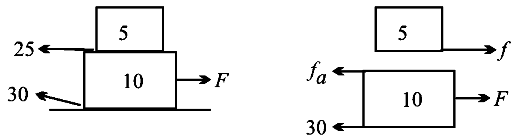
$a_1 = 7, a_2 = 2.5$

Example 28.



Find maximum force for which they can move together.

Solution



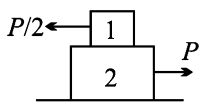
If they are moving together $a_1 = a_2 = a$

$F - f - 30 = 10a$; $\quad$ $f = 5a$; $\quad$ $F - f - 30 = 10 \times \frac{f}{5}$; $\quad$ $F = 30 + 3f$

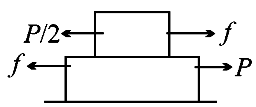
maximum f is 25

$F = 30 + 75$; $\quad$ $F = 105$ **Ans.**

Example 29. Block 1 sits on top of block 2. Both of them have a mass of 1 kg. The coefficients of friction between blocks 1 and 2 are $\mu_s = 0.75$ and $\mu_k = 0.60$. The table is frictionless. A force $P/2$ is applied on block 1 to the left, and force P on block 2 to the right. Find the minimum value of P such that sliding occurs between the two blocks.



Solution



$P - f = 1\,a$

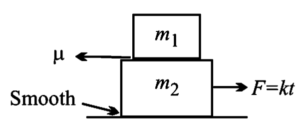
$f - P/2 = 1\,a$

$P/2 = 2a$

$\Rightarrow$ $\quad f = 3P/4 \le \mu_s mg = 0.75 \times 10 = 7.5$

$\Rightarrow$ $\quad P = 10\,N$]

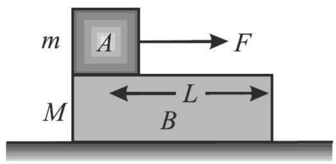
Example 30. Plot a_1 & a_2 as function of time



Example 31. When a body A of mass m is resting on a body B of mass M then two conditions are possible.

1. A force F is applied to the upper body,
2. A force F is applied to the lower body. Discuss the motion of two above cases.

Solution 1. A force F is applied to the upper body, then following four situations are possible.



(i) When there is no friction

(a) The body A will move on body B with acceleration (F/m).

$$a_A = F/m$$

(b) The body B will remain at rest

$$a_B = 0$$

(c) If L is the length of B as shown in figure, A will fall from B after time t

$$t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{F}} \qquad \left[As\ s = \frac{1}{2}at^2\ and\ a = F/m\right]$$

(ii) If friction is present between A and B only and applied force is less than limiting friction $(F < F_{s(max)})$.

(F = Applied force on the upper body, $F_{s(max)}$ = limiting friction between A and B, F_k = Kinetic friction between A and B).

(a) The body A will not slide on body B till $F < F_{s(max)}$ i.e., $F < \mu_s mg$

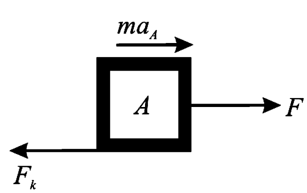
(b) Combined system $(m + M)$ will move together with common acceleration

$$a_A = a_B = \frac{F}{M+m}.$$

(iii) If friction is present between A and B only and applied force is greater than limiting friction $(F > F_{s(max)})$.

In this condition the two bodies will move in the same direction (i.e. of applied force) but with different acceleration. Here force of kinetic friction $\mu_k mg$ will oppose the motion of A while cause the motion of B.

Free body diagram of A



$$F - F_k = ma_A$$

i.e. $$a_A = \frac{F - F_k}{m}$$

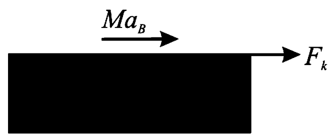
$$a_A = \frac{(F - \mu_k mg)}{m}$$

$$F_k = Ma_B$$

i.e. $a_B = \frac{F_k}{M}$

$\therefore$ $a_B = \frac{\mu_k mg}{M}$

Free body diagram of B



As both the bodies are moving in the same direction.

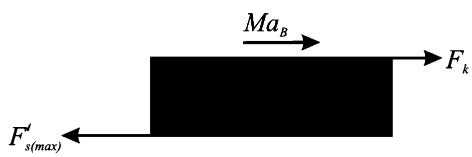
Acceleration of body A relative to B will be $a = a_A - a_B = \frac{MF - \mu_k mg(m+M)}{mM}$

So, A will fall from B after time $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mML}{MF - \mu_k mg\,(m+M)}}$

(iv) If there is friction between B and floor :

(Where $F'_{s\,(max)} = \mu'(M+m)g$ = limiting friction between B and floor, F_k = kinetic friction between A and B)

B will move only if $F_k > F'_{s(max)}$ and then $F_k - F'_{s(max)} = Ma_B$



However if B does not move then static friction will work (not limiting friction) between body B and the floor i.e. friction force = applied force (= F_k) not $F'_{s\,(max)}$.

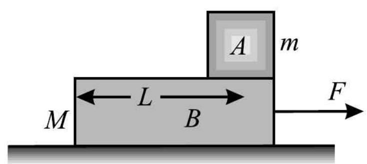
2. A force F is applied to the lower body, then following four situations are possible

(i) When there is no friction

(a) B will move with acceleration (F/M) while A will remain at rest (relative to ground) as there is no pulling force on A.

$$a_B = \left(\frac{F}{M}\right) \text{ and } a_A = 0$$

(b) As relative to B, A will move backwards with acceleration (F/M) and so will fall from it in time t.



$$\therefore \quad t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2ML}{F}}$$

(ii) If friction is present between A and B only and $F' < F_{s(max)}$.

(where F' = Pseudo force on body A and $F_{s(max)}$ = limiting friction between body A and B)

(a) Both the body will move together with common acceleration $a=\dfrac{F}{M+m}$

(b) Pseudo force on the body A,

$$F'=ma=\frac{mF}{m+M} \text{ and } F_{s\,(max)}=\mu_s mg$$

(c) $F'<F_{s(max)} \Rightarrow \dfrac{mF}{m+M}<\mu_s mg \Rightarrow F<\mu_s(m+M)g$

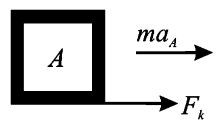
So both bodies will move together with acceleration $a_A=a_B=\dfrac{F}{m+M}$ if $F<\mu_s[m+M]g$

(iii) If friction is present between A and B only and $F<F'_{s(max)}$.

(where $F'_{s\,(max)}=\mu_s mg$ = limiting friction between body A and B)

Both the body will move with different acceleration. Here force of kinetic friction $\mu_k mg$ will oppose the motion of B while will cause the motion of A.

Free body diagram of A



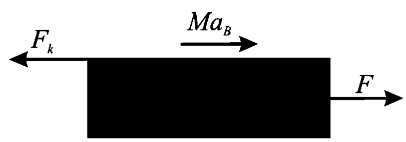
$$ma_A=\mu_k mg$$

i.e. $a_A=\mu_k g$

$$F-F_k=Ma_B$$

i.e. $aB=\dfrac{[F-\mu_k mg]}{M}$

Free body diagram of B



As both the bodies are moving in the same direction.

Acceleration of body A relative to B will be

$$a=a_A-a_B=\left[\frac{F-\mu_k g(m+M)}{M}\right]$$

Negative sign implies that relative to B, A will move backwards and fall it after time

$$t=\sqrt{\frac{2L}{a}}=\sqrt{\frac{2ML}{F-\mu_k g(m+M)}}$$

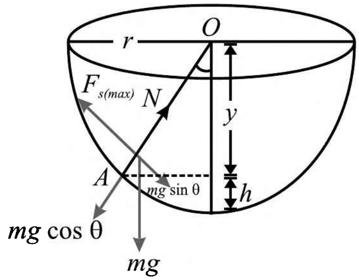
(iv) If there is friction between B and floor and $F>F''_{s(max)}$:

(where $F''_{s\,(max)}=\mu_s(m+M)g$ = limiting friction between body B and surface).

The system will move only if $F>F''_{s\,(max)}$ then replacing F by $F-F''_{s\,(max)}$. The entire case (iii) will be valid.

However if $F>F''_{s\,(max)}$ the system will not move and friction between B and floor will be F while between A and B is zero.

Example **32.** The insect crawling up the inside the rough bowl of radius r, Find the maximum height (h) up to which the insect can crawl.



Let m = mass of the insect, r = radius of the bowl, μ = coefficient of friction

for limiting condition at point A

$N = mg\cos\theta$ ...(i)

and $F_{s(max)} = mg\sin\theta$

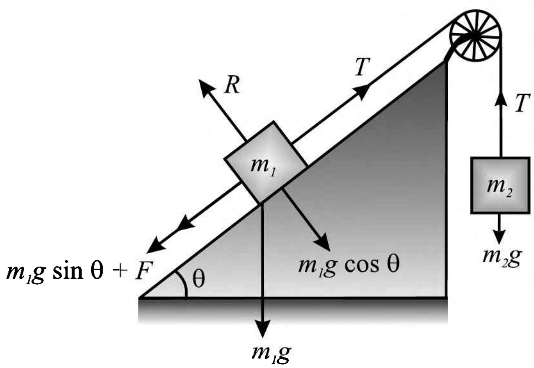
Dividing (ii) by (i)

$$\tan\theta = \frac{F_{s\,(max)}}{N} = \mu \qquad [\text{As } F_{s\,(max)} = \mu N]$$

$$\therefore \frac{\sqrt{r^2 - y^2}}{y} = \mu \quad or \quad y = \frac{r}{\sqrt{1+\mu^2}}$$

$$\text{So, } h = r - y = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right], \therefore h = r\left[1 - \frac{1}{\sqrt{1+\mu^2}}\right]$$

Example 33. A mass m_1 is placed on rough inclined plane and connected with another mass m_2 via a string passing over mass less pulley. Find the minimum value of m_2 so that it start to move.



Solution At limiting condition

For m_2 $T = m_2 g$ ...(i)

For m_1 $T = m_1 g\sin\theta + F$

$\Rightarrow$ $T = m_1 g\sin\theta + \mu R$

$\Rightarrow$ $T = m_1 g\sin\theta + \mu m_1 g\cos\theta$ ...(ii)

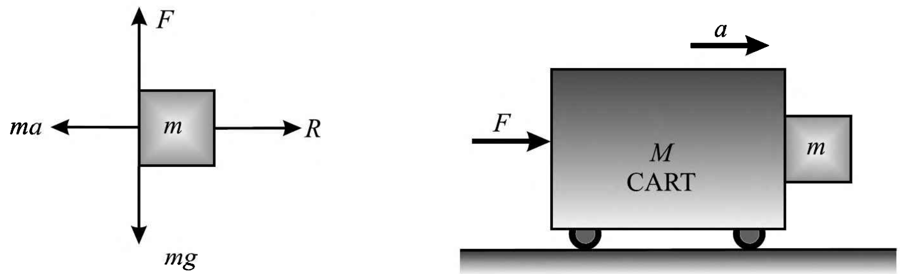
From equation (i) and (ii) $m_2 = m_1$ $[\sin\theta + \mu\cos\theta]$ this is the minimum value of m_2 to start the motion

In the above condition coefficient of friction

$$\mu = \left[\frac{m_2}{m_1\cos\theta} - \tan\theta\right]$$

Example 34. Find the minimum force applied on the cart so that small block (m) remains stationary with respect to cart.

Solution When a cart moves with some acceleration toward right then a pseudo force (ma) acts on block toward left. This force (ma) is action force by a block on cart.



Now block will remain static w.r.t. cart. If friction force $\mu R \geq mg$

$\Rightarrow \quad \mu ma \geq mg \qquad$ [As $R = ma$]

$\Rightarrow \quad a \geq \dfrac{g}{\mu} \qquad \therefore \quad a_{\min} = \dfrac{g}{\mu}$

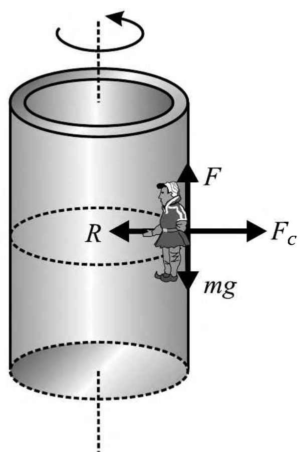
This is the minimum acceleration of the cart so that block does not fall and the minimum force to hold the block together

$$F_{\min} = (M+m)a_{\min}$$

$$F_{\min} = (M+m)\frac{g}{\mu}$$

Example 35. A person with a mass m stands in contact against the wall of a cylindrical drum (rotor). The coefficient of friction between the wall and the clothing is μ. Find the minimum value of $\omega_{\min}$. So that revision remains stationary.

Solution If Rotor starts rotating about its axis, then person thrown away from the center due to centrifugal force at a particulars speed ω, the person stuck to the wall even the floor is removed, because friction force balances its weight in this condition. From the figure.



Friction force (F) – Weight of person (mg)

$\Rightarrow \quad \mu R = mg \Rightarrow \mu F_C = mg$

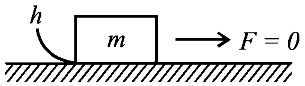
[Here, F_C = centrifugal force]

$\Rightarrow \quad \mu m\omega_{\min}^2 r = mg$

$$\therefore \quad \omega_{\min} = \sqrt{\frac{g}{\mu r}}$$

Example 36. A block of mass m is kept on a horizontal table. If the static friction coefficient is h, find the frictional force acting on the block?

Solution We know that static frictional force is variable force and varies from 0 to ($f_s$max = hN) depending upon the applied force. Since there is no applied force acting on the block. Hence;



$$F = 0$$

$$N = mg$$

$$\Rightarrow \quad f_s = 0$$

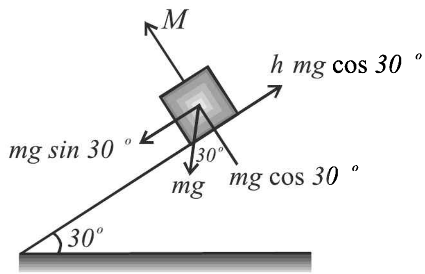
Example 37. A block slides down an inclined plane of inclination 30° with the horizontal. Starting from rest it covers 8 m in the first two seconds. Find the coefficient of kinetic friction between the two.

Solution Net acceleration of the block

$$a = \frac{mg\sin 30° - \mu mg\cos 30°}{m} = g\left(\frac{1}{2} - \frac{h\sqrt{3}}{2}\right)$$

$$a = \frac{10}{2}\left(1 - h\sqrt{3}\right)$$

$$a = 5\left(1 - \sqrt{3}h\right)$$



Since it covers 8 m in the first two seconds;

$$S = 0 + \frac{1}{2}at^2$$

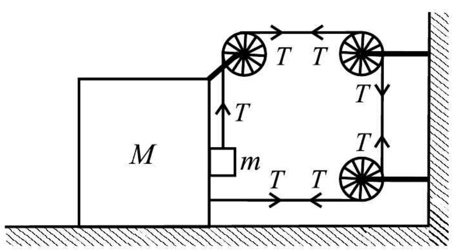
$$8 = \frac{1}{2} \times 5\left(1 - \sqrt{3}h\right) \times 2 \times 2$$

$$1 - \sqrt{3}h = 0.8$$

$$h = \frac{0.2}{\sqrt{3}}$$

$h = 0.11$ **[Ans.]**

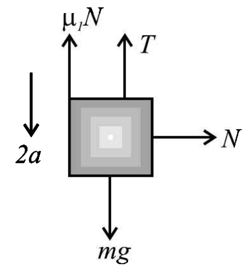
Example 38. Find the acceleration of the block of mass M in the situation of figure. The coefficient of friction between the two blocks is μ_1 and that between the bigger block and the ground is μ_2.



Solution If the bigger block (M) moves a distance x in right side the smaller block moves distance $2x$ in downward direction. Hence we can say that the acceleration of m is twice that of bigger mass (M). Let the acceleration of bigger block is a.

From *F.B.D.* diagram of smaller block (m) Vertical motion

$$mg-(\mu_1 N+T)=2ma \quad ...(i)$$



F.B.D of smaller block *(m)*

Horizontal motion

$$N=ma \quad ...(ii)$$

Combining equation (i) and (ii)

$$mg-\mu_1 ma-T=2ma$$

$$\Rightarrow \quad T=mg-\mu_1 ma-2ma \quad ...(iii)$$

F.B.D. of bigger block Vertical motion

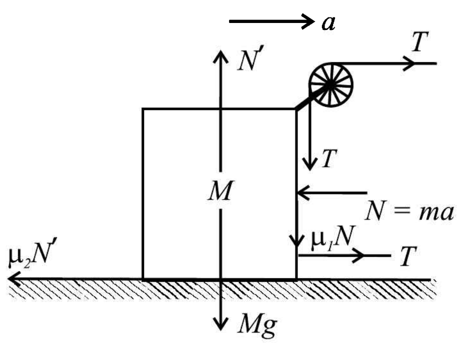
$$N'=Mg+T+\mu_1 N$$

$$N'=Mg+T+\mu_1 ma \quad ...(iv)$$

Horizontal motion

$$2T-(\mu_2 N'+N)=Ma$$

$$\Rightarrow \quad 2T-\mu_2 N'-ma=Ma$$



Putting the value of N' from equation (iv)

$$\Rightarrow \quad 2T-\mu_2(Mg+T+\mu_1 ma)-ma=Ma \quad ...(v)$$

$$\Rightarrow \quad 2T-\mu_2 Mg-\mu_2 T-\mu_1\mu_2 ma-ma=Ma$$

$$\Rightarrow \quad (2-\mu_2)T-\mu_2 Mg-\mu_1\mu_2 ma-ma=Ma$$

Combining equation (iii) and (v)

$$(2+\mu_2)(mg-\mu_1 ma-2ma)-\mu_2 Mg-\mu_1\mu_2 ma-ma=Ma$$

$$\Rightarrow \quad 2mg-2\mu_1 ma-4ma-\mu_2 mg+\mu_1\mu_2 ma+2\mu_2 ma-\mu_2 Mg-\mu_1\mu_2 ma=Ma-ma$$

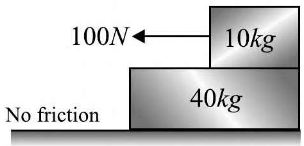
$$\Rightarrow \quad 2mg-2ma(\mu_1-\mu_2)-\mu_2 g(M+m)-5ma=Ma$$

$$\Rightarrow \quad 2mg-\mu_2 g(M+m)=Ma+5ma+2ma(\mu_1-\mu_2)$$

$$\Rightarrow \quad a[M+5m\,(5+2(\mu_1-\mu_2)]=[2m-\mu_2(M+m)]g$$

$$\Rightarrow \quad \boxed{a=\frac{2m-\mu_2(M+m)g}{M+m[5+2(\mu_1-\mu_2)]}}$$

Example 39. A 40 kg slab (B) rests on a smooth floor as shown in figure. A 10 kg block (A) rests on the top of the slab. The static coefficient of friction between slab and block is 0.6 while the kinetic friction coefficient is 0.4. The block (A) is acted upon by a horizontal force 100 N. If $g = 9.8$ m/s^2, the resulting acceleration of the slab (B) will be



(A) 0.98 m/s² (B) 1.47 m/s²
(C) 1.52 m/s² (D) 6.1 m/s²

Solution For a force of 100 N on 10 kg block, relative motion will take place.

∴ The frictional force between 10 kg block and 40 kg block,

$f = \mu mg = 0.4 \times 10\ 0 \times 9.8\ N$

The acceleration of the slab of 40 kg is

$$a = \frac{0.4 \times 10 \times 9.8}{40} = 0.98\ m/s^2$$

∴ **(A)** is the correct answer.

Example 40. A mass of .5 kg is just able to slide down the slope of an inclined rough surface when the angle of inclination is 60°. The minimum force necessary to pull the mass up the incline along the line of greatest slope is (g = 10 m/s²)

(A) 20.25 N (B) 8.66 N
(C) 100 N (D) 1 N

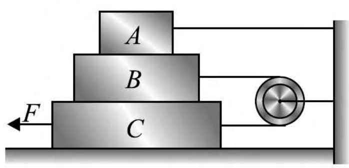
Example Acceleration down the plane = $g(\sin\alpha - \mu\cos\alpha) = 0$

So, it is given $\tan\alpha = \tan 60° = \mu = \sqrt{3}$

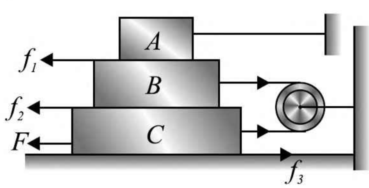
Minimum force necessary = $mg(\sin\alpha + \mu\cos\alpha) = 0.5 \times 10\left(\frac{\sqrt{3}}{2} + \sqrt{3}\frac{1}{2}\right) = 5\sqrt{3} = 8.66N$

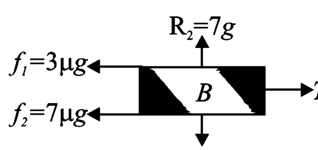
∴ **(B)** is the correct answer.

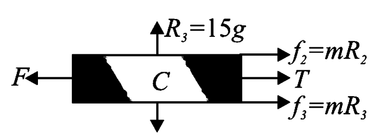
Example 41. In the figure shown, blocks A, B and C weigh 3 kg, 4 kg and 8 kg respectively. The coefficient of sliding friction between any two surfaces is 0.25. A is held at rest by a massless rigid rod fixed to the wall while B and C are connected by a string passing round a frictionless pulley. Find the force needed to drag C along the horizontal surface to left at constant speed. Assume the arrangement shown in figure is maintained all through.



Solution







The free body diagram of B and C are separately shown in figures.

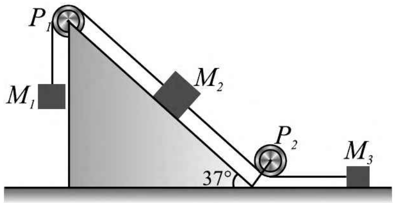
$$T = f_1 + f_2 = 3\mu g + 7\mu g = 10\ \mu g = 10 \times 0.25 \times 9.8 = 24.5\ N$$

Now $F = f_2 + f_3 + T = \mu.7g + \mu.15g + 10\ \mu g$

$$= 0.25 \times 32 \times 9.8 = \mathbf{78.4\ N}$$

Example 42. Masses M_1, M_2 and M_3 are connected by light strings which pass over pulleys P_1 and P_2 as shown. The masses move such that the string between P_1 and P_2 is parallel to incline and the string between P_2 and M_3 is horizontal, $M_2 = M_3 = 4$kg. The coefficient of kinetic friction between masses and the surface is 0.25. The angle of inclination of plane is 37° to the horizontal. If the mass M_1 moves downwards with uniform velocity, find M_1 and the tension in the horizontal string.

Given $g = 9.8$ m/s² and sin37° = 3/5.



Solution Considering the mass M_1, since it moves down with uniform speed

$$T_1 = M_1 g$$

For mass M_2, $T_1 + T_2 + \mu M_2 g\cos 37° + M_2 g\sin 37°$

For mass M_3, $T_2 = \mu M_3 g$

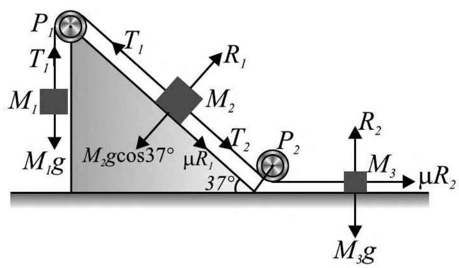
Substituting for T_1 and T_2 from (i) and (ii) respectively in (ii),

$$M_1 g = \mu M_3 g + \mu M_2 g\cos 37° + M_2 g\sin 37°$$

$$M_1 = \mu M_3 + \mu M_2 \cos 37° + M_2 \sin 37°$$

$$\cos 37° = \frac{4}{5};\ \sin 37° = \frac{3}{5};\ \mu = \frac{1}{4}$$

$$\therefore\ M_1 = \frac{1}{4} \times 4 + \frac{1}{4} \times 4 \times \frac{4}{5} + 4 \times \frac{3}{5} = \mathbf{4.2kg}$$



$$T_2 = \mu M_3 g = \frac{1}{4} \times 4 \times 9.8 = \mathbf{9.8\ N}$$

Example 43. A given object taken h times as much time to slide down a 45° rough incline as it takes to slide down a perfectly smooth 45°. The coefficient of kinetic friction between object and incline is given by

(A) $\mu = \dfrac{1}{1 - \eta^2}$ (B) $\mu = 1 - \dfrac{1}{\eta^2}$ (C) $\mu = \sqrt{\dfrac{1}{1 - \eta^2}}$ (D) $\mu = \sqrt{1 - \dfrac{1}{\eta^2}}$

[Ans. (*A*)]

Solution Acceleration without friction $= g\sin\alpha = \dfrac{g}{\sqrt{2}}$

With friction, the acceleration is $g(\sin\alpha - \mu\cos\alpha) = \dfrac{g}{\sqrt{2}}(1 - \mu)$

Since the body starts from rest, the distance is equal to $\frac{1}{2}at^2$.

Thus, $S = \frac{g}{\sqrt{2}}t_1^2 = \frac{g}{\sqrt{2}}(1-\mu)t_2^2$;

Given $t_2 = \eta t_1 \frac{1-\mu}{1} = \frac{t_1^2}{t_2^2} = \frac{1}{\eta^2}$

$\mu = \left(1 - \frac{1}{\eta^2}\right)$

∴ **(*A*)** is the right answer.

Example 44. Two blocks *A* and *B* of masses 5 kg and 3 kg respectively rest on a smooth horizontal surface with block *B* resting over 4. The coefficient of friction between *A* and *B* is 0.5. The maximum horizontal force that can be applied to *A* so that there will be motion of *A* and *B*, without separation is

(g = 10 m/s²)

(A) 15 *N* (B) 25 *N*
(C) 40 *N* (D) 50 *N*

[Ans. (C)]

Solution The blocks *A* and *B* move together with acceleration, $a = \frac{F}{m_1 + m_2}$

$a = \frac{F}{8}$

For mass *B*, $3a = \mu 3g$

$a = \mu g = \frac{F}{8}$

Maximum force required $F = 0.5 \times 8\,g = 40\,N$

∴ **(*C*)** is the right answer.

Example 45. A mass of 0.5 kg is just able to slide down the slope of an inclined rough surface when the angle of inclination is 60°. The minimum force necessary to pull the mass up the incline along the line of greatest slope is (g = 10 m/s²)

(A) 20.25*N* (B) 8.65*N*
(C) 100*N* (D) 1*N*

[Ans. (*B*)]

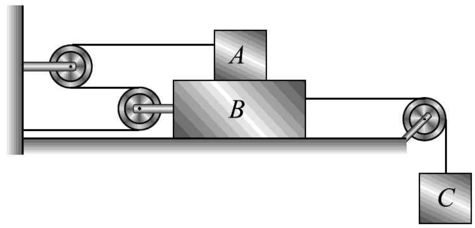
Solution Acceleration down the plane = $g(\sin\alpha - \mu\cos\alpha) = 0$

So, it is given that $\tan\alpha = \tan 60° = \mu = \sqrt{3}$

Minimum force necessary = $mg(\sin\alpha + \mu\cos\alpha) = 0.5 \times 10\left(\frac{\sqrt{3}}{2} + \sqrt{3}\frac{1}{2}\right) = 5\sqrt{3} = 8.66N$

∴ **(B)** is the right answer.

Example 46. The maximum value of mass of block C so that neither A nor B moves is (Given that mass of A is 100 kg and that of B is 140 kg. Pulleys are smooth and friction coefficient between A and B and between B and horizontal surface is $\mu = 0.3$) $g = 10\ m/s^2$



(A) 210 *kg* (B) 190 *kg*
(C) 185 *kg* (D) 162 *kg*

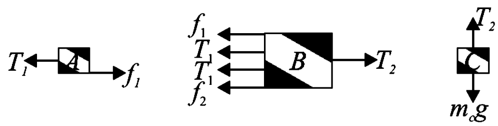
Solution Maximum friction that can be obtained between A and B is

$$f_1 = \mu m_A g = (0.3)(100)(10) = 300N$$

and maximum friction between B and ground is

$$f_2 = \mu(m_A + m_B)g = (0.3)(100 + 140)(10) = 720\ N$$

Drawing free body diagrams of A, B and C in limiting case



Equilibrium of A gives

$$T_1 = f_1 = 300N \qquad ...(1)$$

Equilibrium of B given

$$2T_1 + f_1 + f_2 = 0$$

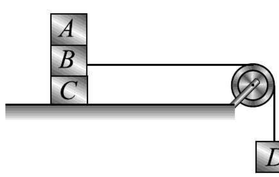
or $$T_2 = 2(300) + 300 + 720 = 1620\ N \qquad ...(2)$$

and equilibrium of C given

$$m_C g = T_2 \qquad \text{or}\ \ 10m_C = 1620 \qquad \text{or} \qquad m_C = 162\ \text{kg}$$

∴ **(D)** is the right answer.

Example 47. Three blocks A, B and C of equal mass m are placed one over the other on a smooth horizontal ground as shown in figure. Coefficient of friction between any two blocks of A, B and C is 1/2. The maximum value of mass of block D so that the blocks A, B and C move without slipping over each other is



(A) 6m (B) 5m
(C) 3m (D) 4m

Solution Blocks *A* and *C* both move due to friction. But less friction is available to *A* as compared to *C* because normal reaction between *A* and *B* is less. Maximum friction between *A* and *B* can be :

$$f_{max} = \mu m_A g = \left(\frac{1}{2}\right) mg$$

∴ Maximum acceleration of *A* can be :

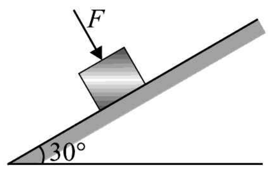
$$a_{max} = \frac{f_{max}}{m} = \frac{g}{2}$$

further $a_{max} = \frac{m_D g}{3m + m_D}$

or $\frac{g}{2} = \frac{m_D g}{3m + m_D}$

∴ **(C)** is the right answer.

Example 48. A block of mass $m = 2$ kg is resting on a rough inclined plane of inclination 30° as shown in figure. The coefficient of friction between the block and the plane is $m = 0.5$. What minimum force *F* should be applied perpendicular to the plane on the block, so that block does not slip on the plane ($g = 10\text{m}/^2$)



(A) zero (B) 6.24 *N*
(C) 2.68 *N* (D) 4.34 *N*

Solution Since $mg\sin 30° > \mu mg\cos 30°$

the block has a tendency to slip downwards. Let *F* be the minimum force applied on it, so that it does not slip. Then

$N = F + mg\cos 30°$

∴ $mg\sin 30° = \mu N = \mu(F + mg\cos 30°)$

or $F = \frac{mg\sin 30°}{\mu} - mg\cos 30° = \frac{(2)(10)(1/2)}{0.5} - (2)(10)\left(\frac{\sqrt{3}}{2}\right)$

or $F = 20 - 17.32 = 2.68\ N$

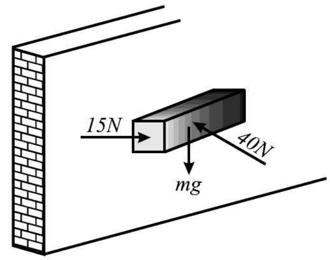
∴ **(C)** is the right answer.

Example 49. A block of mass 2kg is pushed against a rough vertical wall with a force of 40*N*, coefficient of static friction being 0.5. Another horizontal force of 15*N*, is applied on the block in a direction parallel to the wall. Will the block move ? If yes, in which direction and with what minimum acceleration ? If no, find the frictional force exerted by the wall on the block.

Solution The force may cause the tendency of motion or motion in the body is its own weight and the applied horizontal force of 15*N*. The resultant of the forces

$$F = \sqrt{20^2 + 15^2} = 25N$$

In a direction $\tan^{-1}\left(\frac{15}{20}\right) = 37^0$ with the vertical.

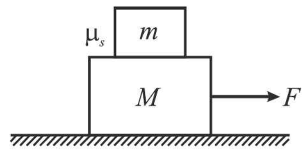


The friction, by its very virtue of opposing tendency, it will act in a direction opposite to the resultant force.

Now, the acceleration (minimum) = $\frac{F - \mu N}{m}$ (as, μN is the maximum frictional force)

$$= \frac{25 - 0.5 \times 40}{2} = \frac{5}{2} N$$

So, Minimum acceleration is $\frac{5}{2}$ = **2.5 m/s²** opposite to resultant force



Example 50. The coefficient of static friction between the two blocks shown in figure is μ and the table is smooth. What maximum horizontal force *F* can be applied to the block of mass *M* so that the blocks move together ?

Solution Two cases are possible either there is relative motion between blocks or not. Let us assume that there is no relative motion, means both blocks move together. The only horizontal force on the upper block of mass *m* is that due to the friction by the lower block of mass *M*. Hence this force on m should be towards right. The force of friction on *M* by *m* should be towards left by Newton's third law.

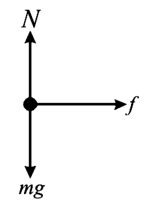
Consider the motion of *m*. The forces on *m* are

(a) *mg* downward by the earth (gravity)

(b) *N* upward by the block *M* (normal force) and

(c) *f* (friction) towards right by the block *M*.

F.B.D. of m



In the vertical direction, there is no acceleration. This gives

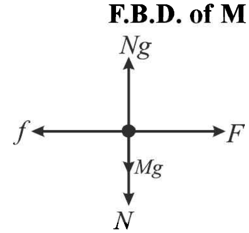
$$N = mg \qquad ...(i)$$

In the horizontal direction, let the acceleration be a, then

$$f = ma$$

For *M*, $F - f = Ma$ ...(ii)

F.B.D. of M



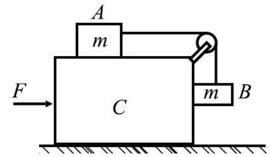
Solving (i) and (ii)

$$a = \frac{F}{M + m}.$$

since *f* is static $f \le \mu N_{mM} = \mu mg$ ...(iii)

from (i) and (ii) we get

$$F \le \mu(M + \mu)g$$



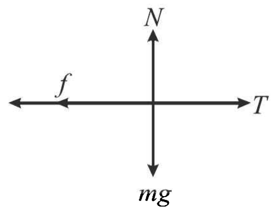
For $F > \mu(M + m)g$ there is relative motion between blocks and friction will become kinetic.

Example 51. Consider the situation shown in figure. The horizontal surface below the bigger block is smooth. The coefficient of friction between the blocks is μ. Find the minimum and the maximum force F that can be applied in order to keep the smaller blocks at rest with respect to the bigger block.

Solution If no force is applied, the block A will slip on C towards right and the block B will move downward. Suppose the minimum force needed to prevent slipping is F. Taking $A + B + C$ as the system, the only external horizontal force on the system is F. Hence, the acceleration of the system is

$$a = \frac{F}{M + 2m} \quad \text{...(i)}$$

F.B.D. of A



Now take the block A as system. The forces on A are,

(i) tention T by the string towards left,

(ii) friction f by the block C towards left,

(iii) weight mg downward and

(iv) normal force N upward.

For vertical equilibrium $N = mg$.

As the minimum force needed to prevent slipping is applied, the friction is limiting. Thus,

$$f = \mu N = \mu\, mg.$$

As the block moves towards right with an acceleration a,

$$T - f = ma$$

or $\quad T - \mu mg = ma \quad$ ...(ii)

Now take the block B as the system. The forces are

(i) tension T upward,

(ii) weight mg downward,

(iii) normal force N' towards right, and

(iv) friction f' upward.

As the block moves towards right with an acceleration a,

$$N' = ma.$$

As the friction is limiting, $f' = \mu N' = \mu ma$

For vertical equilibrium

$$T + f' = mg$$

or $\quad T + \mu\, ma = mg \quad$ ...(iii)

Eliminating T from (ii) and (iii)

$$a_{\min} = \frac{1 - \mu}{1 + \mu} g$$

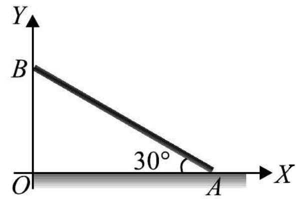
When a large force is applied the blocks *A* slips on *C* towards left and the block *B* slips on *C* in the upward direction. The friction on *A* is towards right and that on *B* is downwards. Solving as above, the acceleration in this case is

$$a_{max} = \frac{1+\mu}{1-\mu}g$$

Thus, a lies betweeen $\frac{1-\mu}{1+\mu}g$ *and* $\frac{1+\mu}{1-\mu}g$

From (i) the force *F* should be between $\frac{1-\mu}{1+\mu}(M+2m)g$ and $\frac{1+\mu}{1-\mu}(M+2m)g$.

Example 52. A rod *AB* rests with the end *A* on rough horizontal ground and the end *B* against a smooth vertical wall. The rod is uniform and of weight *W*. If the rod is in equilibrium in the position shown in figure. Find :



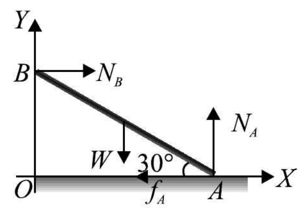
(a) frictional force at *A*

(b) normal reaction at *A*

(c) normal reaction at *B*.

Solution Let length of the rod be 2ℓ. Using the three conditions of equilibrium. Anticlockwise moment is taken as positive.

FBD of rod



(i) $\sum F_x = 0$ $\therefore\ N_B - f_A = 0$

$N_B = f_A$ ...(i)

or (ii) $\sum F_y = 0$

$\therefore\ N_A - W = 0$

or $N_A = W$ ...(ii)

(iii) $\sum \tau_0 = 0$

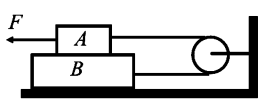
$\therefore\ N_A(2\ell\cos 30°) - W(\ell\cos 30°) = 0$

or $\sqrt{3}\,N_A - N_B - \frac{\sqrt{3}}{2}W = 0$

Solving these three equations, we get

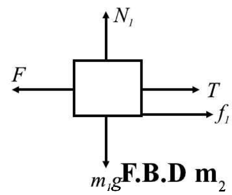
(a) $f_A = \frac{\sqrt{3}}{2}W$ (b) $N_A = W$ (c) $N_B = \frac{\sqrt{3}}{2}W$

Example 53. Two blocks *A* and *B* of masses m_1 and m_2 respectively are placed on each other and their combinations rests on a fixed horizontal surface. A massless string passing over a smooth pulley as shown in the figure is used to connect *A* and *B*. Assuming the coefficient of sliding friciton between all surfaces to be μ, show that both *A* and *B* will move with a uniform speed, if *A* is dragged with a force $F = \mu(3m_1 + m_2)g$ to the left.



Solution For vertical equilibrium

F.B. D. of m_1



$$N_1 - m_1 g = 0$$

$$\Rightarrow \quad N_1 = m_1 g \quad \text{...(1)}$$

as m_1 slides on m_2 hence

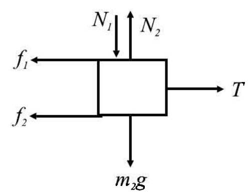
$$f_1 = \mu N_1 = \mu m_1 g \quad \text{...(2)}$$

For horizontal equilibrium

$$F - T - f_1 = 0$$

$$\Rightarrow F - T - \mu m_1 g = 0 \quad \text{...(3)}$$

F.B.D m_2



For vertical equilibrium of m_1

$$N_2 - N_1 - m_2 g = 0$$

$$\Rightarrow N_2 = N_1 + m_2 g = m_1 g + m_2 g \quad \text{...(4)}$$

Since $f_2 = \mu N_2 = \mu(m_1 + m_2)g$ ...(5)

For horizontal equilibrium

$$f_1 + f_2 - T = 0$$

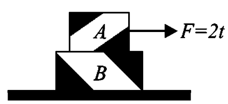
$$\Rightarrow \quad \mu m_1 g + \mu(m_1 + m_2)g - T = 0 \quad \text{...(6)}$$

Subtracting (3) from (6), we get

$$3\mu m_1 g + \mu m_2 g - F = 0$$

$$\Rightarrow \quad F = \mu(3m_1 + m_2)g.$$

Example 54. Two blocks A and B of mass 2 kg and 4 kg are placed one over the other as shown in figure. A time varying horizontal force $F = 2t$ is applied on the upper block as shown in figure. Here t is in second and F is in newton. Draw a graph showing accelerations of A and B on y-axis and time on x-axis. Coefficient of friction between A and B is $\mu = 1/2$ and the horizontal surface over which B is placed is smooth. ($g = 10$ m/s^2)



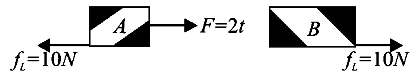
Solution Limiting friction between A and B is

$$f_L = \mu m_A g = \left(\frac{1}{2}\right)(2)(10) = 10 \ N$$

Block B moves due to friction only. Therefore, maximum acceleration of B can be

$$a_{\max} = \frac{f_L}{m_B} = \frac{10}{4} = 2.5 \ m/s^2$$

Thus, both the blocks move together with same acceleration till the common acceleration becomes 2.5 m/s^2, after that acceleration of B will become constant while that of A will go on increasing. To find the time when the acceleration of both the blocks becomes 2.5 m/s (or when slipping will start between A and B) we will write :



for t ≥ 7.5 sec

$$2.5 = \frac{F}{(m_A + m_B)} = \frac{2t}{6}$$

∴ $t = 7.5$ s

Hence, for $t \le 7.5\,s$

$$a_A = a_B = \frac{F}{m_A + m_B} = \frac{2t}{6} = \frac{t}{3}$$

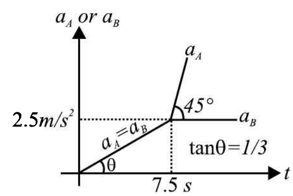
Thus, a_A versus t or a_B versus t graph is a straight line passing through origin of slope 1/3.

For $t \ge 7.5\,s$

$a_B = 2.5$ m/s^2 = constant

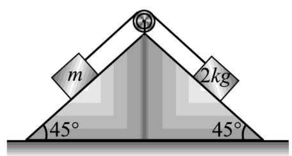
and $a_A = \dfrac{F - f_L}{m_A}$

or $a_A = \dfrac{2t - 10}{2}$ or $a_A = t - 5$



Thus, a_A versus t graph is a straight line of slope 1. While a_B versus t graph is a straight line parallel to t axis. The corresponding graph is as shown in figure.

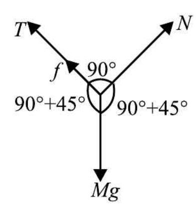
Example 55. Figure shows two blocks connected by a light string placed on the two inclined parts of a triangular wedge. The coefficients of static and kinetic friction are 0.28 and 0.25 respectively at each of the surface. (a) Find the minimum and maximum values of m for which the system remains at rest. (b) Find the acceleration of either block if m is given the minimum value calculated in the first part and is gently pushed up the incline for a short while. $g = 10$ m/s^2.



Solution (a) Consider the 2 kg block as the system. The forces on this block are shown in figure with $M = 2$kg. It is assumed that m has its minimum value so that the 2 kg block has a tendency to slip down. As the block is in equilibrium, the resultant force should be zero.

Applying Lami's theorem

$$\frac{Mg}{\sin 90°} = \frac{T + f}{\sin(90° + 45°)} = \frac{N}{\sin(90° + 45°)}$$



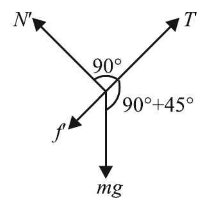
or $N = \dfrac{Mg}{\sqrt{2}}$

and $T = \dfrac{Mg}{\sqrt{2}} - f = \dfrac{Mg}{\sqrt{2}} - \mu_s N = \dfrac{Mg}{\sqrt{2}} - \dfrac{\mu_s Mg}{\sqrt{2}}$

or $T = (1-\mu_s)\dfrac{Mg}{\sqrt{2}}$ ...(i)

Now, consider the other block as the system. The forces acting on this block are shown in figure. Again applying Lami's theorem

$$\frac{mg}{\sin 90^\circ} = \frac{T - f'}{\sin(90^\circ + 45^\circ)} = \frac{N'}{\sin(90^\circ + 45^\circ)}$$



$$N' = \frac{mg}{\sqrt{2}}$$

$$\therefore \quad T = \frac{mg}{\sqrt{2}} + f' = \frac{mg}{\sqrt{2}} + \mu_s N' = \frac{mg}{\sqrt{2}} + \mu_s \frac{mg}{\sqrt{2}}$$

Thus, $\quad T = \dfrac{mg}{\sqrt{2}}(1+\mu_s)$ ...(ii)

From equation (i) and (ii) $m(1+\mu_s) = M(1-\mu_s)$ ...(iii)

or, $\quad m = \dfrac{(1-\mu_s)}{(1+\mu_s)}M = \dfrac{1-0.28}{1+0.28}\times 2 = \dfrac{9}{8}kg$

When maximum possible value of m is required, the directions of friction are reversed because *m* has the tendency to slip down and 2 kg block to slip up. Thus, the maximum value of m can be obtained from (iii) by putting $\mu_s = -0.28$. Thus, the maximum value of m is

$$m = \frac{1+0.28}{1-0.28}\times 2 = \frac{23}{9}kg$$

(b) If $m = 9/8$ kg and the system is gently pushed, kinetic friction will operate. Thus,

$$f = \mu_k \frac{Mg}{\sqrt{2}} \text{ and } f' = \mu_k \frac{mg}{\sqrt{2}},$$

where $\mu_k = 0.25$. If the acceleration is a, Newton's second law for *M* gives

$Mg \sin 45^\circ - T - f = Ma$

or, $\quad \dfrac{Mg}{\sqrt{2}} - T - \dfrac{\mu_k Mg}{\sqrt{2}} = Ma$ ...(iv)

Applying Newton's second law for *m*

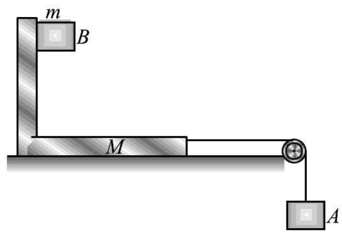
$T - mg\sin 45^\circ - f' = ma$

or, $\quad T - \dfrac{mg}{\sqrt{2}} - \dfrac{\mu_k mg}{\sqrt{2}} = ma$ ...(v)

From equation (iv) and (v)

or, $\quad a = \dfrac{M(1-\mu_k) - m(1+\mu_k)}{\sqrt{2}\,(M+m)}g = \dfrac{2\times 0.75 - 9/8\times 1.25}{\sqrt{2}\,(2+9/8)}g = 0.21 m/s^2$

Example 56. The figure shows an L shaped body of mass M placed on smooth horizontal surface. The block A is connected to the body by means of an inextensible string, which is passing over a smooth pulley of negligible mass. Another block B of mass ms is placed against a vertical wall of the body. Find the minimum value of the mass of block A so that block B remains stationary relative to the wall. Coefficient of friction between the block B and the vertical wall is μ.

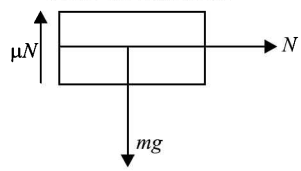


Solution

$$a = \frac{m_A g}{m_A + M + m}$$

For the equalibrium of B,

FBD of block B



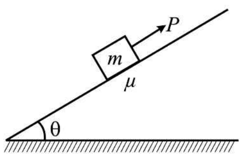
$$mg = \mu N = \mu(ma) = \frac{\mu m m_A g}{m_A + M + m}$$

$$m_A = \frac{(M+m)m}{(\mu - 1)m}$$

$$\therefore \quad m_A = \frac{(M+m)}{\mu - 1}$$

Note : $m_A > 0$ $\therefore \ \mu > 1$

Example 57. A block of mass m is being pulled up the rough incline by an agent delivering constant power P. The coefficient of friction between the block and the incline is μ. The maximum speed of the block during the course of ascent is



(A) $v = \dfrac{P}{mg\sin\theta + \mu mg\cos\theta}$

(B) $v = \dfrac{P}{mg\sin\theta - \mu mg\cos\theta}$

(C) $v = \dfrac{2P}{mg\sin\theta - \mu mg\cos\theta}$

(D) $v = \dfrac{3P}{mg\sin\theta - \mu mg\cos\theta}$

Solution Let at any time the speed of the block along the incline upwards be v.

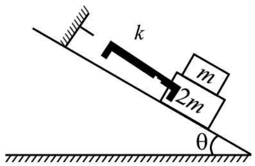
Then from Newton's second law

$\dfrac{P}{v} - mg\sin\theta - \mu mg\cos\theta = \dfrac{mdv}{dt}$ here $\dfrac{P}{v}$ is the force due to pulling agent.

the speed is maximum when $\frac{dv}{dt} = 0$ ∴ $v_{max} = \frac{P}{mg\sin\theta + \mu mg\cos\theta}$

∴ **(A)** is the right answer.

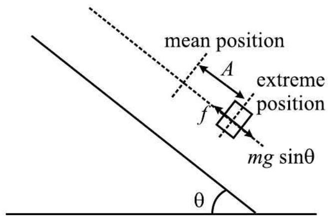
Example 58. The coefficient of friction between block of mass m and $2m$ is $\mu = 2 \tan \theta$. There is no friction between block of mass $2m$ and inclined plane. The maximum amplitude of two block system for which there is no relative motion between both the blocks.



(A) $g \sin \theta \sqrt{\frac{k}{m}}$ (B) $\frac{mg\sin\theta}{k}$

(C) $\frac{3mg\sin\theta}{k}$ (D) None of these

Solution



The maximum static frictional force is

$f = \mu mg \cos\theta = 2 \tan \theta\ mg \cos \theta = 2\ mg \sin\theta$

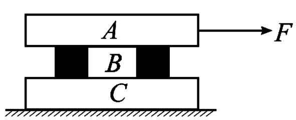
Applying Newton's second law to block at lower extreme position

$f - mg\sin\theta = m\omega^2 A$ $\Rightarrow f = m\omega^2 A + mg \sin \theta$

or $\omega^2 A = g \sin \theta$ or $A = \frac{3mg\sin\theta}{k}$

∴ (c) is the right answer.

Example 59. Given $m_A = 30$ kg, $m_B = 10$ kg, $m_C = 20$ kg. Between A and B $\mu_1 = 0.3$, between B and C $\mu_2 = 0.2$ and between C and ground $\mu_3 = 0.1$. The least horizontal force F to start the motion of any part of the system of three blocks resting upon one another as shown in figure is ($g = 10$ m/s^2)



(A) 60 N (B) 90 N
(C) 80 N (D) 150 N

Solution Limiting friction between A & B = 90 N

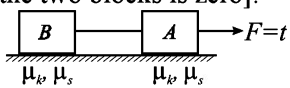
Limiting friction between B & C = 80 N

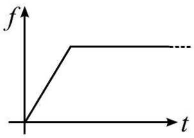
Limiting friction between C & ground = 60 N

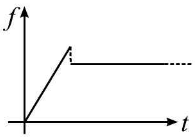
Since limiting friction is least between C and ground, slipping will oceur at first between C and ground. This will occur when $F = 60$ N.

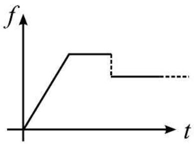
$\therefore$ **(A)** is the correct answer.

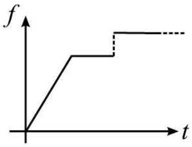
Example 60. A force $F = t$ is applied to a block A as shown in figure, where t is time in seconds. The force is applied at $t = 0$ seconds when the system was at rest. Which of the following graph correctly gives the frictional force between A and horizontal surface as a function of time t.[Assume that at $t = 0$, tension in the string connecting the two blocks is zero].



(A) 

(B) 

(C) 

(D) 

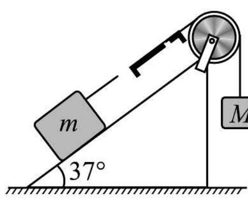
Solution Let m_A and m_B be the mass of blocks A and B respectively.

As the force F increases from 0 to $\mu_s m_A g$, the frictional force f on block A is such that $f=F$. When $F = \mu_s m_A g$, the frictional force f attains maximum value $f = \mu_s mg$.

As F is further increased to $\mu_s(m_A+m_B)g$, the block A does not move. In this duration frictional force on block A remains constant at $\mu_s m_A g$.

Hence (C) is correct choice.

Example 61. A block of mass m is attached with a massless spring of force constant k. The block is placed over a fixed rough inclined surface for which the coefficient of friction is $\mu = \frac{3}{4}$. The block of mass m is initially at rest. The block of mass M is released from rest with spring in unstretched state. The minimum value of M required to move the block up the plane is (neglect mass of string and pulley and friction in pulley.)



(A) $\frac{3}{5}m$

(B) $\frac{4}{5}m$

(C) $\frac{6}{5}m$

(D) $\frac{3}{2}m$

Solution As long as the block of mass m remains stationary, the block of mass M released from rest comes down by $\frac{2Mg}{K}$ (before coming it rest momentanly again).

Thus the maximum extension in spring is

$$x = \frac{2Mg}{K} \quad \text{... (1)}$$

for block of mass m to just move up the incline

$$kx = mg \sin \theta + \mu\, mg \cos \theta \quad \text{... (2)}$$

$$2Mg = mg \times \frac{3}{5} + \frac{3}{4}\, mg \times \frac{4}{5}$$

or $M = \frac{3}{5}\, m$

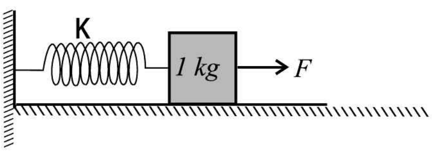
$\therefore$ (*A*) is the right answer.

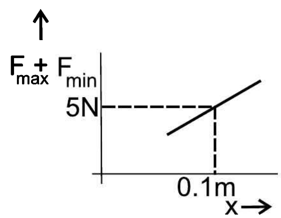
Comprehension Questions

Comprehension-1

A block of mass 1 kg is placed on a rough horizontal surface. A spring is attached to the block whose other end is joined to a rigid wall,as shown in the figure. A horizontal force is applied on the block so that it remains at rest while the spring is elongated by x. $x \geq \frac{\mu mg}{k}$. Let F_{max} and F_{min} be the maximum and minimum values of force F for which the block remains in equilibrium. For a particular x, $F_{max} - F_{min} = 2\,N$.

Also shown is the variation of $F_{max} + F_{min}$ versus x, the elongation of the spring.





Example **1.** The coefficient of friction between the block and the horizontal surface is :

(A) 0.1 (B) 0.2 (C) 0.3 (D) 0.4

Solution

$F_{max} = kx + \mu\,\text{mg}$

$F_{min.} = \text{kx} - \mu\,\text{mg}$

$\therefore F_{max} - F_{min.} = 2\,\mu\,\text{mg}$

or $2 = 2\,\mu\,10$

$\therefore m = 0.1$

(A) is the correct answer.

Example 2. The spring constant of the spring is:

(A) $25\ N/m$ (B) $20\ N/m$ (C) $2.5\ N/m$ (D) $50\ N/m$

Solution $F_{max} + F_{min.} = 2\,kx$ .... (1)

from graph $F_{max} + F_{min.} = 5$

and $x = 0.1$

Putting in equation (1)

$5 = 2\,k(0.1)$

$k = 25$ N/m.

(A) is the right answer.

Example 3. The value of F_{min}, if $x = 3$ cm is :

(A) 0 (B) $0.2N$ (C) $5N$ (D) $1N$

Solution When $x = 0.03$

$kx = 25 \times 0.03$

$= 0.75\ N$, which is less than μ mg $= 0.1 \times 10 = 1\ N$

$\therefore$ The block will be at rest, without applying force F.

gy. tc x = 0.03

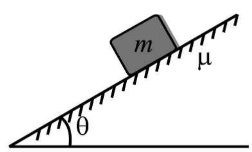
$kx = 25 \times 0.03$

$= 0.75$ N, μ mg $= 0.1 \times 10 = 1\ N$

(A) is the right answer.

Comprehension-2

A block of mass *m* is placed on a rough inclined plane. The coefficient of friction between the block and the plane is μ and the inclination of the plane is θ. Initially $\theta = 0$ and the block will remain stationary on the plane. Now the inclination θ is gradually increased. The block presses the inclined plane with a force mgcosθ. So welding strength between the block and inclined is μ mgcosθ, and the pulling forces is mgsinθ. As soon as the pulling force is greater than the welding strength, the welding breaks and the block starts sliding, the angle θ for which the block starts sliding is called angle of repose (λ). During the contact, two contact forces are acting between the block and the inclined plane. The pressing reaction (Normal reaction) and the shear reaction (frictional force). The net contact force will be resultant of both.



Answer the following questions based on above comprehension :

Example 1. If the entire system, were accelerated upward with acceleration 'a', the angle of repose, would :

(A) increase (B) decrease

(C) remain same (D) increase of $a > g$

Solution Angle (θ′)of repose ;

$$m(g + a)\sin\theta' = f$$

$$m(g + a)\cos\theta' = R$$

$$\therefore \quad \frac{f}{R} = \tan\theta'$$

$$\theta' = \tan^{-1}\left(\frac{f}{R}\right) = \alpha$$

Hence angle of repose does not change. (C) is the correct answer.

Example 2. For what value of θ will the block slide on the inclined plane :

(A) $\theta > \tan^{-1}\mu$ (B) $\theta < \tan^{-1}\mu$ (C) $\theta > \cot^{-1}\mu$ (D) $\theta > \cot^{-1}\mu$

Solution To slide $mg\sin\theta > \mu\, mg\cos\theta$

$$\sin\theta > \mu\cos\theta$$

$$\tan\theta > \mu$$

$$\theta > \tan^{-1}\mu$$

(A) is the correct answer.

Example 3. If μ = 3/4 then what will be frictional force (shear force) acting between the block and inclined plane when $\theta = 30^0$:

(A) $\frac{3\sqrt{3}}{8}$ mg (B) $\frac{mg}{2}$ (C) $\frac{\sqrt{3}}{2}$ mg (D) zero

Shear force = $\mu\, mg\cos\theta$

$$= \frac{3}{4} \times mg \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{8}mg = 00.6 \text{ mg}$$

But, pulling force = $mg\sin\theta = mg\sin 30^0 = 0.5$ mg $< f_{smax}$. ∴ block does not slide.

Hence frictional force (shear force) between the block of the plane at this situation will be

$$= mg\sin 30^0 = \frac{mg}{2} \text{ (not } \frac{3\sqrt{3}}{8}mg)$$

(B) is the correct answer.

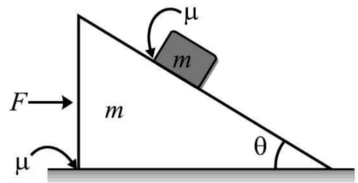
Alternate Sol.

$$\tan\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1.73}{3} = 0.58 < \mu$$

∴ block does not slide. ∴ $fs = mg\sin 30^0$

Comprehension-3

In the situation shown in figure a wedge of mass *m* is placed on a rough surface, on which a block of equal mass is placed on the inclined plane of wedge. Friction coefficient between plane and the block and the ground and the wedge (μ). An external force F is applied horizontally on the wedge. Given that m does not slide on incline due to its weight.

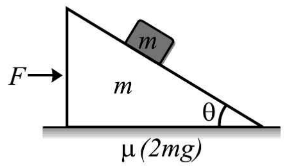


Example The value of F at which wedge will start slipping is :

(A) $= \mu mg$ (B) $= (3/2)\,\mu mg$

(C) $> 2\,\mu mg$ (D) $< \mu\, mg$

Solution Wedge will start slipping when



$F \geq \mu\,(2\,mg)$

(C) is the correct answer.

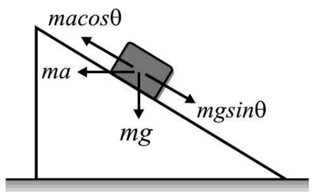
Example The value of F at which no friction will act on block on inclined plane, is :

(A) $2\,\mu\, mg$ (B) $2\,\mu\, mg + 2\, mg \tan\theta$

(C) $2\,\mu\, mg + mg\tan\theta$ (D) $2\,\mu\, mg + mg\sin\theta$

Solution If wedge start's slipping, common acceleration

$$aC = \frac{F - 2\mu mg}{2m}$$



If mg $\sin\theta$ = ma $\cos\theta$ then no force along the plane will be felt by the block and hence friction will be zero.

$$\Rightarrow \qquad mg\sin\theta = m\left(\frac{F - 2\mu mg}{2m}\right)\cos\theta$$

$F = 2mg\tan\theta + 2\,\mu mg$. (B) is the correct answer.

Example The minimum value of acceleration of wedge for which the block starts sliding on the wedge, is :

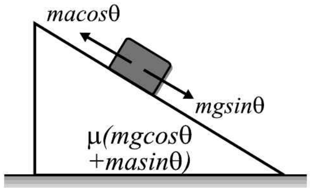
(A) $g\left(\frac{\cos\theta + \mu\sin\theta}{\sin\theta - \mu\cos\theta}\right)$ (B) $g\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)$

(C) $g\left(\frac{\sin\theta - \mu\cos\theta}{\sin\theta + \mu\cos\theta}\right)$ (D) $g\left(\frac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta}\right)$

Solution Block will start sliding if

ma $\cos\theta \geq mg\sin\theta + \mu(mg\cos\theta + ma\sin\theta)$

get a > $g\left(\dfrac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)$



(B) is the correct answer.

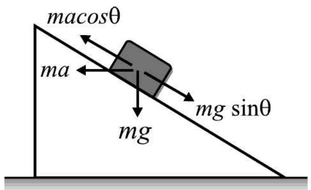
Example The value of F at which no friction will act on block on inclined plane, is :

(A) $2\mu mg$ (B) $2\mu mg + 2mg\tan\theta$

(C) $2\mu mg + mg\tan\theta$ (D) $2\mu mg + mg\sin\theta$

Solution If wedge starts slipping, common acceleration

$$a_C = \frac{F - 2\mu mg}{2m}$$



If $mg\sin\theta = ma\cos\theta$ then no force along the plane will be felt by the bloc and hence friction will be zero.

$$\Rightarrow \quad mg\sin\theta = m\left(\frac{F - 2\mu mg}{2m}\right)\cos\theta$$

$$F = 2mg\tan\theta + 2\mu mg$$

(B) is the correct answer.

Example The minimum value of acceleration of wedge for which the block starts sliding on the wedge, is :

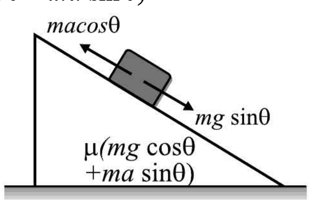
(A) $g\left(\dfrac{\cos\theta + \mu\sin\theta}{\sin\theta - \mu\cos\theta}\right)$ (B) $g\left(\dfrac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)$

(C) $g\left(\dfrac{\sin\theta - \mu\cos\theta}{\sin\theta + \mu\cos\theta}\right)$ (D) $g\left(\dfrac{\cos\theta - \mu\sin\theta}{\sin\theta + \mu\cos\theta}\right)$

Solution Block will start sliding if

$ma\cos\theta \geq mg\sin\theta + \mu(mg\cos\theta + ma\sin\theta)$

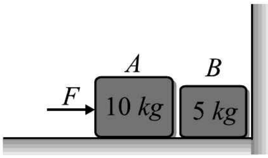
get $a > g\left(\dfrac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)$



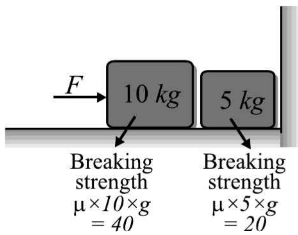
(B) is the correct answer.

Comprehension – 3

Two bodies *A* and *B* of masses 10 kg and 5 kg are placed very slightly separated as shown in figure. The coefficient of friction between the floor and the blocks is $\mu = 0.4$. Block *A* is pushed by an external force *F*. The value of *F* can be changed. When the welding between block *A* and ground breaks, block *A* will start pressing block *B* and when welding of *B* also breaks, block *B* will start pressing the vertical wall –



Example 1. If $F = 20\ N$, with how much force does block A presses the block *B*
(A) 10 *N* (B) 20 *N* (C) 30 *N* (D) Zero

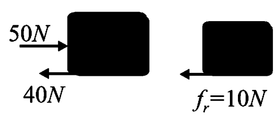


Solution If $F = 20\ N$, 10 kg block will not move and it would not press 5 kg block So $N = 0$.

Example 2. What should be the minimum value of F, so that block B can press the vertical wall
(A) 20 *N* (B) 40 *N* (C) 60 *N* (D) 80 *N*

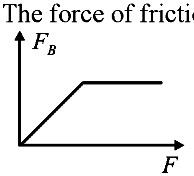
Solution $F_{min} = fA + fB = 60\ N.$
(C) is the correct answer.

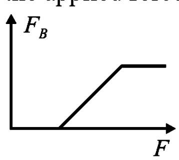
Example 6. If $F = 50\ N$, the friction force (shear force) acting between block *B* and ground will be :
(A) 10 *N* (B) 20 *N* (C) 30 *N* (D) None

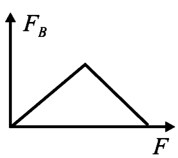


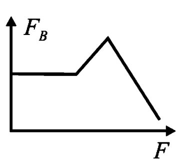
Solution If $F = 50\ N$, force on 5 kg block = 10 *N*
So friction force = 10 *N*. (A) is the correct answer.

Example 4. The force of friction acting on *B* varies with the applied force *F* according to curve :

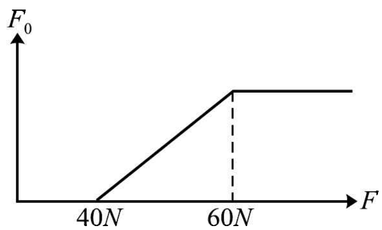
(A) 

(B) 

(C) 

(D) 

Solution Until the 10 kg block is sticked with ground
(... $F = 40\ N$),



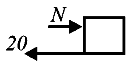
No force will be felt by 5 kg block. After $F = 40\ N$, the friction force on 5 kg increases, till $F = 60\ N$, and after that, the kinetic friction start acting on 5 kg block, which will be constant (20N)

(B) is the correct answer.

Example 5. If the vertical wall is removed and the force applied is 90 N, the pressing force (normal reaction) between block A and block B will be :

(A) 20 N (B) 30 N (C) 40 N (D) None

Solution $F - fA - fB = (mA + mB)a$ $\Rightarrow$ $a = 2\ \text{m/s}^2$,

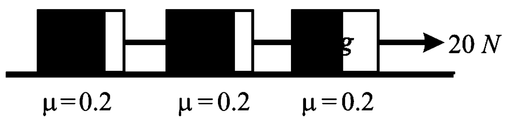


$N - 20 = 5 \times 2$ $N = 30\ N.$

(B) is the correct answer.

Comprehension – 5

Three blocks of masses 6 kg, 4kg & 2 kg are pulled on a rough surface by applying a constant force 20N. The values of cofficient of friction between blocks & surface are shown in figrue.



Example 1. In the arrangement shown tension in the string connecting 4kg and 6kg masses is

(A) 8N (B) 12N (C) 6N (D) 4N

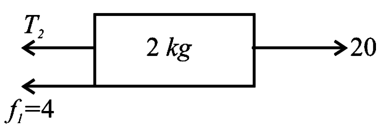
Example 2. Friction force on 4 kg block is

(A) 4N (B) 6 N (C) 12 N (D) 8 N

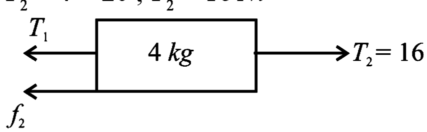
Example 3. Friction force on 6 kg block is

(A) 12 N (B) 8 N (C) 6 N (D) 4 N

Solution

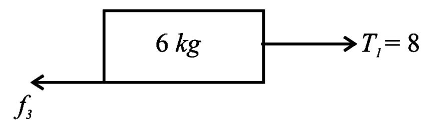


$T_2 + 4 = 20$, $T_2 = 16\ Nt$



$f_2 = 8$, $T_2 = T_1 + f_2$, $T_2 = T_1 + 8$

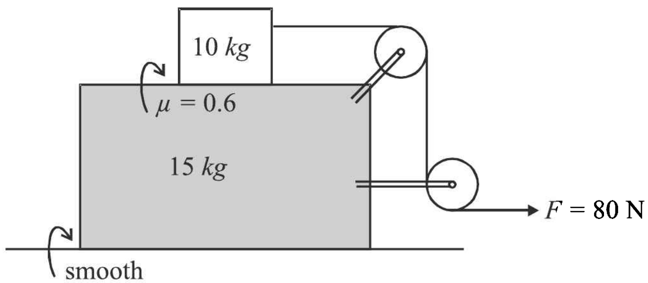
$T_1 = 8$



$f_3 = 8.$

1. (A) is the correct answer.
2. (D) is the correct answer.
3. (B) is the correct answer.

Comprehension-6

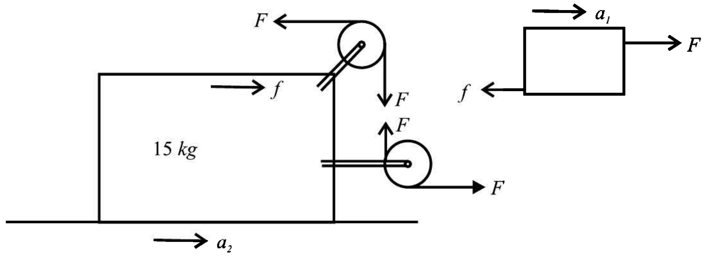


A block of mass 15 kg is placed over a frictionless horizontal surface. Another block of mass 10 kg is placed over it, that is connected with a light string passing over two pulleys fastened to the 15 kg block. A force $F = 80$ N is applied horizontally to the free end of the string. Friction coefficient between two blocks is 0.6. The portion of the string between 10 kg block and the upper pulley is horizontal. Pulley, string & connecting rods are massless. (Take $g = 10$ m/s^2)

Example 1. The magnitude of accelerations of the 10 kg, 15 kg block are :

(A) 3.2 m/s^2, 3.2 m/s^2 (B) 2.0 m/s^2, 4.2 m/s^2 (C) 1.6 m/s^2, 16/3 m/s^2 (D) 0.8 m/s^2, 2.0 m/s^2

Solution First, let us check upto what value of F, both blocks move together. Till friction becomes limiting, they will be moving together. Using the *FBDs*



10 kg block will not slip over the 15 kg block till acceleration of 15 kg block becomes maximum as it is created only by friction force exerted by 10 kg block on it

$$a_1 > a_{2(max)}$$

$$\frac{F - f}{10} = \frac{f}{15}$$ for limiting condition as f maximum is 60 N.

F = 100 N.

Therefore for $F = 80$ N, both will move together.

Their combined acceleration, by applying *NLM* using both as system $F = 25a$

$$a = \frac{80}{25} = 3.2 \text{ m/s}^2$$

(A) is the correct answer.

Example 2. If applied force $F = 120\ N$, then magnitude of acceleration of 15 kg block will be :

(A) 8 m/s² (B*) 4 m/s² (C) 3.2 m/s² (D) 4.8 m/s²

Solution If $F = 120\ N$, then there will be slipping, so using FBDs of both (friction will be 60 N)

For 10 kg block

$120 - 60 = 10\ a$

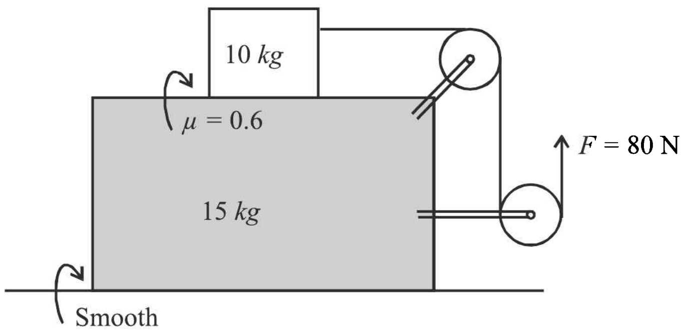
$\Rightarrow\ a = 6 \text{ m/s}^2$

For 15 kg block

$60 = 15a \Rightarrow \quad a = 4 \text{ m/s}^2$

(B) is the correct answer.

Example 3. Continuing with the situation, if the force $F = 80\ N$ is directed vertically as shown in the given figure, the accelerations of the 10 kg, 15 kg block will be :



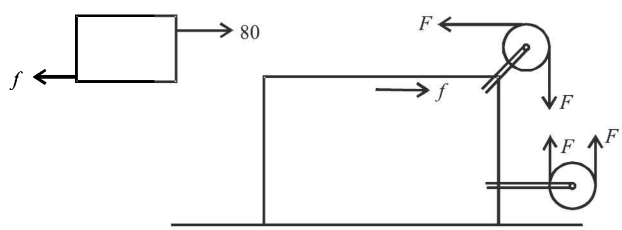
(A) 2 m/s² towards right and 4/3 m/s² towards left

(B) 2 m/s² towards left and 16/5 m/s² towards right

(C) 6 m/s² towards left and 4 m/s² towards right

(D) 16/5 m/s² towards right and 2/3 m/s² towards right

Solution In case 80 N force is applied vertically, then



For 10 kg block $80 - 60 = 10a$

$a = 2 \text{ m/s}^2$

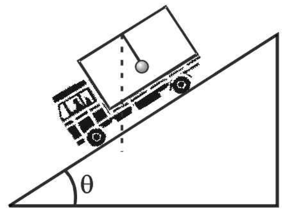
For 15 kg block in horizontal direction.

$F - f = 15a$

$a = 4/3 \text{ m/s}^2$, towards left.

(A) is the correct answer.

Comprehension – 7

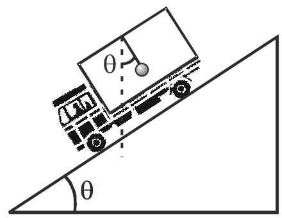


A van accelerates uniformly down an inclined hill going from rest to 30 m/s in 6 s. During the acceleration, a toy of mass m = 0.1 kg hangs by a light string from the van's ceiling. The acceleration is such that string remains perpendicular to the ceiling. (Take g = 10 m/s^2)

Example 1. The angle θ of the incline is :

(A) 30° (B) 60° (C) 90° (D) 45°

Solution Acceleration of the van = $\frac{30}{6}$ = 5 m/s^2



$g \sin\theta = a$

$\Rightarrow \sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 30°$

(A) is the correct answer.

Example 2. The tension in the string is

(A) 1.0 N (B) 0.5 N (C) $\frac{\sqrt{3}}{2}N$ (D) $\sqrt{3}N$

Solution Tension $T = mg\cos\theta = \frac{\sqrt{3}}{2}\ N$

(A) is the correct answer.

Example 3. The friction force on the van is

(A) Zero (B) $mg\cos\theta$ (C) $mg\sin\theta$ (D) $mg\tan\theta$

Solution Since acceleration of the van is $g\sin\theta$, there is no friction.

(A) is the correct answer.

Comprehension – 8

A small mass slides down a fixed inclined plane of inclination θ with the horizontal. The co-efficient of friction is $\mu = \mu_0 x$ where x is the distance through which the mass slides down and μ_0 is a constant.

Example 1. The distance covered by the mass before it stops is:

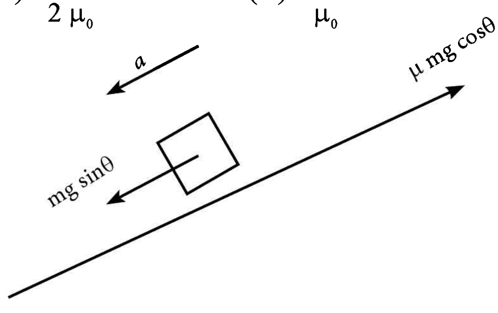
(A) $\frac{2}{\mu_0}\tan\theta$ (B) $\frac{4}{\mu_0}\tan\theta$ (C) $\frac{1}{2\mu_0}\tan\theta$ (D) $\frac{1}{\mu_0}\tan\theta$

Solution $mg\sin\theta - \mu mg\cos\theta = ma$

$a = g[\sin\theta - \mu\cos\theta]$

$\frac{udv}{dx} = g[\sin\theta - \mu_0\cos\theta x]$

$\int_0^0 udv = g(\sin\theta)\int_0^x dx - (g\mu_0\cos\theta)\int_0^x x\,dx$



$\Rightarrow \quad 0 = g(\sin\theta)x - (g\mu_0\cos\theta)\dfrac{x^2}{2}$

$\Rightarrow \quad x = \dfrac{2\tan\theta}{\mu_0}$

$\Rightarrow$ (A) is the correct answer.

Example 2. The heat produced during the half journey of the particle is:

(A) $\dfrac{mg\cos\theta\tan^2\theta}{2\mu_0}$ (B) $\dfrac{mg\cos\theta\tan^2\theta}{4\mu_0}$

(C) $\dfrac{mg\cos\theta\tan^2\theta}{8\mu_0}$ (D) none of these

Solution Heat produed + change in K.E. = Work done

$\Rightarrow$ (A) is the correct answer.

Example 3. The speed of the mass when travelled half the maximum distance is

(A) $\sqrt{\dfrac{g\tan\theta\sin\theta}{\mu_0}}$ (B) $\sqrt{\dfrac{g\tan\theta\sin\theta}{2\mu_0}}$

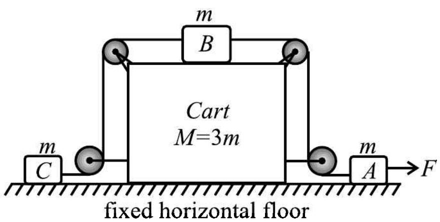
(C) $\sqrt{\dfrac{g\tan\theta\sin\theta}{8\mu_0}}$ (D) none of these

Solution By integration

$\Rightarrow$ (A) is the correct answer.

Comprehension – 9

A block *B* is placed over a cart which in turn lies over a smooth horizontal floor. Block *A* and block *C* are connected to block *B* with light inextensible strings passing over light frictionless pulleys fixed to the cart as shown. Initially the blocks and the cart are at rest. All the three blocks have mass m and the cart has mass $M(M = 3m)$. Now a constant horizontal force of magnitude *F* is applied to block *A* towards right.

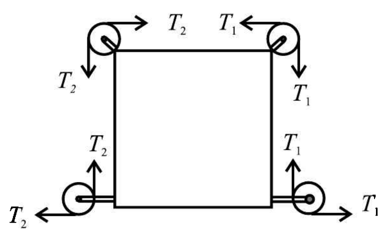


Example 1. Assuming friction to be absent everywhere, the magnitude of acceleration of cart at the shown instant is

(A) $\dfrac{F}{6m}$ (B) $\dfrac{F}{4m}$

(C) $\dfrac{F}{3m}$ (D) 0

Solution The free body diagram of cart is



Hence net horizontal force on cart is zero.

∴ acceleration of cart is zero.

(D) is the correct answer.

Example 7. Taking friction to be absent everywhere the magnitude of tension in the string connecting block B and block C is

(A) $\frac{F}{9}$ (B) $\frac{F}{6}$ (C) $\frac{F}{3}$ (D) $\frac{2F}{3}$

Solution The acceleration of each block is equal and equal to $\frac{F}{3m}$.

∴ Tension in required string can be found by applying Newton's second law to block C.

$T_2 = ma = \frac{F}{3}$.

(C) is the correct answer.

Example 3. Let the coefficient of friction between block B and cart is $\mu(\mu > 0)$ and friction is absent everywhere else. Then the maximum value of force F applied to block A such that there is no relative acceleration between block B and cart is

(A) μmg (B) $2\mu mg$ (C) $3\mu mg$ (D) $4\mu mg$

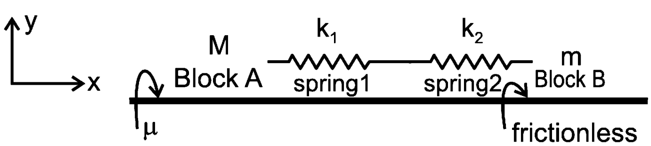
Solution For block B to just slip on the cart, the friction force on cart is μmg. The net force on cart is thus μmg. Hence acceleration of cart is a = $\frac{\mu mg}{3m} = \frac{\mu g}{3}$.

∴ required force $F = (m + m + m + M)\, a = 2\mu$mg

(B) is the correct answer.

Match the following

1. Two blocks A and B of masses m and M are placed on a horizontal surface, both being interconnected with a horizontal series combination of two massless springs 1 and 2, of force constants k_1 and k_2 respectively as shown. Friction coefficient between block A and the surface is m and the springs are initially non-deformed. Now the block B is displaced slowly to the right by a distance x, and it is observed that block A does not slip on the surface. Block B is kept in equilibrium by applying an external force at that position. Match the required information in the left column with the options given in the right column.



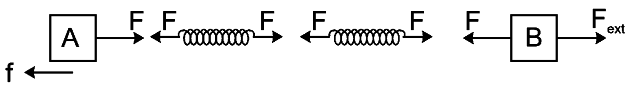
Left column	Right column
(a) Friction force on block A by the surface	(P) $k_1 x (-\hat{i})$
(b) Force by spring 1 on block A	(Q) m $Mg (-\hat{i})$

(c) Force exerted by spring 2 on spring 1. (R) $\dfrac{k_1k_2x}{k_1+k_2}(\hat{i})$

(d) External force on block *B*. (S) $\dfrac{k_1k_2x}{k_1+k_2}(-\hat{i})$

[**Ans.** (a) S, (b) R, (c) R, (d) R]

Solution The free body diagram (FBD) is :



Tension in both springs will be same ($\because$ they are massless)

$F = K_2X_2 = K_1X_1$ and $X_1 + X_2 = X$

$\therefore \quad X_1 = \dfrac{K_2}{K_1+K_2}X, \quad X_2 = \dfrac{K_1}{K_1+K_2}X$

$\therefore \quad F = \dfrac{K_1K_2}{K_1+K_2}X$

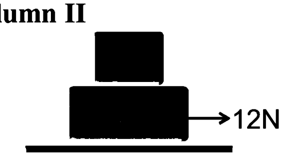
$f = F = \dfrac{K_1K_2}{K_1+K_2}X(-\hat{i})$

$\therefore \quad a \to S, b \to R, c \to R, d \to R$

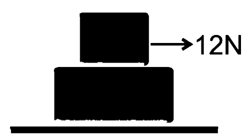
2. Column II gives certain situations involving two blocks of mass 2 kg and 4 kg. The 4 kg block lies on a smooth horizontal table. There is sufficient friction between both the block and there is no relative motion between both the blocks in all situations. Horizontal forces act on one or both blocks as shown. Column I gives certain statement related to figures given in column II. Match the statements in column I with the figure in column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in OMR.

Column I **Column II**

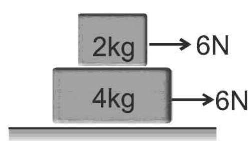
(A) Magnitude of frictional force is maximum. (*p*)



(B) Magnitude of friction force is least. (*q*)



(C) Friction force on 2 kg block is towards right. (*r*)



(D) Friction force on 2 kg block is towards left. (*s*) 

[Ans. (A) *s* (B) *r* (C) *p*, *s* (D) *q*, *r*]

Solution The acceleration of two block system for all cases is $a = 2$ m/s^2

In option (p) the net force on 2 kg block is frictional force

$\therefore$ Frictional force on 2 kg block is

$f = 2 \times 2 = 4N$ towards right

In option (q) the net force on 4 kg block is frictional force

$\therefore$ Frictional force on 4 kg block is

$f = 4 \times 2 = 8N$ towards right

In option (r) the net force on 2 kg block is $2 \times 2 = 4N$

$\therefore$ Friction force f on 2 kg block is towards left.

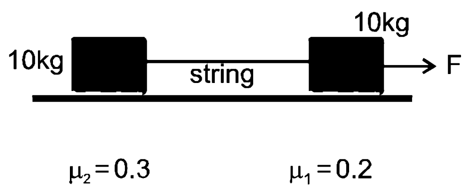
$\therefore$ $6 - f = 2 \times 2$ or $f = 2N$

In option (s) the net force on 2 kg block is ma

$= 2 \times 2 = 4N$ towards right.

$\therefore$ Friction force on 2 kg block is $12N$ towards right.

3. Two blocks of same mass $m = 10$ kg are placed on rough horizontal surface as shown in figure. Initially tension in the massless string is zero and string is horizontal. A horizontal force $F = 40 \sin\left(\frac{\pi}{6}t\right)$ is applied as shown on the block A for a time interval $t = 0$ to $t = 6$ sec. Here F is in Newton and t in second. Friction coefficient between block A and ground is 0.20 and between block B and ground is 0.30. (Take $g = 10$ m/sec^2). Match the statements in column-I with the time intervals (in seconds) in column-II.



Column I	Column II
(A) Friction force between block B and ground is zero in the time interval	(p) $0 < t < 1$
(B) Tension in the string is non zero in the time interval	(q) $1 < t < 3$
(C) Acceleration of block A is zero in the time interval	(r) $3 < t < 5$
(D) Magnitude of friction force between A and ground is decreasing in the time interval	(s) $5 < t < 6$

[Ans.(A) *p*, *s* (B) *q*, *r* (C) *p*, *q*, *r*, *s* (D) *s*]

Solution $F_{max} = 40 < (20 + 30)$

So as long as force F is in the positive x direction, both the block are at rest.

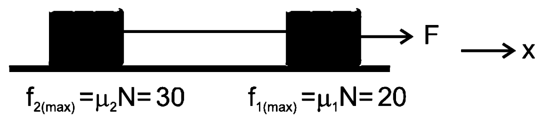
So (C) p, q, r, s

When $F = 20 = 40 \sin\left(\frac{\pi}{6}t\right) \Rightarrow \frac{1}{2} = \sin\frac{\pi t}{6}$

$\Rightarrow \frac{\pi}{6}t = \frac{\pi}{6}, \pi - \frac{\pi}{6} \qquad \Rightarrow t = 1, 5$

When $F = 0$, $t = 0$ and $\frac{\pi t}{6} = p$ i.e. $t = 6$

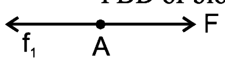
When $F = 40$ i.e. $\frac{\pi t}{6} = \frac{\pi}{2} \Rightarrow t = 3$



For $0 < t < 1$

$0 < F < 20$

FBD of block A (in x-direction)



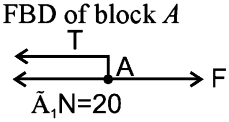
acceleration $a_A = 0$, Tension $T = 0$

$f_1 < 20$ $\qquad a_B = 0$, friction force $f_2 = 0$

So (A) $\rightarrow p$

For $1 < t < 3$ {$20 < F < 40$} and $3 < t < 5$ {$40 > F > 20$}

FBD of block A

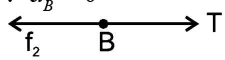


$a_A = 0, T \neq 0$

$T = F - 20$ i.e. $0 < T < 20 \because F < 40$

FBD of block B $\because a_B = 0$

So $B \rightarrow q, r$

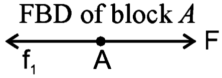


$f_2 \neq 0$

For $5 < t < 6$

$20 > F > 0$

FBD of block A



$a_A = 0$, Tension $T = 0$

$a_B = 0$, Friction force $f_2 = 0$

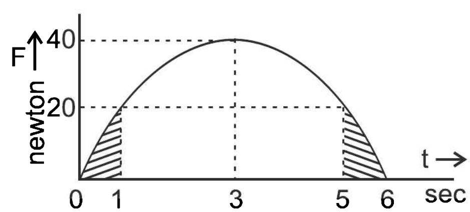
$f_1 = F$

So (D) $\rightarrow s$ and (A) $\rightarrow s$

Alternate :

When $F = 20 = 40 \sin\frac{\pi t}{6} \Rightarrow t = 1$ sec.

$F = 0 = 40 \sin\frac{\pi t}{6} \Rightarrow t = 6$ sec.

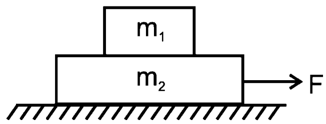


By using symmetry in sine curve

For shaded region $F < 20$, $a_A = 0$, $T = 0$, $a_B = 0$

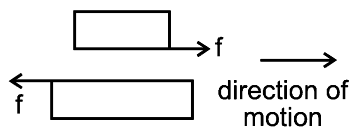
For $1 < t < 5$, $\quad a_A = 0, T \neq 0, a_B = 0$

4. A small block of mass m_1 lies over a long plank of mass m_2. The plank in turn lies over a smooth horizontal surface. The coefficient of friction between m_1 and m_2 is m. A horizontal force F is applied to the plank as shown in figure. Column-I gives four situation corresponding to the system given above. In each situation given in column-I, both bodies are initially at rest and subsequently the plank is pulled by the horizontal force F. Take length of plank to be large enough so that block does not fall from it. Match the statements in column-I with results in column-II.



Column-I	Column-II
(A) If there is no relative motion between the block and plank, the work done by force of friction acting on block in some time interval is	(p) positive
(B) If there is no relative motion between the block and plank, the work done by force of friction acting on plank is some time interval	(r) zero
(C) If there is relative motion between the block and plank, then work done by friction force acting on block plus work done by friction acting on plank is	(q) negative
(D) If there is no relative motion between the block and plank, then work done by friction force acting on block plus work done by friction acting on plank is	(s) is equal to negative of loss in mechanical energy of two block plus plank system.

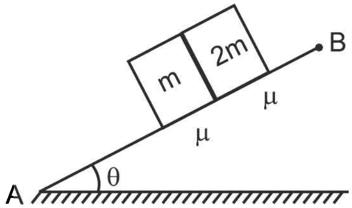
[**Ans.**(A) p (B) q (C) q, s (D) r]



Solution The FBD of block and plank are shown. Work done on block by friction is positive
Work done on plank by friction is negative.
Work done by friction on plank plus block is zero when there is no relative motion between them.
Since there is no rubbing between block and plank, mechanical energy is not lost. (i.e., heat and allied losses are not produced).

Work done by friction on plank + block is negative when there is relative motion between block and plank. This work done is equal loss in mechanical energy of block + plank system.

5. Two blocks of mass m and $2m$ are slowly just placed in contact with each other on a rough fixed inclined plane as shown. Initially both the blocks are at rest on inclined plane. The coefficient of friction between either block and inclined surface is m. There is no friction between both the blocks. Neglect the tendency of rotation of blocks on the inclined surface. Column I gives four situation. Column II gives condition under which statements in column I are true. Match the statement in column I with corresponding conditions in column II and indicate your answer by darkening appropriate bubbles in the 4×4 matrix given in the ORS.



	Column I	Column II
(A)	The magnitude of acceleration of both blocks are same if	(p) $\mu = 0$
(B)	The normal reaction between both the blocks is zero if	(q) $\mu > 0$
(C)	The net reaction exerted by inclined surface on each block make same angle with inclined surface (*AB*) if	(r) $\mu > \tan\theta$
(D)	The net reaction exerted by inclined surface on block of mass $2m$ is double that of net reaction exerted by inclined surface on block of mass m if	(s) $\mu < \tan\theta$

[Ans. (A) *p,q,r,s* (B) *p,q,r,s,* (C) *p,q,r,s,* (D) *p,q,r,s*]

Solution

(A) For $\mu > \tan\theta$, the magnitude of acceleration of both blocks is zero. Hence acceleration of both blocks is same.

For $\mu > \tan\theta$, the acceleration of both blocks is same and equal to ($g \sin\theta + g \cos\theta$)

Hence whatever be the value of μ, the acceleration of both blocks shall be same.

(B) For $\mu > \tan\theta$, both blocks are at rest and their binding with inclined surface is not broken. Hence the blocks cannot exert force on each other. Therefore normal reaction between both blocks is zero.

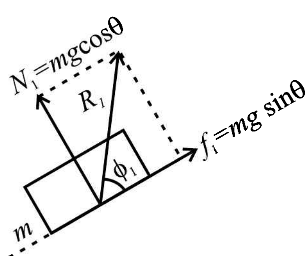
For $\mu > \tan\theta$, both blocks will move down the incline with same acceleration when they are not in contact. Hence they have no tendency to approach.

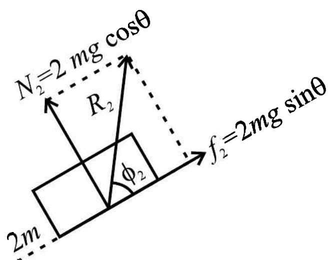
Hence when both blocks are in contact, they will not exert normal reaction no tendency to approach.

Hence whatever be the value of μ, normal reaction between both blocks is zero.

(*C* & *D*) For $\mu > \tan\theta$, both blocks are at rest.

The normal reaction (*N*), friction (*f*) and net reaction on each blocks by inclined surface are as shown.





It is obvious $\phi_1 = \phi_2$ and $R_2 = 2R_1$.

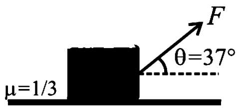
For $\mu < \tan\theta$, both blocks move down the incline.

Again it can be seen that $\phi_1 = \phi_2$ and $R_2 = 2R_1$.

Hence whatever be the value of μ, $R_2 = 2R_1$ and $\phi_2 = \phi_1$.

Assertion Reason Questions

1. **Statement-1:** A block of mass m is placed at rest on rough horizontal surface. The coefficient of friction between the block and horizontal surface is $\mu = \frac{1}{3}$. The minimum force F applied at angle $\theta = 37°$ (as shown in figure) to pull the block horizontally is not equal to μmg. (Take $\sin 37° = \frac{3}{5}$, $\cos 37° = \frac{4}{5}$)



Statement-2 : For a block of mass m placed on rough horizontal surface, the minimum horizontal force required to pull the block is μmg. The minimum force F applied at angle θ (as shown in figure) to pull the block horizontally may be less than μmg. (Where μ is co-efficient of friction).

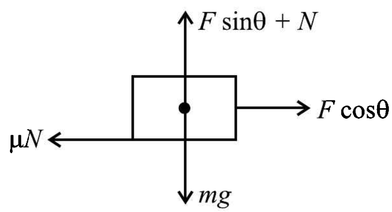
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Solution The *FBD* of block is as shown.



The acceleration of block is given by equn.

$F \cos\theta - \mu N = ma$ .... (1)

where $N = mg - F \sin\theta$ .... (2)

Putting $\mu = \frac{1}{3}$, $\theta = 37°$ and $F = \mu\, mg$

we get $a = 0$

Hence the minimum force F to pull the block at an angle $\theta = 37°$ is $\mu\, mg$.

Hence statement I is false. **[Ans. (D)]**

2. **Statement-1:** A block of mass m = 10 kg lies on rough horizontal surface. The coefficient of friction between block and horizontal surface is $\mu = \frac{3}{4}$. Initially the only force acting on block are its weight and normal reaction due to horizontal surface. An additional force of magnitude 70 N can move the block on horizontal surface.

Statement-2 : The magnitude of minimum force required to move a block of mass m placed on rough horizontal surface is μN. (Where μ is co-efficient of friction and N is normal reaction acting on the block).

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

Solution The minimum force required to pull the block of mass *m* lying on rough horizontal surface is $F = \dfrac{\mu mg}{\sqrt{\mu^2+1}} = 60\,N$, inclined at an angle $\tan^{-1}\mu$ with horizontal (where μ is the coefficient of friction). Hence statement 1 is true and statement 2 is false.

[Ans. (C)]

3. Statement-1 : A body is lying at rest on a rough horizontal surface. A person accelerating with acceleration $a\hat{i}$ (where a is positive constant and $\hat{i}$ is a unit vector in horizontal direction) observes the body. With respect to him, the block experiences a kinetic friction.

Statement-2 : Whenever there is relative motion between the contact surfaces then kinetic friction acts.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

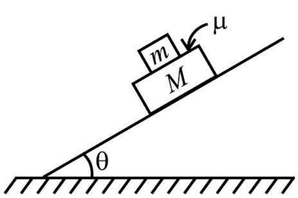
(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Solution Due to pseudo force, the person observes the block to move back. Also the accelerating person does not observe any relative motion between body and the rough surface.

[Ans. (D)]

4. Statement–1: A block of mass *m* is placed on a block of mass *M*, which in turn is placed on smooth fixed inclined plane. The two block system is released from rest as shown. Whatever be the coefficient of friction between both the blocks, the magnitude of friction force between the both the blocks will be zero (As long as they are on inclined surface).



Statement–2 : In the situation of statement-1, there is no tendency of relative motion between the blocks.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

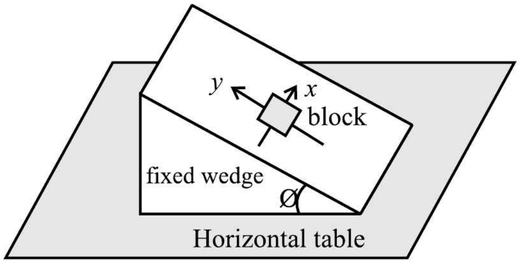
(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

Solution There is no tendency of relative motion between the blocks. Hence Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

[Ans. (A)]

5. **Statement-1 :** A fixed wedge of inclination θ lies on horizontal table. x and y axes are drawn on inclined surface as shown, such that x axis is horizontal and y-axis is along line of greatest slope. A block of mass m is placed (at rest) on inclined surface at origin. The coefficient of friction between block and wedge is μ, such that $\tan\theta = \mu$. Then a force $F > \mu mg\cos\theta$ applied to block parallel to inclined surface and along x-axis can move the block along x-axis.



Statement-2 : To move the block placed at rest on rough inclined surface along the inclined surface, the net force on block (except frictional force) should be greater than μN. (N = normal reaction on block).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

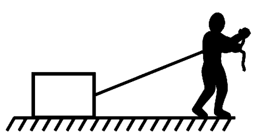
Solution The block cannot move along x-axis by a force applied along x-axis.

For block to move along x-axis, the component of force along y-axis should be equal to mg sinθ, So that net force along y-axis is zero.

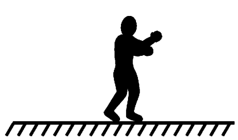
Hence statement-1 is false.

[Ans. (D)]

6. **Statement-1 :** A man and a block rest on smooth horizontal surface. The man holds a rope which is connected to block. The man cannot move on the horizontal surface.



Statement-2 : A man standing at rest on smooth horizontal surface cannot start walking due to absence of friction (The man is only in contact with floor as shown).



(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Solution The man can exert force on block by pulling the rope. The tension in rope will make the man move. Hence statement-1 is false.

[Ans. (D)]

7. **Statement-1 :** The contact force on a rigid spherical body in contact with another rigid body is always directed towards the centre of the rigid body.

   Statement-2 : Whenever two smooth rigid bodies are in contact and press each other, they exert a contact force on each other in direction perpendicular to surface of contact.
   (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
   (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
   (C) Statement-1 is True, Statement-2 is False
   (D) Statement-1 is False, Statement-2 is True

Solution If friction is present contact force between sphere and other rigid body will not be in direction normal to surface of contact. Hence statement 1 is false.

[Ans. (D)]

8. **Statement-1 :** While drawing a line on a paper, friction force acts on paper in the same direction along which line is drawn on the paper.

   Statement-2 : Friction always opposes motion.
   (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
   (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
   (C) Statement-1 is True, Statement-2 is False
   (D) Statement-1 is False, Statement-2 is True

Solution Friction always opposes relative motion.

[Ans. (D)]

True False

1. "When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion. "

   [Ans. False]

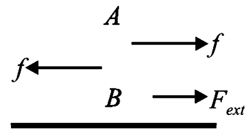
2. The contact force on spherical body is always in radial direction.

Solution If friction acts then contact force will not be radial.

[Ans. False]

3. Whenever a small block is placed (it can move) on the inclined surface of a fixed wedge, the contact force between block and wedge surface is always less than or equal to the weight of the block.

Solution $R = \sqrt{f^2 + N^2} = mg$ if body does not move.



But if it moves then $f < mg \sin\theta$

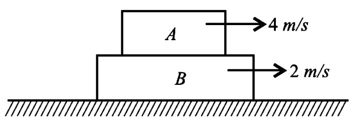
$\therefore \quad R = \sqrt{f^2 + (mg\cos\theta)^2} < mg$

[Ans. True]

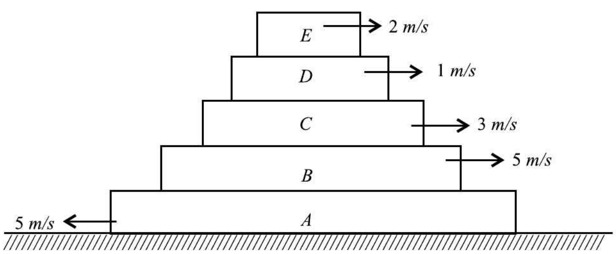
EXERCISE

Exercise–1: Subjective Problems

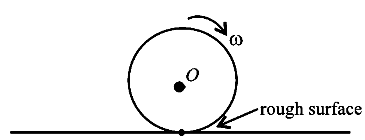
1. All surfaces are rough. Find the direction of friction forces on each block and ground at this instant.



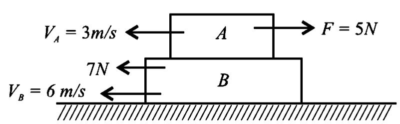
2. Find the direction of friction forces on each block and the ground (Assume all surfaces are rough and all velocities are with respect to ground).



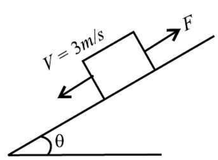
3. The wheel shown in the figure is fixed at 'O' and is in contact with a rough surface as shown. The wheel rotates with an angular velocity ω. What is the direction and nature of friction force on the wheel and on the ground?



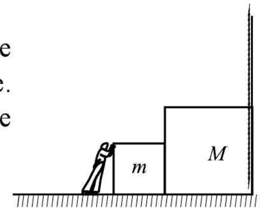
4. In the following figure, find the direction of friction on the blocks and ground .



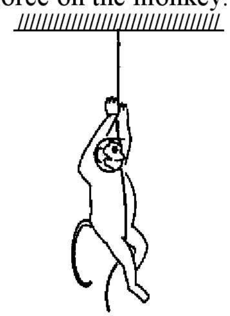
5. In the following figure, find the direction and nature of friction on the block.



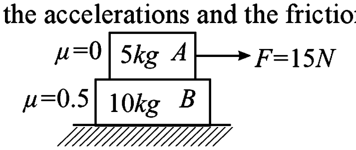
6. An object is slowing down on a rough horizontal plane with a deceleration of $2 m/s^2$. What is the coefficient of kinetic friction?
7. A block is shot with an initial velocity $5ms^{-1}$ on a rough horizontal plane. Find the distance covered by the block till it comes to rest. The coefficient of kinetic friction between the block and plane is 0.1.
8. A block starting from rest slides down 18 m in three seconds on an inclined plane of 30ŗ inclination. Find the coefficient of kinetic friction between the two.
9. A block begins to slide on a rough inclined plane and moves 1 meter in 0.707 seconds. What was the time taken to cover the first half meter on the incline?
10. Suppose the block of the previous problem is pushed down the incline with a force of 4N. How far will the block move in the first two seconds after starting from rest? The mass of the block is 4 kg.
11. The person applies F force on the smaller block as shown in figure. The coefficient of static friction is μ between the blocks and the surface. Find the force exerted by the vertical wall on mass M. What is the value of action-reaction forces between m and M?

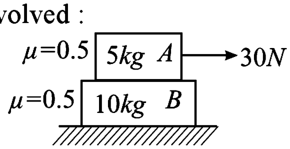


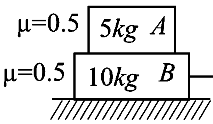
12. Determine the force and its direction on 2 kg block in the above situation. It is known that the two blocks move together. Can we determine the coefficient static friction between the two blocks. If yes, then what is its value?
13. A block of mass 2.5 kg is kept on a rough horizontal surface . It is found that the block does not slide if a horizontal force less than 15 N is applied to it. Also it is found that it takes 5 seconds to slide throughout the first 10 m if a horizontal force of 15 N is applied and the block is gently pushed to start the motion. Taking $g = 10 m/s^2$, calculate the coefficients of static and kinetic friction between the block and the surface.
14. The angle between the resultant contact force and the normal force exerted by a body on the other is called the angle of friction. Show that, if λ be the angle of friction and μ the coefficient of static friction, $\lambda \geq \tan^{-1}\mu$
15. A monkey of mass m is climbing a rope hanging from the roof with acceleration a. The coefficient of static friction between the body of the monkey and the rope is μ. Find the direction and value of friction force on the monkey.

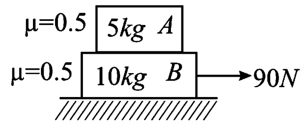


16. Find the accelerations and the friction forces involved :

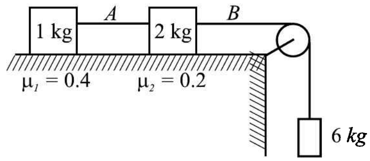
(a) 

(b) 

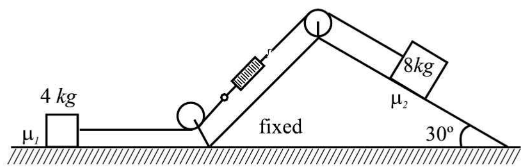
(c) 

(d) 

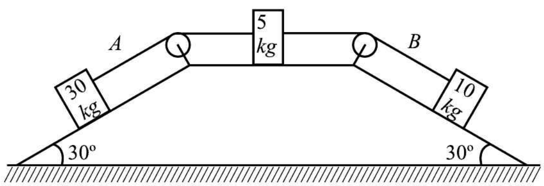
17. Calculate the accelerations of the blocks and the tension in the string *A* & *B*. If the 6 kg block is replaced by a 0.3 kg block, find the new accelerations and tension in the strings *A* & *B*.



18. The reading of spring balance is 32 *N* and the accelerations of both the blocks is 0.5 m/s². Find μ_1 and μ_2.

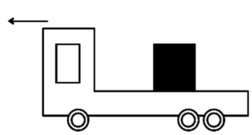


19. A block of mass 15 kg is placed on a long trolley. The coefficient of static friction between the block and the trolley is 0.18. The trolley accelerates from rest with 0.5 m s^{-2} for 20 s and then moves with uniform velocity. Discuss the motion of the block as viewed by (*a*) a stationary observer on the ground. (*B*) an observer fixed with respect to the trolley.
20. The coefficient of friction between 5 kg block and the surface is 0.2. Inclined surfaces are smooth. Find the tension in the strings.

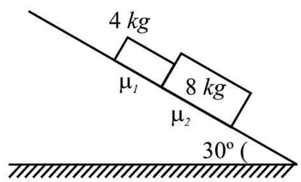


21. The friction coefficient between an athlete's shoes and the ground is 0.90. Suppose a superman wears these shoes and races for 50 m. There is no upper limit on his capacity of running at high speeds. (*a*) Find the minimum time that he will have to take in completing the 50 m starting from rest. (*B*) Suppose he takes exactly this minimum time to complete the 50 m, what minimum time will he take to stop?
22. The rear side of a truck is open and a box of 40 kg mass is placed 5 m away from the open end as shown in figure. The coefficient of friction between the box and the surface below it is 0.15. On a straight road, the truck starts from rest and accelerates with 2 ms^{-2}. At what distance from the starting point of the truck does the box fall off the truck?

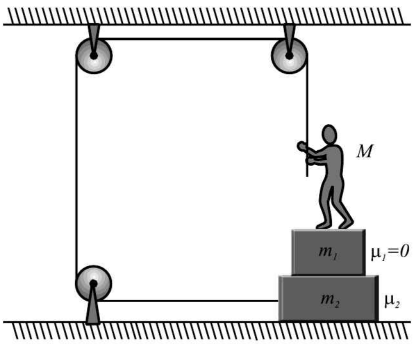
(Ignore the size of the box.)



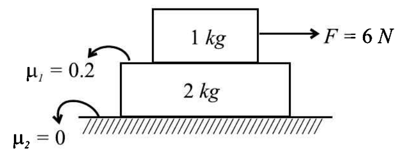
23. In the figure shown below the friction between the 4kg block and the incline as μ_1 and between 8kg and incline is μ_2. Calculate the accelerations of the blocks when (a) $\mu_1 = 0.2$ and $\mu_2 = 0.3$ (b) $\mu_1 = 0.3$ and $\mu_2 = 0.2$. (take $g = 10$ m/s²)



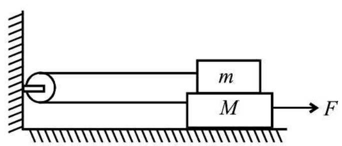
24. What is the minimum value of force required to pull a block of mass M on a horizontal surface having coefficient of friction μ? Also find the angle this force makes with the horizontal.
25. Find the maximum force with which the man can pull the rope such that the mass m_2 does not slide. Find the minimum value of μ_2 if it is known that the blocks do not slide even if the man hangs himself on the rope. Can the value of coefficient of friction be greater than 1?



26. In the situation shown above find the accelerations of the blocks. Also find the accelerations if the force is shifted from the upper block to the lower block.

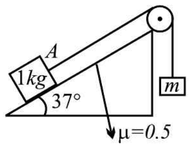


27. A plank of mass m_1 with a bar of mass m_2 placed on it lies on a smooth horizontal plane. A horizontal force growing with time t as $F = kt$ (k is constant) is applied to the bar. Find how the accelerations of the plank a_1 and of the bar a_2 depend on t, if the coefficient of friction between the plank and the bar is equal to μ. Draw the approximate plots of these dependences.
28. In the situation shown below all the surfaces in contact have coefficient μ. (*a*) What is the maximum F that can be applied so that the equilibirium of system is not disturbed. (*b*) If the force exerted is double that of what is found in (*a*), find the accelerations of blocks.

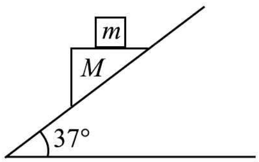


29. If the system of above question is placed in an elevator moving upwards with an acceleration a, repeat the parts (*a*) and (*b*).

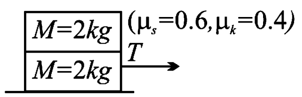
30. A block of mass 2 kg is pushed against a rough vertical wall with a force of 40 N, coefficient of static friction being 0.5. Another horizontal force of 15 N is applied on the block in a direction parallel to the wall. Will the block move ? If yes , in which direction ? If no, find the frictional force exerted by the wall on the block.
31. In the figure, what should be mass m so that block A slide up with a constant velocity?



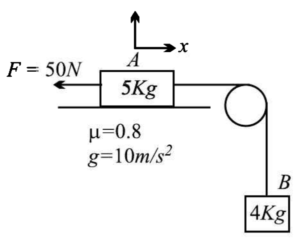
32. A block of mass 1 kg is horizontally thrown with a velocity of 10 m/s on a stationary long plank of mass 2 kg whose surface has a $\mu = 0.5$. Plank rests on frictionless surface. Find the time when m_1 comes to rest w.r.t. plank.
33. Block M slides down on frictionless incline as shown. Find the minimum friction coefficient so that m does not slide with respect to M.



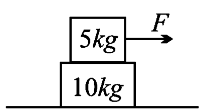
34. The coefficient of static and kinetic friction between the two blocks and also between the lower block and the ground are $\mu_s = 0.6$ and $\mu_k = 0.4$. Find the value of tension T applied on the lower block at which the upper block begins to slip relative to lower block.



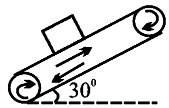
35. Find the acceleration of the blocks and magnitude & direction of frictional force between block A and table, if block A is pulled towards left with a force of 50N.



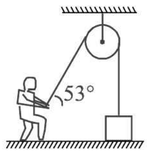
36. Coefficient of friction between 5 kg and 10 kg block is 0.5. If friction between them is 20 N. What is the value of force being applied on 5 kg. The floor is frictionless



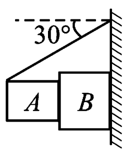
37. A block of mass 1 kg is stationary with respect to a conveyor belt that is accelerating with 1 m/s^2 upwards at an angle of 30° as shown in figure. Determine force of friction on block and contact force between the block & bell.



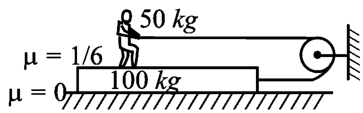
38. A man of mass 63 kg is pulling a mass M by an inextensible light rope passing through a smooth and massless pulley as shown in figure. The coefficient of friction between the man and the ground is $\mu = 3/5$. Find the maximum value of M that can be pulled by the man without slipping on the ground.



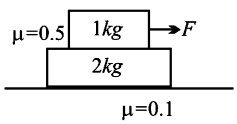
39. Two blocks A and B of mass m 10 kg and 20 kg respectively are placed as shown in figure. Coefficient of friction between all the surfaces is 0.2. Then find tension in string and acceleration of block B. ($g = 10\ \text{m/s}^2$)



40. A man of mass 50 kg is pulling on a plank of mass 100 kg kept on a smooth floor as shown with force of 100 N. If both man & plank move together, find force of friction acting on man.

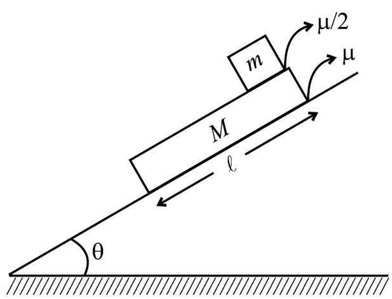


41. What should be minimum value of F so that 2 kg slides on ground but 1 kg does not slide on it? [$g = 10\ \text{m/sec}^2$]

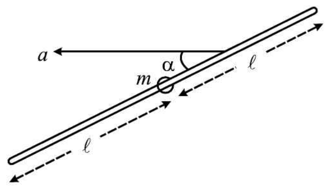


Exercise–2

42. In the above situation it is known that when released the blocks slide. Find the accelerations of the two blocks. Also find the time when the small block will fall off from the larger block.

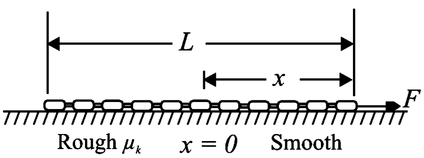


43. A bead of mass 'm' is fitted onto a rod with a length of 2ℓ, and can move on it with friction having the coefficient of friction μ. At the initial moment the bead is in the middle of the rod. The rod moves translationally in a horizontal plane with an acceleration 'a' in the direction forming an angle α with the rod. The time when the bead will leave the rod is : (Neglect the weight of the bead).

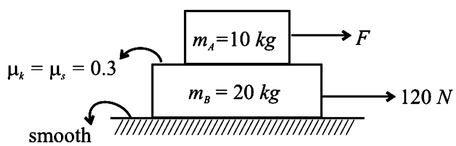


44. A block lying on a long horizontal conveyor belt moving at a constant velocity receives a velocity 5 m/s relative to the ground in the direction opposite to the direction of motion of the conveyor. After $t = 4$ sec, the velocity of the block becomes equal to the velocity of the belt. The coefficient of friction between the block and the belt is 0.2. Calculate the velocity of the conveyor belt.

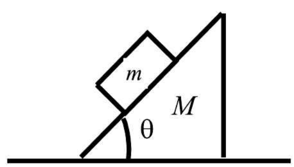
45. A heavy chain with, mass per unit length 'ρ' is pulled by the constant force F along a horizontal surface consisting of a smooth section and a rough section. The chain is initially at rest on the rough surface with $x = 0$. If the coefficient of kinetic friction between the chain and the rough surface is μ_k, then what is the velocity v of the chain when $x = L$, if the force F is greater than μ_k. Find ρgL in order to initiate the motion.



46. In the above situation force F is gradually increased from zero. Discuss the direction and nature of friction and the accelerations of the block at different values of F (Take $g = 10$ m/s^2).



47. A block of mass m lies on wedge of mass M as shown in figure. Answer following parts separately.



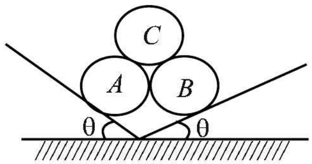
(a) With what minimum acceleration must the wedge be moved towards right horizontally so that block m falls freely?

(b) Find the minimum friction coefficient required between wedge M and ground so that it does not move while block m slips down on it.

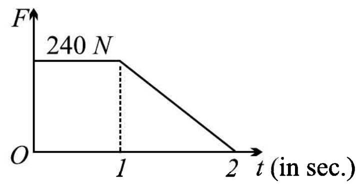
48. A car begins to move at time $t = 0$ and then accelerates along a straight track with a speed given by $V(t) = 2t^2$ ms^{-1} for $0 \le t \le 2$
After the end of acceleration, the car continues to move at a constant speed. A small block initially at rest on the floor of the car begins to slip at $t = 1$sec. and stops slipping at $t = 3$ sec. Find the coefficient of static and kinetic friction between the block and the floor.

49. A thin rod of length 1 m is fixed in a vertical position inside a train, which is moving horizontally with constant acceleration 4 m/s^2. A bead can slide on the rod, and friction coefficient between them is 1/2. If the bead is released from rest at the top of the rod, find the time when it will reach at the bottom.

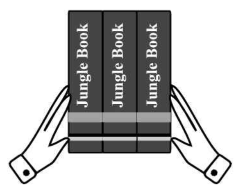
50. Three identical rigid circular cylinders A, B and C are arranged on smooth inclined surfaces as shown in figure. Find the least value of θ that prevent the arrangement from collapse.



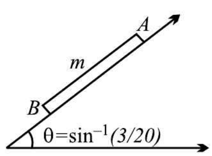
51. A block of mass 50 kg resting on a horizontal surface is acted upon by a force F which varies as shown in the figure. If the coefficient of friction between the block and surface is 0.2, find the time (in second) when the block will come to rest.



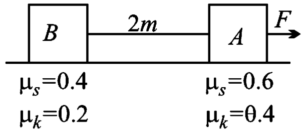
52. Find minimum normal force to be applied by each hand to hold three identical books in vertical position. Each book has mass 'm' and value of coefficient of friction between the books as well as between hand and the book is μ.



53. A plank of mass m is kept on a smooth inclined plane. A man of mass η times the mass of plank moves on the plank, starts from A, such that the plank is at rest, w.r.t. the inclined plane. If he reaches the other end B of the plank in $t = 5$ sec. Then find the acceleration & the value of η, if the length of the plank is 50m.

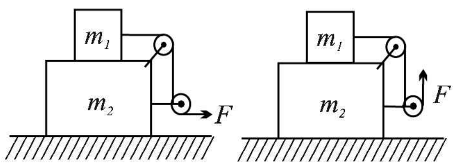


54. Two horizontal blocks each of mass 1/2 kg are connected by a massless, inextensible string of length 2m and placed on a long horizontal table. The coefficient of static & kinetic friction are shown in the figure. Initially the blocks are at rest. If the leading block is pulled with a time dependent horizontal force $F = kt\hat{i}$ where k=1N/sec., determine



(a) The plots of acceleration of each block with time from $t = 0$ to $t = 10$sec.
(b) Velocity of blocks at $t = 10$sec.
(c) Distance transversed by the blocks in the time interval $t = 0$ to $t = 10$sec.
(d) If F stops acting at $t = 10$sec. find after how much further time would B collide with A.

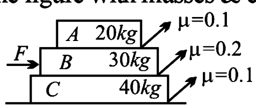
55. $m_1 = 20$kg, $m_2 = 30$kg. m_2 is on smooth surface. Surface between m_1 and m_2 has $\mu_s = 0.5$ and $\mu_k = 0.3$. Find the acceleration of m_1 and m_2 for the following cases



(a) (b)

(a) (i) $F = 160\ N$, (ii) $F = 175\ N$; (b) $F = 160\ N$

56. A system of masses is shown in the figure with masses & co-efficients of friction indicated. Calculate :



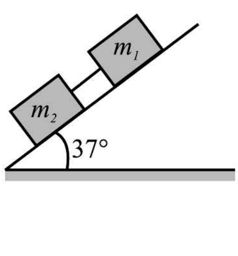
(i) the maximum value of F for which there is no slipping anywhere .
(ii) the minimum value of F for which B slides on C.
(iii) the minimum value of F for which A slips on B.

57. A smooth right circular cone of semi vertical angle $\alpha = \tan^{-1}(5/12)$ is at rest on a horizontal plane. A rubber ring of mass 2.5kg which requires a force of 15N for an extension of 10cm is placed on the cone. Find the increase in the radius of the ring in equilibrium.

Exercise–3

JEE/REE Questions of Previous Years

1. Two blocks $m_1 = 4$kg and $m_2 = 2$kg, connected by a weightless rod on a plane having inclination of 37ŗ. The coefficients of dynamic friction of m_1 and m_2 with the inclined plane are $\mu = 0.25$. Then the common acceleration of the two blocks and the tension in the rod are: **[JEE, 1979]**



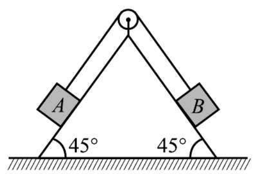
(A) 4 m/s², $T = 0$ (B) 2 m/s², $T = 5\ N$
(C) 10 m/s², $T = 10\ N$ (D) 15 m/s², $T = 9N$

2. A block of mass 2 kg rests on a rough inclined plane making an angle of 30^0 with the horizontal. The coefficient of static friction between the block and the plane is 0.7. The frictional force on the block is: **[IIT, 1980]**
(A) 9.8 N (B) $0.7 \times 9.8\sqrt{3}\ N$
(C) $9.8 \times 7\ N$ (D) $0.8 \times 9.8\ N$

3. A block of mass 1 kg lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is 0.6 If the acceleration of the truck is 5 m/s², the frictional force acting on the block is: **[JEE, 1984]**
(A) 5 N (B) 6
(C) 10 N (D) 15 N

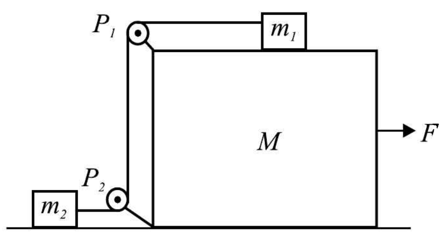
4. A block of mass 0.1kg is held against a wall by applying a horizontal force of 5 N on the block. If the coefficient of friction between the block and the wall is 0.5, the magnitude of the friction force acting on the block is: **[JEE, 1997, 3 marks]**
(A) 2.5 N (B) 0.98 N
(C) 4.9 N (D) 0.49 N

5. Block A of mass m and block B of mass 2 m are placed on a fixed triangular wedge by means of a massless inextensible string and a frictionless pulley as shown in figure. The wedge is inclined at 45ţ to the horizontal on both sides. The coefficient of friction between block A and the wedge is 2/3 and that between block B and the wedge is 1/3. If the system of A and B is released from rest, find **[JEE, 1997,5 Marks]**



(i) the acceleration of A
(ii) tension in the string
(iii) the magnitude and the direction of friction acting on A.

6. In the figure masses m_1, m_2 and M are 20 kg, 5 kg and 50 kg, respectively. The coefficient of friction between M & ground is zero. The coefficient of friction between m_1 & M and that between m_2 & ground is 0.3. The pulleys & the string are massless. The string is perfectly horizontal between P_1 & m_1 and also between P_2 & m_2. The string is perfectly vertical between P_1 & P_2. An external horizontal force F is applied to the mass M. [Take $g = 10$ m/s²] **[JEE, 2000, 2+8 marks]**

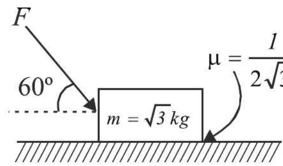


(A) Draw a free–body diagram for mass M, clearly showing all the forces.

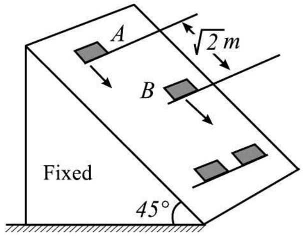
(B) Let the magnitude of the force of friction between m_1 and M be f_1 and that between m_2 and ground be f_2. For a particular F it is found that $f_1 = 2f_2$. Find f_1 and f_2. Write down equations of motion of all the masses. Find F, tension in the string and accelerations of the masses.

7. What is the maximum value of the force F such that the block shown in the arrangement, does not move:

(A) 20 N (B) 10 N
(C) 12 N (D) 15 N **[IIT–JEE (Scr.), 2003]**

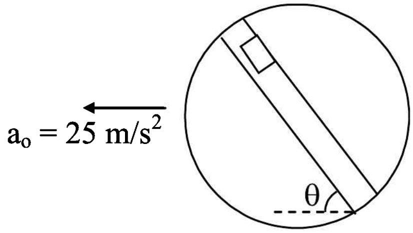


8. Two blocks A and B of equal masses are sliding down along straight parallel lines on an inclined plane of 45°. Their coefficients of kinetic friction are $\mu_A = 0.2$ and $\mu_B = 0.3$ respectively. At $t = 0$, both the blocks are at rest and block A is $\sqrt{2}$ meter behind block B. The time and distance from the initial A position where the front faces of the blocks come in line on the inclined plane as shown in figure. (Use $g = 10\ ms^{-2}$.) **[JEE, 2004, 3 marks]**



(A) 2s, $8\sqrt{2}$m (B) $\sqrt{2}$s, 7m
(C) $\sqrt{2}$s, $7\sqrt{2}$m (D) 2s, $7/\sqrt{2}$m

9. A disc is kept on a smooth horizontal plane with its plane parallel to horizontal plane. A groove is made in the disc as shown in the figure. The coefficient of friction between a mass m inside the groove and the surface of the groove is 2/5 and $\sin\theta = 3/5$. Find the acceleration of mass with respect to the frame of reference of the disc. **[JEE, 2006, 6 marks]**



10. **STATEMENT-1 :** It is easier to pull a heavy object that to push it on a level ground.
and
STATEMENT-2 : The magnitude of frictional force depends on the nature of the two surfaces in contact.

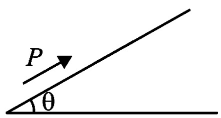
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

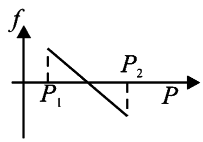
(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

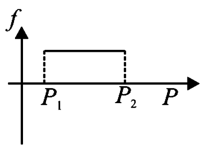
(C) STATEMENT-1 is True, STATEMENT-2 is False

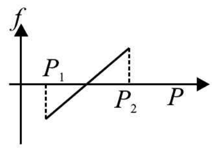
(D) STATEMENT-1 is False, STATEMENT-2 is True **[JEE, 2008]**

11. A block of mass m is on an inclined plane of angle θ. The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force *P* parallel to the plane. The direction of force pointing up the plane is taken to be positive. As *P* is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force *f* versus *P* graph will look like **[JEE, 2010]**

(A) 

(B) 

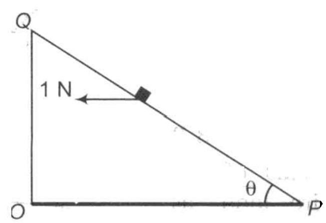
(C) 

(D) 

12. A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ. The force required to just path it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\mu$, then *N* is

[JEE, 2011]

13. A small block of mass of 0.1 kg lies on a fixed inclined plane *PQ* which makes an angle θ with the horizontal. A horizontal force of 1 *N* acts on the block through its centre of mass as shown in the figure. The block remains stationary if (take $g = 10\,\text{m/s}^2$) **[JEE, 2012]**



(A) $\theta = 45°$

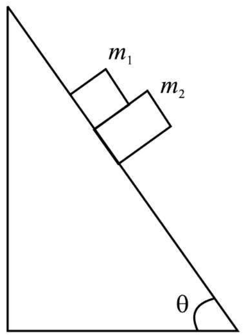
(B) $\theta > 45°$, and a frictional force acts on the block towards *P*

(C) $\theta > 45°$, and a frictional force acts on the block towards *Q*

(D) $\theta < 45°$, and a frictional force acts on the block towards *Q*

14. A block of mass m_1 = 1 kg, another mass m_2 = 2 kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of θ are given in list 1. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In List-II expression for the friction on block m_2 given. Match the correct expression of the friction in List-II with the angles given in List-I, and choose the correct option. The acceleration due to gravity is denotged by g. Useful information tan (5.5°) ≈ 0.1; tan (11.5°) ≈ 0.2; tan (16.5°) ≈ 0.3.

[JEE Advanced, 2014]



List-I		List-II	
P.	$\theta = 5°$	1.	$m_2 g\sin\theta$
Q.	$\theta = 10°$	2.	$(m_1+m_2)\, g \sin\theta$
R.	$\theta = 15°$	3.	$\mu\, m_2\, g \cos\theta$
S.	$\theta = 20°$	4.	$\mu\, (m_1+m_2)\, g \cos\theta$

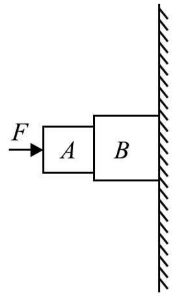
[Ans. (D)]

15. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is **[JEE Main, 2014]**

(A) $\frac{2}{3}m$
(B) $\frac{1}{3}m$
(C) $\frac{1}{2}m$
(D) $\frac{1}{6}m$

[Ans. (D)]

16. Given in the figure are two blocks A and B of weight 20N and 100N respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on the block B is: **[JEE Main, 2015]**



(A) 100 N
(B) 80 N
(C) 120 N
(D) 150 N

[Ans. (C)]

Previous Years' AIEEE Questions

1. The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan\theta = 2\mu$, then the ratio $\frac{F_1}{F_2}$ is **[AIEEE 2011]**

   (A) 4 (B) 1
   (C) 2 (D) 3

EXERCISE

Exercise–4: Objective Problems

1 A force of 98 N is require to just start moving a body of mass 100 kg over ice. The coefficient of static friction is

   (A) 0.6 (B) 0.4
   (C) 0.2 (D) 0.1

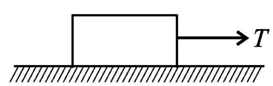
2. The maximum static frictional force is

   (A) Equal to twice the area of surface in contact
   (B) Independent of the area of surface in contact
   (C) Equal to the area of surface in contact
   (D) None of the above

3 Maximum value of static friction is called

   (A) Limiting friction (B) Rolling friction
   (C) Normal reaction (D) Coefficient of friction

4. In the figure shown, a block of weight 10 N resting on a horizontal surface. The coefficient of static friction between the block and the surface $\mu_s = 0.4$. A force of 3.5 N will keep the block in uniform motion, once it has been set in motion. horizontal force of 3 N is applied to the block, then the block will

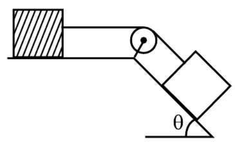


   (A) Move over the surface with constant velocity
   (B) Move having accelerated motion over the surface
   (C) Not move
   (D) First will move with a constant velocity for some time and then will have accelerated motion

5. Starting from rest a body slides down a 45ŗ inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The co-efficient of friction between the body and the inclined plane is

   (A) 0.75 (B) 0.33
   (C) 0.25 (D) 0.80

6. A 60 kg body is pushed with just enough force to start it moving across a floor and the same force continues to act afterwards. The coefficient of static friction and sliding friction are 0.5 and 0.4 respectively. The acceleration of the body is
(A) 6 m/s^2 (B) 4.9 m/s^2
(C) 3.92 m/s^2 (D) 1 m/s^2
7. A 500 kg horse pulls a cart of mass 1500 kg along a level horizontal road with an acceleration of 1 ms^{-2}. If the coefficient of sliding friction between the cart and ground is 0.2, then the force exerted by the horse on the cart in forward direction is (Assume limiting friction is acting)
(A) 3000 N (B) 4500 N
(C) 5000 N (D) 6000 N
8. A fireman of mass 60 kg slides down a pole. He is pressing the pole with a force of 600 N. The coefficient of friction between the hands and the pole is 0.5, with what acceleration will the fireman slide down (g = 10 m/s^2)
(A) 1 m/s^2 (B) 2.5 m/s^2
(C) 10 m/s^2 (D) 5 m/s^2
9. A rope so lies on a table that part of it lays over. The rope begins to slide when the length of hanging part is 25 % of entire length. The co-efficient of friction between rope and table is
(A) 0.33 (B) 0.25
(C) 0.5 (D) 0.2
10. A varying horizontal force $F = at$ acts on a block of mass m kept on a smooth horizontal surface. An identical block is kept on the first block. The coefficient of friction between the blocks is μ. The time after which the relative sliding between the blocks takes place is
(A) $2mg/a$ (B) $2\mu mg$
(C) $\mu mg/a$ (D) none of these
11. The coefficient of friction between a body and ground is $1/\sqrt{3}$ then
(A) The angle of friction can vary from 60^0 to 90^0
(B) The angle of friction can vary from 0^0 to 30^0
(C) The angle of friction can vary from 0^0 to 60^0
(D) The angle of friction can be vary from 30^0 to 90^0
12. Two bodies of identical mass are tied by an ideal string which passes over an ideal pulley. The co-efficient of friction between the bodies and the plane is μ. The minimum value of θ for which the system starts moving is



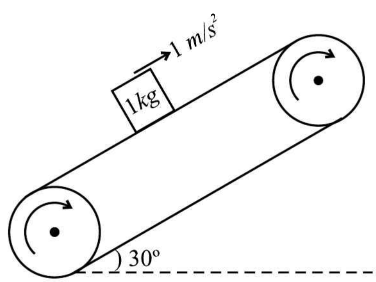
(A) $\cos^{-1}\left(\dfrac{\mu^2 - 1}{\mu^2 + 1}\right)$ (B) $\cos^{-1}\left(\dfrac{\mu^2 + 1}{1 - \mu^2}\right)$
(C) $\cos^{-1}\left(\dfrac{2\mu}{1 + \mu^2}\right)$ (D) $\theta = \cos^{-1}\left(\dfrac{1-\mu^2}{1+\mu^2}\right)$
13. A block of mass M = 5 kg is resting on a rough horizontal surface for which the coefficient of friction is 0.2. When a force F = 40 N is applied, the acceleration of the block will be (g = 10 m/s^2)
(A) 5.73 m/sec^2 (B) 8.0 m/sec^2
(C) 3.17 m/sec^2 (D) 10.0 m/sec^2

14. A body of mass M is kept on a rough horizontal surface (friction coefficient = μ). A person is trying to pull the body by applying a horizontal force but the body is not moving. The force by the surface on A is F where
(A) $F = Mg$
(B) $F = \mu Mg$
(C) $Mg \le F \le Mg\sqrt{1+\mu^2}$
(D) $Mg \ge F \ge Mg\sqrt{1-\mu^2}$

15. A block A kept on an inclined surface just begins to slide if the inclination is 30ţ. The block is replaced by another block B and it is found that it just begins to slide if the inclination is 40ţ.
(A) Mass of A > mass of B
(B) Mass of A < mass of B
(C) Mass of A = mass of B
(D) Insufficient information.

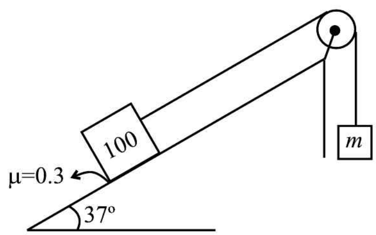
16. A boy of mass M is applying a horizontal force to slide a box of mass M' on a rough horizontal surface. It is known that the boy does not slide . The coefficient of friction between the shoes of the boy and the floor is μ and μ' between the box and the surface. In which of the following cases it is certainly not possible to slide the box ?
(A) $\mu < \mu', M < M'$
(B) $\mu > \mu', M < M'$
(C) $\mu < \mu', M > M'$
(D) $\mu > \mu', M > M$

17. A block of mass 1 kg is stationary with respect to a conveyer belt that is accelerating with 1 m/s^2 upwards at an angle of 30ŗ as shown in figure. Which of the following is/are correct?



(A) Force of friction on block is 6 N upwards.
(B) Force of friction on block is 1.5 N upwards.
(C) Contact force between the block & belt is 10.5 N.
(D) Contact force between the block & belt is $5\sqrt{3}$ N.

18. The value of mass m for which the 100 kg block remains is static equilibrium is

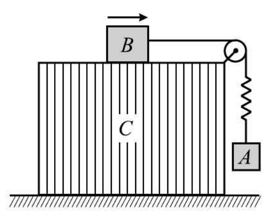


(A) 35 kg
(B) 37 kg
(C) 83 kg
(D) 85 kg

19. Let F, F_N and f denote the magnitudes of the contact force, normal force and the friction exerted by one surface on the other kept in contact. If none of these is zero,
(A) $F > F_N$
(B) $F > f$
(C) $F_N > f$
(D) $F_N - f < F < F_N + f$

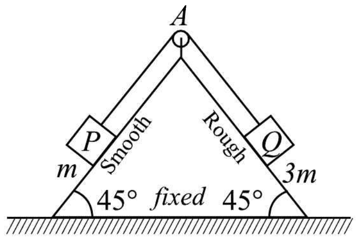
20. The contact force exerted by one body on another body is equal to the normal force between the bodies. It can be said that
(A) The surface must be frictionless.
(B) The force of friction between the bodies is zero.
(C) The magnitude of normal force equals that of friction.
(D) It is possible that the bodies are rough and they do not slip on each other.

21. Out of the following given statements, mark out the correct(s) :
    (A) Static friction is always greater than the kinetic friction.
    (B) Coefficient of static friction is always greater than the coefficient of kinetic friction.
    (C) Limiting friction is always greater than the kinetic friction.
    (D) Limiting friction is never less than the static friction.
22. A block is placed on a rough floor and a horizontal force F is applied on it. The force of friction f by the floor on the block is measured for different values of F and a graph is plotted between them.
    (A) The graph is a straight line of slope 45ŗ
    (B) The graph is straight line parallel to the F-axis.
    (C) The graph is a straight line of slope 45ŗ for small F and a straight line parallel to the F-axis for large F.
    (D) There is a small kink on the graph.
23. A worker wishes to pile a cone of sand into a circular area in his yard. The radius of the circle is r, and no sand is to spill onto the surrounding area. If μ is the static coefficient of friction between each layer of sand along the slope and the sand, the greatest volume of sand that can be stored in this manner is
    (A) $\mu \pi r^3$ (B) $\frac{1}{3}\mu \pi r^3$
    (C) $2 \mu \pi r^2$ (D) $2 \mu \pi r$
24. The upper portion of an inclined plane of inclination α is smooth and the lower portion is rough. A particle slides down from rest from the top and just comes to rest at the foot. If the ratio of the smooth length to rough length is $m : n$, the coefficient of friction is
    (A) $\left[\frac{m+n}{n}\right]\tan\alpha$ (B) $\left(\frac{m+n}{n}\right)\cot\alpha$
    (C) $\left(\frac{m-n}{n}\right)\cot\alpha$ (D) $\frac{1}{2}$
25. Two blocks A & B are connected to each other by a string and a spring. The string passes over a frictionless pulley as shown in the figure. Block B slides over the horizontal top surface of a fixed block C and the block A slides along the vertical side of C with the same uniform speed. The coefficient of friction between the surfaces of the blocks is 0.2. The force constant of the spring is 1960 N m $-^1$. If the mass of the block A is 2 kg, What is the mass of block B, and the extension in the spring is : (g = 9.8 m/s²)



    (A) 5 kg, 5 cm (B) 2 kg, 4 cm
    (C) 10 kg, 1 cm (D) 1 kg, 2 cm
26. A fixed wedge with both surface inclined at 45^0 to the horizontal as shown in the figure. A particle P of mass m is held on the smooth plane by a light string which passes over a smooth pulley A and attached to a particle Q of mass 3m which rests on the rough plane. The system is released from rest. Given that the acceleration of each particle is of magnitude $\frac{g}{5\sqrt{2}}$ then

(a) The tension in the string is



(A) mg (B) $\frac{6}{5\sqrt{2}}$

(C) $\frac{mg}{2}$ (D) $\frac{mg}{4}$

(b) In the above question the coefficient of friction between Q and the rough plane is

(A) $\frac{4}{5}$ (B) $\frac{1}{5}$

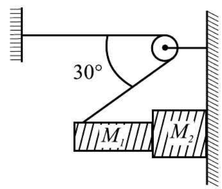
(C) $\frac{3}{5}$ (D) $\frac{2}{5}$

(c) In the above question the magnitude and direction of the force exerted by the string on the pulley is

(A) $\frac{6mg}{5}$ downward (B) $\frac{6mg}{5}$ upward

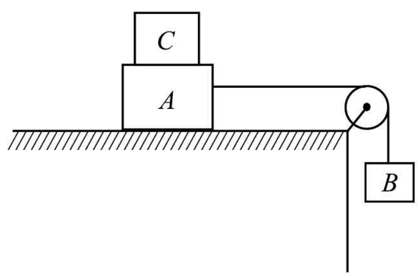
(C) $\frac{mg}{5}$ downward (D) $\frac{mg}{4}$ downward

27. Two blocks with masses m_1 and m_2 of 10 kg and 20 kg respectively are placed as in fig. $\mu_s = 0.2$ between all surfaces, then tension in string and acceleration of m_2 block at this moment will be



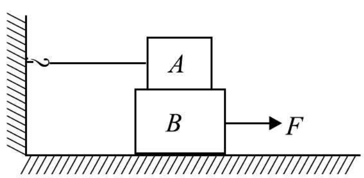
(A) 250 N, 3 m/s^2 (B) 200 N, 6 m/s^2
(C) 306 N, 4.7 m/s^2 (D) 400 N, 6.5 m/s^2

28. Two masses A and B of 10 kg and 5 kg respectively are connected with a string passing over a frictionless pulley fixed at the corner of a table as shown. The coefficient of static friction of A with table is 0.2. The minimum mass of C that may be placed on A to prevent it from moving is



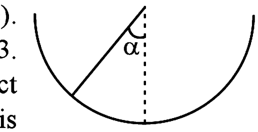
(A) 15 kg (B) 10 kg
(C) 5 kg (D) 12 kg

29. A block A with mass 100 kg is resting on another block B of mass 200 kg. As shown in figure a horizontal rope tied to a wall holds it. The coefficient of friction between A and B is 0.2 while coefficient of friction between B and the ground is 0.3. The minimum required force F to start moving B will be



(A) 900 N (B) 100 N
(C) 1100 N (D) 1200 N

30. An insect crawls up a hemispherical surface very slowly (see figure). The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by



(A) $\cot\alpha = 3$ (B) $\tan\alpha = 3$
(C) $\sec\alpha = 3$ (D) $\text{cosec}\,\alpha = -3$

31. A body takes time t to reach the bottom of an inclined plane of angle θ with the horizontal. If the plane is made rough, time taken now is 2t. The coefficient of friction of the rough surface is

(A) $\frac{3}{4}\tan\theta$ (B) $\frac{2}{3}\tan\theta$ (C) $\frac{1}{4}\tan\theta$ (D) $\frac{1}{2}\tan\theta$

32. A cart weighing 200 N can roll without friction along a horizontal path. The cart carries a block weighing 20 N. The coefficient of friction between the block and the cart is 0.25 and g = 10 m/s^2.

(a) When a force of 2 N is applied to the block then

(i) The force of friction between the block and cart is

(A) $\frac{20}{11}N$ (B) $\frac{10}{11}N$
(C) $\frac{40}{11}N$ (D) $\frac{2}{11}N$

(ii) Acceleration of the block and cart would be respectively

(A) $\frac{1}{11}\text{m/s}^2, \frac{1}{11}\text{m/s}^2$ (B) $\frac{1}{9}\text{m/s}^2, \frac{1}{9}\text{m/s}^2$
(C) $\frac{1}{9}\text{m/s}^2, \frac{1}{11}\text{m/s}^2$ (D) $\frac{1}{6}\text{m/s}^2, \frac{1}{6}\text{m/s}^2$

(b) When a force of 20 N is applied to the block then

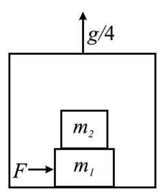
(i) The force of friction between the block and cart is

(A) 2 N (B) 5 N
(C) 8 N (D) 6 N

(ii) Acceleration of the block and cart would be respectively:

(A) 7.5 m/s^2, 0.25 m/s^2 (B) 0.25 m/s^2, 7.5 m/s^2
(C) 7.5 m/s^2, 7.5 m/s^2 (D) 0.25 m/s^2, 0.25 m/s^2

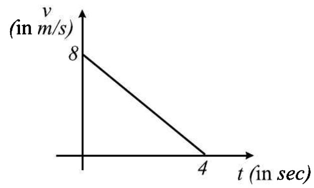
33. A plank of mass M_1 = 8 kg with a bar of mass M_2 = 2 kg placed on its rough surface, lie on a smooth floor of elevator ascending with an acceleration $g/4$. The coefficient of friction is μ = 1/5 between m_1 and m_2. A horizontal force F = 30 N is applied to the plank. Then the acceleration of bar and the plank in the reference frame of elevator are



(A) 3.5 m/s², 5 m/s² (B) 5 m/s², $\frac{50}{8}$ m/s²

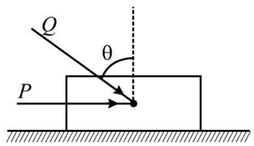
(C) 2.5 m/s², $\frac{25}{8}$ m/s² (D) 4.5 m/s², 4.5m/s²

34. A block of mass 2 kg is given a push horizontally and then the block starts. Sliding over a horizontal plane. The graph shows the velocity-time graph of the motion. The coefficient of kinetic friction between the plane and the block is



(A) 0.02 (B) 0.2
(C) 0.04 (D) 0.4

35. A block of mass m lying on a rough horizontal plane is acted upon by a horizontal force *P* and another force *Q* inclined an at an angle θ to the vertical. The minimum value of coefficient of friction between the block and the surface for which the block will remain in equilibrium is



(A) $\frac{P+Q\sin\theta}{mg+Q\cos\theta}$ (B) $\frac{P\cos\theta+Q}{mg-Q\sin\theta}$

(C) $\frac{P+Q\cos\theta}{mg+Q\sin\theta}$ (D) $\frac{P\sin\theta-Q}{mg-Q\cos\theta}$

36. A block moves down a smooth inclined plane of inclination θ, its velocity on reaching the bottom is *v*. If it slides down a rough inclined plane of same inclination, its velocity on reaching the bottom is v/n, where n is a number greater than one. The coefficient of friction is given by

(A) $\mu = \tan\theta\left(1-\frac{1}{n^2}\right)$ (B) $\mu = \cot\theta\left(1-\frac{1}{n^2}\right)$

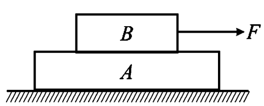
(C) $\mu = \tan\theta\left(1-\frac{1}{n^2}\right)^{1/2}$ (D) $\mu = \cot\theta\left(1-\frac{1}{n}\right)^{1/2}$

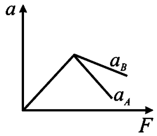
37. A uniform chain of mass *M* and length *L* is lying on a table in such a manner that a part of it is hanging down from an edge of the table. If coefficient of friction is μ, then the maximum length of the chain that can hang without sliding is

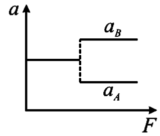
(A) $\frac{L}{\mu}$ (B) $\frac{L}{\mu-1}$

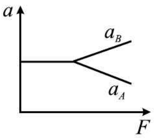
(C) $\frac{\mu L}{\mu-1}$ (D) $\frac{\mu L}{\mu+1}$

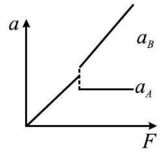
38. A wooden block *A* of mass *M* is placed on a frictionless horizontal surface. On top of *A*, another lead block *B* also of mass *M* is placed. A horizontal force of magnitude *F* is applied to *B*. Force *F* is increased continuously from zero. The graphs below show the dependence of acceleration of the two blocks on the force *F*. Which of the graphs is correct. [$\mu_k < \mu_s$]



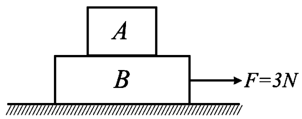
(A) 

(B) 

(C) 

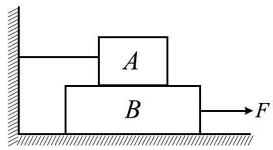
(D) 

39. In the arrangement shown mass of A = 1 kg, mass of B = 2kg and coefficient of friction between A and B is 0.2. There is no friction between B and ground. The frictional force on A is (g = 10 m/s²).



(A) 0 N (B) 2 N
(C) 1.96 N (D) 1 N

40. A is a 100 kg block and B is a 200 kg block. As shown in figure, the block A is attached to a string tied to a wall. The coefficient of friction between A and B is 0.2 and the coefficient of friction between B and floor is 0.3. Then the minimum force required to move the block B will be (g = 10 m/s²)

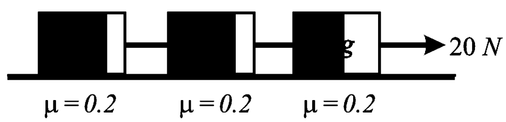


(A) 600 N (B) 800 N
(C) 900 N (D) 1100 N

41. Two similar wooden blocks are tied one behind the other and pulled across a level surface. Friction is not negligible. The force required to pull them at constant speed is F. If one block is stacked upon the other then the new force required to pull then at constant speed will be approximately

(A) $\frac{F}{2}$ (B) F
(C) $\sqrt{2}F$ (D) 2 F

42. (I) In the arrangement shown tension in the string connecting 4kg and 6kg masses is



(A) 8N
(B) 12N
(C) 6N
(D) 4N

(II) Friction force on 4 kg block is

(A) 4N (B) 6 N
(C) 12 N (D) 8 N

(III) Friction force on 6 kg block is

(A) 12 N (B) 8 N
(C) 6 N (D) 4 N

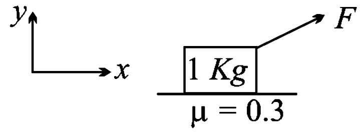
43. A body is placed on a rough inclined plane of inclination θ. As the angle θ is increased from 0° to 90° the contact force between the block and the plane

(A) remains constant (B) first remains constant than decreases
(C) first decreases then increases (D) first increases then decreases

44. A block is projected upwards on an inclined plane of inclination 37° along the line of greatest slope of $\mu = 0.5$ with velocity of 5 m/s. The block 1st stops at a distance of __________ from starting point

(A) 1.25 m (B) 2.5 m
(C) 10 m (D) 12.5 m

45. A force $\vec{F} = \hat{i} + 4\hat{j}$ acts on block shown. The force of friction acting on the block is:



(A) $-\hat{i}$ (B) $-1.8\hat{i}$
(C) $-2.4\hat{i}$ (D) $-3\hat{i}$

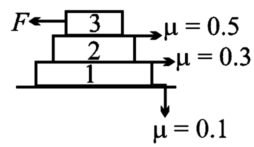
46. A 1.0 kg block of wood sits on top of an identical block of wood, which sits on top of a flat level table made of plastic. The coefficient of static friction between the wood surfaces is μ_1, and the coefficient of static friction between the wood and plastic is μ_2.
A horizontal force F is applied to the top block only, and this force is increased until the top block starts to move. The bottom block will move with the top block if and only if

(A) $\mu_1 < \frac{1}{2}\mu_2$ (B) $\frac{1}{2}\mu_2 < \mu_1 < \mu_2$
(C) $\mu_2 < \mu_1$ (D) $2\mu_2 < \mu_1$

47. A block of mass 2 kg slides down an incline plane of inclination 30°. The coefficient of friction between block and plane is 0.5. The contact force between block and plank is :

(A) 20 *Nt* (B) $10\sqrt{3}$ *Nt*
(C) $5\sqrt{7}$ *Nt* (D) $5\sqrt{15}$ *Nt*

48. If force *F* is increasing with time and at $t = 0$, $F = 0$ where will slipping first start?

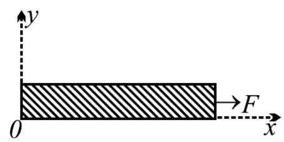


(A) between 3 kg and 2 kg
(B) between 2 kg and 1 kg
(C) between 1 kg and ground
(D) both (*A*) and (*B*)

49. A man is standing on a rough ($\mu = 0.5$) horizontal disc rotating with constant angular velocity of 5 rad/sec. At what distance from centre should he stand so that he does not slip on the disc?

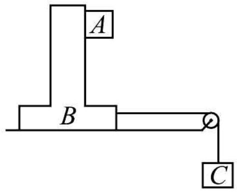
(A) $R \le 0.2$m (B) $R > 0.2$ m
(C) $R > 0.5$ m (D) $R > 0.3$ m

50. A uniform rod of length *L* and mass *M* has been placed on a rough horizontal surface. The horizontal force F applied on the rod is such that the rod is just in the state of rest. If the coefficient of friction varies according to the relation $\mu = Kx$ where *K* is a +ve constant. Then the tension at mid point of rod is



(A) *F*/2 (B) *F*/4
(C) *F*/8 (D) None

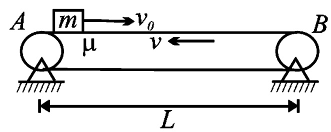
51. In the arrangement shown in the figure, mass of the block B and A is 2m and m respectively. Surface between B and floor is smooth. The block B is connected to the block C by means of a string pulley system. If the whole system is released, then find the minimum value of mass of block C so that block A remains stationary w.r.t. B. Coefficient of friction between A and B is μ



(A) $\frac{m}{\mu}$ (B) $\frac{2m+1}{\mu+1}$

(C) $\frac{3m}{\mu-1}$ (D) $\frac{6m}{\mu+1}$

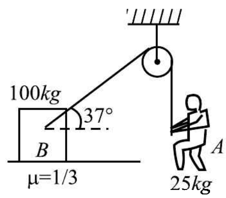
52. With what minimum velocity should block be projected from left end A towards end B such that it reaches the other end B of conveyer belt moving with constant velocity v. Friction coefficient between block and belt is μ.



(A) $\sqrt{\mu g L}$ (B) $\sqrt{2\mu g L}$

(C) $\sqrt{3\mu g L}$ (D) $2\sqrt{\mu g L}$

53. Block B of mass 100 kg rests on a rough surface of friction coefficient μ = 1/3. A rope is tied to block B as shown in figure. The maximum acceleration with which boy A of 25 kg can climbs on rope without making block move is



(A) $\frac{4g}{3}$ (B) $\frac{g}{3}$

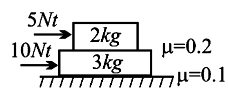
(C) $\frac{g}{2}$ (D) $\frac{3g}{4}$

54. A car moves along a circular track of radius R banked at an angle of 30° to the horizontal. The coefficient of static friction between the wheels and the track is μ. The maximum speed with which the car can move without skidding out is

(A) $\left[2gR(1+\mu)/\sqrt{3}\right]^{1/2}$ (B) $\left[gR(1-\mu)/(\mu+\sqrt{3})\right]^{1/2}$

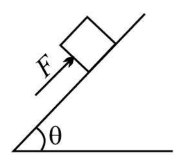
(C) $\left[gR(1+\mu\sqrt{3})/(\mu+\sqrt{3})\right]^{1/2}$ (D) None

55. The system shown in figure is released



(A) $a_1 = 0.35$ m/s^2 ; $a_2 = 4.5$ m/s^2
(B) $a_1 = 3$ m/s^2 ; $a_2 = 0.5$ m/s^2
(C) $a_1 = 2$ m/s^2 ; $a_2 = 2$ m/s^2
(D) $a_1 = 0.5$ m/s^2 ; $a_2 = 3$ m/s^2

56. A block placed on a rough inclined plane of inclination (θ=30°) can just be pushed upwards by applying a force "F" as shown. If the angle of inclination of the inclined plane is increased to (θ = 60°), the same block can just be prevented from sliding down by application of a force of same magnitude. The coefficient of friction between the block and the inclined plane is



(A) $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$ (B) $\dfrac{2\sqrt{3}-1}{\sqrt{3}+1}$

(C) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$ (D) None of these

For Q. 57 to Q.61 refer Figure-1.(5 questions)

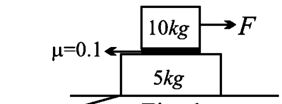
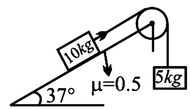


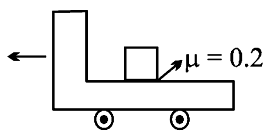
Fig. 1

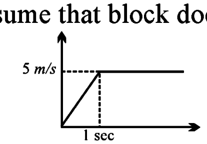
57. When $F = 2N$, the frictional force between 5 kg block and ground is
    (A) $2N$ (B) 0
    (C) $8\,N$ (D) $10\,N$
58. When $F = 2N$, the frictional force between 10 kg block and 5 kg block is
    (A) $2N$ (B) $15\,N$
    (C) $10\,N$ (D) None
59. The maximum "F" which will not cause motion of any of the blocks.
    (A) $10\,N$ (B) $15\,N$
    (C) data insufficient (D) None
60. The maximum acceleration of 5 kg block
    (A) 1 m/s^2 (B) 3 m/s^2
    (C) 0 (D) None
61. The acceleration of 10 kg block when $F = 30N$
    (A) 2 m/s^2 (B) 3 m/s^2
    (C) 1 m/s^2 (D) None
62. The blocks are in equilibrium. The friction force acting on 10 kg block is

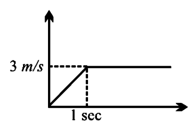


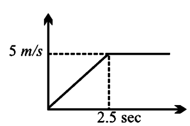
    (A) $10\,N$ down the plane (B) $40\,N$ up the plane
    (C) $10\,N$ up the plane (D) None

63. A truck starting from rest moves with an acceleration of 5 m/s^2 for 1 sec and then moves with constant velocity. The velocity w.r.t ground v/s time graph for block in truck is :
    (Assume that block does not fall off the truck)



(A) 

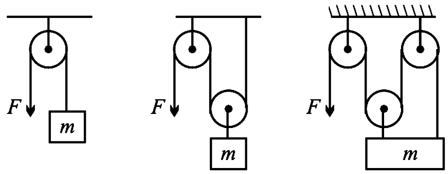
(B) 

(C) 

(D) None of these

64. A small block of mass m is projected horizontally with speed u where friction coefficient between block and plane is given by $\mu = cx$, where x is displacement of the block on plane. Find maximum distance covered by the block.
    (A) $\dfrac{u}{\sqrt{cg}}$ (B) $\dfrac{u}{\sqrt{2cg}}$ (C) $\dfrac{2u}{\sqrt{cg}}$ (D) $\dfrac{u}{2\sqrt{cg}}$

65. Equal force F ($> mg$) is applied to string in all the 3 cases. Starting from rest, the point of application of force moves a distance of 2 m down in all cases. In which case the block has maximum kinetic energy?



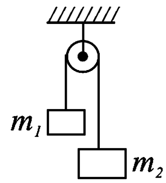
(1) (2) (3)

(A) 1
(B) 2
(C) 3
(D) equal in all 3 cases

One or More than One Option Correct

66. A student calculates the acceleration of m_1 in figure shown as

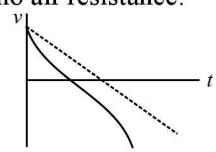
$$a_1 = \frac{(m_1 - m_2)g}{m_1 + m_2}.$$

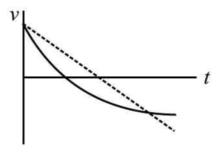


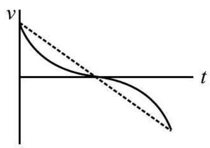
Which assumption is not required to do this calculation.

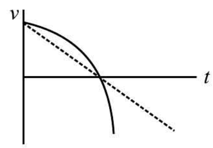
(A) Pulley is frictionless
(B) String is massless
(C) Pulley is massless
(D) String is inextensible

67. Which graph shows best the velocity-time graph for an object launched vertically into the air when air resistance is given by $|D| = bv$? The dashed line shows the velocity graph if there were no air resistance.

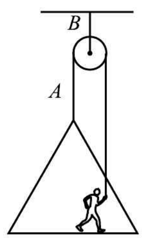
(A) 

(B) 

(C) 

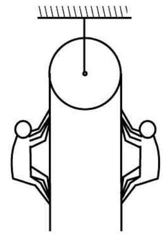
(D) 

68. To paint the side of a building, painter normally hoists himself up by pulling on the rope A as in figure. The painter and platform together weigh 200N. The rope B can withstand 300N. Then



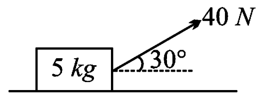
(A) The maximum acceleration that painter can have upwards is 5m/s^2.
(B) To hoist himself up, rope B must withstand minimum 400N force.
(C) Rope A will have a tension of 100 N when the painter is at rest.
(D) The painter must exert a force of 200N on the rope A to go downwards slowly.

69. Two men of unequal masses hold on to the two sections of a light rope passing over a smooth light pulley.
Which of the following are possible?



(A) The lighter man is stationary while the heavier man slides with some acceleration
(B) The heavier man is stationary while the lighter man climbs with some acceleration
(C) The two men slide with the same acceleration in the same direction
(D) The two men move with accelerations of the same magnitude in opposite directions

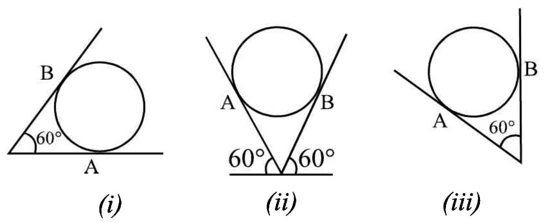
70. Adjoining figure shows a force of 40 N acting at 30° to the horizontal on a body of mass 5 kg resting on a smooth horizontal surface. Assuming that the acceleration of free–fall is 10 ms^{-2}, which of the following statements A, B, C, D, E is (are) correct?



1. The horizontal force acting on the body is 20 N
2. The weight of the 5 kg mass acts vertically downwards
3. The net vertical force acting on the body is 30 N

(A) 1, 2, 3 (B) 1, 2
(C) 2 only (D) 1 only

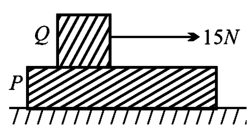
71. An iron sphere weighing 10 N rests in a V shaped smooth trough whose sides form an angle of 60° as shown in the figure. Then the reaction forces are



(A) $R_A = 10\,N$ & $R_B = 0$ in case (i)
(B) $R_A = 10\,N$ & $R_B = 10\,N$ in case (ii)
(C) $R_A = \dfrac{20}{\sqrt{3}}\,N$ & $R_B = \dfrac{10}{\sqrt{3}}\,N$ in case (iii)
(D) $R_A = 10\,N$ & $R_B = 10\,N$ in all the three cases

72. For ordinary terrestrial experiments, which of the following observers below are inertial.
(A) A child revolving in a " giant wheel".
(B) A driver in a sports car moving with a constant high speed of 200 km/h on a straight road.
(C) The pilot of an aeroplane which is taking off.
(D) A cyclist negotiating a sharp turn.

73. A long plank P of the mass 5 kg is placed on a smooth floor. On P is placed a block Q of mass 2 kg. The coefficient of friction between P and Q is 0.5. If a horizontal force 15N is applied to Q, as shown, and you may take g as 10N kg.

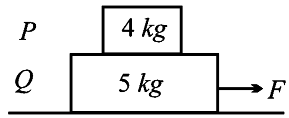


(A) The reaction force on Q due to P is 10N
(B) The acceleration of Q relative to P is 2.5 m/s^2

(C) The acceleration of P relative to the Floor is 2.0 m/s^2

(D) The acceleration of centre of mass of $P+Q$ system relative to the floor is (15/7)m/s^2

74. The coefficient of friction between 4kg and 5 kg blocks is 0.2 and between 5kg block and ground is 0.1 respectively. Choose the correct statements



(A) Minimum force needed to cause system to move is 17N

(B) When force is 4N static friction at all surfaces is 4N to keep system at rest

(C) Maximum acceleration of 4kg block is 2m/s^2

(D) Slipping between 4kg and 5 kg blocks start when F is 17N

75. In a tug–of–war contest, two men pull on a horizontal rope from opposite sides. The winner will be the man who

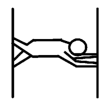
(A) Exerts greater force on the rope

(B) Exerts greater force on the ground

(C) Exerts a force on the rope which is greater than the tension in the rope

(D) Makes a smaller angle with the vertical

76. A man balances himself in a horizontal position by pushing his hands and feet against two parallel walls. His centre of mass lies midway between the walls. The coefficients of friction at the walls are equal. Which of the following is not correct?



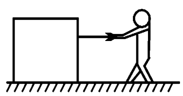
(A) He exerts equal forces on walls

(B) He exerts only horizontal forces on the walls

(C) The forces of friction at the walls are equal

(D) The forces exerted by the walls on him are not horizontal

77. A man pulls a block heavier than himself with a light horizontal rope. The coefficient of friction is the same between the man and the ground, and between the block and the ground



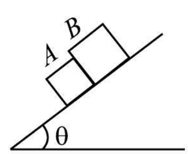
(A) The block will not move unless the man also moves

(B) The man can move even when the block is stationary

(C) If both move, the acceleration of the man is greater than the acceleration of the block

(D) None of the above assertions is correct

78. The two blocks A and B of equal mass are initially in contact when released from rest on the inclined plane. The coefficients of friction between the inclined plane A and B are μ_1 and μ_2 respectively.



(A) If $\mu_1 > \mu_2$, the blocks will always remain in contact.

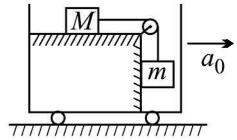
(B) If $\mu_1 < \mu_2$, the blocks will slide down with different accelerations. (if blocks slide)

(C) If $\mu_1 > \mu_2$, the blocks will have a common acceleration $\frac{1}{2}(\mu_1+\mu_2)\, g \sin\theta$.

(D) If $\mu_1 < \mu_2$, the blocks will have a common acceleration $\frac{\mu_1\mu_2 g}{\mu_1+\mu_2}\sin\theta$.

Question No. 79 to 81 (3 questions)

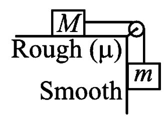
Imagine the situation in which the given arrangement is placed inside a trolley that can move only in the horizontal direction, as shown in figure. If the trolley is accelerated horizontally along the positive x-axis with a_0, then



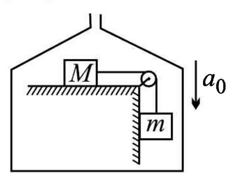
79. Choose the correct statement(s).
   (A) There exists a value of $a_0 = \beta$ at which friction force on block M becomes zero
   (B) There exists two values of $a_0 = (\beta + \alpha)$ and $(\beta - \alpha)$ at which the magnitudes of friction acting on block M are equal
   (C) The maximum value of static friction force acts on the block M at two accelerations a_1 and a_2 such that $a_1 + a_2 = 2\beta$
   (D) The maximum value of friction is independent of the acceleration a_0.
80. If a_{min} and a_{max} are the minimum and maximum values of a_0 for which the blocks remain stationary with respect to the surface, then identify the correct statements
   (A) If $a_0 < a_{min}$, the block m accelerates downward
   (B) If $a_0 > a_{max}$, the block m accelerates upward
   (C) The block m does not accelerate up or down when $a_{min} \leq a_0 \leq a_{max}$
   (D) The friction force on the block M becomes zero when $a_0 = \dfrac{a_{min} + a_{max}}{2}$
81. Identify the correct statement(s) related to the tension T in the string
   (A) No value of a_0 exists at which T is equal to zero
   (B) There exists a value of a_0 at which $T = mg$
   (C) If $T < mg$, then it must be more than μMg
   (D) If $T > mg$, then it must be less than μMg

Question No. 82 to 85 (4 questions)

In figure, two blocks M and m are tied together with an inextensible and light string. The mass M is placed on a rough horizontal surface with coefficient of friction μ and the mass m is hanging vertically against a smooth vertical wall. The pulley is frictionless.



82. Choose the correct statement(s)
   (A) The system will accelerate for any value of m
   (B) The system will accelerate only when $m > M$
   (C) The system will accelerate only when $m > \mu M$
   (D) Nothing can be said
83. Choose the correct statement(s) related to the tension T in the string
   (A) When $m < \mu M$, $T = mg$
   (B) When $m < \mu M$, $T = Mg$
   (C) When $m > \mu M$, $\mu Mg < T < mg$
   (D) When $m > \mu M$, $mg < T < \mu Mg$
84. Imagine a situation in which the given arrangement is placed inside an elevator that can move only in the vertical direction and compare the situation with the case when it is placed on the ground. When the elevator accelerates downward with $a_0 (< g)$, then

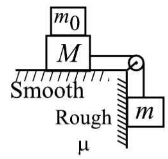


(A) The limiting friction force between the block M and the surface decreases
(B) The system can accelerate with respect to the elevator even when $m < \mu M$
(C) The system does not accelerate with respect to the elevator unless $m > \mu M$
(D) The tension in the string decreases

85. When the downward acceleration of the elevator becomes equal to g, then
(A) Both the blocks remain stationary with respect to the elevator
(B) Both the blocks accelerate vertically downwards with g with respect to ground
(C) The tension in the string becomes equal to zero
(D) The friction force between the block M and the surface is zero

Question No. 86 to 92 (7 questions)

A block of mass M is placed on a horizontal surface and it is tied with an inextensible string to a block of mass, as shown in figure. A block of mass m_0 is also placed on M



86. If there is no friction between any two surfaces, then
(A) The downward acceleration of the block m is $\dfrac{mg}{m+m_0+M}$
(B) The acceleration of m_0 is zero
(C) If the tension in the string is T then $Mg < T < mg$
(D) All the above

If a friction force exist between block M and the horizontal surface with the coefficient of friction μ.

87. The minimum value of μ for which the block m remains stationary is
(A) $\dfrac{m}{M}$ (B) $\dfrac{m}{M+m_0}$
(C) $\dfrac{M+m_0}{M}$ (D) $\dfrac{M}{M+m_0}$

88. If $\mu < \mu_{min}$ (the minimum friction required to keep the block m stationary), then the downward acceleration of m is
(A) $\left[\dfrac{m-\mu M}{m+M}\right]g$ (B) $\left[\dfrac{m-\mu(m_0+M)}{m+m_0+M}\right]g$
(C) $\left[\dfrac{m-\mu(m_0+M)}{m+M}\right]g$ (D) $\left[\dfrac{m-\mu M}{m+m_0+M}\right]g$

89. In previous problem, the tension in the string will be
(A) $\dfrac{mM}{m+M}g$ (B) $\dfrac{m(m_0+M)}{m+m_0+M}g$
(C) $\left[\dfrac{m+\mu(m_0+M)}{m+M}\right]Mg$ (D) $\left[\dfrac{mM+\mu m(m_0+M)}{m+M}\right]g$

90. If μ_0 be the coefficient of friction between the block M and the horizontal surface then the minimum value of m_0 required to keep the block m stationary is

(A) $\dfrac{m}{\mu} - M$ (B) $\dfrac{m-M}{\mu}$

(C) $\dfrac{m}{\mu} + M$ (D) $\dfrac{m+M}{\mu}$

91. If friction force exists between the block M and the block m_0 and not between the block M and the horizontal surface, then the minimum value of μ for which the block m remains stationary is

(A) $\dfrac{m}{m_0}$ (B) $\dfrac{m}{m_0 + M}$

(C) $\dfrac{m - m_0}{M}$ (D) None of these

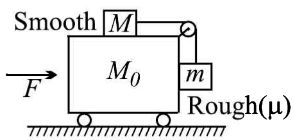
92. The minimum value of μ between the block M and m_0 (taking horizontal surface frictionless) for which all the three blocks move together, is

(A) $\dfrac{m}{m + m_0 + M}$ (B) $\dfrac{m}{m + M}$

(C) $\dfrac{m_0}{m + m_0 + M}$ (D) None of these

Question No. 93 to 96 (4 questions)

Imagine a situation in which the horizontal surface of block M_0 is smooth and its vertical surface is rough with a coefficient of friction μ.



93. Identify the correct statement(s)

(A) If $F = 0$, the blocks cannot remain stationary

(B) For one unique value of F, the blocks M and m remain stationary with respect to M_0

(C) The limiting friction between m and M_0 is independent of F

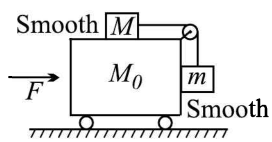
(D) There exist a value of F at which friction force is equal to zero

94. In above problem, choose the correct value(s) of F which the blocks M and m remain stationary with respect to M_0

(A) $(M_0 + M + m)\dfrac{g}{\mu}$ (B) $\dfrac{m(M_0 + M + m)g}{M - \mu m}$

(C) $(M_0 + M + m)\dfrac{mg}{M}$ (D) None of these

95. Consider a special situation in which both the faces of the block M_0 are smooth, as shown in adjoining figure. Mark out the correct statement(s)



(A) If $F = 0$, the blocks cannot remain stationary

(B) For one unique value of F, the blocks M and m remain stationary with respect to block M_0

(C) There exists a range of F for which blocks M and m remain stationary with respect to block M_0

(D) Since there is no friction, therefore, blocks M and m cannot be in equilibrium with respect to M_0

96. In above problem, the value(s) of F for which M and m are stationary with respect to M_0

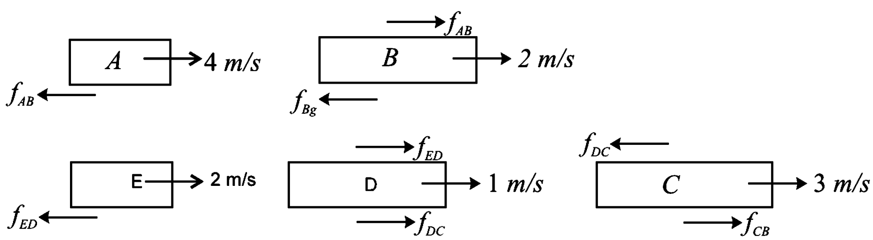
(A) $(M_0 + M + m)g$ (B) $(M_0 + M + m)\dfrac{mg}{M}$

(C) $(M_0 + M + m)\dfrac{Mg}{m}$ (D) None of these

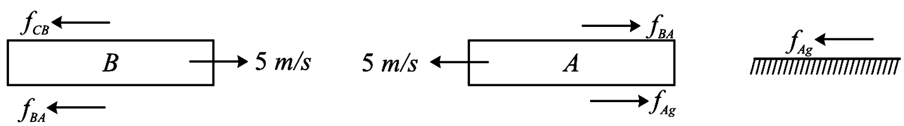
Answer Key

Exercise-1

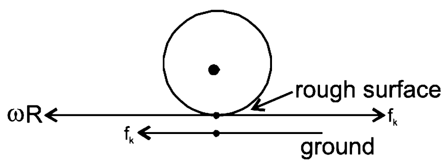
1.



2.

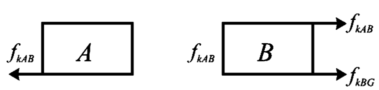


3.



Kinetic friction is involved.

4.



5. Up the incline, kinetic friction.

7. 0.2

8. $V_f^2 - V_i^2 = 2as \Rightarrow \dfrac{25}{2\times 1} = 12.5\text{m}$

9. 0.11

10. 1/2 second

11. 10m

12. $N = 0$ for $F \le \mu(M+m)g$

$N = F - \mu(M+m)g$ for $F > \mu(M+m)g$

action-reaction forces between m and M is $F - \mu mg$ for $F > \mu mg$ and 0 for $F < \mu mg$

13 $2\,N$, $\mu \ge 0.1$

15 $\mu_s = 0.60$, $\mu_k = 0.52$

16 Upwards, $f = m(g+a)$

17 (a) $a_A = 3\ \text{m/s}^2$, $a_B = 0$, $f_A = 0$, $f_B = 0$

(b) $a_A = 1\ \text{m/s}^2$, $a_B = 0$, $f_A = 25N$, $f_B = 25N$

(c) $a_A = 5\ \text{m/s}^2$; $a_B = 10\ \text{m/s}^2$; $f_A = 25N$; $f_B = 75N$

(d) $a_A = 1\text{m/s}^2$; $a_B = 1\text{m/s}^2$; $f_A = 5N$; $f_B = 75N$

18 52/9 m/s², $T_A = 88/9$, $T_B = 76/3\ N$; 0 m/s², $T_A = 0\ N$, $T_B = 3\ N$

19 $\mu_1 = 0.75$, $\mu_2 = 0.06$

20 90 *N* in string *A*, 70 *N* in string *B*.

21 (a) $\frac{10}{3}$ s **(B)** $\frac{10}{3}$ s **22** 20 m

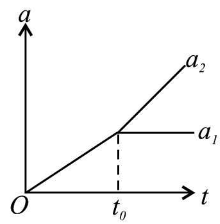
23 (a) 2.4 m/s² both ; (b) 3.2 m/s², 2.4 m/s²

24 $\frac{\mu Mg}{\sqrt{1+\mu^2}}$, $\tan^{-1}\mu$.

25 $\frac{\mu_2(M+m_1+m_2)g}{1+\mu_2}, \frac{M}{m_1+m_2}$

26. Upper block 4 m/s², lower block 1 m/s²; Both blocks 2 m/s²

27. When $t \le t_0$, the accelerations $a_1 = a_2 = kt/(m_1+m_2)$;
when $t \ge t_0$ $a_1 = \mu g m_2/m_1$, $a_2 = (at - \mu m_2 g)/m_2$.
Here $t_0 = \mu g m_2(m_1+m_2)/\text{km}$.



28. (a) $\mu(M+3m)g$, (b) $\frac{\mu(M+3m)g}{M+m}$

29. (a) $\mu(M+3m)(g+a)$, (b) $\frac{\mu(M+3m)(g+a)}{M+m}$

30. It will move at an angle of 53ṛ with the 15 *N* force

31. 1 kg

32. 4/3 sec

33. 3/4

34. 40 *N*

35. $10\hat{i}$

36. 30 *N*

37. contact force between the block and the belt is 10.5 *N*

38. 35 kg

39. 306 *N*, 4.7 m/s²

40. $\frac{100}{3}$ N towards left

41. 3 *N*

42. $a_m = g\sin\theta - \frac{\mu}{2}g\cos\theta$; $a_M = \frac{Mg\sin\theta + \frac{\mu}{2}mg\cos\theta - \mu(M+m)g\cos\theta}{M}$;

$t = \sqrt{\frac{4\ell M}{\mu g\cos\theta(M+m)}}$.

43. $\dfrac{2\ell}{a(\cos\alpha - \mu\sin\alpha)}$

44. 3 m/s

45. $\sqrt{\dfrac{2F}{\rho} - \mu_k gL}$

46. $F = 0 \begin{cases} a_A = 3\text{m/s}^2 \\ a_B = 4.5\text{ m/s}^2 \end{cases}$ *kinetic friction and* $f = 30N$

$0 < F \le 15 \begin{cases} 3 < a_A \le 4.5\text{m/s}^2 \\ a_B = 4.5\text{ m/s}^2 \end{cases}$ kinetic friction in same direction as above and $f = 30N$

$15 < F \le 60$ $\{4.5 < a_A = a_B \le 6\text{ m/s}^2$

static friction, variable and in same direction.

$60 < F \le 105\ N$ $\{\ 6 < a_A = a_B < 7.5\text{ m/s}^2$

static friction, variable and in opposite direction to the previous parts.

$F > 105\ N \begin{cases} a_A > 7.5\text{m/s}^2 \\ a_B = 7.5\text{ m/s}^2 \end{cases}$

Kinetic friction and in opposite direction to the previous parts

47. (a) $a = g\cot\theta$, (b) $m_{\min} = \dfrac{m\sin\theta\cos\theta}{m\cos^2\theta + M}$

48. $\mu_s = 0.4$, $\mu_k = 0.3$

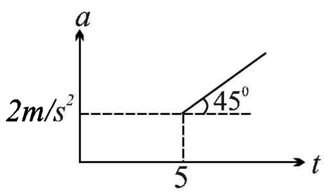
49. 1/2 sec

50. $\tan^{-1}\left(\dfrac{1}{3\sqrt{3}}\right)$

51. 3.60

52. $N = \dfrac{3mg}{2\mu}$

53. (a) $\eta = \dfrac{3}{5}$; (b) acceleration = 4 m/s²

54. (a) 

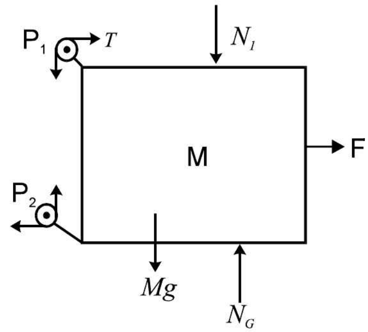
(b) 22.5 m/s ; (c) $\dfrac{275}{6}$ m; (d) $\sqrt{2}$ sec

55. (a) (i) $a_1 = a_2 = 3.2\text{ m/s}^2$, (ii) $a_1 = 5.75\text{m/s}^2$, $a_2 = 2\text{m/s}^2$; (b) $a_1 = 5\text{ m/s}^2$, $a_2 = -10/3\text{ m/s}^2$

56. (i) 90*N*, (ii) 112.5*N* (iii) 150*N*

57. $\Delta r = \dfrac{mg\cot\alpha}{4\pi^2 k}$, 1cm

Exercise-3



1.	*A*	**2.**	*A*
3.	*A*	**4.**	*B*

5. (i) zero, (ii) $\frac{2\sqrt{2}}{3}mg$, (iii) $\frac{mg}{3\sqrt{2}}$, downwards

6. $F = 60\ N,\ T = 18\ N,\ a_{m1} = a_{m2} = a_M = 0.6\ m/s^2$

7.	*A*	**8.**	*A*
9.	10 m/s^2	**10.**	*B*
11.	*A*	**12.**	5
13.	*A, C*		

Previous Years' AIEEE Questions

1. *D*

Exercise-4

1.	*D*	**2.**	*B*	**3.**	*A*	**4.**	*C*
5.	*A*	**6.**	*D*	**7.**	*B*	**8.**	*D*
9.	*A*	**10.**	*D*	**11.**	*B*	**12.**	*D*
13.	*A*	**14.**	*C*	**15.**	*A,B,C*	**16.**	*A*
17.	*A,C*	**18.**	*B,C*	**19.**	*A,B,D*	**20.**	*B,D*
21.	*B,C,D*	**22.**	*C,D*	**23.**	*B*	**24.**	*A*
25.	*C*	**26.**	*(a) B (b) D (c) A*	**27.**	*C*	**28.**	*A*
29.	*C*	**30.**	*A*	**31.**	*A*		
32.	*(a) (i) D (ii) A (b) (i) B (ii) A*			**33.**	*C*		
34.	*B*	**35.**	*A*	**36.**	*A*	**37.**	*D*
38.	*D*	**39.**	*D*	**40.**	*D*	**41.**	*B*
42.	*(I) A (II) D (III) B*	**43.**	*B*	**44.**	*A*		
45.	*A*	**46.**	*D*	**47.**	*D.*	**48.**	*C*
49.	*A*	**50.**	*B*	**51.**	*C*	**52.**	*B*
53.	*B*	**54.**	*D*	**55.**	*C*	**56.**	*C*
57.	*A*	**58.**	*A*	**59.**	*A*	**60.**	*C*
61.	*A*	**62.**	*C*	**63.**	*C*	**65.**	*A*
65.	*C*	**66.**	*C*	**67.**	*B*	**68.**	*A,C*
69.	*A,B,D*	**70.**	*C*	**71.**	*A,B,C*	**72.**	*B*
73.	*C,D*	**74.**	*C*	**75.**	*B*	**76.**	*B*
77.	*A,B,C*	**78.**	*A,B*	**79.**	*A,B,C,D*	**80.**	*A,B,C,D*

81.	*A,B,C*	**82.**	*C*	**83.**	*A,C*	**84.**	*A,C,D*
85.	*A,B,C,D*	**86.**	*B*	**87.**	*B*	**88.**	*C*
89.	*D*	**90.**	*A*	**91.**	*D*	**92.**	*A*
93.	*A,D*	**94.**	*B,C*	**95.**	*A,B*	**96.**	*B*

Chapter

5

Circular Motion

CIRCULAR MOTION

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as the circular motion with respect to that fixed (or moving) point. That fixed point is called centre and the distance is called radius.

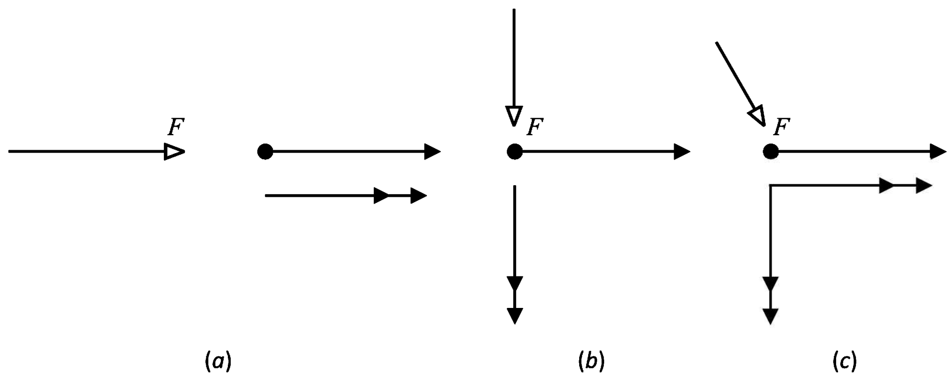
Types of Acceleration

A body has an acceleration whenever its velocity is not constant. Velocity is a vector quantity however and may change either in magnitude (i.e. speed) or in direction or both. In all cases a force must act on the body to produce an acceleration, the direction of the force determing the particular type of acceleration.

(a) A *change in speed* occurs when a force acts *in the direction of motion of* the body to which it is applied. Such a force cannot cause any change in the direction of the velocity.

(b) A *change in direction* at constant speed is caused by a force *perpendicular to the direction of motion* of the body. Such a force will push or pull the body off its previous course but will not affect the speed since there is no force component in the direction of motion.

(c) If both speed and direction of motion are to be changed a force with components both parallel and perpendicular to the direction of motion is required.



Type (*a*) Acceleration of this type has already been studied in Chapter 4 and needs no further analysis here.

Types (*b*) *and* (*c*) A body whose direction of motion is not constant traces out a curved path of some sort. The curve described depends upon the forces which are acting on the body.

In this chapter analysis is concentrated on motion in one particular curve, the circle.

Motion in a Circle with Constant Speed

Consider a particle P describing a circle, centre O and radius r, at constant speed v.

As there is not change in speed, no force component acts in the direction of motion, which is tangential at any instant.

A force must be acting on the particle however as the direction of motion is not constant.

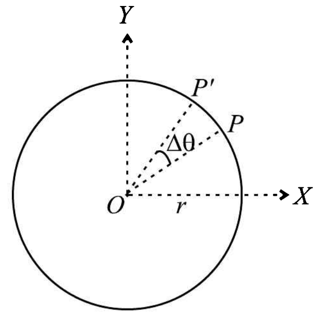
This force must therefore act along the radius, producing a radial acceleration.

KINEMATICS OF CIRCULAR MOTION

Variables of Motion

(A) Angular Position

The angle made by the position vector with given line (reference line) is called angular position. Circular motion is a two dimensional motion of motion in a plane. Suppose a particle P is moving in a circle of radius r and centre O.

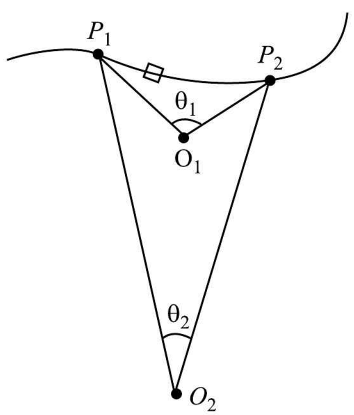


The position of the particle P at a given instant may be described by the angle θ between OP and OX. This angle θ is called the **angular position** of the particle. As the particle moves on the circle its angular position θ change.

Suppose the point rotates an angle $\Delta\theta$ in time Δt.

Explanatory Notes of Angular Position

Lets say you are watching F-1 race Michael Schumacher is going on a curved path and you want to sheet him by your handycam. If you are given two positions to stand O_1 & O_2 which one will you choose?



Ans. O_2 why? $\theta_2 < \theta_1$ hence easy manouvering at camera you have to cover less angle in same time. Although you may not have much idea about circular motion but your decision was based on analysis of angular variables.You decided by thinking that you have to cover smaller angle in same time.(θ_2 and θ_1 are angle substended by $M.S.$ while going from P_1 to P_2 on O_2 and O_1)

(B) Angular Displacement

Definition : Angle rotated by a position vector of the moving particle with some reference line is called as angular displacement.

Important Points

1. It is dimensionless and has proper unit (SI unit) radian while other units are degree or revolution 2π rad = 360° = 1 rev
2. Infinitely small angular displacement is a vector quantity but finite angular displacement is not because the addition of the small angular displacement is commutative while for large is not.

$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \text{ but } \theta_1 + \theta_2 \neq \theta_2 + \theta_1$$

3. Direction of small angular displacement is decided by right hand thumb rule. When the figures are directed along the motion of the point then thumb will represents the direction of angular displacement.
4. Angular displacement can be different for different observers

Explanatory Notes on Angular Displacement

Angle substended by a moving particle on a fixed point is called as angular displacement about the fixed point. Thus in above discussion angular displacement about O_1 is θ_1 & about O_2 is θ_2.

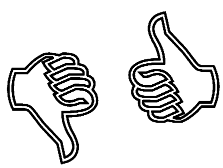
Few Facts: It is a dimensionless quantity.

Units – radian (Never Degree)

Angular displacement depends on reference frame same as linear displacement depends on refrence frame. Angular displacement will be different if we observer the car from another car.

Important Points

- Angular displacement is different for different observers in the same frame. The linear displacement is same for two observers at different positions in same frame e.g. O_1 & O_2 will observe same linear displacement but different angular displacement although both points are in the same ground frame.
- It may be a bit shocking for you but it is a vector quantity. Direction of angular displacement vector is decided by right hand rule.i.e. move your right hand fingers in sense of motion and direction of your thumb will be the direction of angular displacement.



It sounds quite confusing that direction of vector is nowhere near the actual motion. But if you pay attention to a few facts you will understand that perhaps this is the best way to represent angular displacement. If a vector represents angular displacement it holds three informations which can completely describe the angular displacement.

The unique plane perpendicular to the line represents the plane of motion of particle.

Now place your right hand thumb along the vector and direction of your fingers will give you the sense of rotation.

In JEE syllabus plane of circular motion is fixed so the direction of angular variables remains same (although sense may become +ve or –ve). Thus even though they are vectors but we need not give much thought to it as there directions will remain unchanged. Much like one-dimensional motion where variables are vectors but we do not use vector notation, we simply use signs +ve & –ve to represent sense. In short we will study angular kinematics of one-D i.e., will never use vector notation but stick to sign +ve or –ve by defining our +ve sense.

(C) *Angular Velocity* (ω)

(i) Average Angular Velocity

$$\omega_{av} = \frac{\textit{Total Angle of Rotation}}{\textit{Total time taken}};$$

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

where θ_1 and θ_2 are angular position of the particle at time t_1 and t_2

(ii) Instantaneous Angular Velocity

The rate at which the position vector of a particle w.r.t. the centre rotates, is called as instantaneous angular velocity with respect to the centre.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Important Points

1. It is an axial vector with dimensions $[T^{-1}]$ and *SI* unit rad/s.
2. For a rigid body as all points will rotate through same angle in same time, angular velocity is a characteristic of the body as a whole, e.g., angular velocity of all points of earth about its own axis is ($2\pi/24$) rad/hr.
3. If a body makes 'n' rotations in 't' seconds then angular velocity in radian per second will be

$$\omega_{av} = \frac{2\pi n}{2}$$

If T is the period and 'f' the frequency of uniform circular motion

$$\omega_{av} = \frac{2\pi \times 1}{T} = 2\pi f$$

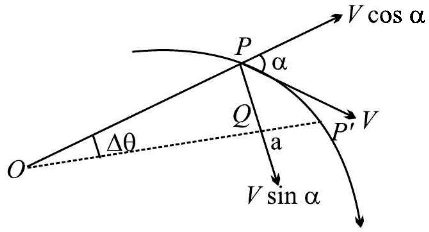
4. If $\theta = a - bt + ct^2$ then $\omega = \frac{d\theta}{dt} = -b + 2ct$

Explanatory Notes on Angular Velocity

Rate of angular displacement is called angular velocity.

$$\omega = \frac{d\theta}{dt}$$

Unit rad s^{-1} / Dimension T^{-1}



Relation between angular velocity & linear velocity.

A particle P is moving with speed V along a curve & observer is located at O α - is angle between line joining OP and velocity.

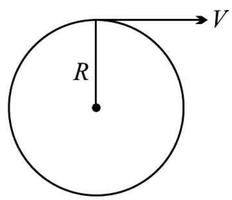
Note that $V \sin \alpha$ (comp. of velocity perpendicular to OP) is the cause of angular displacement. i.e. if only $V \cos \alpha$ existed we need not turn our head to always look at particle. Hence,

$$PQ = (V \sin \alpha)\, \Delta t$$

$$PQ = OP\,(\Delta\theta)$$

Thus $$\frac{\Delta\theta}{\Delta t} = \frac{V \sin\alpha}{(OP)}$$

$$\Rightarrow \quad \omega = \frac{\text{Comp. of velocity perpendicular to line joining}}{\text{length of line joining}}$$



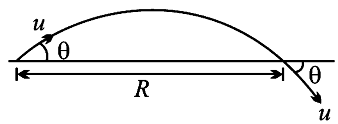
In circular motion find angular velocity of particle moving with speed V wrt center

$$\omega = \frac{V}{R}$$

also, $\vec{v} = \vec{\omega} \times \vec{r}$

Example 1. A projectile (u, θ) is launched from horizontal plane, find angular velocity as observed from the point of projection at the time of landing.

Solution

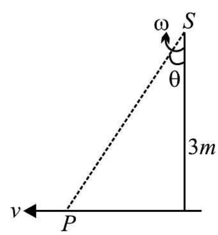


$$\omega = \frac{u \sin\theta}{R}$$

or $$\omega = \frac{g}{2u \cos\theta}$$

Concept

$$\omega = \frac{v_\perp}{R} = \frac{\text{Component of velocity perpendicular to line joining}}{\text{Length of line joining}}$$



Example 2. A spotlight S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/sec. The spot of light P moves along the wall at a distance of $3m$. The velocity of the spot P when $\theta = 45°$ is _____ m/sec.

[Ans. $V = 0.6$m/s]

Solution

$$\frac{d}{\sqrt{2}} = 3$$

$$d = 3\sqrt{2}\, m$$

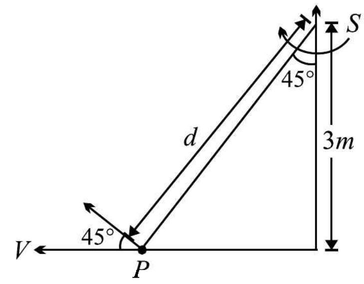
Let velocity of spot at $\theta = 45°$ in V, then

$$\omega = \frac{\text{Component of velocity perpendicular to line joining}}{\text{Length of line joining}}$$

$$\omega = \frac{V_{\perp}}{d}$$

$$0.1 = \frac{\frac{V}{\sqrt{2}}}{3\sqrt{3}}$$

$$V = 0.3\sqrt{2} \times \sqrt{2} \quad \therefore V = 0.6 \text{ m/s}$$



Example 3. Is the angular velocity of rotation of hour hand of a watch greater of smaller than the angular velocity of Earth's rotation about its own axis.

Solution Hourhand completes one rotation in 12 hours while Earth completes one rotation in 24 hours. So, angular velocity of hour hand is double the angular velocity of Earth. $\left(\omega = \frac{2\pi}{T}\right)$.

(D) Angular Acceleration (α)

(i) Average Angular Acceleration

Let ω_1 and ω_2 be the instantaneous angular speeds at times t_1 and t_2 respectively, then the average angular acceleration α_{av} is defined as

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

(ii) Instantaneous Angular Acceleration

It is the limit of average angular acceleration as Δt approaches zero, i.e.,

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$$

Important Points

1. It is also an axial vector with dimension $[T^{-2}]$ and unit rad/s^2.
2. If $\alpha = 0$, circular motion is said to be uniform.
3. As $\omega = \frac{d\theta}{dt}, \alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$,

   i.e., second derivative of angular displacement w.r.t. time gives angular acceleration.

Explanatory Notes on Angular Acceleration

Rate of change angular velocity

$$\alpha = \frac{d\omega}{dt}$$

Unit rad s^{-2} / Dimensions T^{-2}

Direction is along angular velocity if it is increasing otherwise opposite.

For uniform angular acceleration

$$\omega = \omega_0 + \alpha t\ ,\ \theta = \omega_0 t + \frac{1}{2}\alpha t^2\ ,\ \omega^2 = \omega_0^2 + 2\alpha\theta$$

Example 4. $\omega_0 = 10\pi$ rad s^{-1} and $\alpha = -5\pi$ rad s^{-2} (uniform). Find angular displacement and number of turns at t = 6 sec.

Solution $\theta = 60\pi - \frac{1}{2} \times 5p \times 36 = -30\pi$

Number of turns 25.

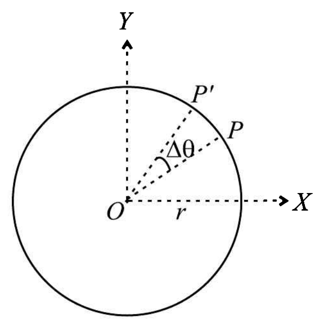
RELATION BETWEEN SPEED AND ANGULAR VELOCITY

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

The rate of change of angular velocity is called the angular acceleration (α). Thus,

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

The linear distance PP' travelled by the particle in time Δt is



$\Delta s = r\Delta\theta$

or $\lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{d\theta}{dt} = r \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$ or $\frac{\Delta s}{\Delta t} = r\frac{d\theta}{dt}$

or $v = r\omega$

Here, v is the linear speed of the particle.

Differentiating again with respect to time, we have

$$a_t = \frac{dv}{dt} = r\frac{d\omega}{dt} \quad \text{or} \quad a_t = r\alpha$$

Here, $a_t = \frac{dv}{dt}$ is the rate of change of speed (not the rate of change of velocity). This is called as the tangential acceleration of the particle. Later, we will see that a_t is the component of net acceleration $\vec{a}$ of the particle moving in a circle along the tangent.

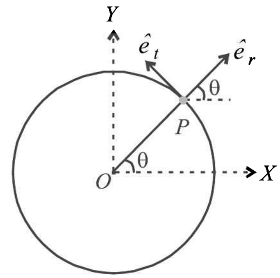
Example 5. A particle travels in a circle of radius 20 cm at a speed that uniform increases. If the speed changes from 5.0 m/s to 6.0 m/s in 2.0s, find the angular acceleration.

Solution The tangential acceleration is given by

$$a_1 = \frac{dv}{dt} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{6.0 - 5.0}{2.0}\ \text{m/s}^2 = 0.5\ \text{m/s}^2.$$

The angular acceleration is $\alpha = a_t / r = \frac{0.5\ \text{m/s}^2}{20\ cm} = 2.5\ \text{rad/s}^2$.

RADIAL AND TANGENTIAL ACCELERATION



Unit vectors along the radius and the tangent: Consider a particle P moving in a circle of radius r and centre at origin O. The angular position of the particle at some instant is say θ. Let us here define two unit vectors, one is $\hat{e}_r$ (called radial unit vector) which is along OP and the other is $\hat{e}_t$ (called the tangential unit vector) which is perpendicular to OP.

Now, since $|\hat{e}_r| = |\hat{e}_t| = 1$

We can write these two vectors as

$$\hat{e}_r = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} \text{ and } \hat{e}_t = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

Velocity and acceleration of particle in circular motion

The position vector of particle P at the instant shown in figure can be written as

$$\vec{r} = \overrightarrow{OP} = r\hat{e}_r \quad \text{or} \quad \vec{r} = r(\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

The velocity of the particle can be obtained by differentiating $\vec{r}$ with respect to time t. Thus,

$$\vec{v} = \frac{d\vec{r}}{dt} = (-\sin\theta\,\hat{i} + \cos\theta\,\hat{j})\,r\omega$$

$$\vec{a} = \frac{d\vec{v}}{dt} = r\left[\omega\frac{d}{dt}(-\sin\theta\hat{i} + \cos\theta\hat{j}) + (-\sin\theta\hat{i} + \cos\theta\hat{j})\frac{d\omega}{dt}\right]$$

$$= -\omega^2 r\,[\cos\theta\,\hat{i} + \sin\theta\,\hat{i}\,] + r\frac{d\omega}{dt}\,\hat{e}_t$$

$$\vec{a} = -\omega^2 r\,\hat{e}_r + \frac{dv}{dt}\,\hat{e}_t$$

Thus, acceleration of a particle moving in a circle has two components one is along $\hat{e}_t$ (along tangent) and the other along $-\hat{e}_r$ (or towards centre). Of these the first one is called the tangential acceleration. (a_t) and the other is called radial or centripetal acceleration (a_r). Thus.

$$a_t = \frac{dv}{dt} = \text{rate of change of speed and } a_r = rw^2 = r\left(\frac{v}{r}\right)^2 = \frac{v^2}{r}$$

Here, the two components are mutually perpendicular. Therefore, net acceleration of the particle will be :

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(r\omega^2)^2 + \left(\frac{dv}{dt}\right)^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

Following three points are important regarding the above discussion:

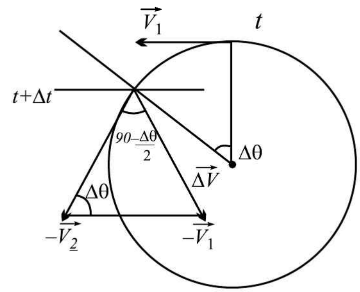
1. In uniform circular motion, speed (v) of the particle is constant, i.e., $\frac{dv}{dt} = 0$. Thus, $a_t = 0$ and

   $a = a_r = r\omega^2$

2. In accelerated circular motion, $\frac{dv}{dt}$ = positive, i.e., a_t is along $\hat{e}_t$ or tangential acceleration of particle is parallel to velocity $\vec{v}$ because $\vec{v} = r\omega\ \hat{e}_t$ and $\vec{a}_r = \frac{dv}{dt}\ \hat{e}_t$

3. In decelerated circular motion, $\frac{dv}{dt}$ = negative and hence, tangential acceleration is anti-parallel to velocity $\vec{v}$.

Alternating Method to Derivation of Centripetal Acceleration for a Particle Moving in a Circle with Constant Speed



$$|\vec{V}_2| = |\vec{V}_1| = V$$

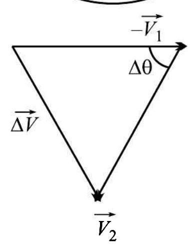
acceleration vector $\vec{a}$, $|\vec{a}| = \left|\frac{\vec{V}_2 - \vec{V}_1}{\Delta t}\right|$

$$|\vec{\Delta V}| = |\vec{V}_2 + (-\vec{V}_1)|$$

$$|\vec{\Delta V}| = 2\ V \sin\left(\frac{\Delta\theta}{2}\right)$$

or $$|\vec{\Delta V}| = V\Delta\theta, \text{ for small } (\Delta\theta)$$

$$|\vec{a}| = a = \frac{V\Delta\theta}{\Delta t} = V\omega = \frac{V^2}{R} = \omega^2 R$$



We can also prove that it is directed towards centre as angle with tangent is $\left(\frac{90 - \Delta\theta}{2}\right)$ in lim $\Delta t \to 0$. lim $\Delta\theta \to 0$ and angle $\to 90°$ i.e., towards the centre $\hat{a} = \Delta\hat{V}$ which is towards the centre.

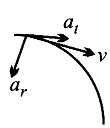
What we observe here is that when magnitude of velocity is constant but only direction is changing acceleration is directed perpendicular to velocity. In other words component of acceleration perpendicular to velocity causes change in direction not magnitude.

Now lets take a case of particle moving in straight line, with changing speed. Its acceleration will be along the line of velocity. This acceleration will change only magnitude.

Conclusion: Components of acceleration along velocity called tangential acceleration changes magnitude and components of acceleration perpendicular to velocity called radial acceleration changes direction of velocity.

Hence $\vec{a} = \vec{a}_r + \vec{a}_t$.

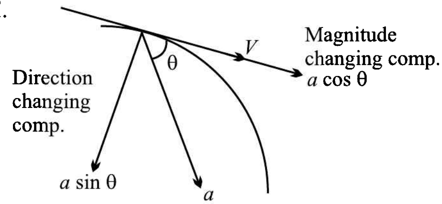
Derivation of tangential acceleration for circular motion.



$$a = \frac{dv}{dt} \Rightarrow \frac{Rd\omega}{dt} = (\alpha)\ R$$

$$\Rightarrow \quad \frac{d(r\omega)}{dt} = \alpha R \quad \Rightarrow \quad \frac{dv}{dt} = \alpha R$$

understand that v is speed or magnitude of velocity, $v = |\vec{v}|$ & rate of change of magnitude of velocity is called tangential acceleration $a_t = \frac{dv}{dt} = \frac{d|\vec{V}|}{dt} = \alpha R$.



or $\quad (\vec{a}_t = \vec{\alpha} \times \vec{R})$

Also note that $\quad \left|\frac{d\vec{V}}{dt}\right| = |\vec{a}| = \sqrt{a_t^2 + a_r^2}$

i.e. $\quad \left|\frac{d\vec{V}}{dt}\right| = \sqrt{(\alpha R)^2 + (\omega^2 R^2)}$

(while $\frac{d|\vec{V}|}{dt} = \alpha R$.)

Note

One is $|\vec{a}|$ magnitude of rate of change of velocity & other is rate of change of magnitude of velocity (a_t).

$$|\vec{a}| = \left|\frac{d\vec{v}}{dt}\right| \quad \text{and} \quad |a_t| = \frac{d|\vec{v}|}{dt}$$

$$\vec{v} = \frac{d\vec{\ell}}{dt} = \frac{d\vec{\theta}}{dt} \times \vec{r} = \vec{\omega} \times \vec{r}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \vec{\omega} \times \frac{d\vec{r}}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{r} = \vec{a}_c + \vec{a}_t$$

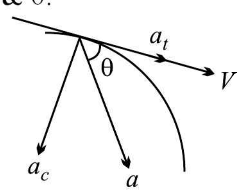
Where, $\vec{a}_c = \vec{\omega} \times \vec{v}$ and $\vec{a}_t = \vec{\alpha} \times \vec{r}$

4. **Centripetal Condition :** When component of acceleration perpendicular to velocity is $\frac{V^2}{R}$ and always directed towards a fixed point then particle will undergo circular motion about that fixed point.

$$a_c = a_r = \frac{V^2}{R}$$

Example 6. If a particle is undergoing circular motion with speed V and radius R angle between acceleration & V is θ. Find magnitude of tangential acceleration in terms of V, R & θ.

Solution



$$a_c = \frac{V^2}{R} = a \sin\theta$$

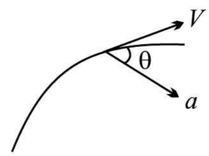
hence $\quad a = \frac{V^2}{R\sin\theta} \quad \Rightarrow \quad a_t = a\cos\theta$

$$a_t = \frac{V^2\cos\theta}{R\sin\theta} = \frac{V^2}{R}\cot\theta$$

Important Points

a_t and a_c are components of a and $\vec{a} \,\&\, \vec{v}$ may be at any angle between 0 to 180°.

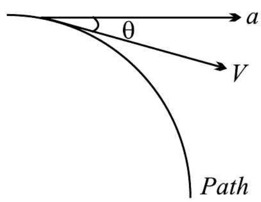
Important Points



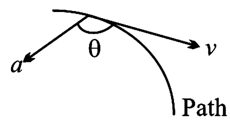
$$\frac{d|\vec{V}|}{dt} = a\cos\theta \text{ (Component of acceleration along velocity)}$$

Valid for any type of motion (circle or not circle)

Is this diagram possible?



Sol. This acceleration is not possible as perpendicular comp. of acceleration is away from curvature. **Is this diagram possible?**



Sol. This is possible but speed will be decreasing.

Example 7. A particle moves in a circle of radius 2.0 cm at a speed given by $v = 4t$, where v is in cm/s and t in seconds.

(a) Find the tangential acceleration at $t = 1$s.

(b) Find total acceleration at $t = 1$s.

Solution (a) Tangential acceleration

$$a_t = \frac{dv}{dt} \quad \text{or} \quad a_t = \frac{d}{dt}(4t) = 4 \text{ cm/s}^2$$

$$a_C = \frac{V^2}{R} = \frac{(4)^2}{2} = 8$$

$$\Rightarrow \quad a = \sqrt{a_t^2 + a_C^2} = \sqrt{(4)^2 + (8)^2} = 4\sqrt{5} \text{ m/s}^2$$

Example 8. A boy whirls a stone in a horizontal circle of radius 1.5 m and at height 2.0 m above level ground. The string breaks, and the stone files off horizontally and strikes the ground after traveling a horizontal distance of 10 m. What is the magnitude of the cetripetal acceleration of the stone while in circular motion?

Solution $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 2}{9.8}} = 6.4\, s$; $v = \frac{10}{t} = 15.63$ m/s; $a = \frac{v_B^2}{R} = 0.45$ m/s^2

Example 9. Find the magnitude of the linear acceleration of a particle moving in a circle of radius 10 cm with uniform speed completing the circle in 4s.

Solution The distance covered in completing the circle is $2\pi r = 2\pi \times 10$ cm. The linear speed is

$$v = 2\pi r/t = \frac{2\pi \times 10\,cm}{4\,s} = 5\pi \text{ cm/s}.$$

The linear acceleration is

$$a = \frac{v^2}{r} = \frac{(5\pi \text{ cm/s})^2}{10\,cm} = 2.5\pi^2 \text{ cm/s}^2.$$

Example 10. A particle moves in a circle of radius 20 am. Its linear speed is given by $v = 2t$ where t is in second and v in meter/second . Find the radical and tangential acceleration at $t = 3$s.

Solution The linear speed at $t = 3$s is $v = 2t = 6$ m/s.

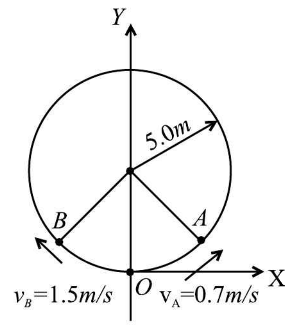
The radical acceleration at $t = 3$s is

$$a_r = v^2 / r = \frac{36 \text{ m}^2/\text{s}^2}{0.20\,m} = 180 \text{ m/s}^2.$$

The tangent acceleration is

$$a_t = \frac{dv}{dt} = \frac{d(2t)}{dt} = 2 \text{ m/s}^2.$$

Example 11. Two particles A and B start at the origin O and travel in opposite directions along the circular path at constant speeds $v_A = 0.7$ m/s and $v_B = 1.5$ m/s, respectively. Determine the time when they collide and the magnitude of the acceleration of B just before this happens.



Solution

$$1.5t + 0.7t = 2\pi R = 10\pi$$

$$\therefore \quad t = \frac{10\pi}{2.2} = 14.3 \text{ s}$$

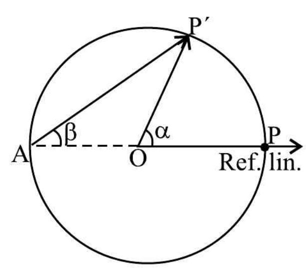
$$a = \frac{v_B^2}{R} = 0.45 \text{ m/s}^2$$

RELATIVE ANGULAR VELOCITY

Angular velocity is defined with respect to the point from which the position vector of the moving particle is drawn

Here angular velocity of the particle w.r.t. 'O' and 'A' will be different

$$\omega_{PO} = \frac{d\alpha}{dt};\ \omega_{PO} = \frac{d\beta}{dt}$$



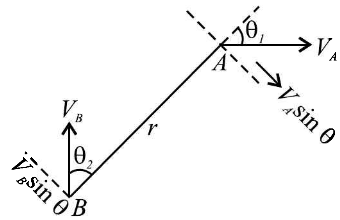
Definition: Relative angular velocity of a particle 'A' with respect to the other moving particle 'B' is the angular velocity of the position vector of 'A' with respect to 'B'. That means it is the rate at which position vector of 'A' with respect to 'B' rotates at that instant

$$\omega_{AB} = \frac{(V_{AB})\perp}{r_{AB}} = \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{Separation between A and B}}$$

$$(V_{AB})_\perp = V_A \sin\theta_1 + V_B \sin\theta_2$$

$$r_{AB} = r$$

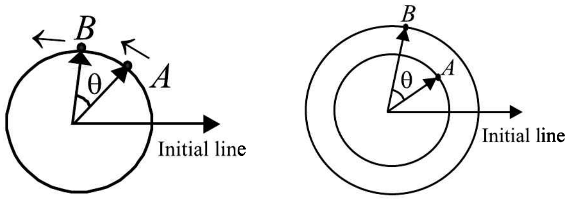
$$w_{AB} = \frac{V_A \sin\theta_1 + V_B \sin\theta_2}{r}$$



Important Points

1. If two particles are moving on the same circle or different coplanar concentric circles in same direction with different uniform angular speed ω_A and ω_B respectively, the angular velocity of *B* relative to *A* for an observer at the center will be

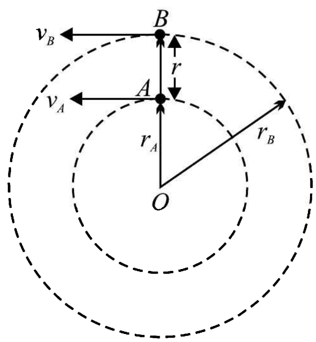
$$\omega_{BA} = \omega_B - \omega_A = \frac{d\theta}{dt}$$



So the time taken by one to complete one revolution around *O* w.r.t. the other

$$T = \frac{2\pi}{\omega_{rel}} = \frac{2\pi}{\omega_2 - \omega_1} = \frac{T_1T_2}{T_1 - T_2}$$

2. If two particles are moving on two different concentric circles with different velocities then angular velocity of *B* relative to A as observed by *A* will depend on their positions and velocities. consider the case when *A* and *B* are closet to each other moving in same direction as shown in figure. In this situation



$$v_{rel} = |\vec{v}_B - \vec{v}_A| = v_B - v_A$$

$$r_{rel} = |\vec{r}_B - \vec{r}_A| = \vec{r}_B - \vec{r}_A$$

so, $$\omega_{BA} = \frac{(v_{rel})_\perp}{r_{rel}} = \frac{v_B - v_A}{r_B - r_A}$$

$(v_{rel})_\perp$ = Relative velocity perpendicular to position vector

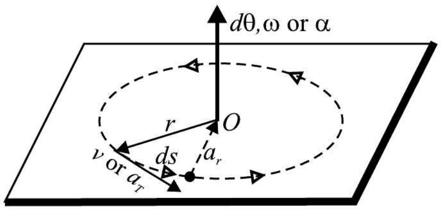
RELATIONS AMONG ANGULAR VARIABLES

These relations are also referred as equations of rotational motion and are :

$$\omega = \omega_0 + \alpha t \quad ...(1)$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \quad ...(2)$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad ...(3)$$



These are valid only if angular acceleration is constant and are analogous to equations of translatory motion, i.e.,

$$v = u + at; \quad s = ut + (1/2)\, at^2 \text{ and } v^2 = u^2 + 2as$$

RADIUS OF CURVATURE

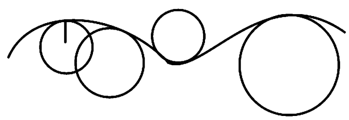
Any curved path can be assume to be made of infinite circular arcs. Radius of curvature at a point is the radius of the circular arc at a particular point which fits the curve at that point.

$$F_c = \frac{mv^2}{R} \quad \Rightarrow \quad R = \frac{mv^2}{F_c} = \frac{mv^2}{F_\perp}$$

$F_\perp$ = Force perpendicular to velocity (centripetal force)

If the equation of trajectory of a particle is given we can find the radius of curvature of the instantaneous circle by using the formula ,

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}.$$



Note

(*1*) *Velocity is always tangential at each and every point on the trajectory and acceleration may have two components.*

(a) Component of acceleration in the direction of velocity known as tangential acceleration.

(*b*) *Component of acceleration perpendicular to the velocity is known as normal or centripetal acceleration.*

STEPS FOR SOLVING PROBLEMS (To find the radius of curvature)

Step 1. At first draw the velocity diagram at the required point. (Velocity must be tangential at that point.)

Step 2. Draw the F.B.D. at the required point.

Step 3. Taking component of all forces in the direction of velocity and perpendicular to the velocity.

Step 4. Apply the Newtons second law of motion along the tangential and normal direction seperately.

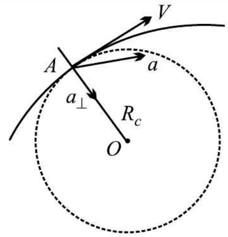
$$\Sigma f_t = ma_t$$

$$\Sigma f_c = ma_c$$

Step 5. Apply the formula of radius of curvature.

$$R_c = \frac{(\text{speed})^2}{\text{comp.of acceleration perpendicular to velocity}}$$

For general curvilinear motion. When the particle crosses this point A, it is satisfying condition of moving on this imaginary circle at this instant, if $a_\perp = \frac{v^2}{R_c}$ / where R_c is radius of curvature at this instant.

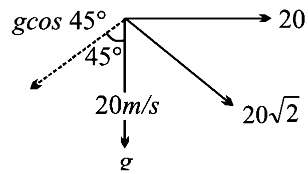


$$R_c = \frac{V^2}{a_\perp}$$

$$R_c = \frac{(\text{speed})^2}{\text{components of acceleration perpendicular to velocity}}$$

Example 12. A projectile is launched horizontally with 20 ms^{-1} from some height. Find R_c at $t = 2$ sec.

Solution 20 m/s after $t = 2$ sec



$$R_C = \frac{V^2}{a_\perp} = \frac{400 \times 2 \times \sqrt{2}}{10}$$

$$R_C = 80\sqrt{2} \text{ m}$$

Concept

$$R_c = \frac{V^2}{a_\perp} = \frac{(\text{speed})^2}{\text{comp.of acceleration perpendicular to velocity}}$$

Note

$\vec{V}, \vec{U} \rightarrow$ Velocity / $v, u \rightarrow$ speed

$$\frac{\vec{V} = \vec{U} + \vec{a}t}{v = u + (a_t)t} \quad \text{but} \quad |\vec{V}| = |\vec{U}| + |\vec{a}|t \text{ is wrong.}$$

DYNAMICS OF CIRCULAR MOTION

In circular motion or motion along any curved path Newton's law is applied in two perpendicular directions one along the tangent and other perpendicular to it . i.e., towards centre. The component of net force along the centre is called as **centripetal force**. The component of net force along the tangent is called as **tangential force**.

$$\text{tangential force } (F_t) = Ma_t = M\frac{dv}{dt} = M\alpha r$$

$$\text{centripetal force } (F_c) = m\omega^2 r = \frac{mv^2}{r}$$

Steps for Solving Problems of Circular Motion

Step 1. Identify the plane of circular motion.

Step 2. Locate the centre and calculate the radius.

Step 3. Make F.B.D.

Step 4. Resolve forces only and always along these three directions:

(a) In the plane along radial direction.

(b) In the plane along tangential direction.

(c) Perpendicular to the plane of circular motion.

Step 5. (a) Add the forces assuming radially inward direction as positive

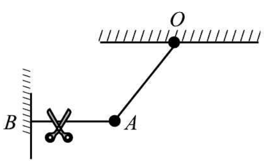
$$\Sigma F_r = \frac{mV^2}{R}$$

(b) ΣF_t

- $\Sigma F_t = 0$ (uniform circular motion) constant ω
- $\Sigma F_t \neq 0$ (Non uniform circular motion) ω changing

(c) If plane is not accelerating then $\Sigma F_\perp = 0$. If plane is accelerating then actually it will not be circular motion from ground frame. But still $F_\perp = ma_\perp$, where $a_\perp$ is acceleration of particle perpendicular to plane.

***Example* 13.** Find tension in *OA* before and after *AB* is cut.

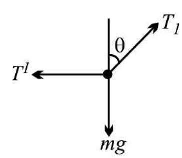


Solution Before cutting $\vec{a} = 0$ in all directions Revolving vertically & horizontally

$$T_1 \cos\theta = mg$$

$$T_1 \sin\theta = T^1$$

$$T_1 = \frac{mg}{\cos\theta}$$



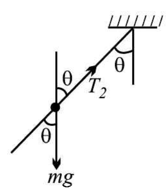
Concept

This discussion is being done to explain what happens when we do not follow step (4) and resolve forces along other directions.

After cutting

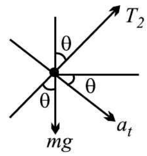
$$T_2 \cos\theta - mg = 0$$

The above equation is wrong because acceleration of a particle in vertical direction is not zero.



while $T_2 - mg\cos\theta = 0$ is true as

$T_2 - mg\cos\theta = m\,\omega^2\,l$ (as $\omega = 0$)



What if we want to write equation in vertical direction.

$a_t = g\sin\theta$ / $a_c = 0$

$mg - T_2\cos\theta = m\,a_t\sin\theta$

$T_2\cos\theta = mg\,(1 - \sin^2\theta)$

$T_2 = mg\cos\theta$

(i) Conical Pendulum: It consists of a string OA whose upper end O is fixed and a bob is tied at the free end. When the bob is drawn aside and given a horizontal push let it describe a horizontal circle with uniform angular velocity ω in such a way that the string makes an angle θ with vertical. As the string traces the surface of a cone of semi-vertical angle θ it is called conical pendulum. Let T be the tension in string, ℓ be the length and r be the radius of the horizontal circle described. The vertical component of tension balances the weight and the horizontal component supplies the centripetal force.

$$T\cos\theta = mg; \qquad T\sin\theta = m\,r\,\omega^2$$

$$\therefore \quad \tan\theta = \frac{r\omega^2}{g} \qquad \omega = \sqrt{\frac{g\tan\theta}{r}} \qquad r = \ell\sin\theta \text{ and } \omega = \frac{2\pi}{T}$$

T being the period i.e. time for one revolution

$$\therefore \quad \frac{2\pi}{T} = \sqrt{\frac{g\tan\theta}{\ell\sin\theta}};\ T = 2\pi\sqrt{\frac{\ell\cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}}, \text{ where } h = \ell\cos\theta$$

(ii) Motion of a Cyclist on a Circular Path: Let a cyclist moving on a circular path of radius r bend away from the vertical by an angle θ.

R is the contact force from the ground which is the resultant of normal reaction and friction force. It can be resolved in the horizontal and vertical directions. The components are respectively equal to $R\sin\theta$ and $R\cos\theta$. The vertical component balances his weight mg. The horizontal component $R\sin\theta$ supplies the necessary force for making the circular path.

$$\therefore \quad R\sin\theta = \frac{mv^2}{r}; \qquad R\cos\theta = mg \qquad \therefore\ \tan\theta = \frac{v^2}{rg}$$

For less bending of the cyclist, v should be small and r should be great.

(iii) Banking of Roads: Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction the roads are banked at the turn so that the outer part of the road is somewhat lifted up as compared to the inner part. The surface of the road makes an angle θ with the horizontal throughout the turn. The figure shows the forces acting on a vehicle when it is moving on the banked road. ABC is the section of the road having a slope θ. R is the normal reaction and mg is the weight.

For vertical equilibrium, $R\cos\theta$ = mg

The horizontal components $R\sin\theta$ is the required centripetal force $\frac{mv^2}{r}$

$$R\sin\theta = \frac{mv^2}{r} \qquad \therefore \quad \tan\theta = \frac{v^2}{rg}$$

Above equation gives the angle of banking required which eliminates the lateral thrust in case of trains on rails or friction in case of road vehicles when rounding a curve.

(iv) Overturning and Skidding of Cars : When a car takes a turn round a bend, whether the car tends to skid or topple depends on different factors. Let us consider the case of a car whose wheels are "2 a" metre apart and whose centre of gravity is "h" metres above the ground. Let the co-efficient of friction between the wheels and the ground be μ.

Figure Represents the Forces on the Car

(a) The weight Mg of the car acts vertically downwards through the centre of gravity G of the car.

(b) The normal reactions of the ground R_1 and R_2 act vertically upwards on the inner and outer wheels respectively.

(c) The force of friction F between the wheels and the ground act towards the centre of the circle of which the road forms a part.

Let the radius of the circular path be r, and the speed of the car be v.

Considering the vertical forces, since there is no vertical acceleration,

$$R_1 + R_2 = Mg \quad \text{... (i)}$$

The horizontal force F provides the centripetal force for motion in a circle

$$\therefore \quad F = \frac{mv^2}{r} \quad \text{... (ii)}$$

Taking moments about G, if there is to be no resultant turning effect about the centre of gravity,

$$Fh = R_1 a + R_2 a \quad \text{...(iii)}$$

Conditions for No Skidding

From equation (ii) it is seen that as the speed increases, the force required to keep the car moving in the circle also increases. However, there is a limit to the frictional force F, because

$$F_{max} = \mu (R_1 + R_2)$$

Substituting from equation (i)

$$F_{max} = \mu Mg$$

Substituting from equation (ii)

$$\frac{Mv^2}{r} = \mu Mg \quad \therefore \quad v^2 = \mu rg \quad \text{or} \quad v = \sqrt{\mu rg}$$

This expression gives the maximum speed v with which the car could take the circular path without skidding.

Conditions for No Overturning

From equation (iii)

$$(R_2 - R_1) a = \mathrm{F} h$$

or $$(R_2 - R_1) = \frac{Fh}{a} = \frac{Mv^2}{r} \cdot \frac{h}{a} \quad \text{... (iv)}$$

But $$R_2 + R_1 = Mg$$

Adding, $$2R_2 = Mg + \frac{Fh}{a} = Mg + \frac{Mv^2}{r} \cdot \frac{h}{a} \quad \text{... (v)}$$

$$2R_2 = M\left(g + \frac{v^2h}{ra}\right) \quad \text{... (vi)}$$

$$R_2 = \frac{1}{2}M\left(g + \frac{v^2h}{ra}\right)$$

Substituting for R_2 in equation (iv)

$$R_2 - R_1 = \frac{1}{2}M\left(g + \frac{v^2h}{ra}\right) - R_1 = \frac{Mv^2h}{ra}$$

$$R_1 = \frac{1}{2}M\left(g + \frac{v^2h}{ra}\right) - \frac{Mv^2h}{ra} = \frac{1}{2}M\left(g + \frac{v^2h}{ra} - \frac{2v^2h}{ra}\right)$$

$$= \frac{1}{2}M\left(g - \frac{v^2h}{ra}\right) \quad \text{...(vii)}$$

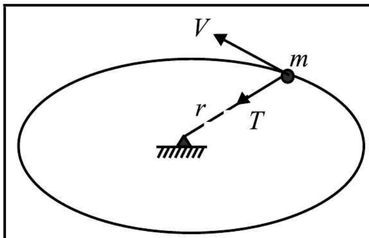
Equation (vi) shows that the reaction R_2 is always positive. However, equation (vii) shows that as the speed "v" increases, the reaction R_1 decreases and when $\frac{v^2h}{ra} = g$, R_1 becomes zero. This means that the inner wheel is no longer in contact with the ground and the car commences to overturn outwards.

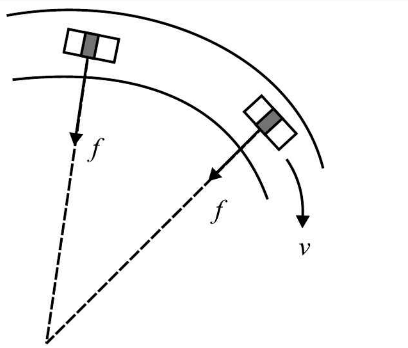
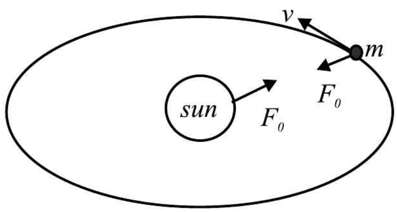
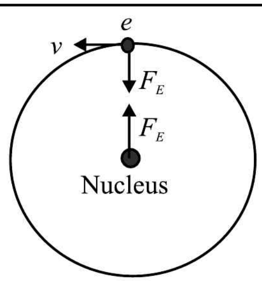
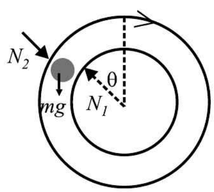
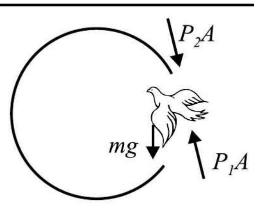
The maximum speed without overturning is given by

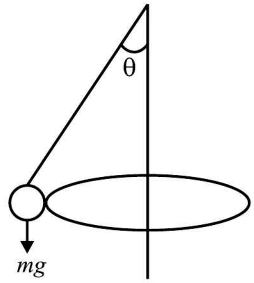
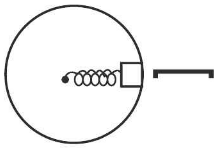
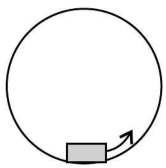
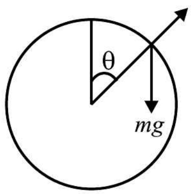
$$g = \frac{v^2h}{ra}; \; v = \sqrt{\frac{gra}{h}}$$

The same expression applies also to the case of a train moving on rails in a circular path of radius "r". Here $2a$ is the distance between the rails and "h" the height of the centre of gravity above the rails.

***Table 5.1** Examples of Circular Motion & Centripetal Force*

	When a stone tied with a string is rotated in a circular path in a horizontal surface, the tension in the string provides the centripetal force. $$T = \frac{mv^2}{r}$$

	When a car takes a turn the force of friction between the tyres and the road provides the necessary centripetal force. $$f = \frac{mv^2}{r}$$
	The gravitational force of attraction between the sun and a planet provides the centripetal force. $$F_g = \frac{mv^2}{r}$$
	The electric force of attraction between the nucleus and the revolving electron provides the centripetal force to the electron. $$F_E = \frac{mv^2}{r}$$
	Here the tube is in vertical plane, the net force towards centre is the component of gravitational force and normal reaction $$mg\cos\theta + N_2 - N_1 = \frac{mv^2}{r}$$
	Here bird is flying along a circle in horizontal plane and here centripetal force is the component of force due to the pressure difference above and below feathers of the bird and other component is balancing its weight.

	Here centripetal force is the component of tension towards the centre $$T \sin\theta = \frac{mv^2}{r}$$
	Here the spring mass system is in horizontal plane and centripetal force is the spring force . $$T = kx = \frac{mv^2}{\ell_0 + x}$$
	Here block is sliding inside a circular wall in horizontal plane and centripetal force is the normal reaction due to wall $$N = \frac{mv^2}{r}$$
	Here a small ring is released from the top of a bigger circular ring in vertical plane $$mg\cos\theta - N = \frac{mv^2}{r}$$

Example 14. A small block of mass 100 g moves with uniform speed in a horizontal circular groove, with vertical side walls, of radius 25 cm. If the block takes 2.0s to complete one round, find the normal contact force by the slide wall of the groove.

Solution The speed of the block is

$$v = \frac{2\pi \times (25\ cm)}{2.0\ s} = 0.785 \text{ m/s}$$

The acceleration of the block is

$$a = \frac{v^2}{r} = \frac{(0.785 \text{ m/s})^2}{0.25} = 2.5 \text{ m/s}^2.$$

towards the center. The only force in this direction is the normal contact force due to the slide walls. Thus from Newton's second law , this force is

$$= ma = (0.100 \text{ kg})\ (2.5 \text{ m/s}^2) = 0.25\ N$$

CENTRIPETAL FORCE

Concepts

This is necessary resultant force towards the centre called the centripetal force.

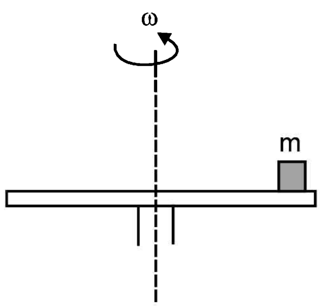
$$F = \frac{mv^2}{r} = m\omega^2 r$$

(i) A body moving with constant speed in a circle is not in equilibrium.

(ii) It should be remembered that in the absence of the centripetal force the body will move in a straight line with constant speed.

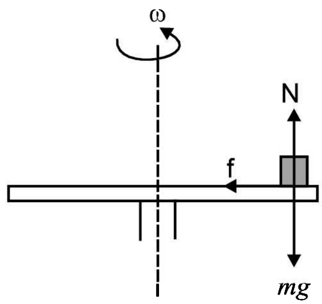
(iii) It is not a new kind of force which acts on bodies. In fact, any force which is directed towards the centre may provide the necessary centripetal force.

A small block of mass m, is at rest relative to turntable which rotates with constant angular speed ω.



(a)

Fig 5.1: A block at rest with respect to a turn table



(b)

Fig 5.2: FBD of block with respect of inertial frame

CIRCULAR MOTION IN HORIZONTAL PLANE

A ball of mass m attached to a light and inextensible string rotates in a horizontal circle of radius r with an angular speed ω about the vertical. If we draw the force diagram of the ball. We can easily see that the component of tension force along the centre gives the centripetal force and component of tension along vertical balances the gravitation force.

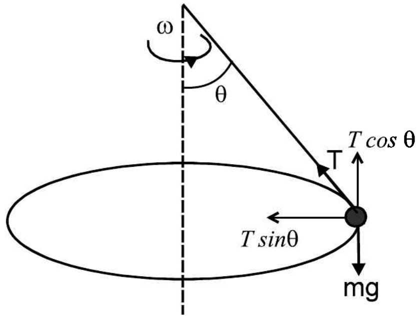


***Fig 5.3:** FBD of ball w.r.t. ground*

Example 15. An aircraft executes a horizontal loop of radius 1 km with a steady speed of 900 km h^{-1}. Compare its centripetal acceleration with the acceleration due to gravity.

Solution $r = 1 \text{ km} = 10^3 \text{ m}$;

$$v = 900 \text{ km } h^{-1} = 900 \times \frac{5}{18} \text{ms}^{-1} = 250 \text{ m s}^{-1}$$

Centripetal acceleration,

$$a_c = \frac{v^2}{r} = \frac{250 \times 250}{10^3} \text{ m s}^{-2} = 62.5 \text{ m s}^{-2}$$

Now, $\frac{a_c}{g} = \frac{62.5}{10} = 6.25$

Example 16. A stone tied to the end of a string 80 cm long is whirled in a horizontal circle with a constant speed. If the stone makes 14 revolutions in 25 s. What is the magnitude and direction of acceleration of the stone?

Solution $r = 80 \text{ cm} = 0.80 \text{ m}; \ \omega = \frac{14 \ revolutions}{25 \ s} \times 2\pi$

Centripetal acceleration of $r\omega^2$

$$= 4 \times \frac{22}{7} \times \frac{22}{7} \times \frac{14}{25} \times \frac{14}{25} \times 0.8 = 9.9 \text{ ms}^{-2}$$

At every point, the acceleration is along the radius and towards the centre.

Example 17. A particle of mass m is suspended from a ceiling through a string of length L. The particle moves in a horizontal circle of radius r. Find (a) the speed of the particle and (b) the tension in the string. Such a system is called a conical pendulum.

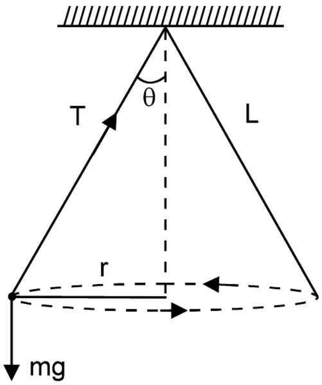
Solution The situation is shown in figure. The angle θ made by the string with the vertical is given by

$$\sin\theta = r / L \qquad ...(i)$$

The forces on the particle are

(a) the tension T along the string and

(b) the weight mg vertically downward.



The particle is moving in a circle with a constant speed v. Thus, the radical acceleration towards the centre has magnitude v^2/r. Resolving the forces along the radial direction and applying . Newton's second law,

$$T\sin\theta = m(v^2/r) \qquad \text{...(ii)}$$

As there is no acceleration in vertical directions,

we have from Newton's law,

$$T\cos\theta = mg \qquad \text{...(iii)}$$

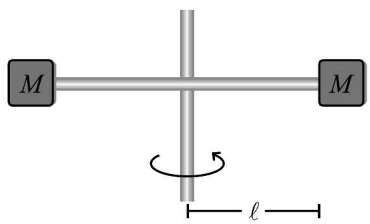
Dividing (ii) by (iii),

$$\tan\theta = \frac{v^2}{rg} \qquad \text{or,} \qquad v = \sqrt{rg\tan\theta}$$

And from (iii), $T = \dfrac{mg}{\cos\theta}$

Using (i), $$v = \frac{r\sqrt{g}}{(L^2 - r^2)^{1/4}} \text{ and } \frac{mgL}{(L^2 - r^2)^{1/2}}.$$

Example 18. Two blocks each of mass M are connected to the ends of a light frame as shown in figure. The frame is rotated about the vertical line of symmetry. The rod brakes if the tension in it exceeds T_0. Find the maximum frequency with which the frame may be rotated without braking the rod.



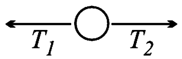
Solution Consider one of the blocks . If the frequency of revolution is f, the angular velocity is $\varpi = 2\pi f$. The acceleration towards the centre is $v^2/1 = \varpi^2 1 = 4\pi^2 f^2 l$. The only horizontal force on the block is the tension of the rod. At the point of braking , this force is T_0. So from Newton's law,

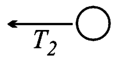
$$T_0 = M.4\pi^2 f^2 1$$

or, $$f = \frac{1}{2\pi}\left[\frac{T_0}{M\ell}\right]^{1/2}$$

Example 19. Two different masses are connected to two *light and inextensible* strings as shown in the figure. Both masses rotate about a central fixed point with constant angular speed of 10 rad s^{-1} on a smooth horizontal plane. Find the ratio of tensions $\dfrac{T_1}{T_2}$ in the strings.

Solution Drawing the *FBDs* for masses M_1 and M_2

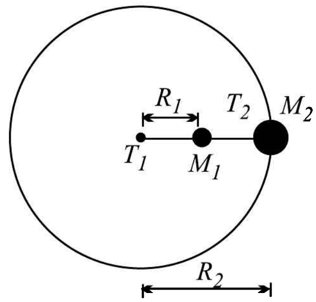




$$T_1 - T_2 = M_1R_1\omega^2 \qquad T_2 = M_2R_2\omega^2$$

$$\therefore \quad \frac{T_1 - T_2}{T_2} = \frac{M_1}{M_2}\cdot\frac{R_1}{R_2} = \frac{1}{4}\cdot\frac{1}{2}$$

$$\therefore \quad \frac{T_1}{T_2} = 1 + \frac{1}{8} = \frac{9}{8}]$$



$M_1 = 0.25$ kg
$M_2 = 1.0$ kg
$R_1 = 5$cm
$R_2 = 10$cm

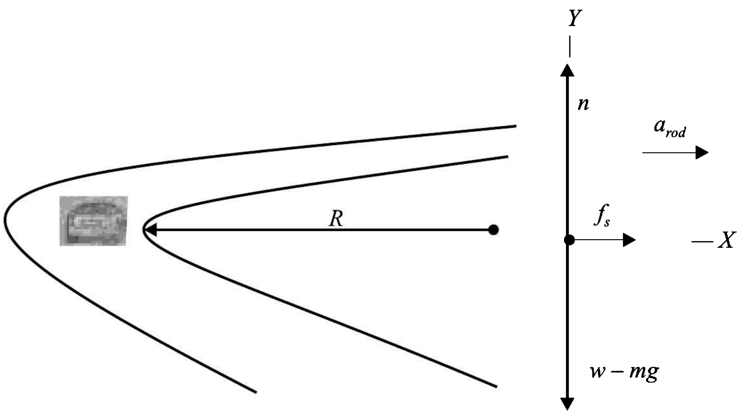
Concept

Centripetal force is the net force directed towards centre.

Example 20. A car is moving in a circular path of radius 50 m, on a flat rough horizontal ground. The mass of the car is 1000 kg. At a certain moment, when the constant speed of the car is 5 m/s, find the force of friction acting on it? [Note : $v \le \sqrt{\mu g r}$ for uniform speed]

(a) The situation (b) Our free-body diagram

(a) Car rounding at curve (b) Free-body diagram for the car



[Ans. 500 *N*]

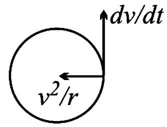
Concept

Only external force which can drive a car is friction. In case of car the direction of friction is not decided by velocity but according to need.

Example 21. A car is moving in a circular path of radius 50 m, on a flat rough horizontal ground. The mass of the car is 1000 kg. At a certain moment, when the speed of the car is 5 m/s, the driver is increasing speed at the rate of 1 m/s^2. Find the value of static friction on tyres (total) at this moment, in Newtons.

Solution $F_{net} = m\sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2} = m\sqrt{\left(\frac{1}{2}\right)^2 + (1)^2}$

$= \frac{m}{2}\sqrt{5} = 500\sqrt{5}\ N$



Concept

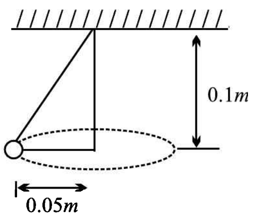
Friction is providing tangential as well as centripetal acceleration both.

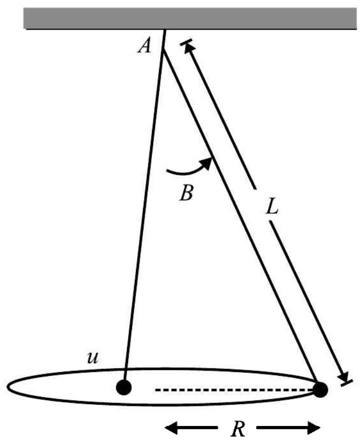
Example 22. A particle suspended from the ceiling by inextensible light string is moving along a horizontal circle of radius 0.05 m as shown. The string traces a cone of height 0.1 m. Find the speed.

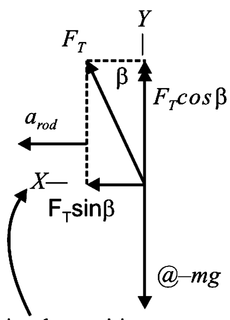
Conical pendulum

(a) The position (b) Our free-body diagram

(a) The situation (b) Free-body diagram for the ball



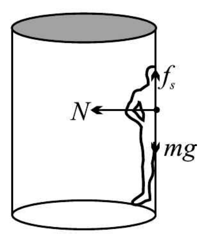




Solution $T\sin\theta = \frac{mv^2}{r}$; & $T\cos\theta = mg$

$\therefore \quad v = \sqrt{g\,r\tan\theta} = 0.5$ m]

Example 23. In a rotor, a hollow vertical cyclindrical structure rotates about its axis and a person rests against the inner wall. At a particular speed of the rotor, the floor below the person is removed and the person hangs resting against the wall without any floor. If athe radius of the rotor is 2m and the coefficient of static friction between the wall and the person is 0.2, find the minimum speed at which the floor may be removed. Take $g = 10$ m/s^2.

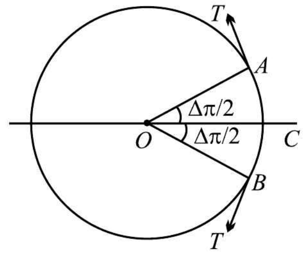


Solution $v = \sqrt{\frac{rg}{\mu_s}} = \sqrt{\frac{2m \times 10m/s^2}{0.2}} = 10$ m/s.

Concept

N is not always equal to mg but any force directed towards centre is called centripetal force.

Example 24. A metal ring of mass m and radius R is placed on a smooth horizontal table and is set rotating about its own axis in such a way that each part of the ring moves with a speed v. Find the tension in the ring.



Solution Consider a small part ACB of the ring that subtends an angle $\Delta\theta$ at the centre as shown in figure. Let the tension in the ring be T.

The forces on this small part ACB are

(a) tension T by the part of the ring left to A,

(b) tension T by the part of the ring right to B,

(c) weight $(\Delta m)g$ and

(d) normal force N by the table.

The tension at A acts along the tangent at A and the tension at B acts along the tangent at B. As the small part ACB moves in a circle of radius R at a constant speed v, its acceleration is towards the centre (along CO) and has a magnitude $(\Delta m)v^2 / R$.

Resolving the forces along the radius CO,

$$T\cos\left(90° - \frac{\Delta\theta}{2}\right) + T\,\text{cozs}\left(90° - \frac{\Delta\theta}{2}\right) = (\Delta m)\left(\frac{v^2}{R}\right)$$

or, $$2T\sin\frac{\Delta\theta}{2} = (\Delta m)\left(\frac{v^2}{R}\right) \qquad \text{... (i)}$$

The length of the part ACB is $R\Delta\theta$. As the total mass of the ring is m, the mass of the part ACB will be

$$\Delta m = \frac{m}{2\pi R}R\Delta\theta = \frac{m\Delta\theta}{2\pi}.$$

Putting Δm in (i),

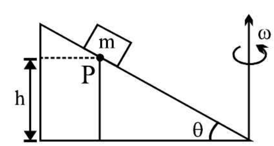
$$2T\sin\frac{\Delta\theta}{2} = \frac{m}{2\pi}\Delta\theta\left(\frac{v^2}{R}\right)$$

or, $$T = \frac{mv^2}{2\pi R}\,\frac{\Delta\theta/2}{\sin(\Delta\theta/2)} \approx 1 \text{ and } T = \frac{mv^2}{2\pi R}]$$

Concept

Net force on the ring is zero but still it has tension.

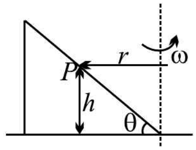
Example 25. A block of mass m is sitting on a rotating frictionless wedge. The wedge rotates with constant angular velocity ω around the axis shown in figure.

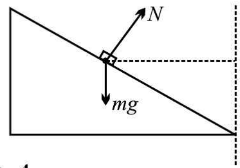


Calculate the value of ω such that the block stays at constant height h. (express your answer in terms of g, h, θ)

Solution The velocity of P point on the wedge. Velocity of wedge at point $P = \omega r = \omega \times \dfrac{h}{\tan\theta}$

Drawing a free body diagram for the block.





Resolving forces as indicated in step 2 .4

$$\frac{h}{r} = \tan\theta \quad \Rightarrow \quad \frac{h}{\tan\theta} = r$$

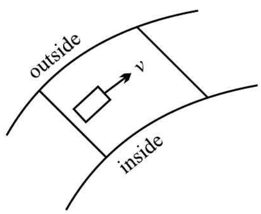
$$N\cos\theta - mg = 0 \qquad (y\text{–direction})$$

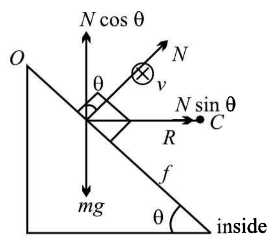
$$N\sin\theta = m\omega^2 r$$

$$\tan\theta = \frac{\omega^2 r}{g} = \frac{\omega^2 h}{g\tan\theta}$$

$$\Rightarrow \qquad \omega^2 = \frac{g\tan^2\theta}{h} \quad \Rightarrow \quad \omega = \sqrt{\frac{g}{h}}\tan\theta$$

What is the speed required to negotiate the turn shown in figure. Frictionless and radius of curvature 'R' and banking 'θ'.



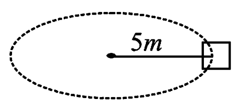


$$N\cos\theta - mg = 0 \qquad N\cos\theta = mg$$

$$N\sin\theta = \frac{mv^2}{R} \; mg \qquad \tan\theta = \frac{mv^2}{R}$$

$$v = \sqrt{Rg\tan\theta}$$

Example **26.** A block of mass 25 kg rests on a horizontal floor ($\mu = 0.2$). It is attached by a 5m long horizontal rope to a peg fixed on floor. The block is pushed along the ground with an initial velocity of 10 m/s so that it moves in a circle around the peg. Find



(a) Tangential acceleration of the block

(b) Speed of the block at time t.

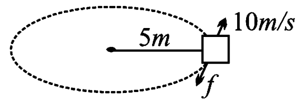
(c) Time when tension in rope becomes zero.

[**Ans.** (a) – 2 m/s^2, (b) $10 - 2t$, (c) 5 sec]

Solution (a) Tangential acceleration is the retardation produced by the friction

$a = -f/m = -\mu mg/ma_t = -0.2 \times 10 = -2$ m/s^2]

(b) $\frac{dv}{dt} = a_t = -2$



$\int_{10}^{v} dv = -2\int_{0}^{t} dt$

$v - 10 = -2t v = 10 - 2t$

(c) Tension in the rope will become zero when centripetal acceleration becomes zero i.e. when speed becomes zero

$v = 0 \quad \Rightarrow \quad 10 - 2t = 0 \quad \Rightarrow \quad t = 5$ sec

Concept

Force directed towards centre is centripetal & force in the velocity direction is tangential force. Tangential acceleration is responsible for change in speed. If friction is kinetic it is opposite to velocity. If there is slipping on surface then net friction is kinetic.

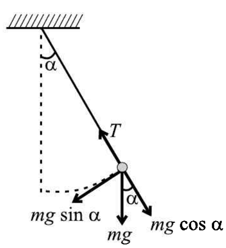
Example 27. A particle of mass 14 g attached to a string of 70 cm length is whirled round in a horizontal circle. If the period of revolution is 2 second, calculate the tension.

Solution 9680 dyne **Ans**.

Example 28. A string breaks under a load of 50 kg. A mass of 1 kg is attached to one end of the string 10 m long and is rotated in horizontal circle. Calculate the greatest number of revolutions that the mass can make without breaking the string.

Solution $n = 1.114$ revolutions per second **Ans.**

Example 29. A simple pendulum is constructed by attaching a bob of mass m to a string of length L fixed at its upper end. The bob oscillates in a vertical circle. It is found that the speed of the bob is v when the string makes an angle α with the vertical. Find the tension in the string and the magnitude of net force on the bob at the instant.



Solution (i) The forces acting on the bob are :

(a) the tension T

(b) the weight mg

As the bob moves in a circle of radius L with centre at O.

A centripetal force of magnitude $\frac{mv^2}{L}$ is required towards O. This force will be provided by the resultant of T and mg cos α. Thus,

or $\quad T - mg\cos\alpha = \frac{mv^2}{L} \quad T = m\left(g\cos\alpha + \frac{v^2}{L}\right)$

(ii) $$|\vec{F}_{net}| = \sqrt{(mg\sin\alpha)^2 + \left(\frac{mv^2}{L}\right)^2} = m\sqrt{g^2\sin^2\alpha + \frac{v^4}{L^2}}$$

Example 30. One end of a string of length 1.4 *m* is tied to a stone of mass 0.4 kg and the other end to a small pivot. Find the minimum velocity of stone required at its lowest point so that the string does not slacken at any point in its motion along the vertical circle?

Solution 8.25 ms^{-1}

Example 31. A particle of mass *m* slides without friction from the top of a hemisphere of radius *r*. At what height will the body lose contact with the surface of the sphere?

Solution At a height of $2r/3$ above the centre of the hemisphere.

CIRCULAR TURNING ON ROADS

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways.

1. By friction only
2. By banking of roads only.
3. By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both. Now let us write equations of motion in each of the three cases separately and see what are the constant in each case.

1. By Friction Only

Suppose a car of mass *m* is moving at a speed *v* in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction *f* acting towards center

Thus, $f = \frac{mv^2}{r}$

Further, limiting value of f is μN

or $f_L = \mu N = \mu mg \qquad (N = mg)$

Therefore, for a safe turn without sliding $\frac{mv^2}{r} \leq f_L$

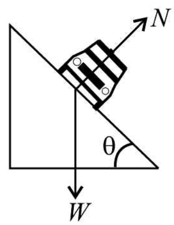
or $\frac{mv^2}{r} \leq \mu mg$ or $\mu \geq \frac{v^2}{rg}$ $or,\ v \leq \sqrt{\mu rg}$

Here, two situations may arise. If μ and r are known to us, the speed of the vehicle should not exceed $\sqrt{\mu rg}$ and if v and r are known to us, the coefficient of friction should be greater than $\frac{v^2}{rg}$.

***Example* 32.** A bend in a level road has a radius of 100 m. Calculate the maximum speed which a car turning this bend may have without skidding. Given : $\mu = 0.8$.

[Ans. 28 ms^{-1}**]**

2. By Banking of Roads Only

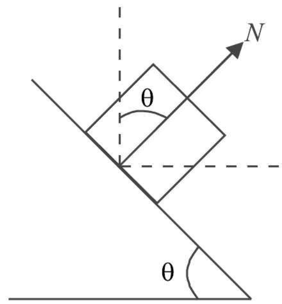


Friction is not always reliable at circular turns if high speeds and sharp turns are involved. to avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.

Applying Newton's second law along the radius and the first law in the vertical direction.

$$N\sin\theta = \frac{mv^2}{r} \qquad \text{or} \qquad N\cos\theta = mg$$

from these two equations, we get $\tan\theta = \dfrac{v^2}{rg}$ or $v = \sqrt{rg\tan\theta}$



***Example* 33.** A circular track of radius 600 m is to be designed for cars at an average speed of 180 km/hr. What should be the angle of banking of the rack ?

Solution Let the angle of banking be θ. The forces on the car are (figure)

(a) weight of the car Mg downward and

(b) normal force N.

For proper banking, static frictional force is not needed.

For vertical direction the acceleration is zero. So,

$$N\cos\theta = Mg \qquad \text{...(i)}$$

For horizontal direction , the acceleration is v^2/r towards the centre, so that $N\sin\theta = Mv^2/r$ ...(ii)

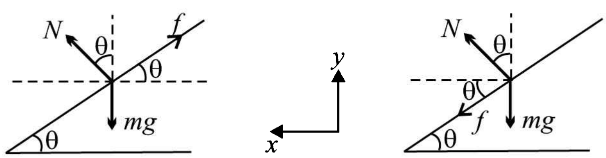
From (i) and (ii), $\tan\theta = v^2/rg$

Putting the values , $\tan\theta = \dfrac{180(km/hr)^2}{(600m)(10m/s^2)} = 0.4167$

θ = 22.6r.

3. By Both Friction and Banking of Roads

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction.



The direction of second force, i.e., normal reaction N is also fixed (perpendicular ro road) while the direction of the third force i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_L = \mu N$). So the magnitude of normal reaction N and directions plus magnitude of friction f are so adjusted that the resultant of the three forces mentioned above is $\frac{mv^2}{r}$ towards the center. Of these m and r are also constant. Therefore, magnitude of N and directions plus magnitude of friction mainly depends on the speed of the vehicle v. Thus, situation varies from problem to problem. Even though we can see that :

(i) Friction f will be outwards if the vehicle is at rest $v = 0$. Because in that case the component weight mg $\sin\theta$ is balanced by f.

(ii) Friction f will be inwards if $v > \sqrt{rg \tan\theta}$

(iii) Friction f will be outwards if $v < \sqrt{rg \tan\theta}$ and

(iv) Friction f will be zero if $v = \sqrt{rg \tan\theta}$

Notes

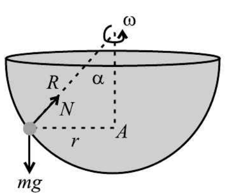
(i) The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle of banking for an aircraft, i.e., the angle through which it should tilt while negotiating a curve, to avoid deviation from the circular path.

(ii) The expression $\tan\theta = \frac{v^2}{rg}$ also gives the angle at which a cyclist should lean inward, when rounding a corner. In this case, θ is the angle which the cyclist must make with the vertical.

Example 34. A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α. Find the angular speed at which the bowl is rotating.

Solution Let ω be the angular speed of rotation of the bowl. Two force are acting on the ball.

1. normal reaction N
2. weight mg



The ball is rotating in a circle of radius r (= $R \sin\alpha$) with centre at A at an angular speed ω. Thus,

$$N \sin\alpha = mr\omega^2 = mRw^2 \sin\alpha \qquad \text{...(i)}$$

and $$N \cos\alpha = mg \qquad \text{...(ii)}$$

Dividing Eqs. (i) by (ii), we get $\frac{1}{\cos\alpha} = \frac{\omega^2 R}{g}$ $\therefore$ $\omega = \sqrt{\frac{g}{R\cos\alpha}}$

NON UNIFORM CIRCULAR MOTION

If the speed of the particle moving in a circle is not constant the acceleration has both radial and tangential components. The radial and tangential accelerations are :

$$a_r = \omega^2 r = \frac{v^2}{r} \qquad a_t = \frac{dv}{dt}$$

The magnitude of the resultant acceleration will be :

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

If the direction of resultant acceleration makes an angle β with the radius, where then

$$\tan\beta = \frac{dv/dt}{v^2/r}$$

Now as acceleration of particle undergoing non-uniform circular motion is

$$a = \sqrt{(\omega^2 R)^2 + \left(R\frac{d\omega}{dt}\right)^2} = \sqrt{\left(\frac{v^2}{R}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

In the direction $\tan^{-1}\left(\frac{dv/dt}{v^2/r}\right)$ with radius it need resultant force of $m\sqrt{\left(\frac{v^2}{R}\right)^2 + \left(\frac{dv}{dt}\right)^2}$ in the direction of acceleration.

Example 35. A car goes on a horizontal circular road of radius R, the speed increasing at a rate $\frac{dv}{dt} = a$. The friction co-efficient between road and tyre is μ. Find the speed at which the car will skid.

Solution Here at any time t, the speed of car becomes V, the net acceleration in the plane of road is

$$\sqrt{\left(\frac{v^2}{R}\right)^2 + a^2}.$$

This acceleration is provided by frictional force. At the moment car will slide

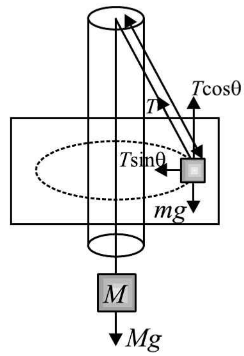
$$M\sqrt{\left(\frac{v^2}{R}\right)^2 + a^2} = \mu M g \quad \Rightarrow \quad v = \left[R^2\left(\mu^2 g^2 - a^2\right)\right]^{1/4}$$

Example 36. A large mass M and a small mass m hang at the two ends of the string that passes through a smooth tube as shown in figure. The mass m moves around in a circular path, which lies in the horizontal plane. The length of the string from the mass m to the top of the tube is ℓ and θ is the angle this length makes with vertical. What should be the frequency of rotation of mass m so that M remains stationary?

Solution The forces acting on mass m and M are shown in figure. When mass M is stationary

$$T = Mg \quad \text{... (i)}$$

where T is tension in string



For the smaller mass, the vertical component of tension $T\cos\theta$ balances mg and the horizontal component $T\sin\theta$ supplies the necessary centripetal force.

$$T\cos\theta = mg \quad \text{... (ii)}$$

$$T\sin\theta = mr\omega^2 \quad \text{... (iii)}$$

ω being the angular velocity and r is the radius of horizontal circular path.

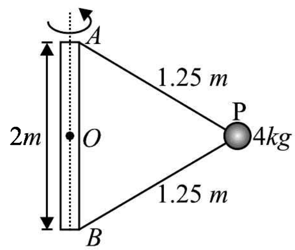
From (i) and (iii), $Mg\sin\theta = mr\omega^2$

$$\omega = \sqrt{\frac{Mg\sin\theta}{mr}} = \sqrt{\frac{Mg\sin\theta}{m\ell\sin\theta}} = \sqrt{\frac{Mg}{m\ell}}$$

$$\text{Frequency of rotation} = \frac{1}{T} = \frac{1}{2\pi\ell\omega} = \frac{\omega}{2\pi}$$

$$\therefore \quad \text{Frequency} = \frac{1}{2\pi}\sqrt{\frac{Mg}{m\ell}}$$

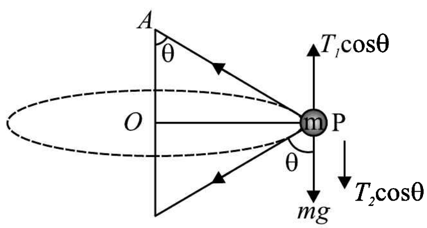
Example 37. The 4 kg block in the figure is attached to the vertical rod by means of two strings. When the system rotates about the axis of the rod, the two strings are extended as indicated in figure. How many revolutions per minute must the system make in order that the tension in upper string is 60 N?



What is tension in the lower string ?

Solution The forces acting on block P of mass 4 kg are shown in the figure.

If θ is the angle made by strings with vertical, T_1 and T_2 tension in strings for equilibrium in the vertical direction



$$T_1\cos\theta = T_2\cos\theta + mg$$

$$(T_1 - T_2)\cos\theta = mg$$

$$\cos\theta = \frac{1}{1.25} = \frac{4}{5} \quad \left[\because \cos\theta = \frac{OA}{AP} = \frac{1}{1.25}\right]$$

$$\therefore \quad T_1 - T_2 = \frac{mg}{\cos\theta} = \frac{5mg}{4} = \frac{5}{4} \times 4 \times 9.8 = 49\,N$$

Given $T_1 = 60\,N$

$$T_2 = T_1 - 49 = 60\,N - 49\,N = 11\,N$$

The net horizontal force $(T_1\sin\theta + T_2\sin\theta)$ provides the necessary centripetal force $m\omega^2 r$.

$$\therefore \quad (T_1 + T_2)\sin\theta = m\omega^2 r \Rightarrow \omega^2 = \frac{(T_1 + T_2)\sin\theta}{mr}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - (4/5)^2} = \frac{3}{5}$$

$$r = OP = \sqrt{1.25^2 - 1^2} = 0.75$$

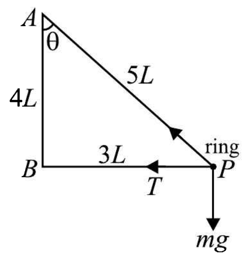
$$\therefore \quad \omega^2 = \frac{(60 + 11)\frac{3}{5}}{4 \times 0.75} = 14.2 \qquad \omega = \sqrt{14.2} = 3.768 \text{ rad/s}$$

$$\text{Frequency revolution} = \frac{\omega}{2\pi} = \frac{3.768}{2 \times 3.14} = 0.6 \text{ rev/s or } 36 \text{ rev/min}$$

Example 38. A small smooth ring of mass m is threaded on a light inextensible string of length $8L$ which has its ends fixed at points in the same verticle line at a distance $4L$ apart. The ring describes horizontal circles at constant speed with both parts of the string taut and with the lower portion of the string horizontal. Find the speed of the ring and the tension in the string. The ring is then tied at the midpoint of the string and made to perform horizontal circles at constant speed of $3\sqrt{gL}$. Find the tension in each part of the string.

Solution When the string passes through the ring, the tension in the string is the same in both the parts. Also from geometry,

$BP = 3L$ and $AP = 5L$



$$T\cos\theta = \frac{4}{5}T = mg \qquad \text{... (i)}$$

$$T + T\sin\theta = T\left(1 + \frac{3}{5}\right) = \frac{8}{5}T$$

$$= \frac{mv^2}{BP} = \frac{mv^2}{3L} \qquad \text{... (ii)}$$

Dividing (ii) by (i)

$$\frac{v^2}{3Lg} = 2$$

$$v = \sqrt{6Lg}$$

From (i) $T = \dfrac{mg}{4/5} = \dfrac{5}{4}mg$

In the second case, ABP is an equilateral triangle

$$T\cos 60^\circ = mg + T_2\cos 60^\circ$$

$$T_1 - T_2 = \frac{mg}{\cos 60^\circ} = 2mg$$

$$T_1\sin 60^\circ + T_2\sin 60^\circ = \frac{mv^2}{r} = \frac{9mgL}{4L\sin 60^\circ}$$

$$T_1 + T_2 = \frac{9mg}{4\sin^2 60^\circ} = 3mg$$

Solving equations (iii) and (iv)

$$T_1 = \frac{5}{2}\, mg \,;\; T_2 = \frac{1}{2}\, mg$$

Example 39. The angular acceleration of a particle moving along a circular path with uniform speed is

(A) uniform but non-zero

(B) zero

(C) variable

(D) such as cannot be predicted from the given information.

Solution As angular speed of the particle is constant and hence angular acceleration is zero.

∴ (B) is the right answer.

Example 40. A particle is projected horizontally from the top of a cliff of height H with a speed $\sqrt{2gH}$. The radius of curvature of the trajectory at the instant of projection will

(A) $H/2$ (B) H

(C) $2H$ (D) ∝.

Solution Since, $\vec{g} \perp \vec{v}$;Radial acceleration $a_r = g$

We know $a_r = v^2/r$

$\Rightarrow \quad \frac{v^2}{r} = g$ where r is the radius of curvature.

$\Rightarrow \quad \frac{2gH}{r} = g \quad (\because v = \sqrt{2gH}\,)$

$\Rightarrow \quad r = 2H$

∴ Hence (C) is the right answer.

Example 41. The angular acceleration of a particle moving along a circular path with uniform speed is

(A) uniform but non-zero

(B) zero

(C) variable

(D) such as cannot be predicted from the given information.

Solution As angular speed of the particle is constant and hence angular acceleration is zero.

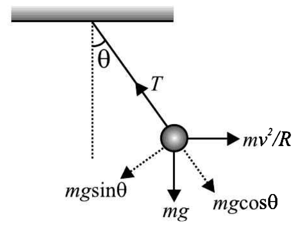
∴ (B) is the right answer.

Example 42. A car is moving in a circular horizontal track of radius 10 m with constant speed of 10 m/s. A plumb bob is suspended from roof by a light rigid rod of length 1 m. The angle made by the rod with the track is :

(A) zero (B) 30°

(C) 45° (D) 60°

Solution The different forces acting on the bob are shown in figure. Resolving the force along the length and perpendicular to the rod, we have



$$mg\cos\theta + \frac{mv^2}{R}\sin\theta = T$$

$$mg\sin\theta + \frac{mv^2}{R}\cos\theta$$

$$\tan\theta = \frac{v^2}{Rg} = \frac{(10)^2}{(10)(10)} = 1$$

$$\theta = \tan^{-1}(1) = 45°$$

Example 43. A particle is revolving with a constant angular acceleration α in a circular path of radius r. Find the time when the centripetal acceleration will be numerically equal to the tangential acceleration.

Solution Le the speed of the particle after time t from starting be v

⇒ The centripetal acceleration

$$a_r = \frac{v^2}{r} = r\omega^2 \text{ \& the corresponding angular speed } \omega = \alpha t.$$

⇒ $a_r = r(\alpha t)^2 = r\alpha^2 t^2$ ...(i)

We known that the tangential acceleration $a_t = r\alpha$ ...(ii)

Since, $a_r = a_t$ (given)

⇒ $r\alpha^2 t^2 = r\alpha$

⇒ $t = \frac{1}{\sqrt{\alpha}}$.

Example 44. Show that the effect of rotation of the earth is to lessen the apparent weight of a body at the equator by 1/303 of itself, the earth being assumed to be a sphere of 6400 km radius. Calculate the proportionate decrease in apparent weight of a train at equator moving east at the rate of 120 km/hour.

Solution Let m be the mass of body. The decrease in its weight is equal to centripetal force on it, i.e., $\frac{mv^2}{R}$. The weight decreases by a fraction of itself

$$= \frac{mv^2}{R \times mg} = \frac{v^2}{Rg} = \frac{(4.65342)^2}{(6400 \times 10^3) \times 9.8} = \frac{1}{303}$$

Velocity of train = 120 km/h = (120 × 1000)/(60 × 60) m/sec

If v' be the velocity of train in space, then

$$v' = 456.42 + \left(\frac{120 \times 1000}{60 \times 60}\right) = 498.72\, m/\text{sec}$$

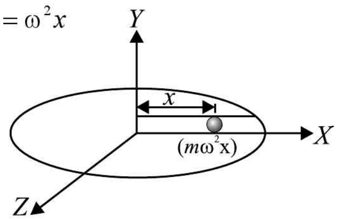
$$\text{Fractional decrease in weight } = \frac{v'^2}{Rg} = \frac{(498.72)^2}{(6400 \times 10^3)(9.8)} = \frac{1}{263}$$

Example 45. A circular table with smooth horizontal surface is rotating at an angular speed ω about its axis. A groove is made on the surface along the radius and a small particle is gently placed inside the groove at a distance ℓ from the centre. Find the speed of the particle with respect to the table as its distance from the centre becomes L.

Solution Here the motion of the particle is confined within the groove only and hence its acceleration is along X-axis. The force along X-axis is centrifugal force = $m\omega^2 x$. The other forces are perpendicular to X-axis and have no components along X-axis.

The acceleration a along X-axis is given by

$$a = \frac{F}{m} = \frac{m\omega^2 x}{m} = \omega^2 x \qquad \text{or} \qquad \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} v = \omega^2 x$$



$\therefore \quad vdv = \omega^2 xdx$

Integrating $\int_0^v vdv = \omega^2 \int_\ell^L xdx$

Solving we get $v = \omega(L^2 - \ell^2)^{1/2}$

Example 46. A particle is moving along a vertical circle of radius r = 20 m with a constant vertical circle of radius r = 20 m with a constant speed v = 31.4 m/s as shown in figure. Straight line ABC is horizontal and passes through the centre of the circle. A shell is fired from point A at the instant when the particle is at C. If distance AB is $20\sqrt{3}$ m and the shell collide with the particle at B, then prove

$$\tan\theta = \frac{(2n-1)^2}{\sqrt{3}}$$

Where n is an integer. Further, show that smallest value of θ is 30°.

Solution As at the time of firing of the shell, the particle was at C and the shell collides with it at B, therefore the number of the revolutions completed by the particle is odd multiple of half i.e., $(2n - 1)/2$, where n is an integer.

Let T be the time period of the particle, then

$$T = \frac{2\pi r}{v} = \frac{2 \times 3.14 \times 20}{31.4} = 4 \text{ second}$$

If t be the time of the flight of the shell, then

t = time of [(2n – 1)/2] revolutions of the particle

$$= \frac{(2n-1)}{2} \times 4 = 2(2n-1) \text{ second}$$

for a projectile, the time of flight is given by $t = \dfrac{2u\sin\theta}{g}$

Hence, $\dfrac{2u\sin\theta}{g} = 2(2n-1)$ ...(i)

The range of the projectile is given by $R = \dfrac{u^2 \sin 2\theta}{g}$

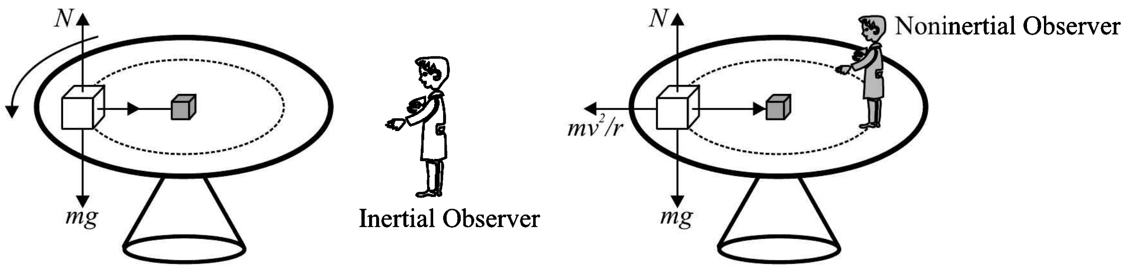
Hence, $\dfrac{u^2 \sin 2\theta}{g} = 20\sqrt{3}$ ...(ii)

From equation (i) and (ii) $\tan\theta = \dfrac{(2n-1)^2}{\sqrt{3}}$

For θ to be smallest, $n = 1$, so $\tan\theta = \dfrac{(2n-1)^2}{\sqrt{3}}$

CENTRIFUGAL FORCE

An observer in a rotating system is another example of a non-inertial observer. Suppose a block of mass m lying on a horizontal frictionless turntable is connected to a string as in figure. According to an inertial observer, if the block rotates uniformly, it undergoes an acceleration of magnitude $\frac{v^2}{r}$ where v is the tangential speed. The inertial observer concludes that this centripetal acceleration is provided by the force exerted by the string T and writes Newton's second law $T = \frac{mv^2}{r}$.



(a) (b)

According to a non-inertial observer attached to the turntable, the block is at rest. Therefore, in applying Newton's second law, this observer introduces a fictitious outward force of magnitude $\frac{mv^2}{r}$. According to the non-inertial observer, this outward force balances the force exerted by the string and therefore $\frac{T - mv^2}{r} = 0$.

In fact, centrigugal force is a sufficient pseudo force only if we were analyzing the particles at rest in a uniformly rotating frame. If we analyze the motion of a particle that moves in the rotating frame we may have to assume other pseudo forces together with the centrifugal force. Such forces are called coriolis forces. The coriolis force is perpendicular to the velocity of the particle and also perpendicular to the axis of rotation of the frame. Once again it should be remembered that all these pseudo forces, centrifugal or Coriolis are needed only if the working frame is rotating. If we work from an inertial frame there is no need to apply any pseudo force. There should not ba a misconception that centrifugal force acts on a particle because the particle describes a circle.

Therefore when we are working from a frame of reference that is rotating at a constant angular velocity ω with respect to an inertial frame. The dynamics of a particle of mass m kept at the distance r from the axis of rotation we have to assume that a force $m\omega^2 r$ acts radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.

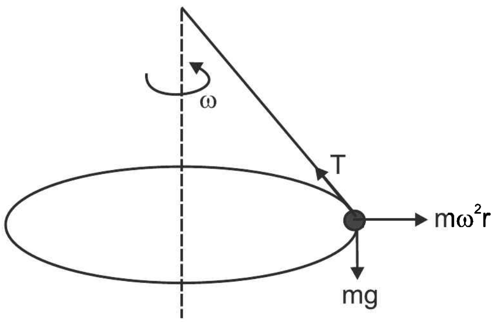
You should be careful when using fictitious forces to describe physical phenomena. Remember that fictitious forces are used only in non-inertial frames of references. When solving problems, it is often best to use an inertial frame.

Explanatory Notes on Centrifugal Force

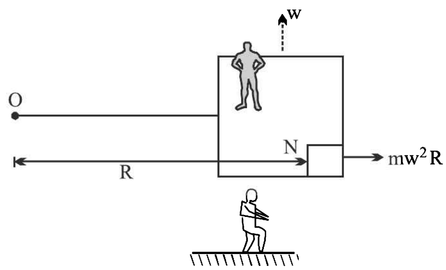
When a body is rotating in a circular path and the centripetal force vanishes, the body would leave the circular path. To an observer A who is not sharing the motion along the circular path, the body appears to fly off tangentially at the point of release. To another observer *B*, who is sharing the motion along the circular path (i.e., the observer *B* is also rotating with the body which is released, it appears to *B*, as if it has been thrown off along the radius away from the centre by some force. This inertial force is called centrifugal force.)

Its magnitude is equal to that of the centripetal force = $\frac{mv^2}{r}$. Centrifugal force is a fictitious force which has to be applied as a concept only in a rotating frame of reference to apply N.L in that frame)

FBD of ball w.r.t. non-inertial frame rotating with the ball.



Suppose we are working from a frame of reference that is rotating at a constant, angular velocity ω with respect to an inertial frame. If we analyse the dynamics of a particle of mass *m* kept at a distance *r* from the axis of rotation, we have to assume that a force $mr\omega^2$ react radially outward on the particle. Only then we can apply Newton's laws of motion in the rotating frame. This radially outward pseudo force is called the centrifugal force.



If reference frame is particle (undergoing circular motion) itself then it will experience pseudo force which will be radially outward and equal to $m(w^2R)$.

For man outside $N = mw^2R$

For man inside $N - mw^2R = 0$.

This is called centrifugal force.

Example **47.** A table with smooth horizontal surface is fixed in a cabin that rotates with angular speed ω in a circular path of radius *R*. A smooth groove *AB* of length *L* (<< *R*) is made on the surface of table as shown in figure. A small particle is kept at the point A in the groove and is released to move. Find the time taken by the particle to reach the point *B*.

Solution Let us analyse the motion of particle with respect to table which is moving with cabin with an angular speed of ω. Along AB centrifugal force of magnitude $m\omega^2R$ will act at A on the particle which can be treated as constant from A to B as $L << R$.

$\therefore$ acceleration of particle along AB with respect to cabin $a = \omega^2R$ (constant)

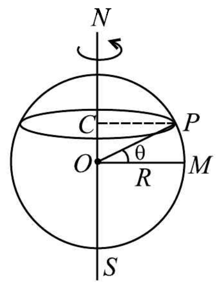
Required time "t" is given by

$$S = ut + \frac{1}{2}at^2 \Rightarrow L = 0 + \frac{1}{2} \times \omega^2 R t^2 \Rightarrow t = \sqrt{\frac{2L}{\omega^2 R}}$$

EFFECT OF EARTH'S ROTATION ON APPARENT WEIGHT

The earth rotates about its axis at an angular speed of one revolution per 24 hours. The line joining the north and the south poles is the axis of rotation.

Every point on the earth moves in a circle. A point at equator moves in a circle of radius equal to the radius of the earth and the centre of the circle is same as the centre of the earth. For any other point on the earth, the circle of rotation is smaller than this.



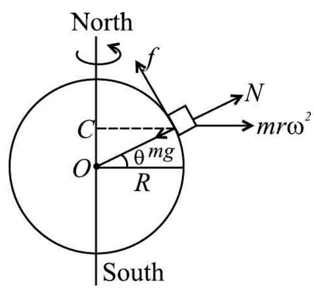
Consider a place P on the earth (figure).

Drop a perpendicular PC from P to the axis SN. The place P rotates in a circle with the centre at C. The radius of this circle is CP. The angle between the line OM and the radius OP through P is called the latitude of the place P. We have

$CP = OP \cos\theta$ or, $r = R\cos\theta$

where R is the radius of the earth.

If we work from the frame of reference of the earth, we shall have to assume the existence of pseudo force. In particular, a centrifugal force mw^2r has to be assumed on any particle of mass m placed at P.



If we consider a block of mass m at point P then this block is at rest with respect to earth. If resolve the forces along and perpendicular the centre of earth then

$$N + mr\omega^2 \cos\theta = mg \Rightarrow N = mg - mr\omega^2\cos\theta \Rightarrow N = mg - mR\omega^2\cos^2\theta$$

Example 48. A body weighs $98N$ on a spring balance at the north pole. What will be its weight recorded on the same scale if it is shifted to the equator? Use $g = GM/R^2 = 9.8$ m/s^2 and the radius of the earth $R = 6400$ km.

Solution At poles , the apparent weight is same as the true weight.

Thus, $98N = mg = m(9.8 \text{ m/s}^2)$

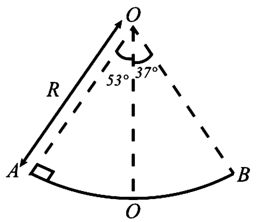
At the equator , the apparent weight is $mg' = mg - m\varpi^2R$

The radius of the earth is 6400 km and the angular speed is

$$\varpi = \frac{2\pi \, rad}{24 \times 60 \times 60 s} = 7.27 \times 10^{-6} \text{ rad/s}$$

$$mg' = 98N - (10 \text{ kg})(7.27 \times 10^{-5} s^{-1})^2 (6400 \text{ km}) = 97.66N$$

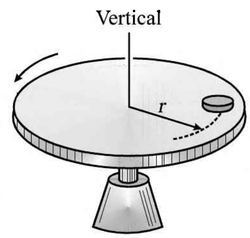
Example 49. A section of fixed smooth circular track of radius R in vertical plane is shown in the figure. A block is released from position A and leaves the track at B. The radius of curvature of its trajectory when it just leaves the track at B is :



(A) R (B) $\frac{R}{4}$ (C) $\frac{R}{2}$ (D) none of these

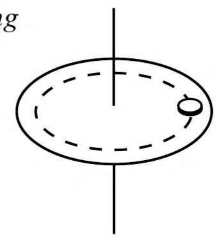
[Ans. (C)]

Example 50. A small coin of mass 40 g is placed on the horizontal surface of a rotating disc. The disc starts from rest and is given a constant angular acceleration α = 2 rad/s^2. The coefficient of static friction between the coin and the disc is μ_s = 3/4 and coefficient of kinetic friction is μ_k = 0.5. The coin is placed at a distance r = 1 m from the centre of the disc. The magnitude of the resultant force on the coin exerted by the disc just before it starts slipping on the disc is :



(A) 0.2 N (B) 0.3 N
(C) 0.4 N (D) 0.5 N

Solution The friction force on coin just before coin is to slip will be : $f = \mu_s mg$



Normal reaction on the coin ; $N = mg$

The resultant reaction by disk to the coin is

$$= \sqrt{N^2 + f^2} = \sqrt{(mg)^2 + \mu_s(mg)^2} = mg\sqrt{1+\mu^2}$$

$$= 40 \times 10^{-3} \times 10 \times \sqrt{1+\frac{9}{16}} = \mathbf{0.5\ N}$$

$\therefore$ (D) is the right answer.

Example 51. A circular road of radius R is banked for a speed v = 40 km/hr. A car of mass m attempts to go on the circular road, the friction co-efficient between the tyre & road is negligible:

(A) the car cannot make a turn without skidding

(B) if the car runs at a speed less than 40 km/hr, it will slip up the slope

(C) if the car runs at the correct speed of 40 km/hr, the force by the road on the car is equal to mv^2/r

(D) if the car runs at the correct speed of 40 km/hr, the force by the road on the car is greater than mg as well as greater than mv^2/r

[Ans. (D)]

Example 52. A ring of mass 2π kg and of radius 0.25m is making 300rpm about an axis through its centre perpendicular to its plane. The tension (in newtons) developed in the ring is:

(A) 50 (B) 100 (C) 175 (D) 250

[Ans. (D)**]**

Example 53. A car driver going at some speed suddenly finds a wide wall at a distance r. To avoid hitting the wall he should

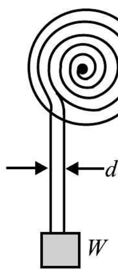
(A) apply the brakes
(B) should turn the car in a circle of radius r.
(C) apply the brakes and also turn the car in a circle of radius r.
(D) jump on the back seat.

[Ans. (A)**]**

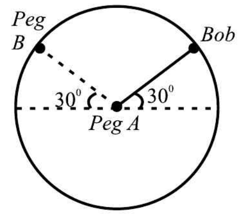
Example 54. A weight W attached to the end of a flexible rope of diameter $d = 0.75$cm is raised vertically by winding the rope on a reel as shown. If the reel is turned uniformly at the rate of 2 r.p.s. What is the tension in rope. The inertia of rope may be neglected.

(A) 1.019W (B) 0.51W
(C) 2.04W (D) W

[Ans. (A)**]**



Example 55. A bob is attached to one end of a string other end of which is fixed at peg A. The bob is taken to a position where string makes an angle of 30^0 with the horizontal. On the circular path of the bob in vertical plane there is a peg 'B' at a symmetrical position with respect to the position of release as shown in the figure. If V_c and V_a be the minimum speeds in clockwise and anticlockwise directions respectively, given to the bob in order to hit the peg 'B' then ratio V_c : V_a is equal to:



(A) 1 : 1 (B) 1 : $\sqrt{2}$
(C) 1 : 2 (D) 1 : 4

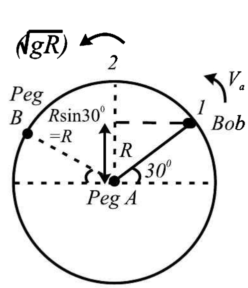
[Ans. (C)**]**

Solution (C) For anti-clockwise motion, speed at the highest point should be $\sqrt{gR}$.

Conserving energy at (1) & (2) :

$$\frac{1}{2}mV_a^2 = mg\frac{R}{2} + \frac{1}{2}m(gR)$$

$\Rightarrow$ $v_a^2 = gR + gR = 2gR$ $\Rightarrow v_a = \sqrt{2gR}$



For clock-wise motion, the bob must have atleast that much speed initially, so that the string must not become loose any where until it reaches the peg B.

At the initial position :

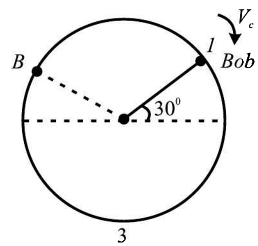
$$T + mg\cos 60^0 = \frac{mv_c^2}{R};$$

V_C being the initial speed in clockwise direction.

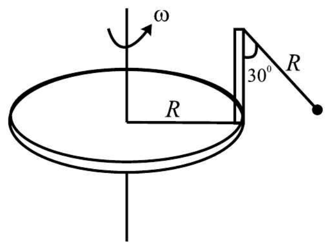
For $V_{C\,min}$: Put $T = 0$;

$\Rightarrow \quad V_C = \sqrt{\dfrac{gR}{2}} \Rightarrow V_C/V_a = \dfrac{\sqrt{\dfrac{gR}{2}}}{\sqrt{2gR}} = \dfrac{1}{2}$

$\Rightarrow \quad V_C : V_a = 1 : 2$



Example 56. A disc of radius R has a light pole fixed perpendicular to the disc at the circumference which in turn has a pendulum of length R attached to its other end as shown in figure. The disc is rotated with a constant angular velocity ω. The string is making an angle 30ŗ with the rod. Then the angular velocity ω of disc is :



(A) $\left(\dfrac{\sqrt{3}g}{R}\right)^{1/2}$ (B) $\left(\dfrac{\sqrt{3}g}{2R}\right)^{1/2}$ (C) $\left(\dfrac{g}{\sqrt{3}R}\right)^{1/2}$ (D) $\left(\dfrac{2g}{3\sqrt{3}R}\right)^{1/2}$

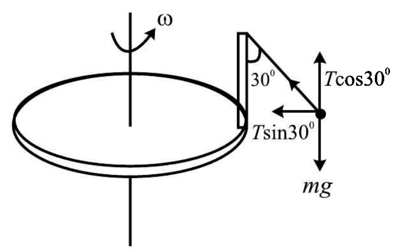
Solution The bob of the pendulum moves in a circle of radius (R + Rsin30ŗ) = $\dfrac{3R}{2}$

Force equations : $T\sin 30^0 = m\left(\dfrac{3R}{2}\right)\omega^2$

$T\cos 30^0 = mg$

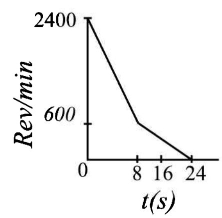
$\Rightarrow \quad \tan 30^0 = \dfrac{3}{2}\dfrac{\omega^2 R}{g} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \quad \omega = \sqrt{\dfrac{2g}{3\sqrt{3}R}}$



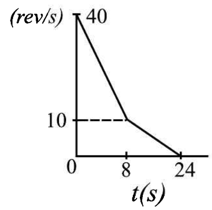
$\therefore$ (D) is the right answer.

Example 57. A table fan rotating at a speed of 2400 rpm is switched off and the resulting variation of the rpm with time is shown in the figure. The total number of revolutions of the fan before it come to rest is:



(A) 420 (B) 280 (C) 190 (D) 16800

Solution The correponding (Rev./sec), graph is :

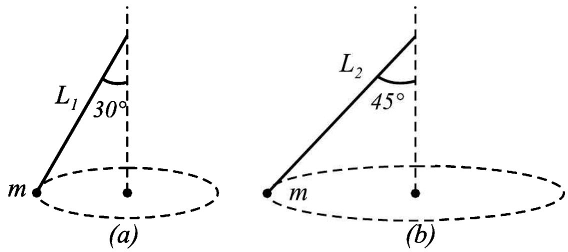


Area under this curve gives the total number revolutions.

$$\Delta = \frac{1}{2}(8)(30) + (10 \times 8) + \frac{1}{2}(16)(10) = 280 \text{ revolutions.}$$

$\therefore$ (B) is the right answer.

Example 58. Two particles tied to different strings are whirled in a horizontal circle as shown in figure. The ratio of lengths of the strings so that they complete their circular path with equal time period is:



(A) $\sqrt{\frac{3}{2}}$ (B) $\sqrt{\frac{2}{3}}$ (C) 1 (D) None of these

Solution since $T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$

$\therefore$ $T_1 = T_2$

$\Rightarrow$ $L_1 \cos\theta_1 = L_2 \cos\theta_2$

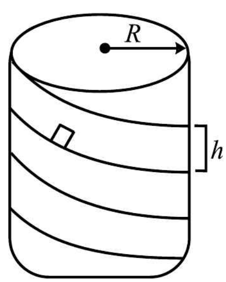
$\therefore$ $\frac{L_1}{L_2} = \frac{\cos\theta_2}{\cos\theta_1} = \frac{\cos 45^\circ}{\cos 30^\circ}$

$\frac{L_1}{L_2} = \frac{\sqrt{2}}{\sqrt{3}}$

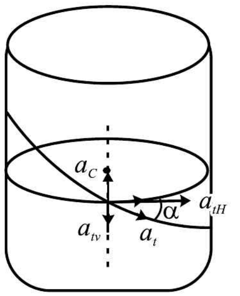
$\therefore$ (B) is the right answer.

Comprehension Questions

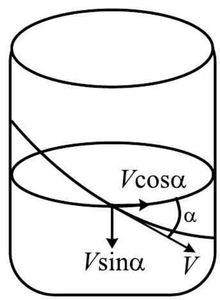
Comprehension- 1



The motion of the body can be considered as a superposition of movement along a circumference with a radius 'R' in a horizontal plane and vertical straight line motion.



Acceleration diagram



Velocity diagram

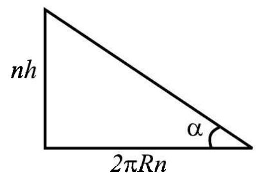
The velocity of the body 'V' at the given moment can be represented as the geometrical sum of the components :

	$V\cos\alpha$:	horizontal velocity
&	$V\sin\alpha$:	vertical velocity

Here 'α' is the angle formed by the helical line of groove with the horizontal plane.

A component of the acceleration of the body is responsible for change in direction and other for the change in speed, i.e. centripetal acceleration and tangential acceleration. The tangential acceleration have two components : one along the circle and one in vertical direction.

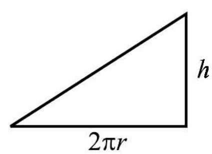
The value of tangential acceleration 'a_t' can be found by mentally developing the surface of the cylinder with the helical groove into a plane. In this case the groove will become an inclined plane with height nh and length of its base $2\pi Rn$, where 'n' is the number of turns in the helix.



Example 59. Distance travelled by the object when it completes one revolution along the groove is :

(A) h (B) $2\pi R$ (C) $\sqrt{h^2+(2\pi R)^2}$ (D) $h\sin\alpha$

Solution During its one revolution the object travels a distance having a component 'h' in the plane vertical and '$2\pi R$' in the horizontal plane as the groove after one revolution becomes an inclined as shown:

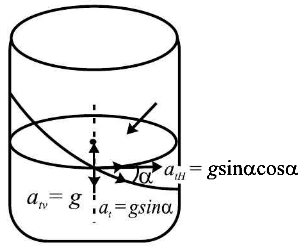


Hence the total distance travelled is $\sqrt{h^2+(2\pi R)^2}$.

Example 60. The angular acceleration of the object moving along the circle will be :

(A) $\dfrac{g\sin\alpha}{R}$ (B) $\dfrac{g\sin\alpha\cos\alpha}{R}$ (C) $\dfrac{g\sin^2\alpha}{R}$ (D) zero

Solution $\alpha = \dfrac{a_{\tan gential}}{R} = \left(\dfrac{g\sin\alpha\cos\alpha}{R}\right).$



Acceleration diagram

(As the object performs a circular motion of radius 'R' with acceleration $a_{\text{tangential}}$.)

Example 61. The speeds of the object at the end of 1st round, 2nd round and 3rd round are in ratio : (Assuming the body starts from rest)

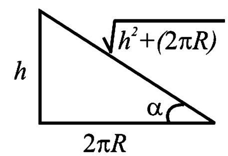
(A) $1:2:3$ (B) $1:3:5$

(C) $\sqrt{1}:\sqrt{2}:\sqrt{3}$ (D) $\sqrt{1}:\sqrt{3}:\sqrt{5}$

Solution Acceleration of the body remains constant through out the motion.

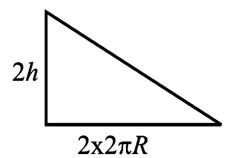
Using :$v^2 = u^2 + 2$ as along the tangential direction :

After 1st round :



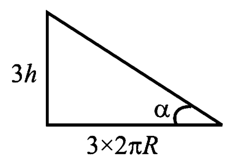
$$v_1^2 = 0^2 + 2a\sqrt{h^2+(2\pi R)^2}$$

After 2nd round :



$$v_2^2 = 0^2 + 2a\left(2\sqrt{h^2+(2\pi R)^2}\right)$$

and After 3rd round :



$$v_3^2 = 2a\left[3\sqrt{h^2+(2\pi R)^2}\right]$$

$\Rightarrow$ $v_1 : v_2 : v_3 = \sqrt{1}:\sqrt{2}:\sqrt{3}$

Example 62. The time taken by the block to complete 1st round, 2nd round and 3rd round are in the ratio:

(A) $\sqrt{1}:\sqrt{2}:\sqrt{3}$ (B) $\sqrt{1}:\sqrt{2}-\sqrt{1}:\sqrt{3}-\sqrt{2}$

(C) $\sqrt{2}-\sqrt{1}:\sqrt{3}-\sqrt{2}:\sqrt{3}-\sqrt{1}$ (D) $1:4:9$

Solution Using $S = ut + \frac{1}{2}at^2$

$u = 0$ and a = constant $\Rightarrow t \propto \sqrt{S}$

$\therefore$ $t_1 \propto \sqrt{(h^2+(2\pi R)^2)}$

$t_2 \propto \sqrt{2}\sqrt{(h^2+(2\pi R)^2)}$

and $t_3 \propto \sqrt{3}\sqrt{(h^2 + (2\pi R)^2)}$

$\therefore$ Time required to complete 1st round is t_1.

Time required to complete 2nd round is $t_2 - t_1$.

and Time required to complete 3rd round is $t_3 - t_2$.

$\Rightarrow$ $t_1 : (t_2 - t_1) : (t_3 - t_2)$

$= \sqrt{1} : \left(\sqrt{2} - \sqrt{1}\right) : \left(\sqrt{3} - \sqrt{2}\right)$ **Ans.**

***Example* 63.** If the speed of the object is v at an instant, then the force exerted by the helical groove at the same instant is :

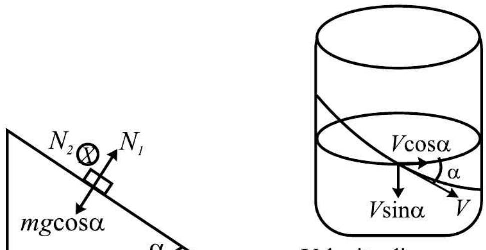
(A) $\dfrac{mv^2}{R}$

(B) $\sqrt{\left(\dfrac{mv^2}{R}\right)^2 + (mg\cos\alpha)^2}$

(C) $\sqrt{\left(\dfrac{mv^2}{R}\right)^2 + (mg\sin\alpha)^2}$

(D) $\sqrt{\left(\dfrac{mv^2\cos^2\alpha}{R}\right)^2 + (mg\cos\alpha)^2}$

Solution The force exerted by the groove as shown in the figure :



Velocity diagram

$\therefore$ $F_{res.} = \sqrt{\left(\dfrac{mv^2(\cos\alpha)^2}{R}\right)^2 + (mg\cos\alpha)^2}$

Comprehension- 2

In a certain experiment to measure the ratio of charge and mass of elementary charged particles, a surprising result was obtained in which two particles moved in such a way that the distance between them remained constant always. It was also noticed that, this two particle system was isolated from all other particles and no force was acting on this system except the force between these two masses. After careful observation followed by intensive calculation it was deduced that velocity of these two particles was always opposite in direction and magnitude of velocity was 10^3 m/s and 2×10^3 m/s for first and second particle respectively and masses of these particles were 2×10^{-30} kg and 10^{-30} kg respectively. Distance between them came out to be 12 Å. (1 Å = 10^{-10} m)

***Example* 64.** Acceleration of the first and the second particle was :

(A) zero

(B) 4×10^{16} m/s^2

(C) 2×10^{16} m/s^2

(D) 2.5×10^{15} m/s^2

Solution (D)

The two particle move in different circles.

The mutual interaction force provides the required centripetal force to the particle.

As magnitude of the interaction force is same

$$\therefore \quad F_{12} = \frac{m_1 v_1^2}{r_1} \text{ and } F_{21} = \frac{m_2 v_2^2}{r_2}$$

$$\because \quad |\vec{F}_1| = |\vec{F}_2|$$

$$\therefore \quad \frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

putting values, we get

$$r_2 = 2\, r_1$$

Also $r_1 + r_2 = 12 \times 10^{-10}$ m (given)

$\Rightarrow$ $r_1 = 4 \times 10^{-10}$ m

and $r_2 = 8 \times 10^{-10}$ m

Acceleration of first particle $= \frac{v_1^2}{r_1} = \frac{(10^3 m/s)^2}{(4 \times 10^{-10} m)} = 2.5 \times 10^{15}$ m/s^2

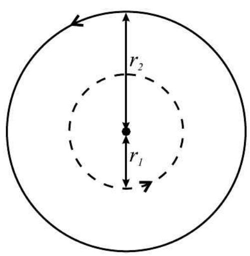
Acceleration of second particle is $= \frac{v_2^2}{r_2} = \frac{(2 \times 10^3)^2}{(8 \times 10^{-10})} = 5 \times 10^{15}$ m/s^2

Example 65. Path of the two particles was

(A) Intersecting straight lines (B) Parabolic

(C) Circular (D) Straight line w.r.t. each other

Solution (C) Since the distance between them always remains constant but move with different velocities. Therefore they must move in different circles with common centre as shown in the figure.



Example 66. Angular velocity of the first particle was :

(A) 2.5×10^{12} rad/s (B) 4×10^{12} rad/s (C) 4×10^{13} rad/s (D) zero

Solution (A)

Angular velocity of the first particle was $\omega_1 = \frac{v_1}{r_1} = \frac{(10^3 m/s)}{(4 \times 10^{-10} m)}$

$\Rightarrow$ $\omega_1 = 2.5 \times 10^{12}$ rad/s

***Example* 67.** If the first particle is stopped for a moment and then released. The velocity of centre of mass of the system just after the release will be :

(A) $\frac{1}{3} \times 10^{-30}$ m/s (B) $\frac{1}{3} \times 10^{3}$ m/s (C) $\frac{2}{3} \times 10^{3}$ m/s (D) None of these

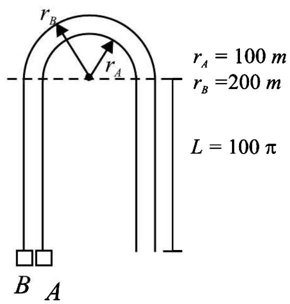
Solution (C)

Just after release :

$$V_{CM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(2\times10^{-30})(0) + (10^{-30})(2\times10^{3})}{3\times10^{-30}} = \frac{2}{3} \times 10^{3} \text{ m/s}$$

Comprehension - 3

Two cars *A* and *B* start racing at the same time on a flat race track which consists of two straight sections each of length 100 π and one circular section as in fig. The rule of the race is that each car must travel at constant speed at all times without ever skidding. (g = 10 m/sec^2)



***Example* 68.** If $\mu_A = 0.1$, $\mu_B = 0.2$ (μ_A is coefficient of friction on track *A* and μ_B is the coefficient of friction on track *B*) then

(A) car *A* completes its journey before car *B*

(B) both cars complete their journey in same time on circular part

(C) speed of car *B* is greater than that of car *A*

(D) car *B* completes its journey before car *A*.

[Ans. (B), (C), (D)]

***Example* 69** If speed of car *A* is 108 kmph and speed of car *B* is 180 kmph, and both tracks are equally rough :

(A) car *A* completes its journey before car *B*

(B) both cars complete their journey in same time

(C) speed of car *A* is greater than that of car *B*

(D) car *B* completes its journey before car *A*.

[Ans. (D)]

***Example* 70** If V_B = 90kmph, the minimum value of μ_A so that car *A* can complete its journey before car *B* is :

(A) $\frac{45}{128}$ (B) $\frac{45}{100}$ (C) $\frac{45}{64}$ (D) None of these

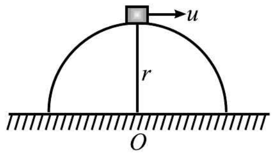
Solution $t = \frac{400\pi\,(m)}{25\,(m/s)} = 16\,\pi$ second ; $v_A = \frac{300\pi(m)}{16\pi(\text{sec})} = \frac{75}{4}$ m/s ; $\mu = \frac{V^2}{rg} = \frac{45}{128}$

[Ans. (A)]

Comprehension - 4

A small block of mass m is projected horizontally from the top of the smooth and fixed hemisphere of radius r with speed u as shown. For values of $u \geq u_0$, ($u_0 = \sqrt{gr}$) it does not slide on the hemisphere.

[i.e. leaves the surface at the top itself]



Example 71. For $u = 2\,u_0$, it lands at point P on ground. Find OP.

(A) $\sqrt{2}\,r$ (B) $2\,r$ (C) $4r$ (D) $2\sqrt{2}\,r$

Solution $mg = \dfrac{mu_0^2}{r} \Rightarrow u_0 = \sqrt{gr}$

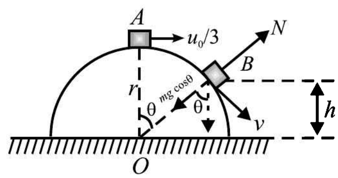
Now, along vertical ; $r = \dfrac{1}{2}gt^2 \Rightarrow t = \sqrt{\dfrac{2r}{g}}$

Along horizontal ; $OP = 2u_0t = 2\sqrt{2}\,r$

Example 72. For $u = u_0/3$, find the height from the ground at which it leaves the hemisphere.

(A) $\dfrac{19\,r}{9}$ (B) $\dfrac{19\,r}{27}$ (C) $\dfrac{10r}{9}$ (D) $\dfrac{10r}{27}$

As at B it leaves the hemisphere,



∴ $N = 0$

$mg\cos\theta = \dfrac{mV^2}{r}$

$mg\dfrac{h}{r} = \dfrac{mV^2}{r}$

$mv^2 = mgh$ ...(1)

By energy conservation between A and B

$$mgr + \frac{1}{2}m\left(\frac{u_0}{3}\right)^2 = mgh + \frac{1}{2}mv^2$$

Put u_0 and mv^2 ∴ $h = \dfrac{19r}{27}$

Example 73. Find its net acceleration at the instant it leaves the hemisphere.

(A) $-g$ (B) $g/2$ (C) g (D) $g/3$

Solution As $a_c = \dfrac{v^2}{r} = g\cos\theta$

∴ $a_t = g\sin\theta$

∴ $a_{net} = g$

Alternate Solution :

when block leave only the force left is mg.

$\therefore \quad a_{net} = g.$

Comprehension - 5

A particle undergoes uniform circular motion. The velocity and angular velocity of the particle at an instant of time is $\vec{v} = 3\hat{i} + 4\hat{j}$ m/s and $\vec{\omega} = x\hat{i} + 6\hat{j}$ rad/sec.

***Example* 74.** The value of x in rad/s is

(A) 8 (B) − 8

(C) 6 (D) can't be calculated

***Example* 75.** The radius of circle in metres is

(A) 1/2 m (B) 1 m

(C) 2 m (D) can't be calculate

***Example* 76.** The acceleration of particle at the given instant is

(A) $-50\hat{k}$ (B) $-42\hat{k}$

(C) $2\hat{i} + 3\hat{j}$ (D) can't be calculated

Sol. 74 to 76. (Moderate)

The angular velocity and linear velocity are mutually perpendicular

$\therefore \quad \vec{v}\cdot\vec{\omega} = 3x + 24 = 0 \quad$ or $\quad x = -8$

The radius of circle $r = \dfrac{v}{\omega} = \dfrac{5}{10} = \dfrac{1}{2}$ meter

The acceleration of particle undergoing uniform circular motion is

$\vec{a} = \vec{\omega} \times \vec{v} = (-8\hat{i} + 6\hat{j}) \times (3\hat{i} + 4\hat{j}) = -50\hat{k}$

$\therefore \quad \vec{v}\cdot\vec{\omega} = 3x + 24 = 0 \quad$ or $x = -8$

Comprehension-6

One end of massless inextensible string of length ℓ is fixed and other end is tied to a small ball of mass m. The ball is performing a circular motion in vertical plane. At the lowest position, speed of ball is $\sqrt{20g\ell}$. Neglect any other forces on the ball except tension force and gravitational force. Acceleration due to gravity is g.

***Example* 77.** Motion of ball is in nature of

(A) circular motion with constant speed

(B) circular motion with variable speed

(C) circular motion with constant angular acceleration about centre of the circle.

(D) none of these

Solution (B) As speed of ball is variable, so motion is non uniform circular motion.

Example 78. At the highest position of ball, tangential acceleration of ball is -

(A) 0 (B) g (C) $5\,g$ (D) $16\,g$

Solution (A) At the highest position of ball, tangential acceleration of ball is zero.

Example 79. During circular motion, minimum value of tension in the string -

(A) zero (B) mg (C) 10 mg (D) 15 mg

Solution (D) Tension in the string is minimum when ball is at the highest position. By conservation of energy

$$\frac{1}{2}mv^2 + mg\,(2\ell) = \frac{1}{2}m(20\,g\ell)$$

$v^2 = 16\,g\ell$ where v is the velocity of ball at the highest point.

$$\text{So } T + mg = \frac{mv^2}{\ell}$$

$$T = \frac{m16\,g\ell}{\ell} - mg = 15\,mg$$

Comprehension - 7

The velocity of a particle moving along positive direction of x – axis varies as, $v = \beta\sqrt{x}$ where β is a positive constant. Assuming that the particle is located at $x = 0$ initially (at $t = 0$). All parameters are in S.I. units. Find:

Example 80. The acceleration of the particle vary with time such that

(A) it is proportional to time (B) it is inversely proportional to time

(C) it is constant with time (D) Cannot be derived from the given information

Solution

$$V = \beta\sqrt{x}$$

$$\frac{dx}{dt} = \beta\sqrt{x} \Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \beta\,dt$$

$$2\sqrt{x} = \beta t$$

$$x = \frac{\beta^2 t^2}{4} \Rightarrow \frac{dx}{dt} = \frac{2\beta^2 t}{4} = V$$

$$a = \frac{dv}{dt} = \frac{\beta^2}{2}$$

i.e. acceleration is constant.

[Ans. (C)]

Example 81. The average velocity of the particle over the time, the particle takes to cover the first ‘s’ meters of its path is :

(A) $\frac{\beta\sqrt{s}}{2}$ (B) $\frac{2\beta\sqrt{s}}{3}$ (C) $\frac{\beta\sqrt{s}}{3}$ (D) $\beta\sqrt{s}$

Solution As $S = \frac{\beta^2 t_0^{\,2}}{4}$

where t_0 is the time to cover first s metre

$$\therefore \quad t_0 = \frac{2\sqrt{s}}{\beta}$$

$$\therefore \quad \langle \vec{v} \rangle = \frac{\int_0^{t_0} v\,dt}{\int_0^{t_0} dt} = \frac{\beta\sqrt{s}}{2}.$$

Example 82. Time at which velocity and acceleration of the particle have same magnitude is

(A) 2 s (B) 3 s (C) $\frac{1}{2}s$ (D) 1s

Solution As ; $V = \frac{\beta^2}{2}t$

$a = \frac{\beta^2}{2}$

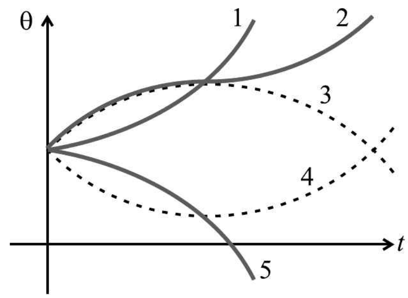
$\therefore$ When $V = a$

$t = 1\ s$

Match the following

Example 83. A particle is moving in circular motion around an axis. The motion of the particle in four different situations is described in the table. In the graph shown. Five curves are plotted and marked, and vertical axis gives angular position θ of the particle. Correctly match the curves with the situations to which they belong.

Situation	I	II	III	IV
Initial θ (rad)	+10	+10	+10	+10
Initial angular Velocity ω (rad/s)	+5	–5	–5	+5
Constant angular acceleration on α (rad/s²)	+2	–2	+2	–2



Situation	Curve
(a) I	(P) 1
(b) II	(Q) 2
(c) III	(R) 3
(d) IV	(S) 4
	(T) 5

Solution

I. Angular velocity is positive and increases. Hence θ is represented by curve 1.

II. Angular velocity is negative and increases in magnitude. Hence θ is represented by curve 5.

III. Angular velocity is negative and becomes positive later on. Hence θ is represented by curve 4.

IV. Angular velocity is positive and becomes negative later on. Hence θ is represented by curve 3.

[Ans. (a) P, (b) T (c) S (D) R]

***Example* 84.** A particle is moving with speed $v = 2t^2$ on the circumference of circle of radius R. Match the quantities given in column-I with corresponding results in column-II.

	Column-I		Column-II
(A)	Magnitude of tangential acceleration of particle	(p)	decreases with time.
(B)	Magnitude of Centripetal acceleration of particle	(q)	increases with time
(C)	Magnitude of angular speed of particle with respect to centre of circle	(r)	remains constant
(D)	Angle between the total acceleration vector and centripetal acceleration vector of particle	(s)	depends on the value of radius R

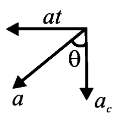
[Ans : (A) q (B) q, s (C) q, s (D) p, s]

Solution $v = 2t^2$

Tangential acceleration $a_t = 4t$

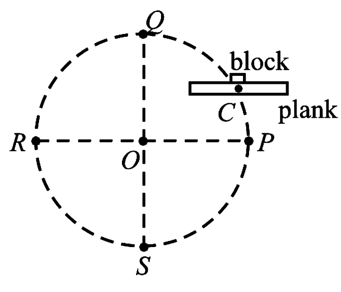
Centripetal acceleration $a_c = \dfrac{v^2}{R} = \dfrac{4t^4}{R}$

Angular speed $\omega = \dfrac{v}{R} = \dfrac{4t}{R}$,



$\tan\theta = \dfrac{a_t}{a_c} = \dfrac{4tR}{4t^4} = \dfrac{R}{t^3}$

***Example* 85.** A small block lies on a rough horizontal platform above its centre C as shown figure. The plank is moved in vertical plane such that it always remains horizontal and its centre C moves in a vertical circle of centre O with constant angular velocity ω. There is no relative motion between block and the plank and the block does not loose contact with the plank anywhere. P, Q, R and S are four points on circular trajectory of centre C of platform. P and R lie on same horizontal level as O. Q is the highest point on the circle and S is the lowest point on the shown circle. Match the statements in column-I with points in column-II.



	Column-I		Column-II
(A)	Magnitude of frictional force on block is maximum	(p)	when block is at position P
(B)	Magnitude of normal reaction on block is equal to mg	(q)	when block is at position Q
(C)	Magnitude of frictional force is zero	(r)	when block is at position R
(D)	Net contact force on the block is directed toward centre	(s)	when block is at position S

[Ans. (A) p,r (B) p,r (C) q,s (D) q,s]

Solution At position P and R frictional force is zero and normal reaction is mg.

At position P and Q frictional force is zero and normal reaction points towards centre.

Example 86. In column-I condition on velocity, force and acceleration of a particle is given. Resultant motion is described in column-II. $\vec{u}$ = initial velocity, $\vec{F}$ = resultant force and $\vec{v}$ = instantaneous velocity.

	Column-I		Column-II
(A)	$\vec{u}\times\vec{F}=0$ and $\vec{F}$ = constant	(p)	path will be circular path
(B)	$\vec{u}\cdot\vec{F}=0$ and $\vec{F}$ = constant	(q)	speed will increase
(C)	$\vec{v}\cdot\vec{F}=0$ all the time and $\|\vec{F}\|$ = constant and the particle always remains in one plane.	(r)	path will be straight line
(D)	$\vec{u}=2\hat{i}-3\hat{j}$ and acceleration at all time $\vec{a}=6\hat{i}-9\hat{j}$	(s)	path will be parabolic

[Ans. (A) *r* (B) *q,s* (C) *p* (D) *q,r*]

Solution

(A) $\vec{F}$ = constant and $\vec{u}\times\vec{F}=0$

Therefore initial velocity is either in direction of constant force or opposite to it. Hence the particle will move in straight line and speed may increase or decrease.

(B) $\vec{u}\cdot\vec{F}=0$ and $\vec{F}$ = constant

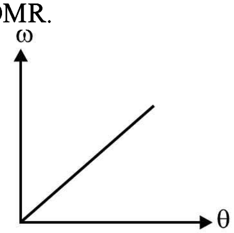
Initial velocity is perpendicular to constant force, hence the path will be parabolic with speed of particle increasing.

(C) $\vec{v}\cdot\vec{F}=0$ means instantaneous velocity is alway perpendicular to force. Hence the speed will remain constant. And also $|\vec{F}|$ = constant. Since the particle moves in one plane, the resulting motion has to be circular.

(D) $\vec{u}=2\hat{i}-3\hat{j}$ and $\vec{a}=6\hat{i}-9\hat{j}$. Hence initial velocity is in same direction of constant acceleration, therefore particle moves in straight line with increasing speed.

Example 87. Each situation in column I gives graph of a particle moving in circular path. The variables ω, θ and t represent angular speed (at any time t) , angular displacement (in time t) and time respectively. Column II gives certain resulting interpretation. Match the graphs in column I with statements in column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

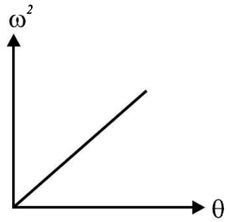
(A)



ω - θ graph

(p) Angular acceleration of particle is uniform

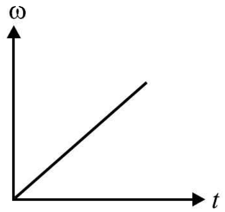
(B)



ω^2 - θ graph

(q) Angular acceleration of particle is non-uniform

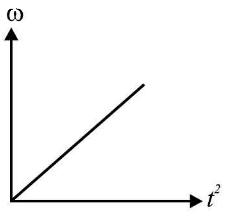
(C)



ω - t graph

(r) Angular acceleration of particle is directly proportional to t.

(D)



ω - t^2 graph

(s) Angular acceleration of particle is directly proportional to θ.

[Ans. (A) *q,s* (B) *p* (C) *p* (D) *q,r*]

Solution From graph (a) $\Rightarrow \omega = k\theta$ where k is positive constant

angular acceleration $= \omega \dfrac{d\omega}{d\theta} = k\theta \times k = k^2\theta$

$\therefore$ angular acceleration is non uniform and directly proportional to θ. $\therefore$ (A) *q, s*

From graph (b) $\Rightarrow \omega^2 = k\theta$. Differentiating both sides with respect to θ.

$2\omega \dfrac{d\omega}{d\theta} = k$ or $\omega \dfrac{d\omega}{d\theta} = \dfrac{k}{2}$ Hence angular acceleration is uniform. $\therefore$ (B) *p*

From graph (c) $\Rightarrow$ $\omega = kt$

angular acceleration $= \dfrac{d\omega}{dt} = k$ Hence angular acceleration is uniform $\Rightarrow$ (C) *p*

From graph (d) $\Rightarrow$ $\omega = kt^2$

angular acceleration $= \dfrac{d\omega}{dt} = 2kt$ Hence angular acceleration is non uniform and directly proportional to t.

$\therefore$ (D) *q,r*

Example 88. Motion of particle is described in column-I. In column-II, some statements about work done by forces on the particle from ground frame is given. Match the particle's motion given in column-I with corresponding possible work done on the particle in certain time interval given in column-II.

	Column-I		Column-II
(A)	A particle is moving in horizontal circle	(p)	work done by all the forces may be positive
(B)	A particle is moving in vertical circle with uniform speed	(q)	work done by all the forces may be negative
(C)	A particle is moving in air (projectile motion without any air resistance) under gravity	(r)	work done by all the forces must be zero
(D)	A particle is attached to roof of moving train on inclined surface.	(s)	work done by gravity may be positive.

[**Ans.** (A) *p,q* (B) *r, s* (C) *p,q,s* (D) *p,q,s*]

Solution

(A) If motion is uniform circular motion (constant speed), change in kinetic energy of particle is zero

$$W_{all} = KE_2 - KE_1$$
$$W_{all} = 0$$

If motion is non uniform circular motion then kinetic energy of particle may decrease or increase. So work done by all the forces may be positive or negative.

(B) The particle's speed is constant, so work done by all the force is zero. For vertical downward displacement, work done by gravity is positive.

(C) In projectile motion, for upward vertical displacement, speed particle decreases, so work done by all the forces will be negative. For vertical downward displacement, speed of particle increases, so work done by all the force will be positive.

(D) If the speed of train is increasing, then work done by all the forces is positive and vice versa. If train is moving downward the incline, work done by gravity on the particle is positive.

(A)

$$W_{all} = KE_2 - KE_1$$
$$W_{all} = 0$$

Assertion Reason Questions

Example 89. **STATEMENT-1 :** When a particle is projected at some angle with horizontal, the radius of curvature of its path during the ascent decreases continuously.

STATEMENT-2 : The radius of curvature of trajectory of a particle in motion at a point is the ratio of square of magnitude of the velocity and the magnitude of acceleration at that point.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Solution (C) As the particle goes up, curve becomes sharper and sharper, and radius of curvature decreases

Statement-2 is wrong. $R = \dfrac{v^2}{a_\perp}$, where $a_\perp$ is acceleration component perpendicular to velocity.

Example 90. STATEMENT–1 : In a circular motion, the force must be directed perpendicular to the velocity all the time.

STATEMENT–2 : A centripetal force is required to provide the centripetal acceleration in a circular motion.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

Solution **(D)** Only in ***uniform*** circular motion force must be directed perpendicular to the velocity all the time. However in ***non-uniform*** circular motion, force will always not be perpendicular to velocity. Hence statement-1 is false.

Example 91. STATEMENT-1 : A ball tied by thread is undergoing circular motion (of radius R) in a vertical plane. (Thread always remains in vertical plane). The difference of maximum and minimum tension in thread is independent of speed (u) of ball at the lowest position (u > $\sqrt{5gR}$)

STATEMENT-2 : For a ball of mass m tied by thread undergoing vertical circular motion (of radius R), difference in maximum and minimum magnitude of centripetal acceleration of the ball is independent of speed (u) of ball at the lowest position ($u > \sqrt{5gR}$).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

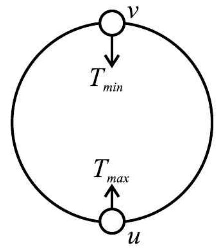
(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Solution **(A)** Let the minimum and maximum tensions be T_{max} and T_{min} and the minimum and maximum speed be u and v.

$$\therefore \quad T_{max} = \frac{mu^2}{R} + mg$$

$$T_{min} = \frac{mv^2}{R} - mg$$



$\therefore \quad \Delta T = m\left(\frac{u^2}{R} - \frac{v^2}{R}\right) + 2\,mg.$

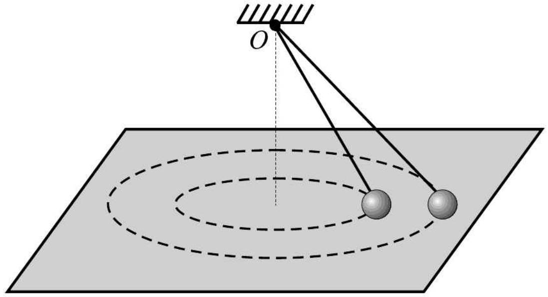
From conservation of energy

$\frac{u^2}{R} - \frac{v^2}{R} = 4g \quad \Rightarrow$ is indepenent of u.

and $\Delta T = 6\,mg$.

$\therefore$ Statement-2 is correct explanation of statement-1.

Example 92. STATEMENT-1 : Two small spheres are suspended from same point O on roof with strings of different lengths. Both spheres move along horizontal circles as shown. Then both spheres may move along circles in same horizontal plane.



STATEMENT-2 : For both spheres in statement-1 to move in circular paths in same horizontal plane, their angular speeds must be same.

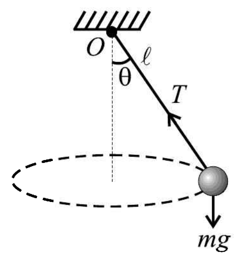
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

Solution (A) For conical pendulum of length ℓ, mass m moving along horizontal circle as shown (conical pendulum)



$T\cos\theta = mg$ ...(1)

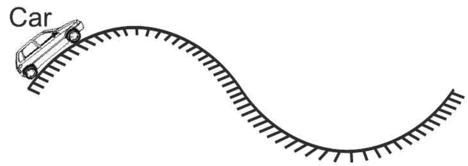
$T\sin\theta = m\omega^2\ell\sin\theta$ ...(2)

From equation 1 and equation 2, $\ell\cos\theta = \frac{g}{\omega^2}$

$\ell\cos\theta$ is the vertical distance of sphere below O point of suspension. Hence if ω of both pendulums are same, they shall move in same horizontal plane.

Hence statement-2 is correct explanation of statement-1.

Example 93. STATEMENT-1 : A car moves along a road with uniform speed. The path of car lies in vertical plane and is shown in figure.The radius of curvature(R) of the path is same everywhere. If the car does not loose contact with road at the highest point, it can travel the shown path without loosing contact with road anywhere else.



STATEMENT-2 : For car to loose contact with road, the normal reaction between car and road should be zero.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Solution **(D)** The normal reaction is not least at topmost point, hence statement 1 is false.

***Example* 94.** **STATEMENT-1 :** For a stone projected horizontally, the magnitude of the tangential acceleration keeps on decreasing even through the speed of particle keeps on increasing.

STATEMENT-2 : A given particle speeds up under action of constant acceleration. The tangential acceleration of any particle is the component of its acceleration in the direction of its velocity. The tangential acceleration of this given particle increases if angle between velocity and acceleration keeps on decreasing.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Solution **(D)**

***Example* 95.** **STATEMENT-1 :** For a particle moving in a circular path, if direction of angular velocity and angular acceleration is same, then angle between its velocity vector and acceleration vector increases.

STATEMENT-2 : For a particle moving in a circular path with speed increasing at constant rate, the centripetal acceleration keeps on increasing

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

Solution **(D)** If the particle speeds up, tangential acceleration may be greater or less than centripetal acceleration. Hence we cannot conclusively say that angle between acceleration and velocity vector increases. Therefore statement-1 is false.

***Example* 96.** **STATEMENT-1 :** A cyclist is cycling on a rough horizontal circular track with increasing speed. Then the net frictional force on cycle is always directed towards centre of the circular track.

STATEMENT-2 : For a particle moving in a circle, component of its acceleration towards centre, that is, centripetal acceleration should exist (except when speed is zero instantaneously).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Solution **(D)** The only horizontal force that acts on cyclist is frictional force. This is also the net force M cyclist. Since the speed is increasing net acceleration and hence net frictional force is not directed towards centre. Hence statement-1 is false.

Example **97.** **STATEMENT-1 :** A particle is moving in circular path. The net work done on the particle is zero.

STATEMENT-2 : For a particle undergoing uniform circular motion, net force acting on the particle and velocity of the particle are always perpendicular.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Solution **(D)** In non-uniform circular motion, particle's kinetic energy changes with time. By work energy theorem, net work done on the particle is non zero. In uniform circular motion, total force on the particle is centripetal in nature.

True False

Example **98.** For motion along a curved path, the velocity and acceleration vectors are never in the same direction.

Solution True

Example **99.** The resultant force acting on a particle moving in a circular path is always directed towards the centre.

Solution False

Example **100.** The time taken by a particle to slide down starting from rest along different smooth chords of a vertical circle starting from highest point of the circle is same. (State True / False with reason.)

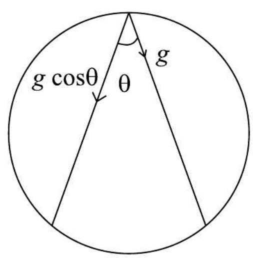
Solution

$$S = ut + \frac{1}{2}at2$$

$$S = 0 + \frac{1}{2}(g\cos\theta)t2$$

$$2R\cos\theta = \frac{1}{2}(g\cos\theta)t^2 \qquad [\because S = 2R\cos\theta]$$

$$t = \sqrt{\frac{4R}{g}}$$



Time is independent to angle θ.

[Ans. True]

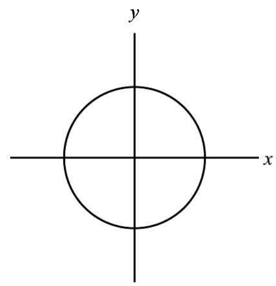
Example 101. A circular curve of a highway is designed for traffic moving at 72 km/h. If the radius of the curved path is 100 *m*, the correct angle of banking of the road should be $\tan^{-1}\frac{1}{5}$. (g = 10 m/s^2)

[**Ans.** False]

Solution $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{(20)^2}{1000}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{2}{5}\right)$

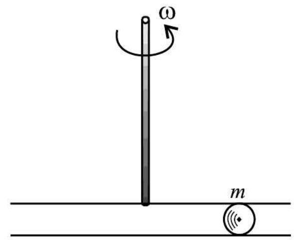
EXERCISE

Exercise–1: Subjective Problems



1. Figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is $v = (2\text{m/s})\hat{i} - (2\text{ m/s})\hat{j}$. Through which quadrant is the particle moving when it is travelling
   (a) clockwise and
   (b) counter clockwise around the circle?
2. A car is moving with speed 30 m/sec on a circular path of radius 500 m. Its speed is increasing at the rate of 2 m/sec^2. What is the acceleration of the car at that moment?
3. A mass is kept on a horizontal frictionless surface. It is attached to a string and rotates about a fixed centre at an angular velocity ω_0. If the length of the stringand angular velocity are doubled, find the tension in the string which wa initially T_0.
4. A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum displacement of the pendulum of the string with respect to the vertical.
5. What is the radius of curvature of the parabola traced out by the projectile in the previous problem in the previous problem at a point where the particle velocity makes an angle $\theta/2$ with the horizontal?
6. A ceiling fan has a diameter (of the circle through the outer edges of the three blades) of 120 cm and rpm 1500 at full speed. Consider a particle of mass 1g sticking at the outer end of a blade. How much forcedoes it experience when the fan runs at full speed ? Who exerts this force on the particle ? How much force does the particle exert on the blade along its surface ?
7. A string breaks if its tension exceeds 10 newtons. A stone of mass 250 gm tied to this string of length 10 cm is rotated in a horizontal circle. Find the maximum angular velocity of rotation.
8. A body of mass *m* hangs at one end of a string of length a, the other end of which is fixed. It is given a horizontal velocity *u* at its lowest position so that the string would just become slack, when it makes an angle of 60° with the upward drawn vertical line. Find the tension in the string at point of projection
9. A ball is moving to and fro about the lowest point *A* of a smooth hemispherical bowl. If it is able to rise up to a height of 20 cm on either side of *A*, find its speed at *A*. (Take = 10 m/s^2, mass of the body 5 *g*).

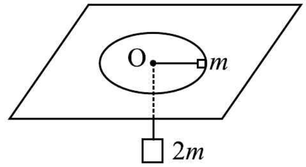
10. A motorcyclist wants to drive on the vertical surface of wooden 'well' of radius 5 m, with a minimum speed of $5\sqrt{5}$ m/s. Find the minimum value of coefficient of friction between the tyres and the wall of the well. (take $g = 10$ m/s^2)
11. A train has to negotiate a curve of radius 400 m. By how much height should the outer rail be raised with respect to inner rail for a speed of 48 km/hr ? The distance between the rails is 1m.
12. A particle of mass m is constrained to move along a groove which is being rotated about a vertical axis through its centre with a constant angular velocity ω. If it starts at a distance R from the axis at $t = 0$, find its velocity relative to the groove when it is at a distance r from the centre



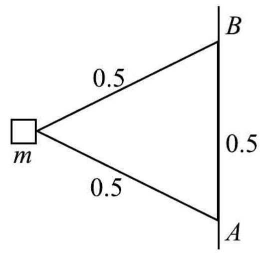
13. A park has a radius of 10m. If a vehicle goes round it at an average speed of 18 km/hr, what should be the proper angle of banking?
14. If the road of the previous problem is horizontal (no banking) , what should be the minimum friction coefficient so that a scotter going at 18 km/hr does not skid.
15. A person stands on a spring balance at the equator.
    (A) By what fraction is the balance reading less than his true weight ?
    (B) If the speed of earth's rotation is increased by such an amount that the balance reading is half the true weight, what will be the length of the day in this case ?
16. A turn of radius 20 m is banked for the vehicles going at a speed of 36 km/h. If the coefficient of static friction between the road and the tyre is 0.4, what are the possible speeds of a vehicle so that is neither slips down nor skids up ?
17. A particle is travelling in a circular path of radius 4m. At a certain instant the particle is moving at 20m/s and its acceleration is at an angle of 37° from the direction to the centre of the circle as seen from the particle
    (a) At what rate is the speed of the particle increasing?
    (b) What is the magnitude of the acceleration?
18. A particle is revolving in a circle of radius 1m with an angular speed of 12 rad/s. At $t = 0$, it was subjected to a constant angular acceleration α and its angular speed increased to$(480/\pi)$ rpm in 2 sec. Particle then continues to move with attained speed. Calculate
    (a) angular acceleration of the particle,
    (b) tangential velocity of the particle as a function of time.
    (c) acceleration of the particle at $t = 0.5$ second and at $t = 3$ second
    (d) angular displacement at $t = 3$ second.
19. A stone is thrown horizontally with the velocity 15m/s. Determine the tangential and normal accelerations of the stone in 1 second after it begins to move.
20. A particle moves in a circle of radius R with a constant speed v. Then, find the magnitude of average acceleration during a time interval $\frac{\pi R}{2v}$.

21. A particle moves in the x-y plane with the velocity $\vec{v} = a\hat{i} + bt\hat{j}$. At the instant $t = a\sqrt{3}/b$ the magnitude of tangential, normal and total acceleration are _____, _______, & _________.
22. A particle is moving in a circle of radius 2m such that its centripetal acceleration is given by $a_c = 2t^2$. Find the angle (in rad.) traversed by the particle in the first two seconds.

23. A mass m rotating freely in a horizontal circle of radius 1 m on a frictionless smooth table supports a stationary mass $2m$, attached to the other end of the string passing through smooth hole O in table, hanging vertically. Find the angular velocity of rotation.

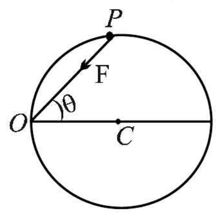


24. Two strings of length $l = 0.5$ m each are connected to a block of mass m = 2 kg at one end and their ends are attached to the point A and B 0.5 m apart on a vertical pole which rotates with a constant angular velocity $\omega = 7$ rad/sec. Find the ratio $\frac{T_1}{T_2}$ of tension in the upper string (T_1) and the lower string (T_2). [Use $g = 9.8$ m/s²]



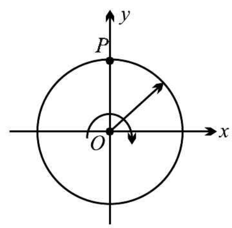
25. A particle P is moving on a circle under the action of only one force acting always towards fixed point O on the circumference.

Find ratio of $\frac{d^2\theta}{dt^2}$ & $\left(\frac{d\theta}{dt}\right)^2$

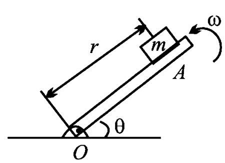


Exercise–2

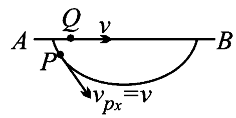
26. A ring rotates about z axis as shown in figure. The plane of rotation is xy. At a certain instant the acceleration of a particle P (shown in figure) on the ring is $(6\hat{i} - 8\hat{j})$ m/s². Find the angular acceleration of the ring & the angular velocity at that instant. Radius of the ring is 2m.



27. A particle is revolving in a circle of radius 1m with an angular speed of 12 rad/s. At $t = 0$, it was subjected to a constant angular acceleration α and its angular speed increased to$(480/\pi)$ rpm in 2 sec. Particle then continues to move with attained speed. Calculate
    (a) angular acceleration of the particle,
    (b) tangential velocity of the particle as a function of time.
    (c) acceleration of the particle at $t = 0.5$ second and at $t = 3$ second
    (d) angular displacement at $t = 3$ second.
28. The member OA rotates in vertical plane about a horizontal axis through O with a constant counter clockwise velocity $\omega = 3$ rad/sec. As it passes the position $\theta = 0$, a small mass m is placed upon it at a radial distance $r = 0.5$ m. If the mass is observed to slip at $\theta = 37ŗ$, find the coefficient of friction between the mass & the member.



29. A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at $t = 0$. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at $t=0$ along the horizontal string AB, with the speed v. Friction between the bead and the string may be neglected. Which bead reaches point B earlier?



30. The blocks are of mass 2 kg shown is in equilibrium. At $t = 0$ right spring in fig (i) and right string in fig (ii) breaks. Find the ratio of instantaneous acceleration of blocks?

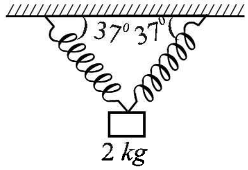


Figure (i)

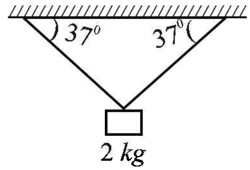
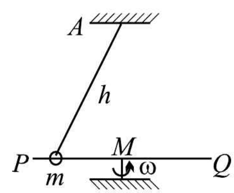


Figure (ii)

31. Two blocks of mass m_1=10kg and m_2=5kg connected to each other by a massless inextensible string of length 0.3m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10rad/sec about a vertical axis passing through its centre. The masses are placed along the diameter of the table on either side of the centre O such that m_1 is at a distance of 0.124m from O. The masses are observed to be at rest with respect to an observer on the turn table.
   (i) Calculate the frictional force on m_1
   (ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position.
   (iii) How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass m_1.

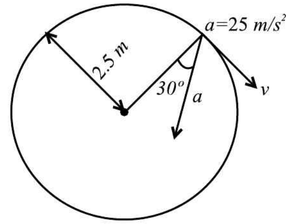
32. A stone is launched upward at 45° with speed v_0. A bee follows the trajectory of the stone at a constant speed equal to the initial speed of the stone.
   (a) Find the radius of curvature at the top point of the trajectory.
   (b) What is the acceleration of the bee at the top point of the trajectory? For the stone, neglect the air resistance.

33. A smooth rod PQ rotates in a horizontal plane about its mid point M which is h=0.1 m vertically below a fixed point A at a constant angular velocity 14 rad/s. A light elastic string of natural length 0.1 m requiring 1.47 N/cm has one end fixed at A and its other end attached to a ring of mass m = 0.3 kg which is free to slide along the rod. When the ring is stationary relative to rod, then inclination of string with vertical, tension in string, force exerted by ring on the rod will be



   (A) $\cos\theta = 3/5$, $T = 9.8\,N$, $N = 2.88\,N$
   (B) $\theta = 60$, $T = 0$, $N = 1.44\,N$
   (C) $\cos\theta = 2/5$, $T = 4.9\,N$, $N = 1.44\,N$
   (D) $\theta = 30$, $T = 0$, $N = 2.88\,N$

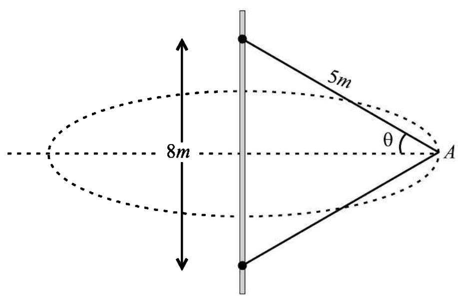
34. Figure shows the total acceleration and velocity of a particle moving clockwise in a circle of radius 2.5 m at a given instant of time. At this instant, find :
(a) the radial acceleration,
(b) the speed of the particle and
(c) its tangential acceleration



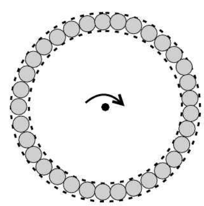
35. Two particle A and B move anticlockwise with the same speed v in a circle of radius R and are diametrically opposite to each other. At $t = 0$, A is given a constant acceleration (tangential) $a_t = \dfrac{72v^2}{25\pi R}$. Calculate the time in which A collides with B, the angle traced by A, its angular velocity and radial acceleration at the time of collision.

36. A motorcycle has to move with a constant speed on an overbridge which is in the form of a circular arc of radius R and has a total length L. Suppose the motorcycle starts from the highest point.
(a) What can its maximum velocity be for which the contact with the road is not broken at the highest point.
(b) If the motorcycle goes at speed $1/\sqrt{2}$ times the maximum found in part (a) where will it lose the contact with the road.
(c) What maximum uniform speed can it maintain on the bridge if it does not lose contact anywhere on the bridge?

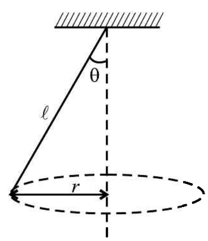
37. A 4 kg block is attached to a vertical rod by means of two strings of equal length. When the system rotates about the axis of the rod, the strings are extended as shown in figure.
(a) How many revolutions per minute must the system make in order for the tension in the upper chord to be 20 kgf?
(b) What is the tension in the lower chord then?



38. A metallic chain with a length ℓ and whose ends are joined together is fitted onto a wooden disc as shown in the figure. The disc rotates with a speed of n revolutions per second. Find the tension of the chain T if its mass is m.

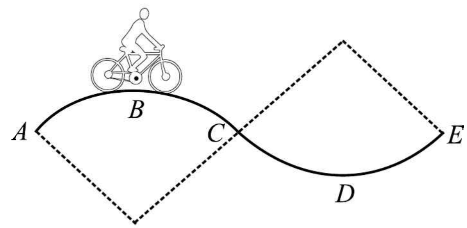


39. A small object of mass m is tied to a string of length l and is whirled round in a horizontal circle of radius r at constant speed v (figure). The centre of the circle is vertically below the point of support.
[This arrangement is called conical pendulum.]
Calculate v in terms of g, r and θ. Also calculate the period of revolution.



40. A car goes on a horizontal circular road of radius R, the speed increasing at a constant rate $\frac{dv}{dt} = a$. The friction coefficient between the road and the tyre is μ. Find the speed at which the car will skid.

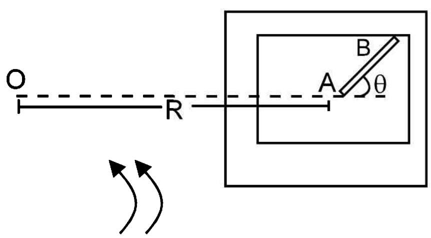
41. A track consists of two circular parts ABC and CDE of equal radius 100 m and joined smoothly as shown in fig. Each part subtends a right angle at its centre. A cycle weighing 100 kg together with the rider travels at a constant speed of 18 km/h on the rack.



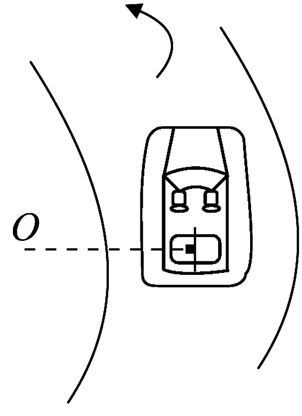
(A) Find the normal contact force by the road on the cycle when it is at B and D.
(B) Find the force of friction exerted by the track on the tyres when the cycle is at B, C and D.
(C) Find the normal force between the road and the cycle just, before and just after the cycle crosses C.
(D) What should be the minimum friction coefficient between the road and the tyre, which will ensure that the cyclist can move with constant speed ? Take $g = 10 \text{m/s}^2$.

42 A block of mass 'm' moves on a horizontal circle against the wall of a cylindrical room of radius R. The floor of the room on which the block moves is smooth but the friction coefficient between the wall and the block is μ. The block is given an initial speed v_0. As a function of the instantaneous speed 'v' write
(A) the normal force by the wall on the block,
(B) the frictional force by the wall and
(C) the tangential acceleration of the block.
(D) obtain the speed of the block after one revolution.

43. A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity ω in a circular path of radius R (figure). A smooth groove AB of length $L(<<R)$ is made on the surface of the table. The groove makes an angle θ with the radius OA of the circle in which the cabin rotates. A small particle is kept at the point A in the groove and is released to move along AB. Find the time taken by the particle to reach the point B.



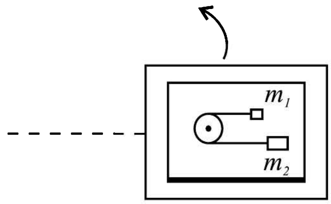
44. A car moving at a speed of 36 km/hr is taking a turn on a circular road of radius 50 m. A small wodden plate is kept on the seat with its plane perpendicular to the radius of the circular road figure. A small block of mass 100g is kept on the seat which rests against the plate. The friction coefficient between the block and the plate is $\mu = 0.58$.



(A) Find the normal contact force exerted by the plate on the block.

(B) The plate is slowly turned so that the angle between the normal to the plate and radius of the angle between the normal to the plate and the radius of the road slowly increases. Find the angle at which the block will just start sliding on the plate.

45. A table with smooth horizontal surface is placed in a cabin which moves in a circle of a large radius R (figure). A smooth pulley of small radius is fastended to the table. Two masses m and $2m$ placed on the table are conneted through a string over the pulley. Initially the masses are held by a person with the string along the outward radius and then the system is released from rest (with respect to the cabin). Find the magnitude of the initial acceleration of the masses as seen from the cabin and the tension in the string.



Exercise–3: Objective Problems

Only One Option is Correct

1. The average acceleration vector for a particle having a uniform circular motion in one complete revolution:
   (A) A constant vector of magnitude $\frac{v^2}{r}$
   (B) $\frac{v^2}{r}$ in magnitude and perpendicular to the plane of circle
   (C) Equal to the instantaneous acceleration vector at the start of the motion
   (D) A null vector
2. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant tangential acceleration. It the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is:
   (A) $160\ \pi$ m/s^2 (B) $40\ \pi$ m/s^2 (C) 40 m/s^2 (D) $640\ \pi$ m/s^2
3. When a particle moves in a circle with a uniform speed
   (A) Its velocity and acceleration are both constant
   (B) Its velocity is constant but the acceleration changes
   (C) Its acceleration is constant but the velocity changes
   (D) Its velocity and acceleration both change
4. An object follows a curved path. The following quantities may remain constant during the motion
   (A) Speed (B) Velocity
   (C) Acceleration (D) Magnitude of acceleration
5. Assume that the earth goes round the sun in a circular orbit with a constant speed of 30 km/s.
   (A) The average velocity of the earth from 1st Jan , 90 to 30th June , 90 is zero
   (B) The average acceleration during the above period is 60 km/s^2.

(C) The average speed from 1st Jan , 90 to 31st Dec, 90 is zero.
(D) The instantaneous acceleration of the earth points towards the sun.

6. The position vector of a particle in a circular motion about the origin sweeps out equal area in equal time. Its
(A) Velocity remains constant
(B) Speed remains constant
(C) Acceleration remains constant
(D) Tangential acceleration remains constant

7. Two particles *P* and *Q* are located at distances r_P and r_Q respectively from the centre of a rotating disc such that $r_P > r_Q$
(A) Both *P* and *Q* have the same acceleration
(B) Both *P* and *Q* do not have any acceleration
(C) *P* has greater acceleration than *Q*
(D) *Q* has greater acceleration than *P*

8. In uniform circular motion, the quantity that remains constant is
(A) Linear velocity
(B) Centripetal force
(C) Acceleration
(D) Speed

9. When a particle moves in a circle with a uniform speed :
(A) Its velocity and acceleration are both constant
(B) Its velocity is constant but the acceleration changes
(C) Its acceleration is constant but the velocity changes
(D) Its velocity and acceleration both change

10. For a particle in a uniformly accelerated (speed increasing uniformly) circular motion:
(A) Velocity is radial and acceleration is tangential only
(B) Velocity is tangential and acceleration is radial only
(C) Velocity is radial and acceleration has both radial and tangential components
(D) Velocity is tangential and acceleration has both radial and tangential components

11. A motor cyclist going round in a circular track at constant speed has
(A) Constant linear velocity
(B) Constant acceleration
(C) Constant angular velocity
(D) Constant force

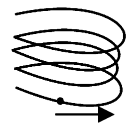
12. For a body in circular motion with a constant angular velocity, the magnitude of the average acceleration over a period of half a revolution is ....times the magnitude of its instantaneous acceleration.
(A) $\frac{2}{\pi}$
(B) $\frac{\pi}{2}$
(C) π
(D) 2

13. A car of mass *m* moves in a horizontal circular path of radius *r* metre. At an instant its speed is *v* m/s and is increasing at a rate a m/s^2, then the acceleration of the car is :
(A) $\sqrt{a\left(\frac{v^2}{r}\right)}$
(B) $\sqrt{a^2+\left(\frac{v^2}{r}\right)^2}$
(C) $\frac{v^2}{r}$
(D) a

14. The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be :

(A) 6.28 & 0 mm/s (B) 8.88 & 4.44 mm/s
(C) 8.88 & 6.28 mm/s (D) 6.28 & 8.88 mm/s

15. A particle is going in a spiral path as shown in figure with constant speed.



(A) The velocity of the particle is constant
(B) The acceleration of the particle is constant
(C) The magnitude of accleration is constant
(D) The magnitude of accleration is decreasing continuously.

16. A particle of mass m is executing uniform circular motion on a path of radius r. If p is the magnitude of its linear momentum. The radial force acting on the particle is

(A) pmr (B) $\frac{rm}{p}$
(C) $\frac{mp^2}{r}$ (D) $\frac{p^2}{rm}$

17. A train is moving towards north. At one place it turns towards north-east, here we observe that
(A) The radius of curvature of outer rail will be greater than that of the inner rail
(B) The radius of the inner rail will be greater than that of the outer rail
(C) The radius of curvature of one of the rails will be greater
(D) The radius of curvature of the outer and inner rails will be the same

18. A stone of mass of 16 kg is attached to a string 144 m long and is whirled in a horizontal circle. The maximum tension the string can withstand is 16 newton. The maximum speed of revolution of the stone without breaking it, will be :
(A) 20 ms^{-1} (B) 16 ms^{-1}
(C) 14 ms^{-1} (D) 12 ms^{-1}

19. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving on a smooth horizontal plane about point O. If the velocity of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is

(A) 3 : 5 : 7 (B) 3 : 4 : 5
(C) 7 : 11 : 6 (D) 3 : 5 : 6

20. A particle is kept fixed on a turnatable rotating uniformly. As seen from the ground, the particle goes in a circle , its speed is 20 cm/s and acceleration is 20 cm/s^2.The particle is now shifted to a new position to make the radius half of the original value.The new values of the speed and acceleration will be
(A) 10 cm/s, 10 cm/s^2 (B) 10 cm/s, 80 cm/s^2
(C) 40 cm/s, 10 cm/s^2 (D) 40 cm/s, 40 cm/s^2

21. A coin placed on a rotating turntable just slips if is placed at a distance of 4 cm from the centre. if the angular velocity of the turntable is doubled , it will just slip at a distance of
(A) 1 cm (B) 2 cm
(C) 4 cm (D) 8 cm

22. A rod of length L is pivoted at one end and is rotated with a uniform angular velocity in a horizontal plane. Let T_1 and T_2 be the tensions at the points $L/4$ and $3L/4$ away from the pivoted ends.
    (A) $T_1 > T_2$
    (B) $T_2 > T_1$
    (C) $T_1 = T_2$
    (D) The relation between T_1 and T_2 depends on whether the rod rotates clockwise or anticlockwise
23. The driver of a car travelling at speed suddenly sees a wall at a distance r directly in front of him. To avoid collision,
    (A) He should apply brakes sharply
    (B) He should turn the car sharply
    (C) He should apply brakes and then sharply turn
    (D) None of these
24. A curved section of a road is banked for a speed v. If there is no friction between road and tyres of the car, then
    (A) Car is more likely to slip at speeds higher than v than speeds lower than v
    (B) Car cannot remain in static equilibrium on the curved section
    (C) Car will not slip when moving with speed v
    (D) None of the above
25. A particle of mass m is observed from an inertial frame of reference and is found to move in a circle of radius r with a unifrom speed v. The centrifugal force on it is
    (A) $\frac{mv^2}{r}$ towards the centre
    (B) $\frac{mv^2}{r}$ away from the centre
    (C) $\frac{mv^2}{r}$ along the tangent through the particle
    (D) zero
26. A train A runs from east to west and another train B of the same mass runs from west to east at the same speed along the equator. A presses the track with a force F_1 and B presses the track with a force F_2.
    (A) $F_1 > F_2$
    (B) $F_1 < F_2$
    (C) $F_1 = F_2$
    (D) the information is insufficient to find the relation between F_1 and F_2.
27. A car of mass M is moving horizontaly on a circular path of radius r. At an instant its speed is v and is increasing at a rate a.
    (A) The acceleration of the car is towards the centre of the path
    (B) The magnitude of the frictional force on the car is greater than $\frac{mv^2}{r}$

(C) The friction coefficient between the ground and the car is not less than a/g.

(D) The friction coefficient between the ground and the car is $\mu = \tan^{-1}\dfrac{v^2}{rg}$

28. A circular road of radius r is banked for a speed $v = 40$ km/hr. A car of mass attempts to go on the circular road. The friction coefficient between the tyre and the road is negligible.
(A) The car cannot make a turn without skidding.
(B) If the car turns at a speed less than 40 km/hr, it will slip down
(C) If the car turns at the current speed of 40 km/hr, the force by the road on the car is equal to $\dfrac{mv^2}{r}$
(D) If the car turns at the current speed of 40 km/hr, the force by the road on the car is greater than mg as well as greater than $\dfrac{mv^2}{r}$

29. A person applies a constant force $\vec{F}$ on a particle of mass m and finds that the particle moves in a circle of radius r with a uniform speed v as seen from an inertial frame of reference.
(A) This is not possible.
(B) There are other forces on the particle
(C) The resultant of the other forces is $\dfrac{mv^2}{r}$ towards the centre.
(D) The resultant of the other forces varies in magnitude as well as in direction.

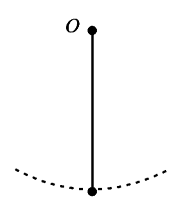
One or More than One Options Correct

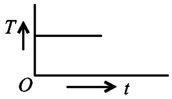
30. A particle of mass m begins to slide down a fixed smooth sphere from the top. What is its tangential acceleration when it breaks off the sphere ?
(A) $\dfrac{2g}{3}$ (B) $\dfrac{\sqrt{5}g}{3}$
(C) g (D) $\dfrac{g}{3}$

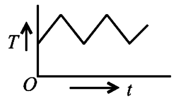
31. A 1 kg stone at the end of 1 m long string is whirled in a vertical circle at constant speed of 4 m/sec. The tension in the string is 6 N when the stone is at ($g = 10$ m/sec^2)
(A) Top of the circle (B) Bottom of the circle
(C) Halfway down (D) None of these

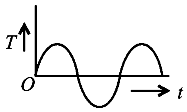
32. A boy whirls a stone in a horizontal circle 1.8 m above the ground by means of a string 1.2 m long. The string breaks and stone flies off horizontally, striking the ground 9.1 m away. The centripetal acceleration during the circular motion was: (use $g = 9.8$ m/s^2)
(A) 94 m/s^2 (B) 141 m/s^2
(C) 188 m/s^2 (D) 282 m/s^2

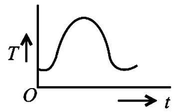
33. A particle of mass m is suspended from a fixed point O by a string of length ℓ. At $t = 0$, it is displaced from equilibrium position and released. The graph, which shows the variation of the tension T in the string with time 't', may be



(A) 

(B) 

(C) 

(D) 

34. A particle P is moving in a circle of radius a with a uniform speed u. C is the centre of the circle and AB is diameter. The angular velocity of P about A and C are in the ratio
(A) 1 : 1 (B) 1 : 2
(C) 2 : 1 (D) 4 : 2

35. The distance of the two planets from the sun are nearly 10_{12} to 10_{13} metres respectively. Then the ratio of their periodic time, if they are supposed to move in a circular orbit is
(A) $\sqrt{10} : 2$ (B) $2 : \sqrt{10}$
(C) $10\sqrt{10} : 1$ (D) $1 : 10\sqrt{10}$

36. A stone of mass 1 kg tied to a light inextensible string of length $L = \frac{10}{3}$ m, whirling in a circular path in a vertical plane. The ratio of maximum tension in the string to the minimum tension in the string is 4, If g is taken to be 10 m/s^2, the speed of the stone at the highest point of the circle is
(A) 10 m/s (B) $5\sqrt{2}$ m/s
(C) $10\sqrt{3}$ m/s (D) 20 m/s

37. Toy cart tied of the end of an unstretched string of length a, when revolved moves in a horizontal circle of radius $2a$ with a time period T. Now the toy cart is speeded up until it moves in a horizontal circle of radius $3a$ with a period T'. If Hook's law holds then
(A) $T' = \sqrt{\frac{3}{2}}\,T$ (B) $T' = \left(\frac{\sqrt{3}}{2}\right)T$
(C) $T' = \left(\frac{3}{2}\right)T$ (D) $T' = T$

38. A small sphere of mass m suspended by a thread is first taken aside so that the thread forms the right angle with the vertical and then released, then :
(i) The total acceleration of the sphere and the thread tension as a function of θ, the angle of deflection of the thread from the vertical will be
(A) $g\sqrt{1+3\cos^2\theta}$, $T = 3mg\cos\theta$ (B) $g\cos\theta$, $T = 3\,mg\cos\theta$.
(C) $g\sqrt{1+3\sin^2\theta}$, $T = 5mg\cos\theta$ (D) $g\sin\theta$, $T = 5\,mg\cos\theta$.
(ii) The thread tension at the moment when the vertical component of the sphere's velocity is maximum will be
(A) mg (B) $mg\sqrt{2}$
(C) $mg\sqrt{3}$ (D) $\frac{mg}{\sqrt{3}}$

(iii) The angle θ between the thread and the vertical at the moment when the total acceleration vector of the sphere is directed horizontally will be

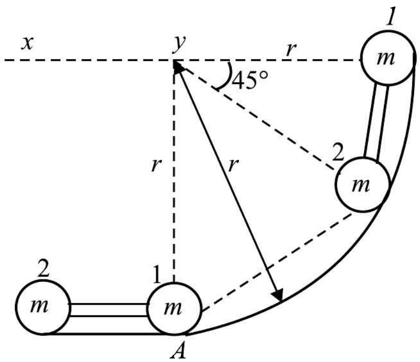
(A) $\cos\theta = \frac{1}{\sqrt{3}}$ (B) $\cos\theta = \frac{1}{3}$

(C) $\sin\theta = \frac{1}{\sqrt{3}}$ (D) $\sin\theta = \frac{1}{\sqrt{2}}$

39. The kinetic energy k of a particle moving along a circle of radius R depends on the distance covered s as $k = as^2$ where a is a constant. The total force acting on the particle is:

(A) $2a\frac{s^2}{R}$ (B) $2as\left(1+\frac{s^2}{R^2}\right)^{1/2}$ (C) $2as$ (D) $2a\frac{R^2}{s}$

40. Two small spheres, each of mass m are rigidly connected by a rod of negligible mass and are released from rest in the position shown and slide down a smooth circular guide in the vertical plane.



(i) Their common velocity as they reach the horizontal dashed position will be :

(A) $\sqrt{2gr - \frac{1}{\sqrt{2}}gr}$ (B) $10\sqrt{2gr - \frac{1}{\sqrt{2}}gr}$

(C) $\sqrt{gr}$ (D) $\sqrt{gr}/2$

(ii) The force R between sphere 1 and the supporting surface at an instant just before the sphere reaches the bottom position A will be :

(A) 22.9 mg (B) 2.29 mg

(C) mg (D) $mg/2$

41. A particle moves with deceleration along the circle of radius R so that at any moment of time its tangential and normal accelerations are equal in moduli. At the initial moment $t = 0$ the speed of the particle equals v_0, then :

(i) the speed of the particle as a function of the distance covered s will be

(A) $v = v_0 e^{-s/R}$ (B) $v = v_0 e^{s/R}$

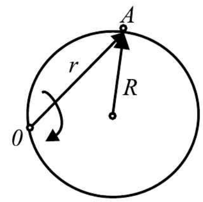
(C) $v = v_0 e^{-R/s}$ (D) $v = v_0 e^{R/s}$

(ii) the total acceleration of the particle as function of velocity and distance covered

(A) $a = \sqrt{2}\dfrac{v^2}{R}$ (B) $a = \sqrt{2}\dfrac{v}{R}$

(C) $a = \sqrt{2}\dfrac{R}{v}$ (D) $a = \dfrac{2R}{v}$

42. A particle A moves along a circle of radius R = 50 cm so that its radius vector r relative to the point O (Fig.) rotates with the constant angular velocity ω = 0.40 rad/s. Then modulus of the velocity of the particle , and the modulus of its total acceleration will be



(A) v = 0.4 m/s, w = 0.4 m/s^2 (B) v = 0.32 m/s, w = 0.32 m/s^2
(C) v = 0.32 m/s, w = 0.4 m/s^2 (D) v = 0.4 m/s, w = 0.32 m/s^2

43. A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance 3 m. What is the velocity of the spot P when $\theta = 45^0$?

(A) 0.6 m/s (B) 0.5 m/s (C) 0.4 m/s (D) 0.3 m/s

JEE Questions from Previous Years

1. A hemispherical bowl of radius r = 0.1m is rotating about its axis (which is vertical) with an angular velocity ω. A particle of mass 10^{-2}kg on the frictionless inner surface of the bowl is also rotating with the same ω. The particle is at a height h from the bottom of the bowl. (a) Obtain the relation between h and ω. What is the minimum value of ω needed in order to have a nonzero value of h. (b) It is desired to measure 'g' using this setup by measuring h accurately. Assuming that r and ω are known precisely and that the least count in the measurement of h is 10^{-4} m. What is minimum error Δ g in the measured value of g. [g = 9.8m/s^2] **[JEE 1993]**

2. A particle of mass m is moving in a circular path of constant radius r such that is centripetal acceleration a_c is varying with time t as $a_c = k^2 rt^2$ where k is a constant. The power delivered to the particle by the force acting on it is- **[JEE 1994]**

(A) $2\pi mk^2 r^2$ (B) $mk^2 r^2 t$ (C) $\dfrac{(mk^4r^2t^5)}{3}$ (D) Zero

3. A smooth semicircular wire track of radius R is fixed in a vertical plane (figure). On end of a massless spring of natural length $3R/4$ is attached to the lowest point O of the wire track. A small ring of mass m which can slide on the track is attached to the other end of the spring. The ring is held stationary at point P such that the spring make an angle 60° with the vertical. The spring constant $K = mg/R$. Consider the instant when the ring is released

(i) Draw the free body diagram of the ring.

(ii) Determine the tangential acceleration of the ring and the normal reaction. **[JEE 1996]**

4. Two blocks of mass m_1 = 10 Kg and m_2 = 5 Kg connected to each other by a massless inextensible string of length 0.3 m are placed along a diameter of turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rad/s about a vertical axis passing through

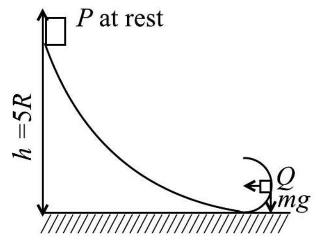
its centre O. The masses are placed along the diameter of the table on either side of the centre O such that the mass m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table.

(i) Calculate the frictional force on m_1.

(ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position.

(iii) How should the masses by placed with the string remaining taut so that there is no frictional force acting on the mass m_1. **[JEE 1997]**

5. A small block of mass m slides along a smooth frictional track as shown in figure. (i) If it starts from rest at P, when is the resultant force acting on it at Q ? (ii) At what height above the bottom of the loop should the block be released so that the force it exerts against the track at the top of the loop equals its weight? **[REE 1997]**

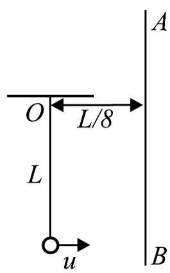


6. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position, where the string is horizontal, is **[JEE 1998]**

(A) $\sqrt{u^2 - 2gL}$ (B) $\sqrt{2gL}$ (C) $\sqrt{u^2 - gL}$ (D) $\sqrt{2(u^2 - gL)}$

7. A particle at rest starts rolling from the top of a large frictionless sphere of radius R. The sphere is fixed on the ground. Calculate that height from the ground at which the particle leaves the surface of the sphere. **[REE 1998]**

8. A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance $L/8$ from O as shown. The object given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find u. **[JEE 1999]**

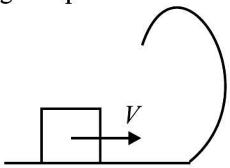


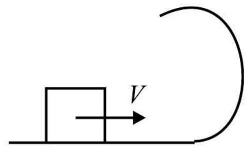
9. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant

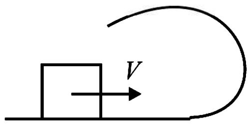
angular acceleration, α . If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is - **[JEE 2000]**

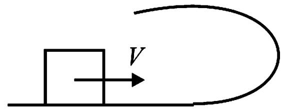
(A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{\mu}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) Infinitesimal

10. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum is- **[JEE 2001]**

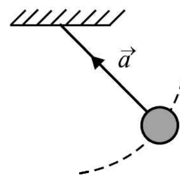
(A) 

(B) 

(C) 

(D) 

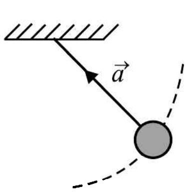
11. An insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the surface and the insect is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given- **[JEE 2001]**

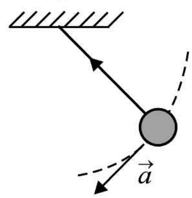


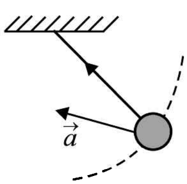
(A) $\cot\alpha = 3$ (B) $\tan\alpha = 3$
(C) $\sec\alpha = 3$ (D) $\operatorname{cosec}\alpha = 3$

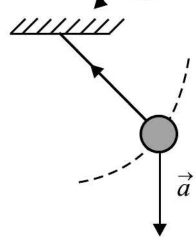
12. A block is placed inside a horizontal hollow cylinder. The cylinder starts rotating with one revolution per second about its axis. The angular position of the block at which it begins to slide is 30° below the horizontal level passing through the center. Find the radius of the cylinder if the coefficient of friction is 0.6. What should be the minimum angular speed of the cylinder so that the block reach the highest point of the cylinder? **[REE 2001]**

13. A simple pendulum is oscillating without damping. When the displacements of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in **[JEE 2002]**

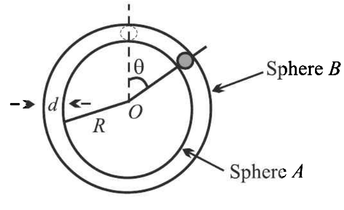
(A) 

(B) 

(C) 

(D) 

14. A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is depended by θ (shown in figure)



[JEE 2002]

(a) Express the total normal reaction force exerted by the spheres on the ball as a function of angle θ.

(b) Let N_A and N_B denote the magnitudes of the normal reaction force on the ball extorted by the spheres A and B, respectively. Sketch the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \leq \theta \leq \pi$ by drawing two separate graphs in your answer book, taking $\cos\theta$ on the horizontal axis

15. A double star system consists of two stars A and B which have time period T_A and T_B. Radius R_A and R_B and mass M_A and M_B. Choose the correct option. **[IIT 2006, (3, –1]**

(A) If $T_A > T_B$ then $R_A > R_B$ (B) If $T_A > T_B$ then $M_A > M_B$

(C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$ (D) $T_A = T_B$

16. **STATEMENT-1:** A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

because

STATEMENT-2: The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

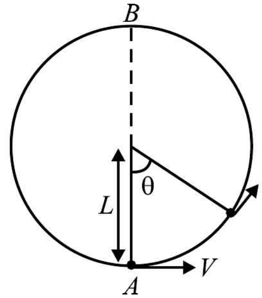
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True **[JEE 2007]**

17. A bob of mass M is suspended by a massless string of length L. The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies **[JEE 2008]**



(A) $\theta = \frac{\pi}{4}$ (B) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

(C) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (D) $\frac{3\pi}{4} < \theta < \pi$

18. **STATEMENT-1 :** For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary. **[JEE 2008]**

and

STATEMENT-2 : If the observer and the object are moving at velocities $\vec{V}_1$ and $\vec{V}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{V}_2 - \vec{V}_1$.

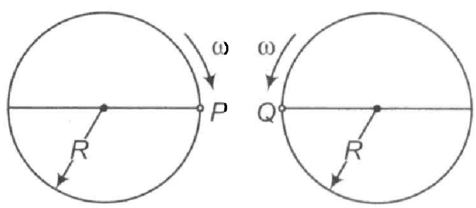
(A) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 is a correct explanation for STATEMENT-1

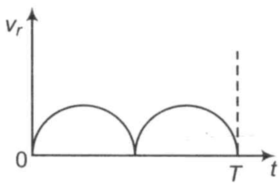
(B) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

19. Two identical discs of same radius R are rotating about their axis in opposite directions with the same constants angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points P and Q are facing each other as shown in the figure. The magnitude of relative velocity between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by **[JEE-2012]**



(A)


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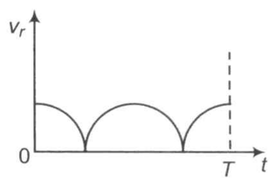
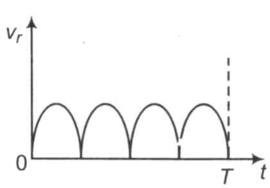
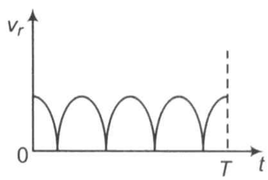
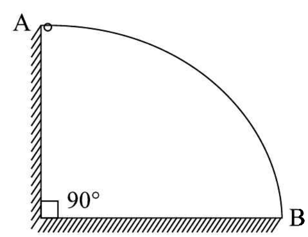


(C)


(D)


20. A wire, which passes through the hold in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in figure. The bead is released from near the top of the wire and its slides along the wire withoug friction. As the bead moves from A to B. The force it applies on the wire is

[JEE Advanced, 2014]



(A) Always radially outwards
(B) Always radially inwards
(C) Radially outwards initially and radially inwards later
(D) Radially inwards initially and radially outwards later.
[Ans. (D)]

Previous Years' AIEEE Questions

1. The maximum velocity (in ms^{-1}) with which a car driver can traverse a flat curve of radius 150 m and cofficient of friction 0.6 to avoid skidding is : **[AIEEE-2002]**
(A) 60 (B) 30 (C) 15 (D) 25
2. Which of the following statements is false for a particle moving in a circle with a constant angular speed ? **[AIEEE - 2004]**
(A) The velocity vector is tangent to the circle
(B) The acceleration vector is tangent to the circle
(C) The acceleration vector point to the center of the circle
(D) The velocity and acceleration vectors are perpendicular to each other
3. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that **[AIEEE - 2004]**
(A) Its velocity is constant (B) Its acceleration is constant
(C) Its kinetic energy is constant (D) It moves in a straight line
4. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when $t = 2$ s is nearly. **[AIEEE - 2010]**
(A) 13 m/s^2 (B) 12 m/s^2 (C) 7.2 m/s^2 (D) 14 m/s^2
5. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P (R, θ) on the circle of radius R is (Here θ is measured from the x-axis) **[AIEEE - 2010]**
(A) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$ (B) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$
(C) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$ (D) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

ANSWER KEY

Exercise-1

1. (a) first (b) third.
2. 2.7 m/sec^2
3. $8T_0$
4. 90°
5. $\frac{u^2\cos^2\theta}{g\cos^3(\theta/2)}$
6. [14.8N, 14.8 N]

7. 20 rad/*s*
8. 4.5 *mg*
9. 2 m/s
10. 0.40
11. 4.5 cm
12. $v = \omega\sqrt{r^2 - R^2}$
13. $\tan^{-1}(1/4)$]
14. 0.25
15. (A) 3.5×10^{-3}, (B) 2.0 hour]
16. Between 14.7 km/h and 54 km/hr]
17. (a) $75 m/s^2$, (b) $125 m/s^2$
18. (a) $2 rad/s^2$, (b) 12+2*t* for $t \le 2s$, 16 for $t \ge 2s$, (c) $\sqrt{28565} \approx 169$, 256 m/s^2 (d) 44 rad
19. $a_t = \dfrac{2g}{\sqrt{13}}, a_n = \dfrac{3g}{\sqrt{13}}$
20. $\dfrac{2\sqrt{2}v^2}{\pi R}$
21. $\sqrt{3}b/2,\ b/2,\ b$
22. 2 rad.
23. $\sqrt{2g}$ rad/s
24. 9
25. 2 tanθ
26. $-3\hat{k}$ rad/s², $-2\hat{k}$ rad/s
27. (a) $2 rad/s^2$, (b) 12+2*t* for $t \le 2s$, 16 for $t \ge 2s$, (c) $\sqrt{28565} \approx 169$, 256 m/s^2 (d) 44 rad
28. 0.1875
29. *P*
30. $\dfrac{25}{24}$
31. (i) 36*N*, (ii) 11.66 rad/sec, (iii) 0.1m, 0.2m
32. (a) $\dfrac{V_0^2}{2g}$, (b) 2*g*
33. A
34. (a) 21.65 m/s² (b) 7.35 m/s (c) 12.5 m/s²
35. $\dfrac{5\pi R}{6v}, \dfrac{11\pi}{6}, \dfrac{17v}{5R}, \dfrac{289v^2}{25R}$
36. (a) $v_{max} = \sqrt{Rg}$, (b) At angle θ = 60ŗ from the vertical position. (c) $v = \sqrt{gR\cos\left(\dfrac{L}{2R}\right)}$
37. (a) 39.6 per min., (b) 150 *N*
38. $T = m\ell n^2$
39. $\sqrt{gr\tan\theta},\ 2\pi\sqrt{\dfrac{r}{g\tan\theta}}$
40. Net force on car = frictional force *f*

$\therefore\ f = m\sqrt{a^2 + \dfrac{v^4}{R^2}}$ (where *m* is mass of the car) ...(1)

For skidding to just occur

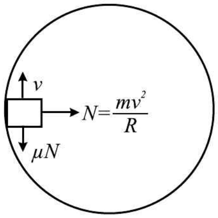
$f = \mu N = \mu mg$ ...(2)

∴ From (1) and (2)

$v = \{R^2[\mu^2 g^2 - a^2]\}^{1/4}\ \left[(\mu^2 g^2 - a^2)R^2\right]^{\frac{1}{4}}$

41. (A) 975N, 1025 N, (B) 0,707N, 0, (C) 682 N, 732 N, (D) 0 1.037]

42. (B)



(i) The normal reaction by wall on the block is $N = \dfrac{mv^2}{R}$

(ii) The friction force on the block by the wall is $F = \mu N = \dfrac{\mu mv^2}{R}$

(iii) The tangential acceleration of the block = $\dfrac{f}{m} = \dfrac{\mu v^2}{R}$

(iv) $\dfrac{dv}{dt} = -\dfrac{\mu v^2}{R}$

or $v\dfrac{dv}{ds} = -\dfrac{\mu v^2}{R} \Rightarrow \int_{v_0}^{v} \dfrac{dv}{v} = -\int_{0}^{2\pi R} \dfrac{\mu}{R} ds$

integrating we get

$\ell n \dfrac{v}{v_0} = -\mu\, 2\pi$ or $v = v_0\, e^{-2\mu\pi}$

Ans. (B) (i) $\dfrac{mv^2}{R}$ (ii) $\dfrac{\mu mv^2}{R}$ (iii) $-\dfrac{\mu v^2}{R}$ (iv) $v_0 e^{-2\pi\mu}$

43. $\sqrt{\dfrac{2L}{\omega^2 R\cos\theta}}$

44. (A) 0.2N, (B) 30ɽ

45. $\dfrac{\omega^2 R}{3}.\dfrac{4}{3}, m\omega^2 R$

Exercise-3

1.	D	2.	C	3.	D	4.	AD
5.	D	6.	BD	7.	C	8.	D
9.	D	10.	D	11.	C	12.	A
13.	B	14.	D	15.	C	16.	D
17.	A	18.	D	19.	D	20.	A
21.	A	22.	A	23.	A	24.	C
25.	D	26.	A	27.	BC	28.	BD
29.	BD	30.	B	31.	A	32.	C
33.	D	34.	B	35.	D	36.	A

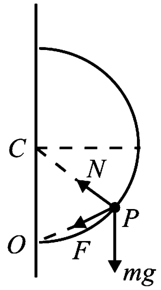
37. *B* 38. (i) *A*, (ii) *C*, (iii) *A* 39. *B* 40. *(i) A (ii) B*

41. *(i) A, (ii) A* 42. *D* 43. A

JEE Questions from Previous Year

1. (*a*) $7\sqrt{2}$ *rad / s* (*b*) *-9.8 x* 10^{-3} *m/s*^{2} 2. *B*

3. (*i*)



(*ii*) $a_T = \dfrac{5\sqrt{3}}{8} g$, $N = \dfrac{3mg}{8}$

4. (*i*) *36 N* (*ii*) *11.67 rad/s* (*iii*) r_1 *= 0.1 m and* r_2 *= 0.2 m*

5. $\left(\sqrt{65}\right)mg$, *3R* 6. *D*

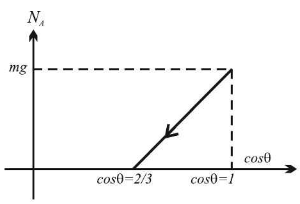
7. *H* = (*5/3*)*R* 8. $u = \sqrt{gL\left(\dfrac{3\sqrt{3}}{2} + 2\right)}$

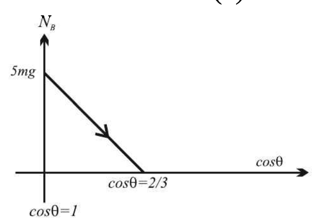
9. *A* 10. *A*

11. *A* 12. *0.24m,6.4rad/sec*

13. *C* 14. (*a*) *N = 3mg cos*θ *- 2mg,*

(*b*)





15. *D* 16. *C*

17. *D* 18. *B*

19. (*A*)

Previous Years' AIEEE Questions

1. (B) 2. (B)

3. (C) 4. (D)

5. (C)

Chapter

6

Work, Power and Energy

Whatever we have learnt till now is more or less sufficient for solving any problem involving particles. By doing force analysis we can find acceleration and then we can find velocity, position, etc. That means we should be finished with particle dynamics.

But it is not so. We will learn a new technique and concept which will make problem solving faster and will give simple solution to complicated problems. Work Energy Theorem is an extension to Newton's Laws but much simpler to use (mainly because of absence of vectors).

WORK DONE BY CONSTANT FORCE

The term 'work' as understood in everyday life has a different meaning in scientific sense. If a coolie is carrying a load on his head and waiting for the arrival of the train, he is not performing any work in the scientific sense. In the present study, we shall have a look into the scientific aspect of this most commonly used term i.e., work.

The physical meaning of the term work is entirely different from the meaning attached to it in everyday life. In everyday life, the term 'work' is considered to be synonym of 'labour', 'toil', effort' etc. In physics, there is a specific way of defining work.

Work is said to be done by a force when the force produces a displacement in the body on which it acts in any direction except perpendicular to the direction of the force.

For work to be done, following two conditions must be fulfilled:

(i) A force must be applied.

(ii) The applied force must produce a displacement in any direction except perpendicular to the direction of the force.

Suppose a force $\vec{F}$ is applied on a body in such a way that the body suffers a displacement $\vec{S}$ in the direction of the force. Then the work done is given by

$$W = FS$$

However, the displacement does not always take place in the direction of the force. Suppose a constant force $\vec{F}$, applied on a body, produces a displacement $\vec{S}$ in the body in such a way that $\vec{S}$ is inclined to $\vec{F}$ at an angle θ. Now the work done will be given by the dot product of force and displacement.

$$W = \vec{F} \cdot \vec{S}$$

Since, work is the dot product of two vectors therefore it is a scalar quantity.

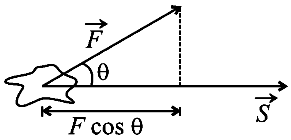
$W = FS\cos\theta$ or $W = (F\cos\theta)S$

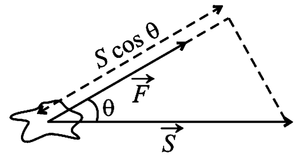
$\therefore$ W = component of force in the direction of displacement × magnitudes of displacement.

So, work is the product of the component of force in the direction of displacement and the magnitude of the displacement.

Also, $W = F(S\cos\theta)$

or work is product of the component of displacement in the direction of the force and the magnitude of the displacement.





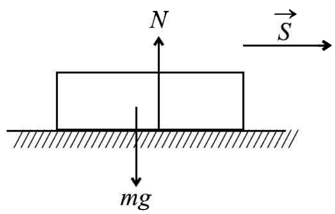
Special Cases

Case (i): When θ = 90ŗ, then $W = FS$ cos 90ŗ = 0

So, work done by a force is zero if the body is displaced in a direction perpendicular to the direction of the force.

Examples:

1. Consider a body sliding over a horizontal surface. The work done by the force of gravity and the reaction of the surface will be zero. This is because both the force of gravity and the reaction act normally to the displacement.



   The same argument can be applied to a man carrying a load on his head and walking on a railway platform.

2. Consider a body moving in a circle with constant speed. At every point of the circular path, the centripetal force and the displacement are mutually perpendicular (Figure 6.1). So, the work done by the centripetal force is zero. The same argument can be applied to a satellite moving in a circular orbit. In this case, the gravitational force is always perpendicular to displacement. So, work done by gravitational force is zero.

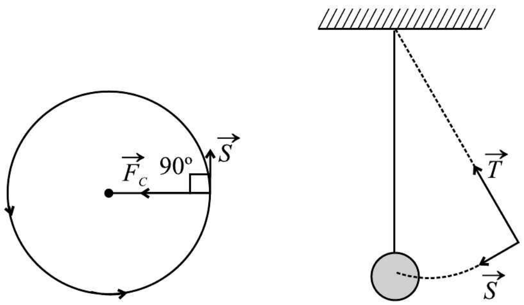


Fig 6.1

Example 1. The tension in the string of a simple pendulum is always perpendicular to displacement. (Figure). So, work done by the tension is zero.

Case (ii): When S = 0, then $W = 0$.

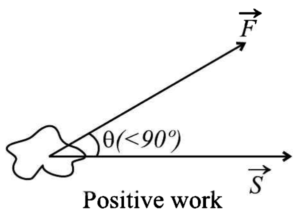
So, work done by a force is zero if the body suffers no displacement on the application of a force.

Example 2. A person carrying a load on his head and standing at a given place does no work.

Case (iii): When $0^\circ \le \theta < 90^\circ$ [Figure 6.2], then $\cos\theta$ is positive. Therefore.

$W (= FS \cos\theta)$ is positive.

Work done by a force is said to be positive if the applied force has a component in the direction of the displacement.



Positive work

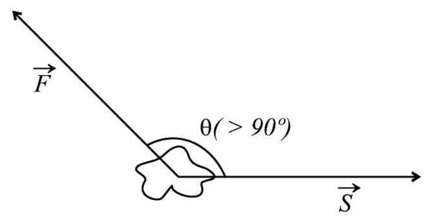
Fig 6.2

Examples:

1. When a horse pulls a cart, the applied force and the displacement are in the same direction. So, work done by the horse is positive.
2. When a load is lifted, the lifting force and the displacement act in the same direction. So, work done by the lifting force is positive.
3. When a spring is stretched, both the stretching force and the displacement act in the same direction. So, work done by the stretching force is positive.

Case (iv): When $90^\circ < \theta \le 180^\circ$ (Figure), then $\cos\theta$ is negative. Therefore $W (= FS \cos\theta)$ is negative.

Work done by a force is said to be negative if the applied force has component in a direction opposite to that of the displacement.

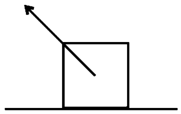


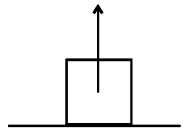
Negative work

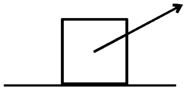
Examples:

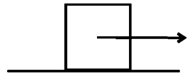
1. When brakes are applied to a moving vehicle, the work done by the braking force is negative. This is because the braking force and the displacement act in opposite directions.
2. When a body is dragged along a rough surface, the work done by the frictional force is negative. This is because the frictional force acts in a direction opposite to that of the displacement.
3. When a body is lifted, the work done by the gravitational force is negative. This is because the gravitational force acts vertically downwards while the displacement is in the vertically upwards direction.

Example 3. Figure shows four situations in which a force acts on a box while the box slides rightward a distance d across a frictionless floor. The magnitudes of the forces are identical, their orientations are as shown. Rank the situations according to the work done on the box during the displacement, from most positive to most negative.

(A) 

(B) 

(C) 

(D) 

[Ans. D, C, B, A]

Explanation:

In (D), θ = 0ţ, cos θ = 1 (maximum value). So, work done is maximum.

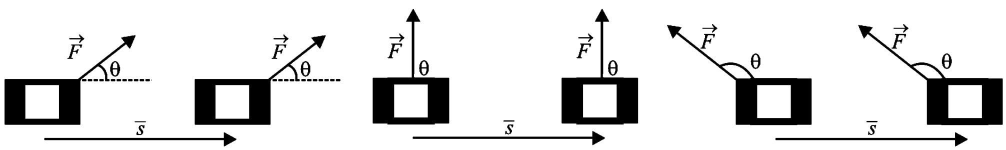
In (C), θ = 90ţ, cos θ is positive. Therefore, *W* is positive.

In (B), θ = 90ţ, cos θ is zero. *W* is zero.

In (A), θ is obtuse, cos θ is negative. *W* is negative.

Work Done by a Constant Force

Let $\vec{F}$ be a constant force acting on a body. If the body goes through a displacement $\bar{s}$, then the work done by the force $\vec{F}$ is given by $W = F\ s \cos\theta$; where θ = angle between force vector $\vec{F}$ and displacement vector $\bar{s}$.

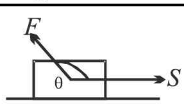
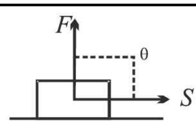
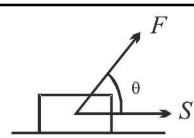


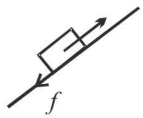
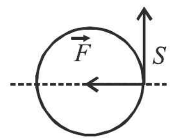
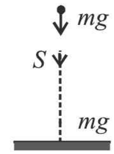
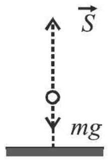
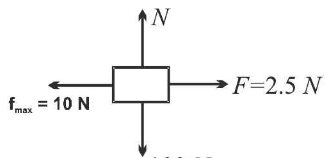
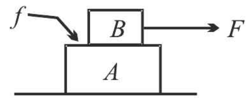
F = magnitude of force $\vec{F}$ and *s* = magnitude of displacement $\bar{s}$.

- If θ is acute, *W* is positive (force tries to increase the speed of the body).
- If θ = 90° i.e., force is perpendicular to displacement, *W* = 0
- If θ is obtuse, *W* is negative (force tries to decrease the speed of the body).

Note

1. If $\vec{F}$ is in same direction as $\bar{s}$, $W = F\,s$
2. If $\vec{F}$ is opposite to $\bar{s}$, $W = -F\,s$

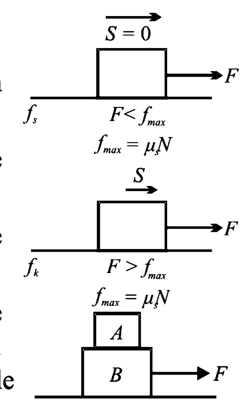
Negative Work	Zero Work	Positive Work
 (θ>90°)	 (θ=90°)	 (θ<90°)

Negative Work	Zero Work	Positive Work
 Work done by friction force ($\theta = 180°$)	 Motion of particle on circular path ($\theta = 90°$)	 Motion under gravity ($\theta = 0°$)
 Work done by gravity ($\theta = 180°$)	 As $f = F$, hence $S = 0$	 Work done by friction ($\theta = 0°$)

Work Done by Gravity

The work done by the force of gravity on a particle depends on the initial and final vertical coordinates (because gravity is a vertical force). The work done by gravity is positive when the body moves downward and it is negative when the body moves upward.

Work Done by Friction



The work done by friction may be zero, positive or negative depending upon the situation.

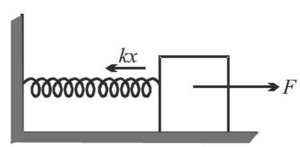
Case 1: Where a block is pulled by a force F which is insufficient to over come the friction, $F < f_{max}$. Here the work done by the friction force is zero.

Case 2: Where a block is pulled by a force F which is sufficiently large to over come friction. $F > f_{max}$. Here the work done by the friction force is negative.

Case 3: Block *A* is placed on the block *B*, when the block *A* is pulled with a force *F*, the friction force does negative work on block *A* and positive work on block *B*. The friction force and displacement are oppositely directed in case of block *A*, while in case of *B* they are in same direction.

Work Done by a Spring Force

When a spring is stretched or compressed, the spring force always tend to restore it to the equilibrium position. If *x* be the displacement of the free end of the spring from it equilibrium position, then the magnitude of the spring force is $F_s = -kx$. The negative sign indicates that the force is restoring. The work done by the spring force for a displacement from x_i to x_f is given by



$$W_s = \int_{x_i}^{x_f} F_s dx = \int_{x_i}^{x_f} kx dx \; ; \quad W_s = -\frac{1}{2}k(x_f^2 - x_i^2)$$

Work Done by Multiple Forces

If several forces act on a particle, then we can replace $\vec{F}$ in equation $W = \vec{F} \cdot \vec{S}$ by the net force $\Sigma\vec{F}$ where

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$$

$$\therefore \quad W = [\Sigma\vec{F}]\cdot\vec{S} \qquad ...(i)$$

This gives the work done by the net force during a displacement $\vec{S}$ of the particle.

We can rewrite equation (i) as:

$$W = \vec{F}_1\cdot\vec{S} + \vec{F}_2\cdot\vec{S} + \vec{F}_3\cdot\vec{S} + ... \text{ or } W = W_1 + W_2 + W_3 + ...$$

So, the work done on the particle is the sum of the individual works done by all the forces acting on the particle.

Points to Remember

1. Work is defined for an interval or displacement. There is no term like instantaneous work similar to instantaneous velocity.
2. For a particular displacement, work done by a force is independent of type of motion i.e., whether it moves with constant velocity, constant acceleration or retardation etc.
3. For a particular displacement work is independent of time. Work will be same for same displacement whether the time taken is small or large.
4. When several forces act, work done by a force for a particular displacement is independent of other forces.
5. A force is independent from reference frame. Its displacement depends on frame so work done by a force is frame dependent therefore work done by a force can be different in different reference frame.
6. Effect of work is change in kinetic energy of the particle or system.
7. Work is done by the source or agent that applies the force.

Units of Work

I. In cgs system, the unit of work is erg.

One erg of work is said to be done when a force of one dyne displaces a body through one centimetre in its own direction.

$\therefore \quad 1 \text{ erg} = 1 \text{ dyne} \times 1 \text{ cm} = 1g \text{ cm s}^{-2} \times 1 \text{ cm} = 1\, g \text{ cm}^2 \text{ s}^{-2}$

Note

Erg is also called dyne centimetre.

II. In SI i.e., International System of units, the unit of work is joule (abbreviated as J). It is named after the famous British physicist James Personal Joule (1818 – 1869).

One joule of work is said to be done when a force of one newton displaces a body through one metre in its own direction.

$1 \text{ joule} = 1 \text{ newton} \times 1 \text{ metre} = 1 \text{ kg} \times 1 \text{ m/s}^2 \times 1 \text{ m} = 1\text{kg m}^2 \text{ s}^{-2}$

Note

Another name for joule is newton metre.

Relation Between Joule and ERG

1 joule = 1 newton × 1 metre

1 joule = 10^5 dyne × 10^2 cm = 10^7 dyne cm

1 joule = 10^7 erg

1 erg = 10^{-7} joule

Dimensions of Work

[Work]= [Force] [Distance] = $[MLT^{-2}]\ [L] = [ML^2T^{-2}]$

Work has one dimension in mass, two dimensions in length and '–2' dimensions in time,

On the basis of dimensional formula, the unit of work is kg m^2 s^{-2}.

Note that 1 kg m^2 s^{-2} = (1 kg m s^{-2}) m = 1 N m = 1 J.

Example 4. A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposite to the motion.

(a) How much work does the road do on the cycle?

(b) How much work does the cycle do on the road?

Solution

(a) The work done on the cycle by the road is the work done by the frictional force exerted by the road on the cycle.

Now, $W = \vec{F} \cdot \vec{S} = FS \cos 180^\circ$

or $W = -FS$

or $W = -200\ N \times 10\ m$

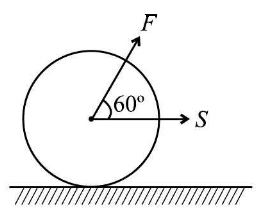
or $W = \mathbf{-2000\ J}$

It is this negative work which brings the cycle to rest. This is clearly in accordance with work-energy theorem.

(b) The displacement of the road is zero. So, work done by the cycle on the road is zero.

(Using Newton's third law of motion, an equal and opposite force acts on the road due to the cycle. The magnitude of this force is 200 N.)

Example 5. A gardener moves a lawn roller through a distance of 100 metre with a force of 50 newton. Calculate his wages if he is to be paid 10 paise for doing 25 joule of work. It is given that the applied force is inclined at 60° to the direction of motion.



Solution Force, $F = 50\ N$; Distance, $S = 100$ m ; Angle , $\theta = 60^\circ$

$$W = FS \cos\theta = 50 \times 100 \times \cos 60^\circ \text{ joule}$$

$$W = 50 \times 100 \times \frac{1}{2}\ J = 2500\ J\ [\because \cos 60° = \frac{1}{2}]$$

$$\text{Wages} = \frac{2500}{25} \times 10 \text{ paise} = \mathbf{10\ rupees}$$

Example 6. Calculate the work done in raising a stone of mass 5 kg and specific gravity 3 lying at the bed of a lake through a height of 5 metre.

Solution When a body is immersed in water, its apparent weight is decreased in accordance with the Archimedes' principle.

$$\text{Loss of weight in water} = \frac{\textit{weight in air}}{\textit{specific gravity}} = \frac{5 \textit{ kg wt}}{3}$$

$$\therefore \quad \text{Weight of stone in water} = \left(5 - \frac{5}{3}\right) \text{kg wt} = \frac{10}{3} \text{ kg wt}$$

$$\text{Force, } F = \frac{10}{3} \text{kg wt} = \frac{10}{3} \times 9.8\ N = \frac{98}{3}\ N$$

$$\text{Work done, } W = \frac{98}{3} \times 5\ J = 163.3\ J.$$

Work in Terms of Rectangular Components

In terms of rectangular components, $\vec{F}$ and $\vec{S}$ may be written as:

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \text{ and } \vec{S} = S_x\hat{i} + S_y\hat{j} + S_z\hat{k}$$

$$\vec{F}\cdot\vec{S} = (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(S_x\hat{i} + S_y\hat{j} + S_z\hat{k})$$

$$= F_xS_x(\hat{i}.\hat{i}) + F_xS_y(\hat{i}.\hat{j}) + F_xS_z(\hat{i}.\hat{k}) + F_yS_x(\hat{j}.\hat{i}) + F_yS_y(\hat{j}.\hat{j}) + F_yS_z(\hat{j}.\hat{k})$$

$$+ F_zS_x(\hat{k}.\hat{i}) + F_zS_y(\hat{k}.\hat{j}) + F_zS_z(\hat{k}.\hat{k})$$

But $\hat{i}.\hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{i} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = \hat{k}.\hat{j} = 0$

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$ $\quad\therefore\ [\vec{F}\cdot\vec{S} = F_xS_x + F_yS_y + F_zS_z]$

Example 7. A body constrained to move along the z-axis of a coordinate system is subjected to a constant force $\vec{F}$ given by $\vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}\,N$

where $\hat{i}$, $\hat{j}$, $\hat{k}$ are unit vectors along the *x*, *y* and *z*-axis of the system respectively. What is the work done by this force in moving the body a distance of 4m along the *z*-axis?

Solution Since the body is displaced 4 m along z-axis only,

$$\therefore \quad \vec{S} = 0\hat{i} + 0\hat{j} + 4\hat{k}$$

$$\text{Also, } \vec{F} = -\hat{i} + 2\hat{j} + 3\hat{k}$$

Work done, W = $\vec{F}\cdot\vec{S} = (-\hat{i} + 2\hat{j} + 3\hat{k}).(0\hat{i} + 0\hat{j} + 4\hat{k}) = 12(\hat{k}.\hat{k})$ joule = **12 joule.**

Example 8. An object is displaced from point A(2*m*, 3*m*, 4*m*) to a point B(1*m*, 2*m*, 3*m*) under a constant force $\vec{F} = (2\hat{i} + 3\hat{j} + 4\hat{k})N$. Find the work done by this force in this process.

Solution

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}\cdot d\vec{r} = \int_{(2m,3m,4m)}^{(1m,2m,3m)} (2\hat{i} + 3\hat{j} + 4\hat{k})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= (2x + 3y + 4z)_{(2m,3m,4m)}^{(1m,2m,3m)} = -9 \text{ J } \textbf{Ans.}$$

Work Done by a Variable Force

When the magnitude and direction of a force vary in three dimensions, it can be expressed as a function of the position. For a variable force work is calculated for infinitely small displacement and for this displacement force is assumed to be constant

$$dW = \vec{\mathbf{F}}\cdot d\vec{\mathbf{s}}$$

The total work done will be sum of infinitely small work

$$W_{A\to B} \int_A^B \vec{F}\cdot d\vec{s} = \int_A^B (\vec{F}\cos\theta) d\vec{s}$$

In terms of rectangular components,

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}; \qquad d\vec{s} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$W_{A\to B} \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

When the force is an arbitrary function of position, we need the techniques of calculus to evaluate the work done by it. The figure shows F_x as some function of the position x. We begin by replacing the actual variation of the force by a series of small steps. The area under each segment of the curve is approximately equal to the area of a rectangle. The height of the rectangle is a constant value of force, and its width is a small displacement Δx. Thus, the step involves an amount of work $\Delta W_n = F_n \Delta x_n$. The total work done is approximately by the sum of the areas of the rectangles:

$$W \approx \Sigma F_n \Delta x_n.$$

As the size of the steps is reduced, the tops of the rectangle more closely trace the actual curve shown in figure. In the limit $\Delta x \to 0$, which is equivalent to letting the number of steps tend to infinity, the discrete sum is replaced by a continuous integral.

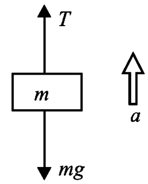
$$\lim_{\Delta x_n \to 0} \sum F_n \Delta x_n = \int F_x dx$$

Thus, the work done by a force F_x from an initial point A to final point B is

$$W_{A\to B} = \int_{x_A}^{x_B} F_x dx$$

Example 9. A block of mass 5 kg is being raised vertically upwards by the help of a string attached to it. It rises with an acceleration of 2 m/s². Find the work done by the tension in the string if the block rises by 2.5 m. Also find the work done by the gravity and the net work done.

Solution Let us first calculate the tension.



From force diagram:

$$T - mg = 5a$$

$$T = 5\,(9.8 + 2) = 59\,N.$$

As the T and displacement $\bar{s}$ are in same direction (upwards), work done by the tension T is W.

$$W = Ts = 59\,(2.5) = 147.5\,J$$

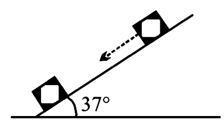
Work done by the gravity $= -mgs = -5\,(9.8)\,(2.5) = -122.5\,J$

Net work done on block = work done by T + work done by mg

$$= 147.5 + (-122.5) = 25\,J$$

***Example* 10.** A block of mass 4 kg slides down a plane inclined at 37° with the horizontal. The length of plane is 3 m. The coefficient of sliding friction between the block and the plane is 0.2. Find the work done by the gravity, the frictional force and the normal reaction between the block and the plane.

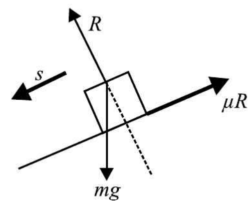
Solution As the normal reaction is perpendicular to the displacement,



work done by the normal reaction $R = R\,s \cos 90° = 0$

The magnitude of displacement $= s = 3$ m

and the angle between force of gravity (mg) and displacement is equal to $(90° - 37°)$



work done by the gravity $= mgs \cos (90° - 37°)$

$mg \sin 37° = 4 \times 9.8 \times 3 \times 3/5 = 70.56\,J$

⇒ work done by friction

$$= -(\mu R)s = -(\mu\, mg \cos 37°)\, s$$

$$= -0.2 \times 4 \times 9.8 \times 4/5 \times 3 = -18.816\,J$$

***Example* 11.** A chain of mass $m = 0.80$ kg and length $\ell = 1.5$ m rests on a rough - surfaced table so that one of its ends hangs over the edge. The chain starts sliding off the table all by itself provided the over hanging part equals $n = 1/3$ of the chain length. What will be the total work performed by the friction forces acting on the chain by the moment it slides completely off the table ?

Solution Slipping occurs when the weight of hanging part is just sufficient to overcome the frictional force exerted by the table. Let μ be the coefficient of friction between chain and table.

Weight of hanging part = μ(weight of horizontal part)

$$nmg = \mu\,(1 - n)\,mg; \qquad \mu = \frac{n}{1-n}$$

Let x be the length of the hanging part at some time instant.

$$\text{frictional force } f(x) = \mu \text{ (normal reaction)} = \frac{\mu(\ell - x)\,mg}{\ell}$$

The work done by the frictional force if the hanging part increases to $(x + dx)$ is:

$$dW = -f(x)\,dx$$

$$W = \int dW = -\int_{n\ell}^{\ell} \frac{\mu(\ell - x)mg}{\ell}$$

$$W = -\frac{\mu mg}{\ell}\left[\ell x - \frac{x^2}{2}\right]_{n\ell}^{\ell}$$

$$W = -\mu\, mg\left[\ell(1-n) - \frac{\ell}{2}(1-n^2)\right]$$

Substituting the value of μ from (I), we get:

$$W = -\frac{n(1-n)mg\ell}{2}$$

Example 12. An object is displaced from position vector $\vec{r}_1 = (2\hat{i} + 3\hat{j})m$ to $\vec{r}_2 = (4\hat{j} + 6\hat{k})m$ under a force $\vec{F} = (3x^2\hat{i} + 2y\hat{j})N$. Find the work done by this force.

Solution $$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}\cdot d\vec{r} = \int_{\vec{r}_1}^{\vec{r}_2} (3x^2\hat{i} + 2y\hat{j})\cdot(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

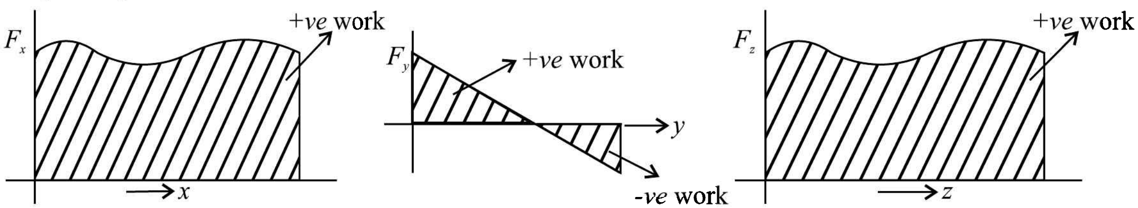
$$= \int_{\vec{r}_1}^{\vec{r}_2} (3x^2dx + 2ydy) = [x^3 + y^2]_{(2,3)}^{(4,6)} = 83J \ \textbf{Ans.}$$

Example 13. An object is displaced from a point $A(0, 0, 0)$ to $B(1m, 1m, 1m)$ under a force $\vec{F} = (y\hat{i} + x\hat{j})N$. Find the work done by this force in this process.

[**Ans.** $W = 1\ J$]

Area under Force Displacement Curve

Graphically area under the force-displacement is the work done



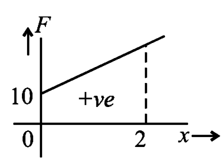
The work done can be positive or negative as per the area above the x-axis or below the x-axis respectively.

Example 13. A force $F = 0.5x + 10$ acts on a particle. Here F is in newton and x is in metre. Calculate the work done by the force during the displacement of the particle from $x = 0$ to $x = 2$ metre.

Solution Small amount of work done dW in giving a small displacement $\vec{dx}$ is given by

$$dW = \vec{F}\cdot\vec{dx}$$

or $dW = Fdx \cos 0ŗ$ or $dW = Fdx$ [∵ cos 0ŗ = 1]



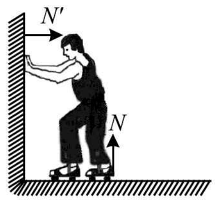
Total work done, $W = \int_{x=0}^{x=2} Fdx = \int_{x=0}^{x=2} (0.5\,x + 10)dx$

$$= \int_{x=0}^{x=2} 0.5x\,dx + \int_{x=0}^{x=2} 10\,dx = 0.5\left|\frac{x^2}{2}\right|_{x=0}^{x=2} + 10|x|_{x=0}^{x=2}$$

$$= \frac{0.5}{2}[2^2 - 0^2] + 10[2 - 0] = (1 + 20) = 21 \text{ J}$$

INTERNAL WORK

Suppose that a man sets himself in motion backward by pushing against a wall. The forces acting on the man are his weight 'W', the upward force N exerted by the ground and the horizontal force N' exerted by the wall. The works of 'W' and of N are zero because they are perpendicular to the motion. The force N' is the unbalanced horizontal force that imparts to the system a horizontal acceleration. The work of N', however, is zero because there is no motion of its point of application. We are therefore confronted with a curious situation in which a force is responsible for acceleration, but its work, being zero, is not equal to the increase in kinetic energy of the system.



The new feature in this situation is that the man is a composite system with several parts that can move in relation to each other and thus can do work on each other, even in the absence of any interaction with externally applied forces. Such work is called internal work. Although internal forces play no role in acceleration of the composite system, their points of application can move so that work is done; thus the man's kinetic energy can change even though the external forces do no work.

Basic concept of work lies in following lines:

- Draw the force at proper point where it acts that give proper importance to the point of application of force.
- Think independently for displacement of point of application of force, Instead of relating the displacement of applicant point with force relate it with the observer or reference frame in which work is calculated.

W = (Force Vector) × (displacement vector of point of application of force as seen by observer)

Explanatory Notes on Work

When we say 'work' in physics it is different from word 'work' we use in daily life. Work done by force $\vec{F}$ is defined as $dW = \vec{F}\cdot d\vec{S}$,

There are two interpretations of $d\vec{s}$.

1. If the body is moving as a complete unit, $d\vec{s}$ is the displacement of the body e.g., when we walk on earth, there is a force of friction on earth. The earth moves as a unit. Each point has same displacement. So $d\vec{s}$ is displacement of earth.
2. If the different points of body have different displacement, $d\vec{s}$ is the displacement of point of application of force. In the example above displacement of different parts of our body are different. So $d\vec{s}$ is displacement of point of contact i.e., the foot in contact with the ground. So we can say clearly that work by earth on us is also zero.

Actually, there are no 2 divisions dW = $\vec{F}\cdot d\vec{S}$, where $d\vec{s}$ is point of application of force on body. With this, you can explain both answers.

(i) We can note that work is a scalar quantity.

(ii) $dW = |\vec{F}||d\vec{S}|Cos\theta$ i.e. if component of force is along displacement ($\theta < 90°$) work is positive otherwise work is negative ($\theta > 90°$)

(1) True or False?

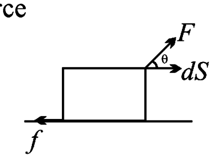
A boy jumps up into the air by applying a force downward on the ground. The work-kinetic energy theorem $W = \Delta K$ can be applied to the boy to find the speed with which he leaves the ground.

(2) True or False?

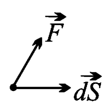
A balloon is compressed uniformly from all sides. Because there is no displacement of the balloon's center of mass, no work is done on the balloon.

Both of these claims are false. Question (1) refers to a simple, everyday experience that unfortunately cannot be analyzed by means of traditional physics teaching without the introduction of additional work-like quantities and energy-like equations. The upward force on the boy that projects him into the air is the normal force on his feet from the ground. The center of mass of the boy indeed moves through an upward displacement. The normal force, however, goes through no displacement in the reference frame of the ground, and therefore no work is done by this force on the boy. The change in the boy's kinetic energy does not come from work done on the system of the boy. This is a case of a deformable system. Other cases include a person climbing stairs or a ladder, a girl pushing off a wall while standing on a skateboard, and a piece of putty slamming into a wall. In all of these cases, no work is done by the contact force, because there is no displacement of the point of application of the force

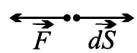
Example 15. A block kept on rough surface is being pulled by force F, as shown



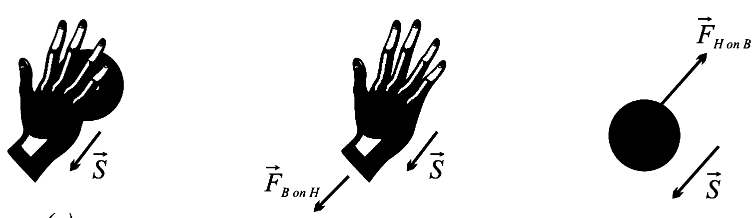
Work by F is positive



Work by friction is negative



Meaning of Negative and Positive Work



(a) (b) (c)

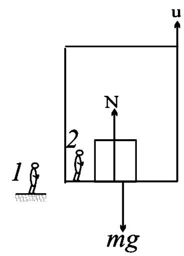
Ball does positive work on hand	Hand does negative work on ball

When you catch a ball as in figure (a), your hand and the ball move together with the same displacement $\vec{s}$ (figure *b*). The ball exerts a force $\vec{F}_{B\,on\,H}$ on your hand in the same direction as the hand's displacement, so the work done by the ball on your hand is positive. But by Newton's third law your hand exerts an equal and opposite force $\vec{F}_{H\,on\,B} = -\vec{F}_{B\,on\,H}$ on the ball (figure c). This force, which slows the ball to a stop opposite to the ball's displacement. Thus the work done by your hand on the ball is negative. Because your hand and the ball have the same displacement, the work that your hand does on the ball is just the negative of the work that the ball does on your hand.

Caution: Always specify exactly which force is doing the work, and on what. When you lift a book, you exert an upward force on the book and the book's displacement is upward, so the work done by the lifting force on the book is positive. But the work done by the gravitational force (weight) on a book being lifted is negative because the downward gravitational force is opposite to the upward displacement.

(iii) Work depends on reference frame because displacement is relative. (Remember force is not dependent on reference frame)

***Example* 16.** A lift is going up with constant velocity. We will calculate work from two reference frames. In both reference frame



$N\text{-}mg = 0 \Rightarrow N = mg$

Work Done in 2nd ref. frame = 0

Work Done in 1st ref. frame = N·ut

(We can observe that work is dependent on reference frame.)

3. **Calculation of work**

Case 1. When Force is uniform

$$dW = \vec{F}\cdot d\vec{S}$$

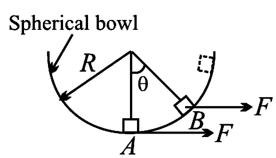
$$W = \int dW = \int \vec{F}\cdot d\vec{S} = \vec{F}\cdot\int d\vec{S}$$

$$W = \vec{F}\cdot\vec{S} \text{ (This is true only for uniform forces)}$$

$$W = F(S\cos\theta) = (F\cos\theta)S$$

Component of $\vec{S}$ along $\vec{F}$ Component of $\vec{S}$ along $\vec{F}$

***Example* 17.** A block of mass *m* is taken from *A* to *B* along spherical bowl.

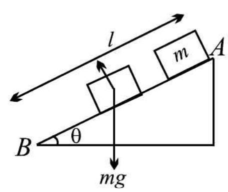


Work Done by gravity $= -mgR(1-\cos\theta)$

Work Done by force $F = FR(\sin\theta)$

Work Done by normal = 0

***Example* 18.** Find work done by gravity and normal when block comes from *A* to *B*



$W_g = mg(l\sin\theta) = (mg\sin\theta)l$

$W_N = 0$ *(because displacement is perpendicular to Force)*

Example 19. A particle is moving along a straight line from point A to point B with position vectors $2\hat{i}+7\hat{j}-3\hat{k}$ and $5\hat{i}-3\hat{j}-6\hat{k}$ respectively. One of the force acting on the particle is $\vec{F}=20\hat{i}-30\hat{j}+15\hat{k}$. Find the work done by this force.

Solution $\vec{W}=\vec{F}.\vec{d}$ $\quad \vec{d}=3\hat{i}-10\hat{j}-3\hat{k}$

$F=20\hat{i}-30\hat{j}+15\hat{k}$

$\vec{W}=60+300-45=315$ **Ans.**

Case 2. When Force is non–uniform (either magnitude or direction)

$dW=\vec{F}\cdot d\vec{S} \Rightarrow W=\int \vec{F}\cdot d\vec{S}$ ($\vec{F}$ cannot come out of integer since it is varying)

Example 20 $\vec{F}=x\hat{i}+y^2\hat{j}$. Particle moves from (1, 2) to (–3,4)

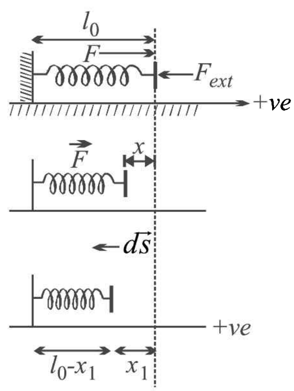
$dW=\vec{F}\cdot d\vec{S}$ $\quad (d\vec{S}=dx\hat{i}+dy\hat{j})$

$dW=xdx+y^2dy$

$$W=\int_1^{-3} xdx+\int_2^4 y^2dy=\left.\frac{x^2}{2}\right|_1^{-3}+\left.\frac{y^3}{3}\right|_2^4=\frac{68}{3}$$

* If force is not expressed as function of (x,y,z) then also we can solve problem by expressing force and displacement in same format.

SPRING FORCE



Natural length of spring is l_0.

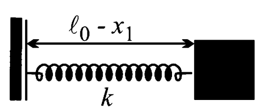
Similarly, when we compress spring by x_1 from natural length, then work done by spring force.

$\vec{F}=kx\hat{i}$

$d\vec{S}=(dx)(-\hat{i})$ {dx is +ve as x is increasing}

$dW=\vec{F}\cdot d\vec{S}$

$$\int dW=-\int_0^{x_1} kx\,dx=-\frac{1}{2}k\,x_1^2$$

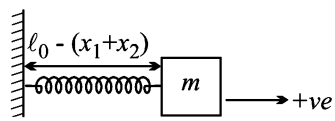


Example 21. Find work done by spring if we compress it further by x_2.

Solution $\vec{F}=kxi$ $\quad d\vec{S}=dx(-\hat{i})$

$dW=\vec{F}\cdot d\vec{S}=-kx\,dx$

$$W=-k\int_{x_1}^{(x_2+x_1)} x\,dx=-\frac{1}{2}k[(x_2+x_1)^2-x_1^2]$$



Basic Concept of Work Lies in Following Lines–Draw the force at proper point where it acts that give proper importance to the point of application of force.

Think independently for displacement of point of application of force, Instead of relating the displacement of applicant point with force relate it with the observer or reference frame in which work is calculated.

W = (Force vector)·(Displacement vector of point of application of force as seen by observer)

KINETIC ENERGY

***Definition*:** Kinetic energy is the internal capacity of doing work of the object by virtue of its motion. Kinetic energy is a scalar property that is associated with state of motion of an object. An aeroplane in straight and level flight has kinetic energy of translation and a rotating wheel on a machine has kinetic energy of rotation. If a particle of mass m is moving with speed 'v' much less than the speed of the light than the kinetic energy 'K' is given by

$$K = \frac{1}{2}mv^2$$

Important Points for K.E.

1. As mass m and v^2 ($\vec{v}.\vec{v}$) are always positive, kinetic energy is always positive scalar i.e, kinetic energy can never be negative.
2. The kinetic energy depends on the frame of reference,

$$K = \frac{p^2}{2m} \text{ and } P = \sqrt{2mk}\ ; P = \text{linear momentum}$$

The speed v may be acquired by the body in any manner. The kinetic energy of a group of particles or bodies is the sum of the kinetic energies of the individual particles. Consider a system consisting of n particles of masses m_1, m_2, ..., m_n. Let v_1, v_2, ... , v_n be their respective velocities. Then, the total kinetic energy E_k of the system is given by

$$E_k = \frac{1}{2} m_1v_1^2 + \frac{1}{2}m_2v_2^2 + ... + \frac{1}{2}m_nv_n^2$$

If n is measured in gram and v in cm s^{-1}, then the kinetic energy is measured in erg. If m is measured in kilogram and v in $m\ s^{-1}$, then the kinetic energy is measured in joule. It may be noted that the units of kinetic energy are the same as those of work. Infact, this is true of all forms of energy since they are inter-convertible.

Table 6.1: *Typical kinetic energies (K)*

S.No.	Object	Mass (kg)	Speed (m s–1)	K(J)
1	Air molecule	»10–26	500	» 10^{-21}
2	Rain drop at terminal speed	3.5×10^{-5}	9	1.4×10^{-3}
3	Stone dropped from 10 m	1	14	10^2
4	Bullet	5×10^{-5}	200	10^3
5	Running athlete	70	10	3.5×10^3
6	Car	2000	25	6.3×10^5

Relation Between Momentum and Kinetic Energy

Consider a body of mass m moving with velocity v. Linear momentum of the body, $p = mv$

Kinetic energy of the body, $E_k = \frac{1}{2}mv^2$

$$E_k = \frac{1}{2}(m^2v^2)$$

or $$E_k = \frac{p^2}{2m} \quad \text{or} \quad p = \sqrt{2mE_k}$$

Example 22. The kinetic energy of a body is increased by 21%. What is the percentage increase in the magnitude of linear momentum of the body?

Solution

$$E_{k2} = \frac{121}{100}E_{k1} \quad \text{or} \quad \frac{1}{2}m\,v_2^2 = \frac{121}{100}\frac{1}{2}mv_1^2$$

$$\text{or} \quad v_2 = \frac{11}{10}v_1 \quad \text{or} \quad mv_2 = \frac{11}{10}mv_1$$

$$\text{or} \quad p_2 = \frac{11}{10}p_1 \quad \text{or} \quad \frac{p_2}{p_1} - 1 = \frac{11}{10} - 1 = \frac{1}{10}$$

$$\text{or} \quad \frac{p_2 - p_1}{p_1} \times 100 = \frac{1}{10} \times 100 = 10$$

So, the percentage increase in the magnitude of linear momentum is 10%.

Example 23. The linear momentum of a body is increased by 10%. What is the percentage change in its kinetic energy?

[**Ans.** Percentage increase in kinetic energy = 21%]

$$\left[\textbf{Hint}: mv_2 = \frac{110}{100}mv_1,\ v_2 = \frac{11}{10}v_1,\ \frac{E_1}{E_2} = \left(\frac{11}{10}\right)^2 = \frac{121}{100}\right.$$

$$\left.\textit{Percentage increase in kinetic energy} = \frac{E_1 - E_1}{E_1} \times 100\right] = 21\%$$

POTENTIAL ENERGY

Definition: Potential energy is the internal capacity of doing work of a system by virtue of its configuration.

In case of conservative force (field) potential energy is equal to negative of work done by the conservative force in shifting the body from some reference position to given position.

Therefore, in case of conservative force

$$\int_{U_1}^{U_2} dU = \int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} \text{ i.e.,} \qquad U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} = -W$$

Whenever and wherever possible, we take the reference point at ∞and assume potential energy to be zero there, i.e., If we take $r_1 = \infty$ and $U_1 = 0$ then

$$U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r} = -W$$

Important Points for K.E.

1. Potential energy can be defined only for conservative forces. It has no relevance for non-conservative forces.
2. Potential energy can be positive or negative, depending upon choice of frame of reference.
3. Potential energy depends on frame of reference but change in potential energy is independent of reference frame.
4. Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.
5. It is a function of position and does not depend on the path.

Types of Potential Energy

(a) **Elastic Potential Energy:** It is the energy associated with state of compression or expansion of an elastic (spring like) object and is given by:

$$U = \frac{1}{2}ky^2$$

where k is force constant and '*y*' is the stretch or compression. Elastic potential energy is always positive.

(b) **Electric Potential Energy:** It is the energy associated with charged particles that interact via electric force. For two point charges q_1 and q_2 separated by a distance '*r*',

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

As charge can be positive or negative, therefore electric potential energy can also be positive or negative.

(c) **Gravitational Potential Energy:** It is due to gravitational force. For two particles of masses m_1 and m_2 separated by a distance '*r*', it is given by:

$$U = -G\frac{m_1 m_2}{r}$$

which for a body of mass '*m*' at height '*h*' relative to surface of the earth reduces to $U=mgh$

Gravitational potential energy can be positive or negative.

MECHANICAL ENERGY

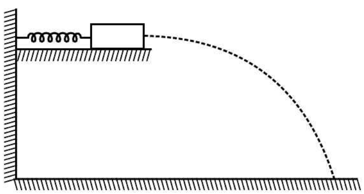
Definition: Mechanical energy '*E*' of an object or a system is defined as the sum of kinetic energy '*K*' and potential energy '*U*', i.e.,

$$E = K + U$$

Important Points for K.E.

1. It is a scalar quantity having dimensions $[ML^2T^{-2}]$ and SI units joule.
2. It depends on frame of reference.
3. A body can have mechanical energy without having either kinetic energy or potential energy. However, if both kinetic and potential energies are zero, mechanical energy will be zero. The converse may or may not be true, i.e., if $E = 0$ either both PE and KE are zero or PE may be negative and KE may be positive such that $KE + PE = 0$.
4. As mechanical energy $E = K + U$, i.e., $E - U = K$. Now as K is always positive, $E - U \geq 0$,i.e., for existence of a particle in the field, $E \geq U$.
5. As mechanical energy $E = K + U$ and K is always positive, so, if 'U' is positive 'E' will be positive. However, if potential energy U is negative, 'E' will be positive if $K > |U|$ and E will be negative if $K < |U|$ i.e., mechanical energy of a body or system can be negative, and negative mechanical energy means that potential energy is negative and in magnitude it is more than kinetic energy. Such a state is called bound state, e.g., electron in an atom or a satellite moving around a planet are in bound state.

Example 24. A small block of mass 100 g is pressed against a horizontal spring fixed at one end to compress the spring through 5.0 cm (figure). The spring constant is 100 N/m. When released, the block moves horizontally till it leaves the spring. Where will it hit the ground 2 m below the spring ?



Solution When block released, the block moves horizontally with speed V till it leaves the spring.

By energy conservation $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$V^2 = \frac{kx^2}{m} \quad \Rightarrow \quad V = \sqrt{\frac{kx^2}{m}}$$

Time of flight $t = \sqrt{\frac{2H}{g}}$

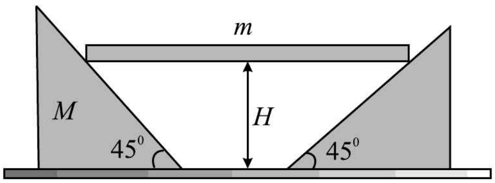
So, horizontal distance travelled from the free end of the spring is $V \times t$

$$= \sqrt{\frac{kx^2}{m}} \times \sqrt{\frac{2H}{g}}$$

$$= \sqrt{\frac{100 \times (0.05)^2}{0.1}} \times \sqrt{\frac{2 \times 2}{10}} = 1\,m$$

So, At a horizontal distance of 1 m from the free end of the spring.

Example 25. A rigid body of mass m is held at a height H on two smooth wedges of mass M each of which are themselves at rest on a horizontal frictionless floor. On releasing the body it moves down pushing aside the wedges. The velocity of recede of the wedges from each other when rigid body is at a height h from the ground is

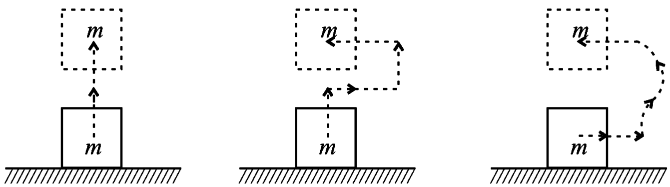


(A) $\sqrt{\dfrac{2mg(H-h)}{m+2M}}$ (B) $\sqrt{\dfrac{2mg(H-h)}{2m+M}}$ (C) $\sqrt{\dfrac{8mg(H-h)}{m+2M}}$ (D) $\sqrt{\dfrac{8mg(H-h)}{2m+M}}$

[Ans. (C)]

CONSERVATIVE FORCES

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.



(a) (b) (c)

Consider a body of mass m being raised to a height h vertically upwards as show in above figure. The work done is mgh. Suppose we take the body along the path as in (b). The work done during horizontal motion is zero. Adding up the works done in the two vertical parts of the paths, we get the result mgh once again. Any arbitrary path like the one shown in (c) can be broken into elementary horizontal and vertical portions. Work done along the horizontal parts is zero. The work done along the vertical parts add up to mgh. Thus, we conclude that the work done in raising a body against gravity is independent of the path taken. It only depends upon the initial and final positions of the body. We conclude from this discussion that the force of gravity is a conservative force.

Examples of Conservative forces

(i) Gravitational force, not only due to the Earth but in its general form as given by the universal law of gravitation, is a conservative force.

(ii) Elastic force in a stretched or compressed spring is a conservative force.

(iii) Electrostatic force between two electric charges is a conservative force.

(iv) Magnetic force between two magnetic poles is a conservative forces.

Forces acting along the line joining the centres of two bodies are called central forces. Gravitational force and Electrostatic forces are two important examples of central forces. Central forces are conservative forces.

Properties of Conservative Forces

(i) Work done by or against a conservative force depends only on the initial and final positions of the body.

(ii) Work done by or against a conservative force does no depend upon the nature of the path between initial and final positions of the body.

If the work done a by a force in moving a body from an initial location to a final location is independent of the path taken between the two points, then the force is conservative.

(iii) Work done by or against a conservative force in a round trip is zero.

If a body moves under the action of a force that does no total work during any round trip, then the force is conservative; otherwise it is non-conservative.

The concept of potential energy exists only in the case of conservative forces.

(iv) The work done by a conservative force is completely recoverable.

Complete recoverability is an important aspect of the work of a conservative force.

Conservative Force & Potential Energy

$F_s = -\partial U/\partial s$, i.e. the projection of the field force, the vector $\boldsymbol{F}$, at a given point in the direction of the displacement dr equals the derivative of the potential energy U with respect to a given direction, taken with the opposite sign. The designation of a partial derivative $\partial/\partial s$ emphasizes the fact of deriving with respect to a definite direction.

So, having reversed the sign of the partial derivatives of the function U with respect to x, y, z, we obtain the projection F_x, F_y and F_z of the vector $\boldsymbol{F}$ on the unit vectors $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$. Hence, one can readily find the vector itself: $\vec{F} = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$, or

$$\vec{F} = -\left(\frac{\partial U}{\partial x}i + \frac{\partial U}{\partial y}j + \frac{\partial U}{\partial z}k\right).$$

The quantity in parentheses is referred to as the scalar gradient of the function U and is denoted by grad U or ∇U. We shall use the second, more convenient, designation where ∇ ("nabla") signifies the symbolic vector or operator

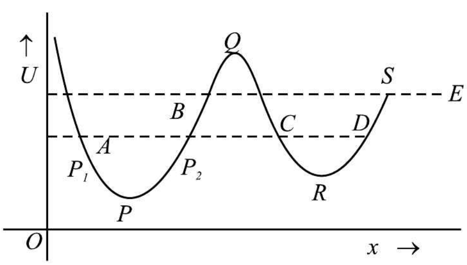
$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

Potential Energy Curve

(a) A graph plotted between the *PE* a particle and its displacement from the centre of force field is called *PE* curve.

(b) Using graph, we can predict the rate of motion of a particle at various positions.

(c) Force on the particle is $F_{(x)} = -\dfrac{dU}{dx}$



Case 1. On increasing x, if U increases, force is in (–) ve x direction i.e. attraction force.

Case 2. On increasing x, if U decreases, force is in (+) ve x-direction i.e. repulsion force.

Different Positions of a Particle

Position of Equilibrium

If net force acting on a body is zero, it is said to be in equilibrium. For equilibrium $\frac{dU}{dx} = 0$. Points P, Q R and S are the states of equilibrium positions.

Types of Equilibrium

(a) Stable equilibrium

When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.

Necessary conditions: $-\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} = +\text{ve}$

(b) Unstable Equilibrium

When a particle is displaced slightly from a position and force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.

Condition: $-\frac{dU}{dx} = 0$ potential energy is maximum i.e. $= \frac{d^2U}{dx^2} = -\text{ ve}$

(c) Neutral equilibrium

In the neutral equilibrium potential energy is constant. When a particle is displaced from its position it does not experience any force acting on it and continues to be in equilibrium in the displaced position. This is said to be neutral equilibrium.

Explanatory Notes on Potential Energy

Definition: It is defined as negative of work done by conservative forces.

Formula: $\vec{F}$ represents force for which we are writing potential energy.

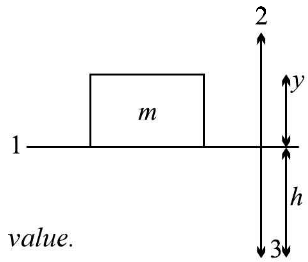
$$dU = -\vec{F}\cdot d\vec{S}$$

$$\int_1^2 dU = -\int_1^2 \vec{F}\cdot d\vec{S} \qquad \text{thus} \qquad U_2 - U_1 = -\int_1^2 \vec{F}\cdot d\vec{S}$$

Purpose: By defining PE we can avoid repeated calculation of work for conservative forces and since PE depends only on position (initial and final), we can directly write effect of conservative forces in terms of their respective PE's.

We will define PE for gravity and spring.

Gravity



$$U_2 - U_1 = -\int_0^h mg(-\hat{j})dh(\hat{j})$$

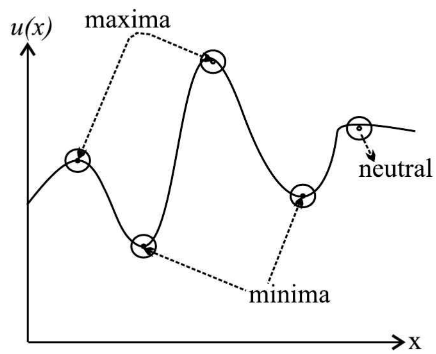
$$U_2 - U_1 = mgh$$

$$U_3 - U_1 = -mgh \text{(similarly)}$$

Emphasise that by definition we can only find difference of PE *not absolute value.*

If we assume $U_1 = 0$ then $U_2 = mgh$; $U_3 = -mgh$

EQUILIBRIUM



If F and U are dependent only on one variable $\vec{F} = -\frac{dU}{dx}(\hat{x})$

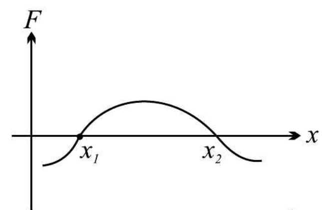
Thus, if we say equilibrium $\vec{F} = 0 \Rightarrow \frac{dU}{dx} = 0$

i.e., maxima or minima of *PE* represents equilibrium. Maxima is unstable equilibrium and minima is stable equilibrium.

i.e. $\frac{d^2U}{dx^2} < 0 \Rightarrow$ maxima unstable equilibrium

$\frac{d^2U}{dx^2} > 0 \Rightarrow$ minima stable equilibrium

$\frac{d^2U}{dx^2} = 0 \Rightarrow$ neutral equilibrium



x_1 is unsatble and x_2 is stable.

***Example* 26.** The potential energy between two atoms in a molecule is given by, $U_{(x)} = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are positive constants and x is the distance between the atoms. The system is in stable equilibrium when

(A) $x = 0$ (B) $x = \frac{a}{2b}$ (C) $x = \left(\frac{2a}{b}\right)^{1/6}$ (D) $x = \left(\frac{11a}{5b}\right)$

Solution (C) Given that, $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

We, know $F = -\frac{du}{dx} = (-12)\, a\, x^{-13} - (-6b)\, x^{-7} = 0$

or $\frac{-6b}{x^7} = \frac{12a}{x^{13}}$ or $x^6 = 12a/6b = 2a/b$ or $x = \left(\frac{2a}{b}\right)^{1/8}$

***Example* 27.** The potential energy of a particle in a certain field has the form $U = a/r^2 - b/r$, where a and b are positive constant, r is the distance from the centre of the field. Find the value of r_0 corresponding to the equilibrium position of the particles ; examine whether this position is stable.

Solution $U(r) = a/r^2 - b/r$

$$\text{Force} = F = -\frac{dU}{dr} = -\left(\frac{-2a}{r^3} + \frac{b}{r^2}\right); \qquad F = -\frac{(br - 2a)}{r^3}$$

$$\text{At equilibrium } F = \frac{dU}{dr} = 0$$

Hence $br - 2a = 0$ at equilibrium

$r = r_0 = 2a/b$ corresponds to equilibrium

At stable equilibrium, the potential energy is minimum and at unstable equilibrium, it is maximum. From calculus, we know that for minimum value around a point $r = r_0$, the first derivative should be zero and the second derivative should be positive.

For minimum potential energy,

$$\frac{dU}{dr} = 0 \text{ and } \frac{d^2U}{dr^2} > 0 \quad \text{at} \quad r = r_0$$

we have already se $dU/dr = 0$ to get $r = r_0 = 2a/b$.

Let us investigate the second derivative.

$$\frac{d^2U}{dr^2} = \frac{d}{dr}\left(\frac{dU}{dr}\right) = \frac{d}{dr}\left(-\frac{2a}{r^3} + \frac{b}{r^2}\right) = \frac{6a}{r^4} - \frac{2b}{r^3} \text{ At } r = r_0 = 2a/b,$$

$$\frac{d^2U}{dr^2} = \frac{6a - 2br_0}{r_0^4} = \frac{2a}{r_0^4} > 0.$$

Hence the potential energy function $U(r)$ has a minimum value at $r_0 = 2a/b$. The system has a stable equilibrium at minimum potential energy state.

NON-CONSERVATIVE FORCES

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

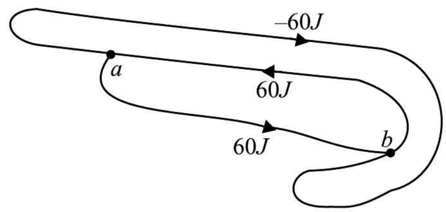
The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does no depend only on the initial and final positions. Note that the work done by frictional force in a round trip is not zero.

The velocity-dependent forces such as air resistance, viscous force, magnetic force etc., are non conservative forces.

S. No.	Conservative forces	Non-Conservative forces
1	Work done does not depend upon path	Work done depends on path.
2	Work done in round trip is zero.	Work done in a round trip is not zero.
3	Central in nature.	Forces are velocity-dependent and retarding in natural.

S. No.	Conservative forces	Non-Conservative forces
4	When only a conservative force acts within a systream, the kinetic engry and potential energy can change. However, their sum, the merchanical energy of the system, does not change.	Work done against a non-conservative force may be dissipated as heat energy.
5	Work done is completely recoverable.	Work done in not completely recoverable.

Example 28. The figure shows three paths connecting points a and b. A single force F does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force F conservative?



Solution No

Explanation: For a conservative force, the work done in a round trip should be zero.

Example 29. The potential energy of a conservative system is given by $U = ax^2 - bx$ where a and b are positive constants. Find the equilibrium position and discuss whether the equilibrium is stable, unstable or neutral.

Solution In a conservative field $F = -\dfrac{dU}{dx}$

$$\therefore \quad F = -\frac{d}{dx}(ax^2 - bx) = b - 2ax$$

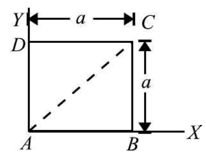
For equilibrium $F = 0$

$$\text{or} \quad b - 2ax = 0 \quad \therefore \quad x = \frac{b}{2a}$$

From the given equation we can see that $\dfrac{d^2U}{dx^2} = 2a$ (positive), i.e., U is minimum.

Therefore, $x = \dfrac{b}{2a}$ is the stable equilibrium position. **[Ans.]**

Example 30. A force $F = x^2y^2\hat{i} + x^2y^2\hat{j}$ (N) acts on a particle which moves in the XY plane.



(a) Determine if F is conservative and

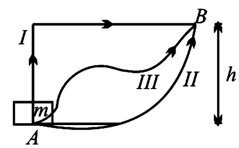
(b) Find the work done by F as it moves the particle from A to C (fig.) along each of the paths ABC, ADC, and AC.

[Ans. (b) $W_{ABC} = W_{ADC} = \dfrac{a^5}{3}$ (J), $W_{AC} = \dfrac{2a^5}{5}$ (J) **]**

To check whether a force is conservative. Explain this by the concept of potential energy.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} = 0 \qquad \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{i} + \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right)\hat{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k} = 0$$

Explanatory Notes on Nature of Forces



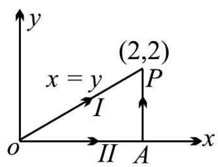
1. ***Conservative forces***: Forces for which calculation of work is independent of path taken by body. e.g. gravity, spring.

   In this case work done by force of gravity is same for taking body from A to B by any path (It can be shown mathematically)

   In conservative forces total work done for around closed path = 0

2. ***Non–conservative forces***: Forces for which calculation of work depends on path not just on initial and final position eg. friction.

   $W_{fI} = -\mu mgl$; $W_{fII} = -\mu mg(3l)$



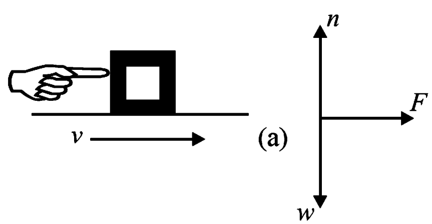
Example 31. $\vec{F} = xy\hat{i} + xy\hat{j}$

Calculate work required to take particle from (0,0) to (2,2) (give it w/o path) Then remind them it cannot be calculated w/o path then show

$$W_{I\,OP} \neq W_{II\,OAP}$$

WORK-ENERGY THEOREM

The total work done on a body by external forces is related to the body's displacement that is, to changes in its position.

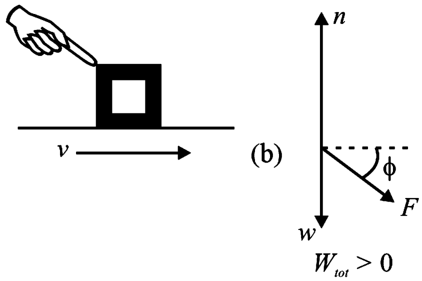


$W_{tot} > 0$

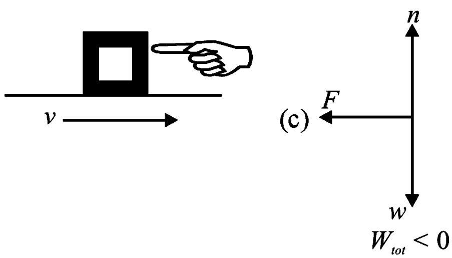
But the total work is also related to changes in the speed of the body. To see this, consider figure, which shows several example of a block sliding on a frictionless table. The forces acting on the block are its weight $\vec{w}$, the normal force $\vec{n}$ and the force $\vec{F}$ exerted on it by the hand.

In figure (a) the net force on the block is in the direction of its motion. From Newton's second law, this means that the block speeds up; from equation, this also means that the total work W_{tot} done on the block is positive. (a) The total work is also positive in figure (b), but only the component

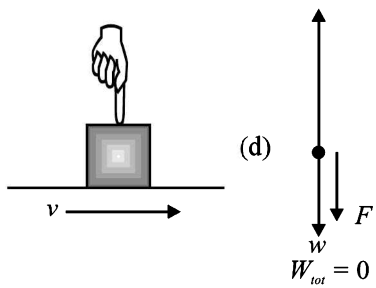
$F \cos \phi$ contributes to W_{tot}. The block again speed up, and this same component $F \cos \phi$ is what causes the acceleration.



The total work is negative in figure (c) because the net force opposes the displacement ; in this case the block slows down.



The net force is zero in figure (d), so the speed of the block stays the same and the total work done on the block is zero . We can conclude that when a particle undergoes a displacement, it speed up if $W_{tot} > 0$, slow down if $W_{tot} < 0$, and maintains the same speed if $W_{tot} = 0$.



A block sliding on a frictionless table.

(a) The net force causes the speed to increase and does positive work.

(b) Again the net force causes the speed to increase and does positive work.

(c) The net force opposes the displacement, causes the speed to decrease, and does negative work.

(d) The net force is zero and does no work, and the speed is constant.

Derivation:

For a particle: $\Sigma \vec{F} = m\,\vec{a}$

$$\Sigma \vec{F} \cdot d\vec{S} = m \frac{d\vec{V}}{dt} \cdot d\vec{S}$$

Work Done by resultant Force = $\int(\Sigma\vec{F})d\vec{S} = m\int_{v_i}^{v_f}\vec{v}\,d\vec{v}$

Summation of work by all the forces = $\Sigma(\int\vec{F}\cdot d\vec{S}) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$$\Sigma W = k_f - k_i$$

i.e., sum of work done by all the forces on a particle is equal to change in kinetic energy of the particle

So, the kinetic energy of a particle is equal to the total work that was done to accelerate it from rest to its present speed.

The kinetic energy of a particle is equal to the total work that particle can do in the process of being brought to rest. This is why you pull your hand and arm backward when you catch a ball. As the ball comes to rest, it does an amount of work (force times distance) on your hand equal to the ball's initial kinetic energy. By pulling your hand back, you maximize the distance over which the force acts and so minimize the force on your hand.

According to work-energy theorem, the work done by all the forces on a particle is equal to the change in its kinetic energy.

$W_C + W_{NC} + W_{PS} = \Delta K$

Where, W_C is the work done by all the conservative forces.

W_{NC} is the work done by all non-conservative forces.

W_{PS} is the work done by all psuedo forces.

Modified Form of Work-Energy Theorem

We know that conservative forces are associated with the concept of potential energy, that is

$$W_C = -\Delta U$$

So, Work-Energy theorem may be modified as

$$W_{NC} + W_{PS} = \Delta K + \Delta U$$

$$W_{NC} + W_{PS} = \Delta E$$

When a particle is acted upon by various forces and undergoes a displacement, then its kinetic energy changes by an amount equal to the total (net) work W_{net} done on the particle by all the forces.

i.e. $W_{net} = K_f - K_i = \Delta K$ ...(i)

The above expression is called the work-energy theorem.

The expression (i) is valid whether the forces are constant or varying and whether the path followed by the particle is straight or curved.

The expression (i) can be further elaborated as:

$W_C + W_{NC} + W_{Oth} = \Delta K$ ...(ii)

where W_C = work done by conservative forces

W_{NC} = work done by non-conservative forces

W_{Oth} = work done by all other forces which are not included in the category of conservative, non-conservative and pseudo forces.

Since $W_C = -\Delta U$ (definition of potential energy), therefore, the expression (ii) can be modified as

$$W_{NC} + W_{Oth} = \Delta K + \Delta U = \Delta(K + U) \quad ...(iii)$$

The term $K + U = E$ is called the mechanical energy of the system.

Conservation of Mechanical Energy

If no forces other than the conservative do work on a particle (body), then the sum total of kinetic and potential energy (also called mechanical energy) is conserved.

That is, $E = K + U = \text{constant}$

or $\Delta K + \Delta U = 0$

or $K_i + U_i = K_f + U_f$ ...(iv)

the equation (iv) is a special case of work-energy theorem when $W_{NC} + W_{Oth} = 0$.

Example 32. A 60 gm tennis ball thrown vertically up at 24 m/s rises to a maximum height of 26 m. What was the work done by resistive forces?

Solution $w_g + w_{res} = (0 - \frac{1}{2}mu^2)$ $\quad -mgh + w_{res} = -\frac{1}{2}mu^2$

$w_{res} = 0.06 \times 10 \times 26 \quad -\frac{1}{2} \times 0.06 \times 24 \times 24 = -1.68$ J]

Example 33. A force of $(3\hat{i} - 1.5\hat{j})$ N acts on a 5 kg body. The body is at a position of $(2\hat{i} - 3\hat{j})$ m and is travelling at 4 ms^{-1}. The force acts on the body until it is at the position $(\hat{i} + 5\hat{j})$ m. Assuming no other force does work on the body, the final speed of the body.

Solution Given Mass of the body = 5 kg

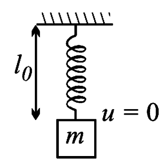
Force $\vec{F} = 3\hat{i} - 1.5\hat{j}$

Now displacement $\overrightarrow{\Delta s} = \{(\hat{i} + 5\hat{j}) - (2\hat{i} - 3\hat{j})\}\, m = (-\hat{i} + 8\hat{j})\, m$

From Work Energy principle

$W = \vec{F} \cdot \overrightarrow{\Delta s} = \frac{1}{2}m(v^2 - u^2) \quad \Rightarrow \quad v = \sqrt{10}$ m/s]

Example 34. A block is connected to spring while spring is in relaxed state. Find maximum extension of spring.

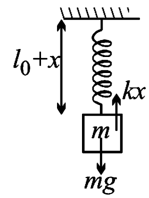


Solution Forces acting on block are spring and gravity

Work done for x displacement

$$W = mgx - \frac{1}{2}kx^2$$

for max. displacement velocity should become zero



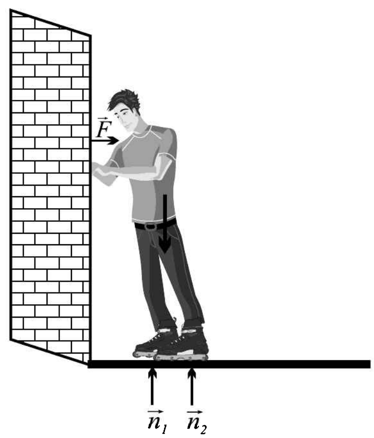
$$k_i = 0 \qquad k_f = 0$$

$$mgx - \frac{1}{2}kx^2 = 0$$

$$x = \frac{2mg}{k}$$

Applying Work Energy Theorem on System

We have been careful to apply the work energy theorem only to bodies that we can represent as particles - that is, as moving point masses. The reason is that new complexeties appear for more complex systems that have to be represented in terms of many particles with different motions.Here's an exmaple.



Consider a man standing on frictionless roller skates on a level surface, facing a rigid wall. He pushes against the wall, setting himself in motion to the right. The forces acting on his are his weight $\vec{w}$, the upward normal forces $\vec{n}_1$ and $\vec{n}_2$ exerted by the ground on his skates, and the horizontal force $\vec{F}$ exerted on him by the wall. There is no vertical displacement, so $\vec{w}$, $\vec{n}_1$ and $\vec{n}_2$ do not work. The force $\vec{F}$ is the horizontal force that accelerates him to the right, but the parts of his body where that force is applied (the man's hands) do not move. Thus the force $\vec{F}$ also does no work. So where does the man's kinetic energy come from ?

The difficulty is that it's simply not correct to represent the man as a single point mass. For the motion as we've described, different parts of the man's body must have different motions; his hands are stationary against the wall while his torso(upper body) is moving away from the wall. The various parts of his body interact with each other, and one part can exert forces and do work on another part. Therefore the total kinetic energy of this composite system of body parts can change, even though no work is done by forces applied by bodies(such as the wall) that are outside the system. This would not be possible with a system that can be repesented as a single point particle.

While using Work Energy Theorem for a system or applying work energy equation on many particles together we must remember that work due to all the forces (external & internal) must be written.

Otherwise use this equation seperately on individual particles. (Although total work done by static friction, tension and normal contact force i.e. by action and reaction on a system will always be zero.)

According to newton's laws, the body moves under the influence of the external force only. Internal forces do not accelerate the body. But work can still be done by them, even if they do not accelerate the body. In the case of work energy theorem, we should be cautious. Do not forget to take into account the work done by the internal forces.

Example 35. To illustrate these points about work, let us take a simple daily life example. Suppose a boy of mass 40 kg is walking on a rough ground with a uniform acceleration of 1 m/s^2. We wish to find the work done on the boy when he moves a distance of 1 m starting from rest.

Solution It is clear that the horizontal acceleration of the boy is possible only by an external force. This force is friction force between him and the ground. We can imagine trying to walk on a smooth surface. We will slip!

This friction force $f = ma = 40\ N$.

But who does work on the boy? Obviously not the friction force. Then none of us need to eat anything, cars need not be supplied any petrol! Friction cannot do any work. But we studied that work is

$$W = \vec{F}.\vec{S} = FS\cos\theta$$

What is wrong here? Let us examine the motion of the boy more closely. When the boy walks, he does not slip his feet on the ground. Rather, he places one foot on the ground, lifts another foot and moves that foot further. The force of friction is not acting on the moving foot. It is acting on the foot which is in contact with the ground. So this is the case of a body where different parts of the body having different displacements.

Just now we said that in such cases, the work done by the force is $W = \vec{F}.\vec{S} = FS\cos\theta$. where S is the displacement of the point of application of the force. So although the boy as a whole is moving, the point at which friction force is being applied is not moving. On close examination, we can say that it is the muscles of the body who are rotating the legs, imparting the energy to the boy. Their work can be estimated with the help of the work energy theorem.

$$V^2 = U^2 + 2as = 0 + 2 \times 1 \times 1$$

$$W_{int} = \Delta K = \frac{1}{2} \times 40 \times 2 = 40\ J$$

Observe the same thing from the frame of the boy and explain it.

The work done by internal forces should be same as before. This should be! After all the work done by the boy reflects in his food consumption. This should be the same from every frame of reference.

Frictional Work

Consider a block sliding across a horizontal table and eventually coming to rest due to the frictional force exerted by the table.

As the kinetic energy of the block decreases, there is a corresponding increase in the internal energy of the system of block and the table. This increase in internal energy might be observed as a slight increase in the temperature of the surfaces of the block and the table. It is a common observation that kinetic friction between two surfaces causes an increases in the temperature, as for example in the case of holding a piece of metal against a grinding wheel or applying the brackes to an automobile or a bicycle (in which case both the brakes and the sliding tires can become warmer). You can even observe that effect by rubbing your hands together.

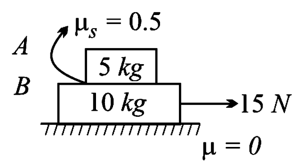
We might be tempted to write the magnitude of the work done by the frictional force as the product of the frictional force times the displacement through which the object moves: $|W_f| = fs$.

If $\Delta\mathbf{r}$ in the definition of work is identified as "the displacement of the object," it often follows in textbook and lecture discussions that the work done by friction on a block sliding on a surface is $W = -f_k d$, where f_k is the force of kinetic friction on the block and d is the distance through which the block moves relative to the surface. The negative sign indicates that the friction force is in the opposite direction to the displacement. This expression for work is then incorporated into the work-kinetic energy theorem for the block.

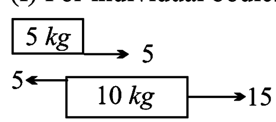
This approach ignores the fact that the displacement of the block is not the same as the many displacements of the friction force at a large number of contact points. This latter displacement is complicated and involves deformations of the lower surface of the block. It has to be noted that the work done by friction cannot be calculated because we cannot find the displacement at large number of contact point. even in the case of static friction, there is very small displacement at the contact point so the work done by static friction is also not zero. but for teaching in jee, we will assume the following for finding the work done by frction.

A student may have little difficulty with $W = -f_k d$, based on his or her understanding of evaluating the work done by any force by performing a path integral over the path followed by the object. In the case of a block sliding over a stationary surface, the friction force is always oppositely directed to each infinitesimal displacement of the block. For a constant friction force, this integral reduces to the product of the force and the length of the path (not the displacement).

Example 36. The force 15 N pulls the lower block for 2m. Find final speed.



Solution (I) For individual bodies



$w_A = 5 \times 2$

$w_B = 15 \times 2 + (-5) \times 2$

$$\Sigma W = 30 = \Delta kE_{sys} = \frac{1}{2} \times 10v^2 + \frac{1}{2} \times 5v^2; \; v = 2$$

(II) We know work done by static friction will be zero because action–reaction will be in opposite direction but displacement of contact point will be same.

Thus $\vec{f}_A \cdot d\vec{S}_A + \vec{f}_B \cdot d\vec{S}_B = 0$ because $d\vec{S}_A = d\vec{S}_B$ but $\vec{f}_A = -\vec{f}_B$

Thus $15 \times 2 = (1/2) \times 10v^2 + (1/2) \times 5v^2$

$v = 2$

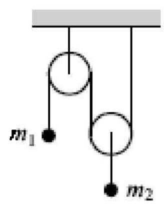
METHOD OF VIRTUAL WORK

The method of virtual work for finding constraint relation is very useful in complicated situations where visual inspection is difficult and number of strings is more.

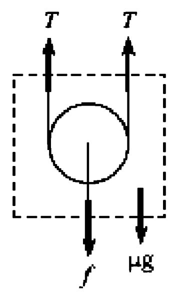
Step 1. Constraint forces are those forces whose work on the entire system is zero. To apply this method we should write the tension acting on each block.

Step 2. Displace each of the movable bodies in +ve direction by $S_A.S_B$ etc. Here we need not bother whether these displacements are physically possible or not. Automatically the analysis will tell the relationship between them.

Step 3. Find the work done by tension on each of the bodies. The sum total of all these works should be zero.



Assume that m_1 moves a distance S_1 down and m_2 moves a distance S_2 down.{ This is not physically possible, but we are dealing with vectors here. If the displacements are in opposite directions, the answer will be negative for them.



$$W_1 = \vec{F}.\vec{S} = Fs\cos\theta$$

$= TS_1.\cos 180$ŗ $= -TS_1$

Since the pulley is massless, the tension in the string connecting m_2 to the pulley can be found out

Newton's law for pulley

$$f = 2T$$

$W_2 = 2TS_2.\cos 180$ŗ $= -2TS_2$

- $W_1 + W_2 = 0$

  This principle that the work done by the string is 0 is called the principle of virtual work.

- Here we are actually using the fact that the work done by the two strings on the total system is 0. But that is as good, becuase sum of two zeroes will also be zero.

  $\Rightarrow \quad -TS_1 - 2\,TS_2 = 0$

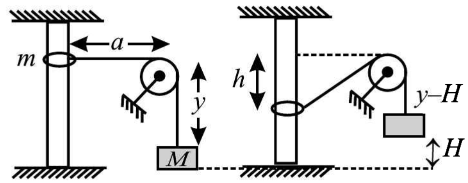
  $\Rightarrow \quad S_1 + 2S_2 = 0$

  $\Rightarrow \quad V_1 + 2V_2 = 0$

  $\Rightarrow \quad a_1 + 2a_2 = 0$

Principle of virtual work seems to be more complicated, but once we get an understanding of it, it becomes a very easy tool.

Example 37. In the figure shown, the ring starts moving down from rest. What will be the relation between the velocity of the ring and the velocity of the block at any position? What will be the distance that the ring moves before coming to rest?



Solution To find the relationship between the velocities of the block and the ring, we will use the concept of virtual work. We have already studied that the total work done by tension on a system is always zero. Assuming small displacements of the bodies when the angle made by the string is θ (displacement is assumed to be small so that the angle made by the string does not change appreciably)

$$-TS_R\cos\theta - TS_B = 0$$

$$S_R\cos\theta + S_B = 0$$

$$\Rightarrow \quad v_R\cos\theta + v_B = 0$$

Here we should be careful, the relationship between the small displacements is same as that of the velocities because we have divided the entire relationship by small time interval dt. But to obtain the relationship between accelerations, we have to differentiate this expression which will involve derivative of cosθ also because θ is also a variable.

Since tension does not do any work, only work here is done by the force of gravity.

$$W_g = \Delta K_1 + \Delta K_2$$

$$mgh - MgH = \frac{1}{2}mv_R^2 + \frac{1}{2}mv_B^2$$

The interesting thing to note here is that even if mass of the ring is less than that of the block, the ring will go down. Explain.

At the position of rest, $v_B = 0$. So from the equation of constrained motion, v_R is also 0.

$$mgh - MgH = 0.$$

also from the geometry, the original length of the string is

$$a + y = \ell.$$

$$\sqrt{a^2 + h^2} + y - H = \ell; \ \sqrt{a^2 + h^2} = a + H$$

$$h^2 = H^2 + 2aH$$

$$\Rightarrow \quad \left(\frac{MH}{m}\right)^2 = H^2 + 2aH$$

$$H = \frac{2aM^2}{M^2 - m^2}; \quad h = \frac{2aMm}{M^2 - m^2}$$

It is clear from the equation that the physically admissible solution is available only when $M > m$. If $M < mh$ comes out to be negative which is not possible. This means that if $M<m$, the block and the ring will never come to rest.

Example 38. A particle of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$ where $a = 5\ m^{-1/2}\ s^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2m$?

Solution $m = 0.5\ kg,\ v = ax^{3/2},\ a = 5\ m^{-1/2}\ s^{-1},\ W = ?$

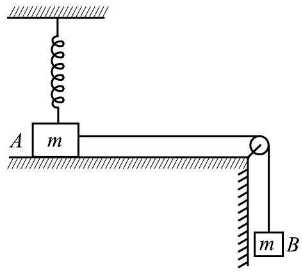
Initial velocity at $x = 0,\ v_0 = a \times 0 = 0$

Final velocity at $x = 2,\ v_2 = a \times 2^{3/2} = 5 \times 2^{3/2}$

Work done = Increase in kinetic energy

$$= \frac{1}{2} m\,(v_2^2 - v_0^2) = \frac{1}{2} \times 0.5\ [(5 \times 2^{3/2})^2 - 0] = 50\ \text{J}.$$

Example 39. Figure shows two blocks A and B, each having a mass of 320 g connected by a light string passing over a smooth light pulley. The horizontal surface on which the block A can slide is smooth. The block A is attached to a spring of spring constant 40 N/m whose other end is fixed to a support 40 cm above the horizontal surface. Initially, the spring is vertical and unstretched when the system is released to move. Find the velocity of the block A at the instant it breaks off the surface below it.

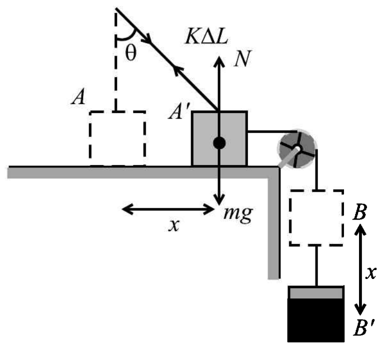


Take $g = 10\ \text{m/s}^2$.

Solution Let the block A start loosing contact with the surface below it at A' after travelling a distance x as shown in figure.

In this process the block B will shift from B to B' such that $BB' = AA' = x$ (as string is inextensible) and so there is a loss of gravitational potential energy = mgx.

This energy is partly stored as elastic potential energy in the spring which is stretched by ΔL and partly appears as kinetic energy of blocks A and B. So, by conservation of mechanical energy, we have



$$mgx = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}k(\Delta L)^2$$

or
$$v^2 = gx - \frac{k}{2m}(\Delta L)^2$$

...(i)

Now, for vertical equilibrium of block A at A',

$$N + F\cos\theta = mg$$

But as for spring $F = k\Delta L$ and for breaking off $N = 0$ the above equation reduces to

$$k\Delta L\cos\theta = mg \qquad ...(ii)$$

So, substituting the value of ΔL from Eq. (iii) in (ii) and solving for $\cos\theta$, we get

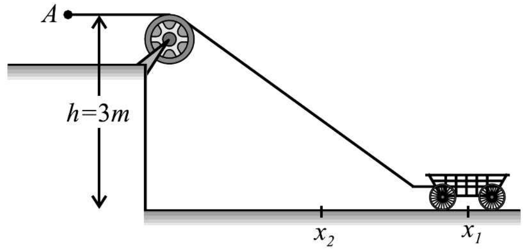
$$\cos\theta = 1 - \frac{mg}{kL} = 1 - \frac{0.32 \times 10}{10 \times 0.40} = \frac{4}{5}$$

So, that
$$\Delta L = \left(\frac{L}{\cos\theta} - L\right) = \frac{0.4 \times 5}{4} - 0.4 = 0.1\,m \text{ and } x = L\tan\theta = 0.4 \times \frac{3}{4} = 0.3\ \text{m}$$

Substituting these value of ΔL and x in Equation (ii),

$$v = \left[10 \times 0.3 - \frac{40 \times (0.1)^2}{2 \times 0.32}\right]^{1/2} = \sqrt{3 - 0.625} = 1.54 \text{ m/s} \textbf{ Ans.}$$

Example 40. Figure shows a light, inextensible string attached to a cart that can slide along a frictionless horizontal rail aligned along an x axis. The left end of the string is pulled over a pulley, of negligible mass and friction and fixed at height $h = 3$m from the ground level. The cart slides from $x_1 = 3\sqrt{3}$ m to $x_2 = 4$ m and during the move, tension in the string is kept constant 50 N. Find change in kinetic energy of the cart in joules. (Use $\sqrt{3} = 1.7$)



Solution Displacement of the point of 'A' of the string

$$= \sqrt{(3\sqrt{3})^2 + (3)^2} - \sqrt{4^2 + 3^2} = 6 - 5 = 1 \text{ m}$$

$$\Delta k = \text{Work done by tension} = 50 \times 1 = 50 \text{ Joule.}$$

Example 41. A bullet leaving the muzzle of a rifle barrel with a velocity v penetrates a plank and loses one fifth of its velocity. It then strikes second plank, which it just penetrates through. Find the ratio of the thickness of the planks supposing average resistance to the penetration is same in both the cases.

Solution Let R = resistance force offered by the planks,

t_1 = thickness of first plank,

t_2 = thickness of second plank.

For first plank:

Loss in KE = work against resistance

$$\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{4}{5}v\right)^2 = Rt_1 \Rightarrow \frac{1}{2}mv^2\left(\frac{9}{25}\right) = Rt_1 \quad \text{...(i)}$$

For second plank

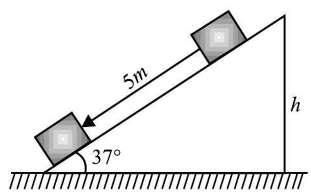
$$\frac{1}{2}m\left(\frac{4}{5}v\right)^2 - 0 = Rt_2 \quad \Rightarrow \quad \frac{1}{2}mv^2\left(\frac{16}{25}\right) = Rt_2 \quad \text{... (ii)}$$

dividing (I) & (II) $\Rightarrow \dfrac{t_1}{t_2} = \dfrac{9}{16}$

Example 42. A block is placed on the top of a plane inclined at 37° with horizontal. The length of the plane is 5 m. The block slides down the plane and reaches the bottom.

(a) Find the speed of the block at the bottom if the inclined plane is smooth.

(b) Find the speed of the block at the bottom if the coefficient of friction is 0.25



Solution Let h be the height of inclined plane

$\Rightarrow$ $h = 5 \sin 37° = 3$ m

(a) As the block slides down the inclined plane, it loses *GPE* and gains *KE*.

loss in *GPE* = gain in *KE*

mg (loss in height) $= KE_f - KE_i$

$$\Rightarrow \quad mgh = \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \quad v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 3} = 7.67 \text{ m/s}$$

Note

1. loss in energy - initial energy - final energy
2. gain in energy - final energy - initial energy

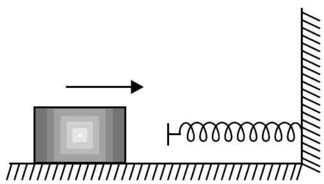
(b) As the block comes down, it loses *GPE*. It gains *KE* and does work against friction. loss in *GPE* = gain in *KE* + work done against friction

$$\Rightarrow \quad mgh = (1/2\ mv^2 - 0) + (\mu\ mg \cos 37°)\ s$$

$$\Rightarrow \quad 3mg = 1/2\ mv^2 + (0.25) \times mg \times 4/5 \times 5$$

$$\Rightarrow \quad v = \sqrt{4g} = 6.26 \text{ m/s}$$

Example 43. A 1.0 kg block collides with horizontal light spring of force constant 2 N/m. The block compresses the spring 4 m from the rest position. Assuming that the coefficient of kinetic friction between the block and the horizontal surface is 0.25, what was the speed of the block at the instant of collision ?



Solution When the block compresses the spring, let x m be the amount of compression, i.e. $x = 4$m.

Let v = velocity of the block when it collides with the spring.

Loss in *KE* of the block = (gain in elastic potential energy of the spring) + (work done against friction)

$$\Rightarrow \quad \frac{1}{2}mv^2 - 0 = \frac{1}{2}kx^2 + \mu\ mg\ x$$

$$\frac{1}{2}mv^2 = \frac{1}{2}(2)\ (4)^2 + 0.25 \times 1 \times 9.8 \times 4$$

$$v^2 = 51.6 \quad \Rightarrow \quad v = \sqrt{51.6} = 7.18 \text{ m/s}$$

Example 44. A pump is required to lift 1000 kg of water per minutes from a well 20 m deep and eject it at a rate of 20 m/s.

(a) How much work is done in lifting water ?

(b) How much work is done is giving it a *KE* ?

(c) What HP (horse power) engine is required for the purpose of lifting water?

Solution Work done in lifting water = gain in *PE* (potential energy)

work = 1000 × g × 20 = 1.96×10^5 *J* per minute

Work done (per minute) in giving it $KE = 1/2\ mv^2$

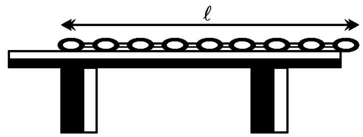
$= 1/2\ (1000)\ (20)^2 = 2 \times 10^5$ J per minute

Power of the engine = work done per second

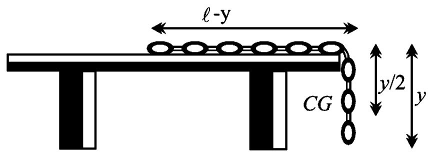
$1/60\ (1.96 + 2) \times 10^5$ J = $6./6 \times 10^3$ *W* (watts)

Since 1 *HP* = 746 *W*, *HP* required = 8.85

Example **45.** A uniform chain of length ℓ and mass m lies on a smooth table. A very small part of this chain hangs from the table, if begins to fall under the weight of hanging end. Find the velocity of chain when the length of hanging part becomes *y*.



Solution As the chain slips down from the table, the gravitational potential energy of hanging part decreases and gets converted into kinetic energy.



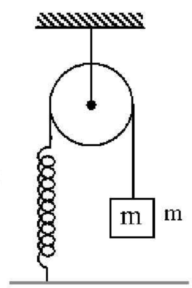
The part of the chain lying on the table does not lose any *GPE*.

Loss in *GPE* of hanging part = gain in *KE* of the chain = $1/2\ mv^2$

(mass of hanging part) *g* (loss in height of C.G) = $1/2\ mv^2$

$$\Rightarrow \quad \left(\frac{my}{\ell}\right) g \left(\frac{y}{2}\right) = \frac{1}{2} mv^2 \quad \Rightarrow \quad v^2 = \frac{gy^2}{\ell} \Rightarrow v = y\sqrt{\frac{g}{\ell}}$$

Example **46.** In the figure shown stiffness is *k* and mass of the block is *m*. The pulley is fixed. Initially the block *m* is held such that, the elongation in the spring is zero and then released from rest. Find the maximum elongation in the spring. Neglect the mass of the spring, pulley and that of the string.



Solution Let the maximum elongation in the spring be *x*, when the block is at position 2.

The displacement of the block *m* is also *x*. If E_1 and E_2 are the energies of the system when the block is at position 1 and 2 respectively. Then

$$E_1 = U_{1g} + U_{1s} + T_1$$

where U_{1g} = gravitational *P.E.* with respect to surface S.

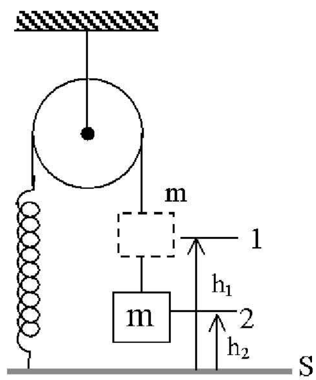
U_{1S} = *P.E.* stored in the spring.

T_1 = initial K.E. of the block.

$\Rightarrow \quad E_1 = mgh_1 + 0 + 0 = mgh_1$...(i)

and $\quad E_2 = U_{2g} + U_{2s} + T_2$

$= mgh_2 + \frac{1}{2}kx^2 + 0$ ...(ii)



From conservation of energy $E_1 = E_2$

$$\Rightarrow \qquad mgh_1 = mgh_2 + \frac{1}{2}kx^2$$

$$\Rightarrow \qquad \frac{1}{2}kx^2 = mg(h_1 - h_2) = mgx$$

$$\Rightarrow \qquad x = 2mg/k$$

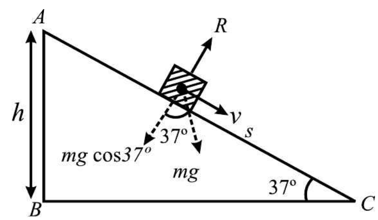
Example 47. A block is placed on the top of a plane inclined at 37° with horizontal. The length of the plane is 5 m. The block slides down theplane and reaches the bottom.

(a) Find the speed of the block at the bottom if the inclined plane is smooth.

(b) Find the speed of the block at the bottom if the coefficient of friction is 0.25

Solution Let h be the height of inclined plane

$$\Rightarrow \qquad h = 5 \sin 37° = 3 \text{ m}$$



(a) As the block slides down the inclined plane, it loses *GPE* and gains *KE*.

Loss in *GPE* = gain in *KE*

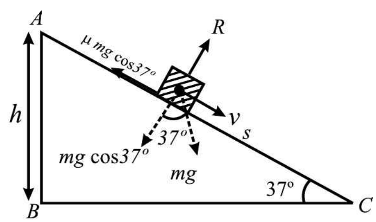
mg (loss in height) = $KE_f - KE_i$

$$\Rightarrow \qquad mgh = \frac{1}{2}mv^2 - 0$$

$$\Rightarrow \qquad v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 3} = 7.67 \text{ m/s.}$$

Note

1. Loss in energy = initial energy – final energy
2. gain in energy = final energy – initial energy



(b) As the block comes down, it loses GPE.

It gains *KE* and does work againstfriction.

loss in *GPE* = gain in *KE* + work done against friction

$$\Rightarrow \qquad mgh = (1/2\, mv^2 - 0) + (\mu\, mg \cos 37°)\, s$$

$$\Rightarrow \qquad 3mg = 1/2\, mv^2 + (0.25) \times mg \times 4/5 \times 5$$

$$\Rightarrow \qquad v = \sqrt{4g} = 6.26 \text{ m/s}$$

Example 48. A 1.0 kg block collides with horizontal light spring of force constant 2 N/m. The block compresses the spring 4 m from the rest position. Assuming that the coefficient of kinetic friction between the block and the horizontal surface is 0.25, what was the speed of the block at the instant of collision ?

Solution When the block compresses the spring, let x m be the amount of compression, i.e. $x = 4m$.

Let v = velocity of the block when it collides with the spring.

Loss in KE of the block = (gain in elastic potential energy of the spring) + (work done against friction)

$$\Rightarrow \quad \frac{1}{2} mv^2 - 0 = \frac{1}{2} kx^2 + \mu \, mg \, x$$

$$\frac{1}{2} mv^2 = \frac{1}{2}(2)(4)^2 + 0.25 \times 1 \times 9.8 \times 4$$

$$v^2 = 51.6 \quad \Rightarrow \quad v = \sqrt{51.6} = 7.18 \text{ m/s}$$

Example 49. A pump is required to lift 1000 kg of water per minute from a well 20 m deep and eject it at a rate of 20 m/s.

(a) How much work is done in lifting water?

(b) How much work is done in giving it a KE?

Solution

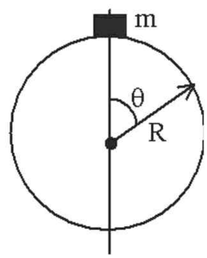
(a) Work done in lifting water = gain in PE (potential energy)

work = $1000 \times g \times 20 = 1.96 \times 10^5 \, J$ per minute

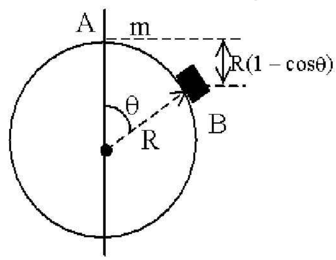
(b) Work done (per minute) in giving it $KE = 1/2 \, mv^2$

$= 1/2 \, (1000)\,(20)^2 = 2 \times 10^5 \, J$ per minute

Example 50. A block of mass m starts from rest and slides down the surface of a frictionless solid sphere of radius R as shown in figure. Measure angles from the vertical and potential energy from the top. Find the change in potential energy of the mass with angle.



Solution Consider the mass when it is at the point B.



U_A($P.E.$ at A) $= 0$

U_B (P.E. at B) $= -\text{mgR}\,(1 - \cos\theta)$

$$\Rightarrow \quad U = U_B - U_A$$

$$\Rightarrow \quad U = -mgR(1 - \cos\theta)$$

Negative sign indicates that $P.E.$ decreases as particle slides down.

Example 51. In fig. (a) and (b), AC, DG and GF are fixed inclined planes. $BC = EF = x$ and $AB = DE = y$. A small block of mass M is released from rest from the point A. It slides down AC and reaches C with a speed V_C. The same block is released from rest from point D. It slides down DGF and reaches the point F with speed V_F. The coefficients of kinetic friction between the block and both the surfaces AC and GDF are μ. Calculate V_C and V_F.

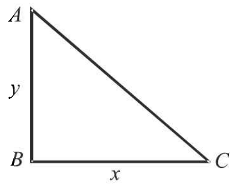


Fig. (a)

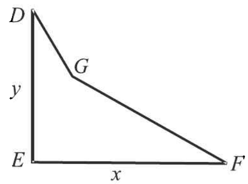


Fig. (b)

Solution

(i) Let $\angle ABC = \alpha$, then $\cos\alpha = \dfrac{x}{AC}$ and $\sin\alpha = \dfrac{y}{AC}$

Potential energy of the block at point $A = M\,g\,y$ ...(1)

Kinetic energy of the block at point $C = \dfrac{1}{2}MV_C^2$ ...(2)

Friction force $F = \mu R = \mu\, Mg\cos\alpha = \dfrac{\mu Mgx}{AC}$

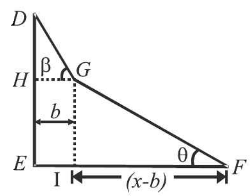
Work done against friction $= F \times AC = \mu\, M\, g\, \dfrac{x}{AC} + AC = \mu Mgx$ ...(3)

Applying the law of conservation of energy, we have $Mg\,y = \dfrac{1}{2}MV_C^2 + \mu Mgx$ ...(4)

$$2gy = V_C^{\,2} + 2\mu gx$$

$\therefore$ $$V_C = \sqrt{[2g(y - mx)]}$$ ...(5)

(ii) In this case, the work has to be done against friction along the inclined planes DG and GF. Let $\angle DGH = \beta$ and $\angle GFE = \theta$



Potential energy of block at $D = Mgy$ ...(6)

Kinetic energy of block at $F = \dfrac{1}{2}MV_F^2$ ...(7)

Work done due to friction F_1 for

$DG = \mu\, M\, g\cos\beta DG = \mu Mg\, \dfrac{b}{DG} DG = \mu Mgb$ ...(8)

Work done due to friction F_2 for

$GF = \mu\, M\, g\cos\theta.\ GF = \mu\, M\, g\, \dfrac{(x-b)}{GF} GF = \mu(Mg)\,(x-b)$ ...(9)

Applying the law of conservation of energy, we have

$$Mgy = \frac{1}{2}MV_F^2 + \mu Mgb + \mu Mg(x-b)$$ ...(10)

or $$2gy = V_F^2 + 2\mu Mgx$$

$$\therefore \quad V_F = \sqrt{[2g(y - \mu x)]} \quad ...(11)$$

The velocity V_F is the same as V_C as is quite obvious from the law of conservation of energy.

Example 52. A 20 kg body is released from rest so as to slide in between vertical rails and compresses a vertical spring of force constant $k = 1920\ N/m$ placed at a height $h = 1.0$ m from the starting position of the body. The rail offers a frictional force of 36 N opposing the motion of the body. Find:

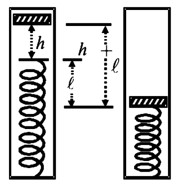
(i) the velocity v of the body just before striking with the spring,

(ii) the distance ℓ through which the spring is compressed, and

(iii) the distance h' through which the body is rebounded up.

Solution

(i) As shown in figure, the body Slides a distance h against the frictional force before striking the spring. Now Loss of gravitational potential energy of the body = gain in $K.E.$ + Work done against friction.



$$\therefore \quad mgh = \frac{1}{2}mv^2 + \mu h$$

$$20 \times 9.8 \times 1.0 = \frac{1}{2} \times 20 \times v^2 + 36 \times 1.0$$

$$196 = 10v^2 + 36$$

$$v^2 = \frac{196 - 36}{10} = 16 \qquad v = 4 \text{ m/sec.}$$

(ii) Let the maximum compression of the spring be ℓ as shown in figure. Applying the law of conservation of energy in this position, we get

$$mg(h + \ell) = \frac{1}{2}k\ell^2 + \mu(h + \ell)$$

$$mgh + mg\ell = \frac{1}{2}k\ell^2 + \mu h + \mu\ell$$

$$\frac{2mgh}{k} + \frac{2mg\ell}{k} = \ell^2 + \frac{2\mu h}{k} + \frac{2\mu\ell}{k}$$

$$\therefore \quad \ell^2 + \ell\left(\frac{2\mu}{k} - \frac{2mg}{k}\right) + \left(\frac{2\mu h}{k} - \frac{2mgh}{k}\right) = 0$$

$$\ell = \frac{\left(\frac{2mg}{k} - \frac{2\mu}{k}\right) + \sqrt{\left[\left(\frac{2\mu h}{k} - \frac{2mg}{k}\right)^2 - 4\left(\frac{2\mu h}{k} - \frac{2mgh}{k}\right)\right]}}{2}$$

Substituting the given values and solving, we get

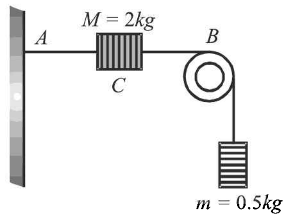
$\ell = 0.5$ m (leaving the negative value).

(iii) Let the spring rebounds to a height *h*'. In this case

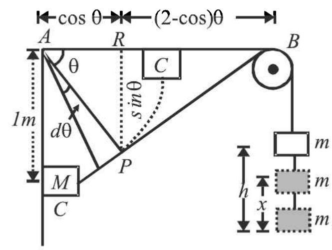
$$\frac{1}{2}k\ell^2 = mgh' + \mu h' \qquad \frac{1}{2}\times 1920\times(0.5)^2 = 20\times 9.8\times h' + 36h'$$

$$240 = 232\,h' \qquad \therefore\ h' = \frac{240}{232} = 1.03\,m$$

Example 53 A string with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2m from the wall, has a point mass $M = 2$ kg attached to it at a distance of 1m from the wall. A mass $m = 0.5$ kg attached at the free end is held at rest so that string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass *m* is released?



Solution When the mass *m* is gently released, the mass *M* begins to descend along the arc of circle *CC*'. The radius of the arc being *AC* with centre *A* as shown in figure.



The mass *M* strikes the wall at *C*' and the string supporting m is pulled upward by a distance *h*.

Now

$$h = BC' - BC = \sqrt{(AB)^2 + (AC)^2} - BC = \sqrt{(2)^2 + (1)^2} - 1 = (\sqrt{5} - 1)$$

Let *P* be the intermediate position of mass *M* at certain time *t*. Suppose θ be the angle subtended by *M* at this instant. The displacement of mass *m* (when *M* is at *P*) is shown by *x*. Now the angular speed of mass *M* i.e., $\omega = \frac{d\theta}{dt}$.

So the instantaneous speed of mass *M* at *P* i.e., $v_p - r_\omega = 1\left(\frac{d\theta}{dt}\right) = \left(\frac{d\theta}{dt}\right)$.

Now, $$x = BP - BC = \sqrt{[(BR)^2 + (RP)^2]} - BC$$

$\therefore$ $$x = \sqrt{[(\sin\theta)^2 + (2-\cos\theta)^2]} - 1 = \sqrt{(5 - 4\cos\theta)} - 1$$

Hence, the velocity *v* of mass *m* (when *M* is at *P*) is given by

$$v = \frac{dx}{dt} = \frac{1}{2}\cdot\frac{4\sin\theta}{\sqrt{(5-4\cos\theta)}}\cdot\frac{d\theta}{dt} = \frac{2\sin\theta}{\sqrt{(5-4\cos\theta)}}\times v_p$$

Let *V* be the speed of mass *M* at *C*'. In this position θ = 90°, i.e., sinθ = sin90° = 1

$$\cos\theta = 0 \text{ and } V = v_p \qquad \therefore\ v = \frac{2}{\sqrt{5}}V$$

According to the law of conservation of energy

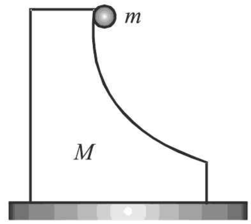
Loss of potential energy of *M* = gain of potential energy of *m* + gain of *K.E.* of *M* + gain *K.E.* of *m*

$$\therefore\quad Mg\times 1 = mg\,[(\sqrt{5}) - 1] + \frac{1}{2}MV^2 + \frac{1}{2}mv^2$$

$$2 \times 9.8 \times 1 = 0.5 \times 9.8\ [(\sqrt{5}) - 1] + \frac{1}{2} \times 2 \times V^2 + \frac{1}{2} \times 0.5\left(\frac{2}{\sqrt{5V}}\right)^2$$

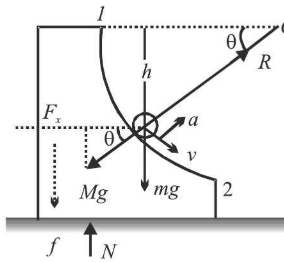
Solving for V, we get V = **3.36 m/s**

Example 54. A wedge of mass M with a smooth quarter circular plane, is kept on a rough horizontal surface. A particle of mass m is released from rest from the top of the wedge as shown in the figure. When the particle slides along the quarter circular plane, it exerts a force on the wedge. If the wedge begins to slide when the particle exerts a maximum horizontal force on it, find the coefficient of friction between the wedge and the horizontal surface.



Solution $\mu = \dfrac{f_{max}}{N}$ ...(i)

Where, f_{max} = maximum static friction between the wedge and ground. (limiting friction) that must be equal to the maximum horizontal force $(F_x)_{max}$ for prevalence of sliding of the wedge and N = normal force offered by the horizontal surface on the wedge.



⇒ $(F)_{max}$ and N can be calculated as follows.

Let the particle attain a speed v at the angular position θ as shown in the free body diagram.

Since the particle accelerates towards the centre O with an acceleration $a = \dfrac{v^2}{r}$, the force exerted on it must be radially inwards.

$$\Rightarrow R - mg\sin\theta = ma = \frac{mv^2}{r} \quad \Rightarrow \quad R = mg\sin\theta + \frac{mv^2}{r} \text{ ...(ii)}$$

Conserving energy of the particle between position 1 and 2 we obtain

$$(\Delta KE)_{1\to 2} = (\Delta PE)_{1\to 2} \Rightarrow \frac{1}{2}mv^2 = mgh = mr\sin\theta.G$$

$$\Rightarrow \quad v = \sqrt{2gr\sin\theta} \quad \text{...(iii)}$$

Elimination of v between (ii) and (iii) yields

$$R = 3\,mg\sin\theta \quad \text{...(iv)}$$

⇒ The horizontal force acting on the wedge = $F_x = R\cos\theta = \dfrac{3}{2}mg\sin\theta$ ...(v)

For F_x to be maximum sin2θ = 1 ⇒ θ = 45° Putting θ in equation (v) horizontally

we obtain $(F_x)_{max} = \dfrac{3}{2}mg$ ...(vi)

Resolving forces acting on the wedge for its equilibrium along horizontal and vertical we obtain.

$$f_{max} - (F_x)_{max} = Ma = 0 \text{ and } N - Mg - R\sin\theta = Ma_y = 0$$

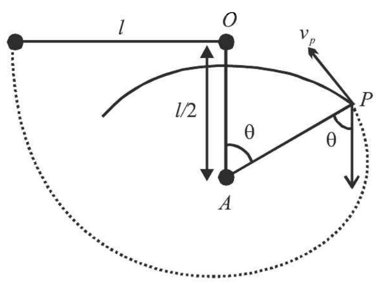
$$\Rightarrow \quad f_{max} = (F_x)_{max} = \frac{3}{2}mg \text{ and } N = Mg + R\sin\theta\,|_{\theta=\pi/4} = \left(M + \frac{3}{2}\right)g$$

using (i) the values of f_{max} and N we obtain $\mu = \dfrac{3m}{2M+3m}$

Example 55. A small ball is suspended from point O by a thread of length ℓ. A nail is driven into the wall at a distance of $\frac{\ell}{2}$ below O, at A. The ball is drawn aside so that the thread takes up a horizontal position at the level of point O and then released. Find:

(i) At what point of the ball's trajectory, will the tension in the thread disappear?

(ii) What will be the highest point to which it will rise?



Solution If at point P, tension is zero.

Then, $mg\cos\theta = \dfrac{mv^2}{r}$

Using conservation of energy, $v^2 = g\ell(1-\cos\theta)$

$\therefore \quad mg\cos\theta = \dfrac{mg\ell}{\ell/2}(1-\cos\theta) \quad \Rightarrow \quad \theta = \cos^{-1}\left(\dfrac{2}{3}\right)$

$\therefore \quad$ Height of point $P = \dfrac{\ell}{2} + \dfrac{\ell}{2}\cos\theta = \dfrac{5\ell}{6}$, from lowest point.

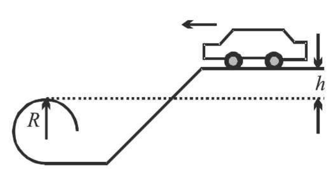
$$v^2 = g\ell\left(1-\frac{2}{3}\right) = \frac{g\ell}{3} \Rightarrow v\sqrt{\frac{g\ell}{3}}$$

Now the particle describes parabolic path. The height attained by the particle, from point P.

$$h = \frac{(v\sin\theta)}{2g} = \frac{5\ell}{54}$$

$\therefore \quad$ Highest point from lowest point will be $\left(\dfrac{5\ell}{6} + \dfrac{5\ell}{54}\right) = \dfrac{50\ell}{54}$

Example 56. Figure show a loop-the-loop track of radius R. A car (without engine) starts from a platform at a distance h above the top of the loop and goes around the loop without falling off the track. Find the minimum value of h for a successful looping.



Neglect friction.

Solution Let the gravitational potential energy be zero at the platform and the car starts with a negligible speed. Suppose the speed of the car at the top most point of the loop be v. Now applying the law of conservation of energy, we have

$$0 = mgh - \frac{1}{2}mv^2$$

or $\quad mv^2 = 2mgh$

or $\quad v^2 = 2gh \qquad \text{...(1)}$

Applying Newton's law at the top of the circular path, we get

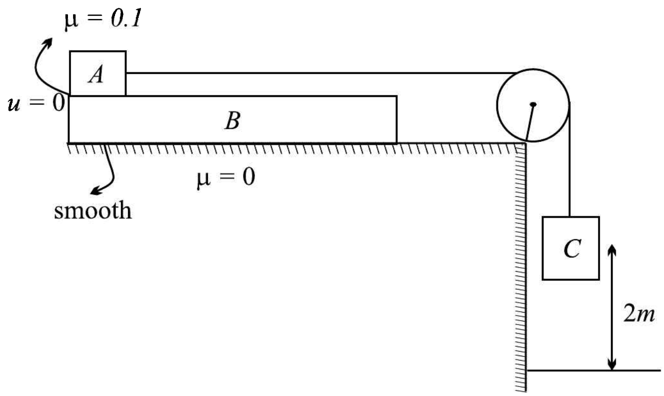
$$mg + N = \frac{mv^2}{R} \quad \text{or} \quad mg + N = \frac{2mgh}{R} \qquad ...(2)$$

For h to be minimum, N should assume the minimum value i.e., zero.

Hence, $$mg = \frac{2mgh_{min}}{R} \quad \text{or} \quad h_{min} = \frac{R}{2}$$

Example 57. $m_A = 1$ kg, $m_B = 2$ kg, $m_C = 10$kg Find velocity of A, B & C when C has descended 2m

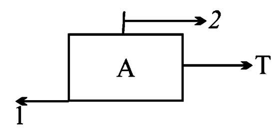
Solution Here work is done by kinetic friction between A & B so it will not cancel out. But by tension on A & C will cancel out.



$$w_A = T \times 2 - 1 \times 2$$

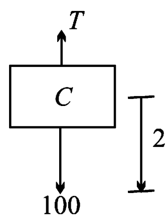
$$w_C = 100 \times 2 - T \times 2$$

$$\text{Total work} = 100 \times 2 - 1 \times 2$$



$$99 \times 2 = \frac{1}{2} \times 10v^2 + \frac{1}{2} \times 1 \times v^2$$

$$v^2 = \frac{99 \times 2 \times 2}{11} \mid v = 6 \text{ m/s} \mid A \text{ and } C$$

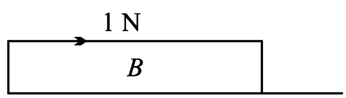


Finding displacement of B

$$a_B = 0.5 \text{ ms}^{-2}, u = 0, t \text{ from } A \text{ and } C$$

$$t = 2/3$$

$$s = \frac{1}{2} \times \frac{1}{2} \times \frac{4}{9} = \frac{1}{9} m$$



$\vec{F} \cdot \vec{S}$

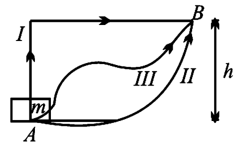
$$1 \times \frac{1}{9} = \frac{1}{2} \times 2 \times v^2 \text{ or } v_B = \frac{1}{3} \text{ms}^{-1}$$

you can see that work done by kinetic friction on A & B is not cancelling out completely.

Note

Thus except tension, normal and static friction even if we write work because of action and reaction on a system it is not necessary that total work will be zero.

Nature of Forces



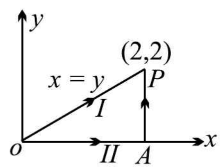
1. ***Conservative forces***: Forces for which calculation of work is independent of path taken by body. e.g. gravity, spring.

   In this case work done by force of gravity is same for taking body from A to B by any path (You can show it mathematically)

   * In conservative forces total work done for around closed path = 0

2. ***Non–conservative forces***: Forces for which calculation of work depends on path not just on initial and final position eg. friction.

$$W_{fI} = -\mu mgl$$
$$W_{fII} = -\mu mg(3l)$$



Example **57.** $\vec{F} = xy\hat{i} + xy\hat{j}$. Calculate work required to take particle from (0,0) to (2,2) (give it w/o path) Then remind them it cannot be calculated w/o path then show $w_{I\,OP} \neq w_{II\,OAP}$

POTENTIAL ENERGY

It can be defined only for conservative forces.

Definition: It is defined as negative of work done by conservative forces

Formula: $\vec{F}$ represents force for which we are writing potential energy

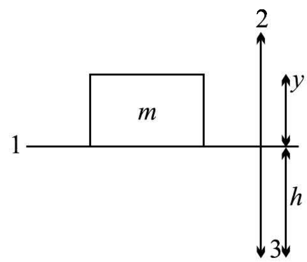
$$dU = -\vec{F}\cdot d\vec{S}$$

$$\int_1^2 dU = -\int_1^2 \vec{F}\cdot d\vec{S}$$

thus $$U_2 - U_1 = -\int_1^2 \vec{F}\cdot d\vec{S}$$

Purpose: By defining PE we can avoid repeated calculation of work for conservative forces and since PE depends only on position (initial and final), we can directly write effect of conservative forces in terms of their respective PE's

We will define PE for gravity and spring



Gravity: $U_2 - U_1 = -\int_0^h mg(-\hat{j})dh(\hat{j})$

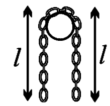
$U_2 - U_1 = mgh$

$U_3 - U_1 = -mgh$ (similarly)

Emphasise that by definition we can only find difference of PE not absolute value.

If we assume $U_1 = 0$ then $U_2 = \text{mgh}U_3 = -mgh$

Example 59. If chain starts slipping, find its *KE* when chain becomes completely straight

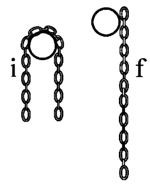


Hint: $W_g = (KE_f - KE_i)$

But $W_g = -(U_f - U_i)$

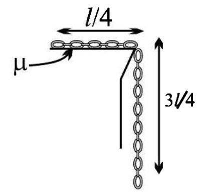
$$-U_f + U_i = KE_f - KE_i$$

$KE_f + U_f = KE_i + U_i$ find U by using calculus



emphasise that if we have tried to find work due to gravity directly then it would have been very difficult as compared to the solution we are giving.

Example 60. Chain is on the verge of slipping, find the velocity of the chain, when it has slipped.



Solution

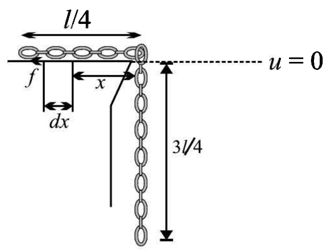
$$f = \frac{3Mg}{4}$$

$$\mu\frac{3Mg}{4} = \frac{3Mg}{4} \quad \Rightarrow \quad \mu = 3$$

work done by friction force when chain completely slip off the table.

$$df = \mu dmg$$

$$dw = df\,x \int_0^{L/4} \mu\frac{M}{L}\,dx\,gx$$



$$w_f = 3\frac{Mg}{L}\left(\frac{x^2}{2}\right)_0^{L/4} = \frac{3Mg\text{l}}{32}$$

Now decrease in PE = inc. in $KE + w_f$

$$PE_i - PE_f = \frac{1}{2}mv^2 + w_f$$

$$\left(-\frac{9Mg\text{l}}{32}\right) - \left(-\frac{Mg\text{l}}{2}\right) = \frac{1}{2}Mv^2 + \frac{3Mg\text{l}}{32}$$

$$\frac{7Mg\text{l}}{32} = \frac{1}{2}Mv^2 + \frac{3Mg\text{l}}{32}$$

$$\frac{1}{2}Mv^2 = \frac{4Mg\text{l}}{32}$$

$$v = \frac{1}{2}\sqrt{g\text{l}}$$

As we have learnt from previous problem if some forces are acting on a body

$$W_1 + W_2 + \ldots + W_n = KE_f - KE_i$$

if some of them are conservative and others are non–conservative then for conservative forces we can write *PE*

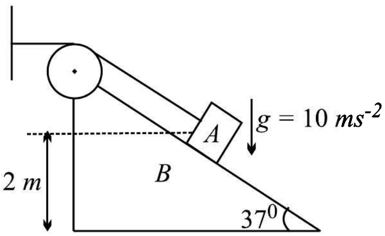
$$\Sigma W_c + \Sigma W_{nc} = KE_f - KE_i$$

$$\Sigma\{-(U_f - U_i)\} + \Sigma W_{nc} = KE_f - KE_i$$

$$\Sigma W_{nc} = KE_f - KE_i + \Sigma(U_f - U_i)$$

Term on *RHS* is often called **mechanical energy**. Emphasise that effect of a force can either be written as work on *LHS* or it can come as *PE* on *RHS*

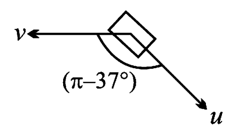
Example 61. Find velocity of *A* and *B* when *A* is about to touch the ground. Also verify that work done by tension on the whole system and *N* between *A* and *B* is zero.



$m_A = 5$ kg $m_B = 10$ kg

Solution $|\vec{V}| = |\vec{u}|$

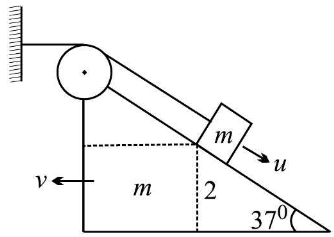
Net speed of block



$$V_b = \sqrt{u^2 + u^2 - 2u^2\cos 37°} = \sqrt{2u^2 - 2u^2\frac{4}{5}} = v\sqrt{\frac{2}{5}}$$

By energy conservation

Decline in *P.E.* of block = Increase in *KE* of wedge + block



$$mgh. = \frac{1}{2}mv^2 + \frac{1}{2}mv_b^2$$

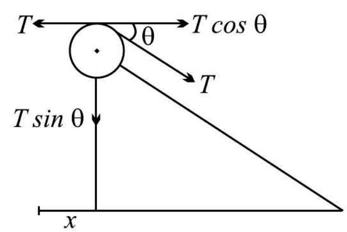
$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv^2\frac{2}{5}$$

$$5 \times 10 \times 2 = \frac{1}{2}10v^2 + \frac{1}{2}5 \times \frac{2}{5}v^2$$

$$5 \times 10 \times 2 = \frac{12}{2}v^2$$

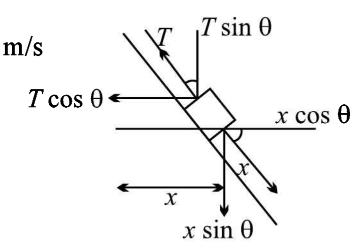
$$5 \times 10 \times 2 = \frac{12}{2}v^2$$

$$v = \sqrt{\frac{50}{3}}$$



velocity of wedge $= 5\sqrt{\frac{2}{3}}$ m/s

velocity of block $= v\sqrt{\frac{2}{5}} = \sqrt{\frac{50}{3} \times \frac{2}{5}} = \sqrt{\frac{20}{3}} = 2\sqrt{\frac{5}{3}}$ m/s



work done by tension

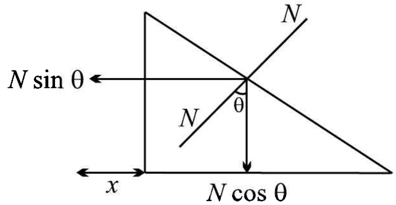
(1) on wedge

$w = (T - T\cos\theta)\,x$

(2) on the block

$T\cos\theta\,(x - x\cos\theta) - T\sin\theta\, x\sin\theta$

$T \times \cos\theta - Tx$



Net $w = TX - Tx\cos\theta + Tx\cos\theta - Tx = 0$

By normal reaction between A & B

(1) on the wedge

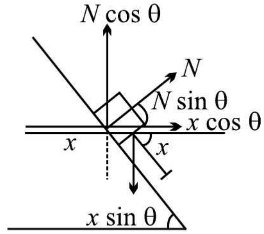
$N\sin\theta \, . \, x$

(2) on the block

$-N\sin\theta\,(x - \cos\theta) + (-N\cos\theta\, x\sin\theta)$

$= -N\sin\theta\, x + N x\sin\theta\cos\theta - N x\sin\theta\cos\theta$

$= -N\sin\theta\, x$



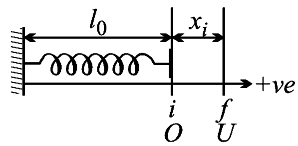
Net work done by normal raction

$N\sin\theta\, x - N\sin\theta\, x = 0$

Spring

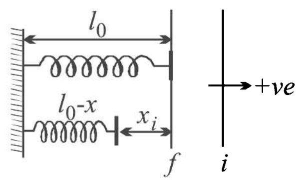
In case of spring natural length of spring is assumed to be reference point and always assigned zero potential energy (This is a universal assumption). In gravity we can take any point as reference and assign it any value of potential energy.

$$U_f - U_i = -\int_i^f \vec{F}\cdot d\vec{S}$$



$$U_f - 0 = -\int_o^{x_i} kx(-\hat{i})(dx)\hat{i}$$

$$U = \frac{1}{2}kx_1^{\,2}$$



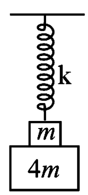
for compression: $U_f - U_i = -\int_i^f \vec{F}\cdot d\vec{S} = -\int_o^{x_i} kx\hat{i}\,(dx)(-\hat{i})$

$$U = \frac{1}{2}kx^2$$

Thus if spring is either stretched or compressed from natural length by x the potential energy is $\frac{1}{2}kx^2$.

Emphasise that for solving problems of spring always measure distances from nature length.

Example 62. Find how much m will rise if $4m$ falls away. Blocks are at rest and in equilibrium



Solution Applying WET on block of mass m

$$w_g + w_{sp} = k_f - k_i$$

Let finally displacement of block from equilibrium is x

$$-mg\left(\frac{5mg}{k} + x\right) + \frac{1}{2}k\left(\frac{25m^2g^2}{k^2}\right) - \frac{1}{2}kx^2 = 0$$

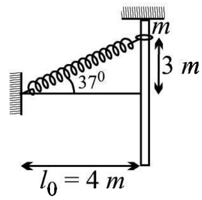
$$\frac{1}{2}kx^2 + mgx - \frac{15m^2g^2}{2k} = 0;\ x = \frac{3mg}{k}$$

displacement from initial is $\frac{5mg}{k} + \frac{3mg}{k} = \frac{8mg}{k}$

Example 63. Find velocity of ring when spring becomes horizontal

$m = 10$ kg

$k = 400$ Nm^{-1}



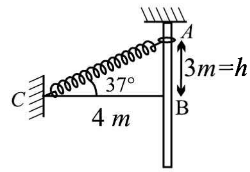
Solution

$m = 10$ kg

$k = 400$ N/m

natural length of spring = 4m

decreasing in PE = inc. in KE



$$\frac{1}{2}\ k \times 1 + mgh = \frac{1}{2}\ mv^2$$

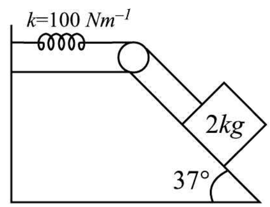
$$\frac{1}{2} \times 400 \times 1^2 + 10 \times 10 \times 3 = \frac{1}{2} \times 10V^2$$

$200 + 300 = 5V^2$

$5V^2 = 500;\ V = \sqrt{100}$ m/s = 10 m/s

Example 64.

(a) A 2 kg block situated on a smooth fixed incline is connected to a spring of negligible mass, with spring constant $k = 100$ Nm^{-1}, via a frictionless pulley. The block is released from rest when the spring is unstretched. How far does the block move down the incline before coming (momentarily) to rest? What is its acceleration at its lowest point?



(b) The experiment is repeated on a rough incline. If the block is observed to move 0.20 m down along the incline before it comes to instantaneous rest, calculate the coefficient of kinetic friction.

Solution

(a) Applying work-energy theorem

$$mgs \sin 37° = \frac{1}{2}ks^2$$

$$2\times10\times s\times\frac{3}{5} = \frac{1}{2}\times100\times s^2 \text{ on solving } s = 0.24 \text{ m}$$

accelerating at its lowest point

$$a = \frac{ks - mg\sin 37°}{m}$$

$$= \frac{100 \times 0.24 - 2 \times 10 \times \frac{3}{5}}{2}$$

$= 6$ m/s^2 $a = 6$ m/s^2

(b) Work done by gravity + work done by friction = Energy stored in spring

$$mg\,s\sin 37° - \mu mg\cos 37° \times s = \frac{1}{2}ks^2$$

$$mg\sin 37° - \frac{1}{2}ks = \mu mg\cos 37°$$

$$2 \times 10 \times \frac{3}{5} - \frac{1}{2} \times 100 \times s = \mu \times 2 \times 10 \times \frac{4}{5}$$

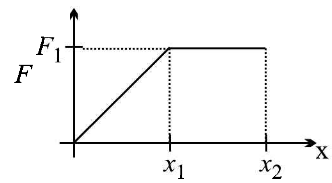
given $s = 0.20\ m$

$$\frac{12 - 50s}{16} = \mu$$

$$\therefore \qquad \mu = \frac{1}{8}$$

[**Ans.** (a) $s = 0.24\ m$, $a = 6$ m/s^2, (b) $x = 1/8$]

Example 65. Draw U–x graph



Finding F from U

$\vec{F} = -\vec{\nabla}(U)$

$\vec{\nabla}$ represents $\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

Example 66. If $U = 4x^2y + 2yz^2$ find force

Hint: If U depends on only one variable lets say r, then $\vec{F} = -\left(\frac{dU}{dr}\right)\hat{r}$

Example 67. $U = 4r^3$ find force

Solution $\vec{F} = 12\,r^2(-\hat{r})$

ENERGY DIAGRAMS

Diagram, in which the total energy E and the potential energy U are plotted as functions of positions is called as energy diagram.

The kinetic energy $K = E - U$ is easily found by inspection. Since kinetic energy can never be negative, the motion of the system is constrained to regions where $U \leq E$.

(a) Energy Diagram for a Harmonic Oscillator

$$U = \frac{Kx^2}{2}$$

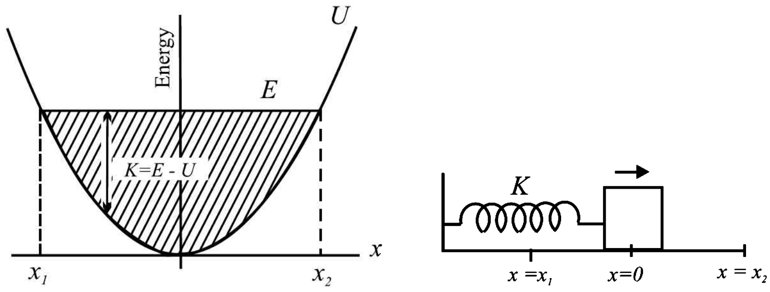


Fig 6.3

The potential energy of the block is $U = \dfrac{Kx^2}{2}$ is a parabola centered at the origin. Since, the total energy is constant for a conservative system, E is represented by a horizontal straight line. Motion is limited to the shaded region where $E \geq U$; the limits of the motion x_1 and x_2 in the sketch, are sometimes called the turning points. The kinetic energy, $K = E - U$, is greatest at the origin. As the particle flies past to a complete rest at one of the turning points x_1, x_2. The particle then moves towards the origin with increasing kinetic energy, and the cycle is repeated.

The harmonic oscillator provides a good example of bounded motion. As E increases, the turning point moves farther and farther off, but the particle can never move away freely. If E is decreased, the amplitude of motion decreases, until finally for $E = 0$ the particle lies at rest at $x = 0$.

(b) Energy Diagram for a Particle Acted on by a Repulsive Force

$$F = \frac{A\hat{r}}{r^2}; \; U = \frac{A}{r}$$

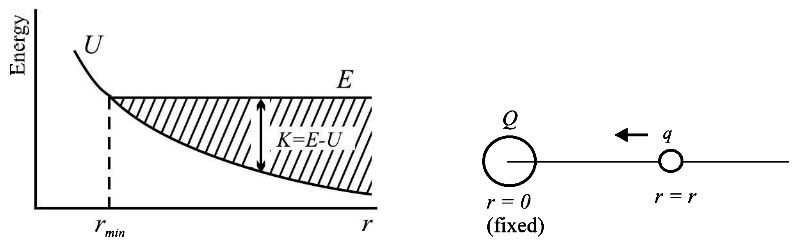


Fig 6.4

There is a distance of closest approach, r_{min}, as shown in the figure 6.4, but the motion is not bounded for large r since U decreases with distance. If the particle is shot toward the origin, it gradually loses kinetic energy until it comes momentarily to rest at r_{min}. The motion then reverses and the particle moves out toward infinity. The final and initial speeds at any point are identical; the collision merely reverses the velocity.

(c) Energy Diagram for a Typical Attractive Two Atom System

Consider the interaction between two atoms. At large separations, the atoms attract each other weakly with the Vander Wall's force, which varies as $1/r^7$. As the atoms approach, the electron clouds begin to overlap, producing strong forces. In this intermediate region the force is either attractive or repulsive depending on the details of the electron configuration.

If the force is attractive, the potential energy decreases with decreasing r. At very short distances the atoms always repel each other strongly, U increases rapidly as r becomes small.

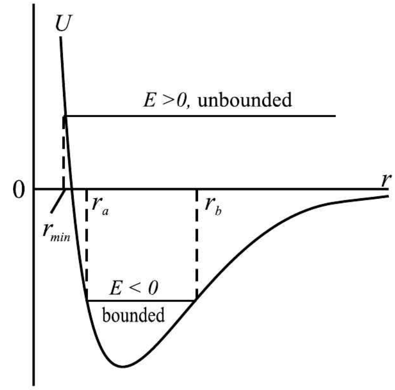


Fig 6.5

For positive energy, $E > 0$, the motion is unbounded, and the atoms are free to fly apart. As the figure 6.5 indicates, the distance of closest approach, r_{min}, does not change appreciably as E is increased. The steep slope of the potential energy curve at small r means that the atoms behave like hard spheres and r_{min} is not sensitive to the energy of collision.

The situation is quite different if E is negative. Then the motion is bounded for both small and large separations; the atoms never approach closer than r_a or move farther apart than r_b. A bound system of two atoms is, of course, a molecules energy diagram.

If two atoms collide with positive energy, they cannot form a molecule unless some means is available for losing enough energy to make E negative.

In general, a third body is necessary to carry off the excess energy. Sometimes the third body is a surface, which is the reason surface catalysts are used to speed certain reactions.

A third atom can also carry off the excess energy, but for this to happen the two atoms must collide when a third atom is nearby. This is a rare event at low pressures, but itbecomes increasingly important at higher pressures. Another possibility is for the two atoms to lose energy by the emission of light.

Let us summarise the concept developed so far.

Energy

Definition: Energy is defined as internal capacity of doing work. When we say that a body has energy we mean that it can do work.

Energy appears in many forms such as mechanical, electrical, chemical, thermal (heat), optical (light), acoustical (sound), molecular, atomic, nuclear, etc., and can change from one form to the other.

Kinetic Energy

***Definition*:** Kinetic energy is the internal capacity of doing work of the object by virtue of its motion.

Kinetic energy is a scalar property that is associated with state of motion of an object. An aeroplane in straight and level flight has kinetic energy of translation and a rotating wheel on a machine has kinetic energy of rotation. If a particle of mass m is moving with velocity 'v' much less than the velocity of the light than the kinetic energy 'K' is given by

$$K = \frac{1}{2}mv^2$$

Impotant Points for K.E.

1. As mass m and v^2 $(\vec{v}.\vec{v})$ are always positive, kinetic energy is always positive scalar i.e., kinetic energy can never be negative.
2. The kinetic energy depends on the frame of reference,

$$K = \frac{p^2}{2m} \text{ and } P = \sqrt{2mk} \text{ ; } P = \text{linear momentum}$$

Potential Energy

Definition: Potential energy is the internal capacity of doing work of a system by virtue of its configuration.

In case of conservative force (field) potential energy is equal to negative of work done by conservative force in shifting the body from some reference position to given position.

In case of conservative force as:

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} \qquad \text{i.e.,} \qquad U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} = -W$$

Whenever and wherever possible, we take the reference point at ∞ and assume potential energy to be zero there, i.e., If we take $r_1 = \infty$ and $U_1 = 0$ then

$$U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r} = -W$$

Impotant Points for K.E.

1. Potential energy can be defined only for conservative forces. It does not exist for non-conservative forces.
2. Potential energy can be positive or negative.
3. Potential energy depends on frame of reference but change in potential energy is independent of reference frame.
4. Potential energy should be considered to be a property of the entire system, rather than assigning it to any specific particle.
5. It is a function of positions and does not depend on the path.

Types of Potential Energy

(a) **Elastic Potential Energy:** It is the energy associated with state of compression or expansion of an elastic (spring like) object and is given by:

$$U = \frac{1}{2} k y^2$$

where k is force constant and 'y' is the stretch or compression. Elastic potential energy is always positive.

(b) **Electric Potential Energy:** It is the energy associated with charged particles that interact via electric force. For two point charges q_1 and q_2 separated by a distance 'r',

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

As charge can be positive or negative, electric potential energy can be positive or negative

(c) **Gravitational Potential Energy:** It is due to gravitational force. For two particles of masses m_1 and m_2 separated by a distance 'r', it is given by:

$$U = -G\frac{m_1 m_2}{r}$$

which for a body of mass 'm' at height 'h' relative to surface of earth reduces to $U = mgh$

Gravitational potential energy can be positive or negative.

Mechanical Energy

***Defenition*:** Mechanical energy 'E' of a particle, object or system is defined as the sum of kinetic energy 'K' and potential energy 'U', i.e., $E = K + U$

Impotant Points for K.E.

1. It is a scalar having dimensions $[ML^2T^{-2}]$ and *SI* units joule.
2. It depends on frame of reference.
3. A body can have mechanical energy without having either kinetic energy or potential energy. However, if both kinetic and potential energies are zero, mechanical energy will be zero. The converse may or may not be true, i.e., if $E = 0$ either both *PE* and *KE* are zero or *PE* negative and *KE* positive such that $KE + PE = 0$.
4. As mechanical energy $E = K + U$, i.e., $E - U = K$. Now as K is always positive, $E - U \geq 0$,i.e., for existence of a particle in the field, $E \geq U$.
5. As mechanical energy $E = K + U$ and K is always positive, so, if 'U' is positive 'E' will be positive. However, if potential energy U is negative, 'E' will be positive if $K > |U|$ and E will be negative if $K < |U|$

i.e., mechanical energy of a body or system can be negative, and negative mechanical energy means that potential energy is negative and in magnitude it is more than kinetic energy. Such a state is called bound state, e.g., electron in an atom or a satellite moving around a planet are in bound state.

Conservative Force & Potential Energy

$$F_s = -\partial U/\partial s,$$

i.e., the projection of the field force, the vector $\boldsymbol{F}$, at a given point in the direction of the displacement dr equals the derivative of the potential energy U with respect to a given direction, taken with the opposite sign. The designation of a partial derivative $\partial/\partial s$ emphasizes the fact of deriving with respect to a definite direction.

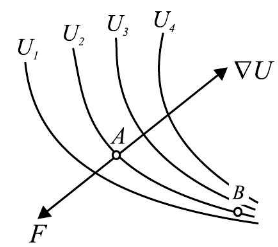
So, having reversed the sign of the partial derivatives of the function U with respect to x, y, z, we obtain the projection F_x, F_y and F_z of the vector $\boldsymbol{F}$ on the unit vectors $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$. Hence, one can readily find the vector itself: $F = F_x\boldsymbol{i} + F_y\boldsymbol{j} + F_z\boldsymbol{k}$, or

$$F = -\left(\frac{\partial U}{\partial x}\mathrm{i} + \frac{\partial U}{\partial y}\mathrm{j} + \frac{\partial U}{\partial z}\mathrm{k}\right).$$

The quantity in parentheses is referred to as the scalar gradient of the function U and is denoted by grad U or ∇U. We shall use the second, more convenient, designation where ∇ ("nabla") signifies the symbolic vector or operator

$$\nabla = \mathrm{i}\frac{\partial}{\partial x} + \mathrm{j}\frac{\partial}{\partial y} + \mathrm{k}\frac{\partial}{\partial z}$$

Therefore, ∇U may be formally regarded as the product of a symbolic vector ∇ by a scalar U.



Consequently, the relationship between the force of a field and the potential energy, expressed as a function of coordinates, can be written in the following compact form.

$F = -\nabla U,$

the vector $\boldsymbol{F}$ is oriented in the direction of decreasing U values. As $\boldsymbol{F}$ is directed oppositely to the vector ∇U, we may conclude that the gradient of U is a vector oriented along a normal to an equipotential surface in the direction of increasing values of potential energy U.

It shows a system of equipotentials ($U_1 < U_2 < U_3 < U_4$), a gradient of the potential energy ∇U and the corresponding vector of the force $\boldsymbol{F}$ at the point A of the field. It pays to consider how these two vectors are directed, for example, at the point B of the given field.

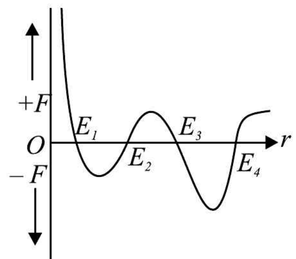
Translatory Equilibrium

Definition: When several forces act on a body simultaneously in such a way that the resultant force on the body is zero, the body is said to be in translotry equilibrium.

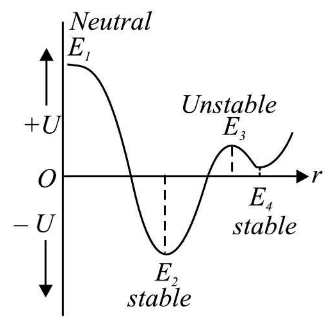
Graphically: If we plot graphs between F/r and U/r, at equilibrium F will be zero while U will be optimum(max. or min. or constant). If

U = min.	i.e.,	(d^2U/dr^2) = positive, equilibrium is stable
U = max.	i.e.,	(d^2U/dr^2) = negative, equilibrium is unstable
U = const.	i.e.,	(d^2U/dr^2) = zero, equilibrium is neutral

S.No.	Stable Equilibrium	Unstable Equilibrium	Neutral Equilibrium
1.	Net force is zero	Net force is zero	Net force is zero
2.	$\frac{dU}{dr}=0$ or slope of U–r graph is zero.	$\frac{dU}{dr}=0$ or slope of U–r graph is zero.	$\frac{dU}{dr}=0$ or slope of U–r graph is zero.
3.	When displaced slightly, from its equilibrium position a net restoring force starts acting on the body which has a tendency to bring the body back to its equilibrium position.	When displaced slightly, from its equilibrium position a net force starts acting on the body which moves the body in the direction of displacement or away from the equilibrium position.	When displaced slightly, from its equilibrium position the body has neither the tendency to come back to original position nor to move away from the original position.
4.	Potential energy in equilibrium position is minimum as compared to its neighbouring points or $\frac{d^2U}{dr^2}$ = positive	Potential energy in equilibrium position is maximum as compared to its neighbouring points or $\frac{d^2U}{dr^2}$ = positive	Potential energy remains constant even if the body is displaced from its equilibrium position or $\frac{d^2U}{dr^2}=0$
5.	When displaced from equilibrium position the center of gravity of the body goes up.	When displaced from equilibrium position the center of gravity of the body comes down.	When displaced from equilibrium position the center of gravity of the body remains at the same level.



(A)



(B)

1. *Work-Energy Theorem*

According to work-energy Theorem, the work done by all the forces on a particle is equal to the change in kinetic energy of it.

$$W_C + W_{NC} + W_{PS} + W_{all} = \Delta K$$

Where W_C is the work done by conservative force

W_{NC} is the work done by non-conservative force

W_{PS} is the work done by psuedo force

2. Modified Form of Work-Energy Theorem

We know that conservative forces are associated with the concept of potential energy, that is $W_C = -\Delta U$

So, Work-Energy theorem may be modified as

$$W_{NC} + W_{PS} + W_{all} = \Delta K + \Delta U; \qquad W_{NC} + W_{PS} + W_{all} = \Delta E$$

3. Mechanical Energy Conservation

In an inertial frame, if only conservative forces are present then, the mechanical energy of a system is conserved.

Alternatively,

if $W_{NC} = 0$, $W_{PS} = 0$ Then $\Delta E = \Delta(K + U) = 0$

or $E = K + U = \text{constant}$

POWER

Power is defined as the time rate of doing work

When the time taken to complete a given amount of work is important, we measure the power of the agent doing work.

The average power ($\bar{P}$ or p_{av}) delivered by an agent is given by

$$\bar{P} \text{ or } p_{av} = \frac{W}{t}$$

where W is the amount of work done in time t.

Power is the ratio of two scalars- work and time. So, power is a scalar quantity. If time taken to complete a given amount of work is more, then power is less. For a short duration dt, if P is the power delivered during this duration, then

$$P = \frac{\vec{F} \cdot d\vec{S}}{dt} = \vec{F} \cdot \frac{d\vec{S}}{dt} = \vec{F} \cdot \vec{v}$$

By definition of dot product, $P = Fv \cos \theta$

where θ is the smaller angle between $\vec{F}$ and $\vec{v}$.

This P is called as instantaneous power if dt is very small.

Explanatory Notes on Power

From a particle viewpoint, it is interesting to know not only the work done on an object but also the rate at which the work is being done. The time rate of doing work is called power.

If an external force is applied to an object (which we assume as a particle), and if the work done by this force is ΔW in the time interval Δt, then the average power during this interval is defined as

$$P = \frac{\Delta W}{\Delta t}$$

The work done on the object contributes to increasing the energy of the object. A more general definition of power is the time rate of energy transfer. The instantaneous power is the limiting value of the average power as Δt approaches zero.

i.e., $$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

where we have represented the infinitesimal value of the work done by dW (even though it is not a change and therefore not a differential).

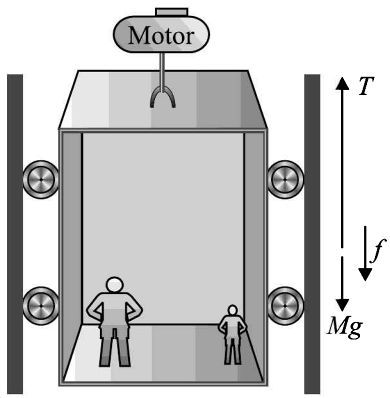
$$P = \frac{dW}{dt} = \vec{F}.\frac{d\vec{s}}{dt} = \vec{F}.\vec{v}$$

where we have used the fact that $\vec{v} = \frac{d\vec{s}}{dt}$.

This *SI* unit of power is joule per second (*J/s*), also called watt(*W*)' (after James Watt);

$1W = 1\ J/s = 1\ kg.m^2/s^3$.

Example 68. An elevator has a mass of 1000 kg and carries a maximum load of 800 kg. A constant frictional force of 4000 *N* retards its upward motion, as shown in the figure.



(a) What must be the minimum power delivered by the motor to lift the elevator at a constant speed of 3 m/s?

(b) What power must the motor deliver at any instant if it is designed to provide an upward acceleration of 1 m/s²?

Solution

(a) The motor must supply the force T that pulls the elevator upward. From Newton's second law and from the fact that $a = 0$ since v is constant, we get

$T - f - Mg = 0$, where M is the total mass (elevator plus load), equal to 1800 kg.

$\Rightarrow \quad T = f + Mg = 4 \times 10^3\ N + (1.80 \times 10^3\ \text{kg})\ (9.80\ \text{m/s}^2) = 2.16 \times 10^4\ N$

Using equation (7) and the fact that T is the same direction as v, we have gives

$P = Tv = (2.16 \times 10^4\ N)\ (3\ \text{m/s}) = 64.9\ kW = 87\ hp$

(b) Application of Newton's second law to the elevator gives

$T - f - Mg = Ma$

$\Rightarrow \quad T = M(a + g) + f = (1.80 \times 10^3\ \text{kg})\ (1 + 9.80)\ \text{m/s}^2 + 4 \times 10^3\ N = 2.34 \times 10^4\ N$

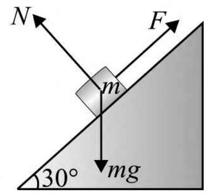
Therefore, using equaiton (7) we get the required power

$P = Tv = \mathbf{(2.34 \times 10^4\ v)}W$

where v is the instantaneous speed of the elevator in metres per second. Hence, the power required increases with increasing speed.

Example 69. A block of mass 2 kg is pulled up on a smooth incline of angle 30° with horizontal. If the block moves with an acceleration of 1 m/s², find the power delivered by the pulling force at a time

4 seconds after motion starts. What is the average power delivered during these four seconds after the motion starts?



Solution The forces acting on the block are shown in figure.

Resolving forces parallel to incline,

$$F - mg\sin\theta = ma$$

$\Rightarrow$ $$F = mg\sin\theta + ma = 2 \times 9.8 \times \sin 30° + 2 \times 1 = 11.8\,N$$

The velocity after 4 seconds = $u + at = 0 + 1 \times 4 = 4$ m/s

Power delivered by force at $t = 4$ seconds

$$= \text{Force} \times \text{Velocity} = 11.8 \times 4 = \mathbf{47.2\ W}$$

The displacement during 4 seconds is given by the formula

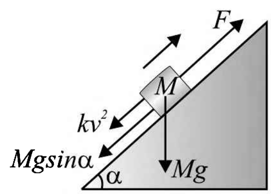
$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 1 \times S \quad \therefore\ S = 8\,m$$

Work done in four seconds = Force × distance = $11.8 \times 8 = 94.4\,J$

$\therefore$ average power delivered $= \dfrac{workdone}{time} = \dfrac{94.4}{4} = \mathbf{23.6\ W}$

Example 70. A motorcar of mass 1000 kg attains a speed of 64 km/hr when running down an inclined of 1 in 20 with the engine shut off. It can attain a speed of 48 km/hr up the same incline when the engine is switched on. Assuming that the resistance varies as the square of the velocity, find the power developed by engine.



Solution When the motor car is moving down the plane there is a force $Mg\sin\alpha$ down the plane. This is opposed by the resistance, which is proportional to square of the velocity. That is

$$Mg \sin\alpha \propto V^2$$

$$Mg \times \frac{1}{20} = kV^2, \text{ where } k \text{ is a constant.}$$

$$\therefore \quad \frac{1000 \times g}{20} = k\left(64 \times \frac{5}{18}\right)^2$$

$$k = \frac{1000 \times g}{20} \times \left(\frac{18}{64 \times 5}\right)^2 \qquad ...(1)$$

When the engine is on, the tractive force (froce exerted by engine) be F. This is used to overcome the force due to incline and the resistance offered.

$$\therefore \quad F = k(48 \times 5/18)^2 + \frac{Mg}{20} \quad F = k(48 \times 5/18)^2 + \frac{1000 \times g}{20}$$

Substituting the value of k from equation (1)

$$F = \frac{1000 \times g}{20} \times \left(\frac{18}{64 \times 5}\right)^2 \times \left(48 \times \frac{5}{18}\right)^2 + \frac{1000 \times g}{20}$$

$$= \frac{1000 \times 9.8}{20}\left[\frac{9}{16} - 1\right] = \frac{50 \times 9.8 \times 25}{16} = 765.6\ N$$

Power developed = Force × Velocity = 765.6 × 48 × 5/18 = 10208 W = **10.2 W**

$$\text{Power}_{\text{avg}} = \frac{\Delta W}{\Delta t} = \frac{total\ work}{total\ time}$$

$$\text{Power}_{\text{inst}} = \frac{dW}{dt} = \frac{d}{dt}(\vec{F}\cdot d\vec{S}) = \vec{F}\cdot\frac{d\vec{S}}{dt} = \vec{F}\cdot\vec{V}$$

$P = (|\vec{F}|\cos\theta)|\vec{V}|$ = (comp. of $\vec{F}$ along $\vec{V}$)·Speed

Example 71. If power delivered by net force if P_0 find velocity as function of time ($t = 0$, vel. $= u$).

Solution $P = \left(m\frac{dv}{dt}\right)v \Rightarrow \int P dt = \int mv\, dv$

Example 72. A block moves in uniform circular motion because a cord tied to the block is anchored at the centre of a circle. Is the power of the force exerted on the block by the cord position, negative, or zero?

Solution Zero

Explanation. $\vec{F}$ and $\vec{v}$ are perpendicular.

∴ Power = $\vec{F}\ .\ \vec{v} = Fv \cos 90ş$ = Zero.

Unit of Power

A unit power is the power of an agent which does unit work in unit time.

The power of an agent is said to be one watt if it does one joule of work in one second.

1 watt = 1 joule/section = 10^7 erg/second

Also, 1 watt $= \frac{1\ newton \times 1\ metre}{1\ \text{second}} = 1\ \text{N m s}^{-1}$.

Dimensional formula of power

$$[\text{Power}] = \frac{[Work]}{[\text{Time}]} = \frac{[ML^2T^{-2}]}{[\text{T}]} = [ML^2\ T^{-3}]$$

Power has 1 dimension in mass, 2 dimensions in length and – 3 dimensions in time.

S.No.	Human Activity	Power (W)
1	Heart beat	1.2
2	Sleeping	83
3	Sitting	120
4	Riding in a car	140
5	Walking (4.8 km h^{-1})	265
6	Cycling (15 km h^{-1})	410
7	Playing Tennis	440

S.No.	Human Activity	Power (W)
8	Swimming (breaststroke, 1.6 km h^{-1})	475
9	Skating	535
10	Climbing Stairs (116 steps min^{-1})	685
11	Cycling (21.3 km h^{-1})	700
12	Playing Basketball	800
13	Tube light	40
14	Fan	60

Example 73. What is represented by the slope of the work-time graph?

Solution Instantaneous power.

Example 74. What is represented by area under power-time graph?

Solution Work.

Example 75. What is the power of an engine which can lift 20 metric ton of coal per hour from a 20 metre deep mine?

Solution Mass, m = 20 metric ton = 20 × 1000 kg; Distance, S = 20 m; Time, t = 1 hour = 3600 s

$$\text{Power} = \frac{Work}{\text{Time}} = \frac{mg \times S}{t} = \frac{20 \times 1000 \times 9.8 \times 20}{3600} \text{ watt} = \mathbf{1.09 \times 10^3}\ W$$

Example 77. A one kilowatt motor pumps out water from a well 10 metre deep. Calculate the quantity of water pumped out per second.

Solution Power, P = 1 kilowatt = 10^3 watt

S = 10 m ; Time, t = 1 second; Mass of water, m = ?

$$\text{Power} = \frac{mg \times S}{t} \quad \therefore\ 10^3 = \frac{m \times 9.8 \times 10}{1} \quad \text{or } m = \frac{10^3}{9.8 \times 10} \text{ kg} = 10.204 \text{ kg}$$

Example 77. The blades of a windmill sweep out a circle of area A. (a) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through in time t ? (b) What is the kinetic energy or the air? (c) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that A = 30 m^2, v = 36 km h^{-1} and the density of air is 1.2 kg m^{-3}. What is the electrical power produced?

Solution

(a) Volume of wind flowing per second = Av

Mass of wind flowing per second = $Av\rho$

Mass of air passing in t second = $Av\rho t$

(b) Kinetic energy of air $= \frac{1}{2} mv^2 = \frac{1}{2} (Av\rho t)v^2 = \frac{1}{2} Av^2\rho t$

(c) Electrical energy produced $= \frac{25}{100} \times \frac{1}{2} Av^3\rho t = \frac{Av^3\rho t}{8}$

$$\text{Electrical power} = \frac{Av^3\rho t}{8t} = \frac{Av^3\rho}{8}$$

Now, $A = 30\text{ m}^2$, $v = 36\text{ km h}^{-1} = 36 \times \frac{5}{18}\text{ m s}^{-1} = 10\text{ m s}^{-1}$, $\rho = 1.2\text{ kg ms}^{-1}$

$$\therefore \quad \text{Electrical power} = \frac{30 \times 10 \times 10 \times 1.2}{8}\ W = 4500\ W = \mathbf{4.5\ kW}$$

Example 78. One coolie takes one minute to raise a box through a height of 2 metre. Another one takes 30 second for the same job and does the same amount of work. Which one of the two has greater power and which one uses greater energy?

Solution Power of first coolie $= \frac{Work}{Time} = \frac{M \times g \times S}{t} = \frac{M \times 9.8 \times 2}{60}\ J\,s^{-1}$

Power of second coolie $= \frac{M \times 9.8 \times 2}{30}\ J\,s^{-1} = 2\left(\frac{M \times 9.8 \times 2}{60}\right)\ J\,s^{-1}$

$= 2 \times$ Power of first coolie

So, the power of the second coolie is double that of the first.

Both the coolies spend the same amount of energy.

Aliter, We know that $W = Pt$

For the same work, $W = p_1t_1 = P_2t_2$

or $\frac{P_2}{P_1} = \frac{t_1}{t_2} = \frac{1\text{ minute}}{30\ s} = 2$ or $P_2 = 2P_1$

Example 79. A large family uses 8 kW of power. Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square metre. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8kW?

Solution If $A\ m^2$ be the area, then power = 200 . A watts

Useful electrical energy produced/s $= \frac{20}{100}(200\,A) = 40\,.\,A$ watts

But $40\,A = 8000$ or $A = 200\text{ m}^2$

Example 80. An elevator of total mass (elevator + passenger) 1800 kg is moving up with a constant speed of 2 ms^{-1}. A frictional force of 4000 N oppose its motion. Determine the minimum power delivered by the motor to the elevator. Take $g = 10\text{ m s}^{-2}$.

Solution Weight of (elevator + passenger) $= mg = 1800 \times 10\ N = 18000\ N$

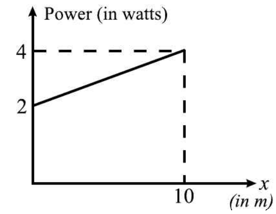
Frictional force = 4000 N

Total downward force on the elevator = $(18000 + 4000)\ N = 22000\ N$

Clearly, the motor must have enough power to balance this force.

Now, power, $P = Fv = 2200\ N \times 2\ m\ s^{-1} = 4400\ W = \frac{44000}{746}\ hp = 58.98\ hp$

Example 81. A particle A of mass $\frac{10}{7}$ kg is moving in the positive direction of x. Its initial position is $x = 0$ and initial velocity is 1 m/s. The velocity at $x = 10$ is: (use the graph given)



(A) 4 m/s (B) 2 m/s (C) $3\sqrt{2}$ m/s (D) 100/3 m/s

Solution Area under P–x graph = $\int \rho\, dx = \int mv \frac{dv}{dt}\, dx = \int_1^v mv^2\, dV = \left[\frac{mv^3}{3}\right]_1^v = \frac{10}{7\times 3}(v^3 - 1)$

from graph ; area = $\frac{1}{2}(2 + 4) \times 10 = 30$

∴ $\frac{10}{7\times 3}(v^3 - 1) = 30$

∴ $v = 4$ m/s

∴ (A) is the right answer.

Alternate:

from graph

$P = 0.2\,x + 2$

or $mv\frac{dv}{dx}\, v = 0.2\,x + 2$ or $mv^2\, dv = (0.2\,x + 2)\, dx$

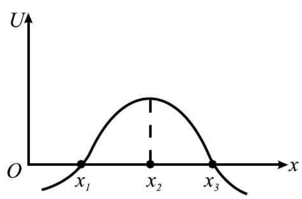
Now integrate both sides, $\int_1^v mv^2 dv = \int_1^{10} (0.2x + 2)\, dx$

Example 82. Work done by static friction on an object:

(A) May be positive (B) Must be negative
(C) Must be zero (D) None of these

[Ans. (A)]

Example 83. In the figure shown the potential energy U of a particle is plotted against its position 'x' from origin. Then which of the following statement is correct. A particle at :



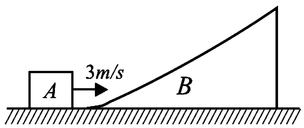
(A) x_1 is in stable equilibrium (B) x_2 is in stable equilibrium
(C) x_3 is in stable equilibrium (D) None of these

Solution $x = x_1$ and $x = x_3$ are not equilibrium positions because $\dfrac{du}{dx} \neq 0$ at these points.

$x = x_2$ is unstable, as u is max. at this point.

∴ (D) is the right option.

Example 84. In the figure shown A is of mass 1 kg and B is mass 2 kg. A moves with velocity 3 m/s and rises on B. All the surfaces are smooth. By the time A reaches the highest point on B:



(A) work done by A on B is zero
(B) work done by gravity on B is positive
(C) work done by A on B is 1 Joules
(D) work done by B on A is 1 Joule

[Ans. (C)]

Example 85. Power delivered to a body varies as $P = 3\,t^2$. Find out the change in kinetic energy of the body from $t = 2$ to $t = 4$ sec.

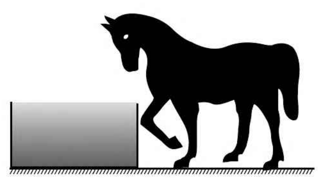
Solution Applying work energy theorem to body

ΔKE = work done by forces delivering power P

$$= \int_{t=2}^{4} P dt = \int_{2}^{4} 3t^2\, dt = 56\,J$$

[Ans. 56 J]

Example 86. A horse drinks water from a cubical container of side 1 m. The level of the stomach of horse is at 2 m from the ground. Assume that all the water drunk by the horse is at a level of 2 m from the ground. Then minimum work done by the horse in drinking the entire water of the container is (Take $\rho_{water} = 1000$ kg/m^3 and $g = 10$ m/s^2):

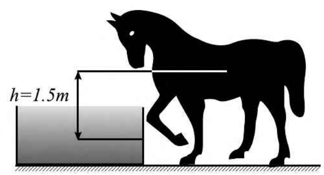


(A) 10 kJ
(B) 15 kJ
(C) 20 kJ
(D) zero

Solution The mass of water is

$m = 1 \times 10^3$ kg

∴ The increase in potential energy of water is



$= mgh = (1 \times 10^3)\,(10)\,1.5 = 15\,kJ$

∴ (B) is the right option.

Example 87. A particle is projected vertically upwards with a speed of 16 m/s, after some time, when it again passes through the point of projection, its speed is found to be 8 m/s. It is known that the work done by air resistance is same during upward and downward motion. Then the maximum height attained by the particle is (Take $g = 10$ m/s^2):

(A) 8 m
(B) 4.8 m
(C) 17.6 m
(D) 12.8 m

Solution From work energy theorem

for upward motion

$$\frac{1}{2}m(16)^2 = mgh + W \text{ (work by air resistance)}$$

for downward motion

$$\frac{1}{2}m(8)^2 = mgh - W$$

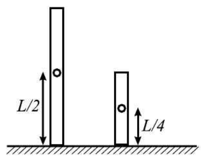
$$\frac{1}{2}[(16)^2 + (8)^2] = 2gh \qquad \text{or} \qquad h = 8\,m$$

∴ (A) is the right option.

Example 88. A man places a chain (of mass 'm' and length 'ℓ') on a table slowly. Initially the lower end of the chain just touches the table. The man drops the chain when half of the chain is in vertical position. Then work done by the man in this process is:

(A) $-mg\frac{\ell}{2}$ (B) $-\frac{mg\ell}{4}$ (C) $-\frac{3mg\ell}{8}$ (D) $-\frac{mg\ell}{8}$

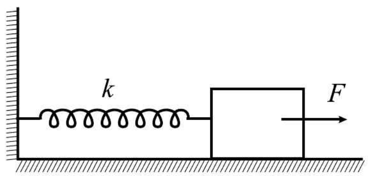
Solution The work done by man is negative of magnitude of decrease in potential energy of chain



$$\Delta U = mg\frac{L}{2} - \frac{m}{2}g\frac{L}{4} = 3mg\frac{L}{8} \quad \therefore -\frac{3mg\ell}{8}$$

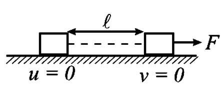
∴ (C) is the right option.

Example 89. A block attached to a spring, pulled by a constant horizontal force, is kept on a smooth surface as shown in the figure. Initially, the spring is in the natural state. Then the maximum positive work that the applied force F can do is: [Given that string does not break]



(A) $\frac{F^2}{K}$ (B) $\frac{2F^2}{K}$ (C) ∞ (D) $\frac{F^2}{2K}$

Solution Applying work energy theorem on block



$$F\ell - \frac{1}{2}k\ell^2 = 0 \therefore \qquad \ell = \frac{2F}{k} \text{ or work done} = F\ell = \frac{2F^2}{k}$$

∴ (B) is the right option.

Example 90. The potential energy for a force field $\vec{F}$ is given by $U(x,y) = \cos(x + y)$. The force acting on a particle at position given by coordinates $\left(0, \frac{\pi}{4}\right)$ is

(A) $-\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (B) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ (C) $\left(\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j}\right)$ (D) $\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{2}\hat{j}\right)$

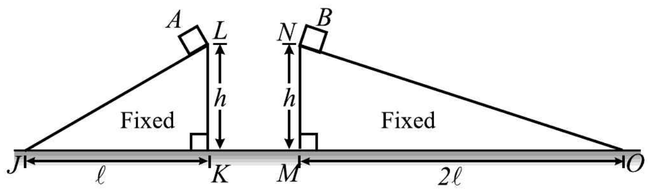
Solution $F_x = -\frac{\partial U}{\partial x} = \sin(x+y)$ $F_y = -\frac{\partial v}{\partial y} = \sin(x+y)$

$F_x = \sin(x+y)]_{(0,\,\pi/4)} = \frac{1}{\sqrt{2}}$ $F_y = \sin(x+y)]_{(0,\,\pi/4)} = \frac{1}{\sqrt{2}}$

$\therefore$ $F = \frac{1}{\sqrt{2}}\left[\hat{i}+\hat{j}\right]$

$\therefore$ (B) is the right option.

Example 91. Two identical blocks *A* and *B* are placed on two inclined planes as shown in diagram. Neglect air resistance and other friction



Read the following statements and choose the correct options.

Statements I: Kinetic energy of '*A*' on sliding to J will be greater than the kinetic energy of *B* on falling to *M*.

Statements II: Acceleration of '*A*' will be greater than acceleration of '*B*' when both are released to slide on inclined plane

Statements III: Work done by external agent to move block slowly from position *B* to *O* is negative

(A) only statement I is true (B) only statement II is true
(C) only I and III are true (D) only II and III are true

Solution

Statements I: Work done by gravity is same for motion from *A* to *Z* and *B* to *M* for equal mass. So K.E. will be equal.

Statements II: Acceleration = $g \sin\theta$

$\tan\theta_A > \tan\theta_B$

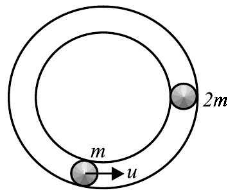
$\frac{h}{\ell} > \frac{h}{2\ell}$

Statement III: $W_g + W_{ext} = 0$ (Because moved slowly)

$W_{ext} = -W_g$

W_g is positive so $W_{ext} < 0$

$\therefore$ (D) is the right option.



Example 92. Two masses 'm' and '$2m$' are placed in fixed horizontal circular smooth hollow tube as shown. The mass 'm' is moving with speed 'u' and the mass '$2m$' is stationary. After their first collision, the time elapsed for next collision. (coefficient of restitution $e = 1/2$)

(A) $\frac{2\pi r}{u}$ (B) $\frac{4\pi r}{u}$

(C) $\frac{3\pi r}{u}$ (D) $\frac{12\pi r}{u}$

Solution Let the speeds of balls of mass m and $2m$ after collision be v_1 and v_2 as shown in figure. Applying conservation of momentum

$$mv_1 + 2mv_2 = mu \quad \text{and} \quad -v_1 + v_2 = \frac{u}{2}$$

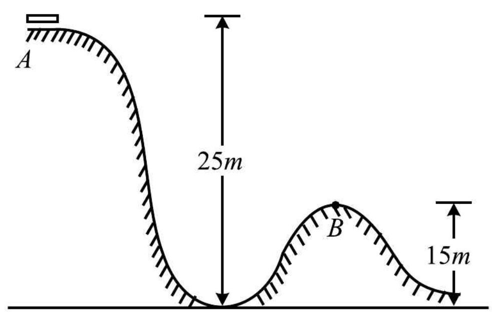
$$\text{solving we get } v_1 = 0 \quad \text{and} \quad v_2 = \frac{u}{2}$$

Hence the ball of mass m comes to rest and ball of mass $2m$ moves with speed $\frac{u}{2}$.

$$t = \frac{2\pi r}{u/2} = \frac{4\pi r}{u}$$

$\therefore$ (B) is the right option.

Example 93. Figure shows the roller coaster track. Each car will start from rest at point A and will roll with negligible friction. It is important that there should be at least some small positive normal force exerted by the track on the car at all points, otherwise the car would leave the track. With the above fact, the minimum safe value for the radius of curvature at point B is ($g = 10$ m/s^2):



(A) 20 m
(B) 10 m
(C) 40 m
(D) 25 m

Solution (A) $V_B = \sqrt{2\times10\times10}$; $\frac{mv_B^2}{R} \le mg$; $R \ge \frac{v_B^2}{g} \Rightarrow R \ge 20$ m

Example 94. A fire hose has a diameter of 2.5 cm and is required to direct a jet of water to a height of at least 40 m. The minimum power of the pump needed for this hose is:

(A) 21.5 *kW* (B) 40 *kW* (C) 36.5 *kW* (D) 48 *kW*

Solution The speed of the water leaving the hose must be $\sqrt{2gh}$ if it is to reach a height h when directed vertically. If the diameter is d, the volume of water ejected at this speed is $\frac{1}{4}\pi d^2 \times \sqrt{2gh}\ \frac{m^3}{s}$.

Mass ejected is $\frac{1}{4}\pi d^2 \times \sqrt{2gh} \times \rho\ \frac{kg}{s}$.

The kinetic energy of this water leaving the hose = $\frac{1}{2}mv^2 = \frac{1}{8}\pi d^2 \times (2gh)^{3/2} \times \rho = 21.5\ kW.$

$\therefore$ (A) is the right answer.

Example 95 A particle with total energy E moves in one dimension in a region where the potential energy is $U(x)$. The acceleration of the particle is zero where:

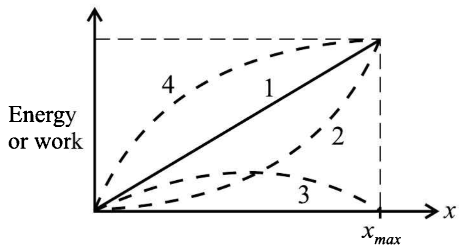
(A) $U(x) = E$ (B) $U(x) = 0$ (C) $\frac{dU(x)}{dx} = 0$ (D) $\frac{d^2U(x)}{dx^2} = 0$

Solution $F = 0$ when $\frac{dU(x)}{dx} = 0$

$\therefore$ (C) is the right option.

Answer Q.no.96 to 98 based on the paragraph given.

A spring lies along an x axis attached to a wall at one end and a block at the other end. The block rests on a frictionless surface at $x = 0$. A force of constant magnitude F is applied to the block that begins to compress the spring, until the block comes to a maximum displacement x_{max}.



Example 96. During the displacement, which of the curves shown in the graph best represents the kinetic energy of the block.

(A) 1 (B) 2 (C) 3 (D) 4

[Ans. (C)]

Example 97. During the displacement, which of the curves shown in the graph best represents the work done on the spring block system by the applied force.

(A) 1 (B) 2 (C) 3 (C) 4

[Ans. (A)]

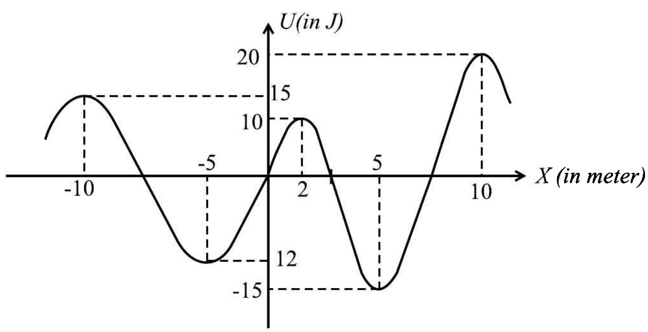
Example 98. During the first half of the motion, applied force transfers more energy to the

(A) kinetic energy (B) potential energy

(C) equal to both (D) depends upon mass of the block

[Ans. (C)]

Example 99. In the figure the variation of potential energy of a particle of mass $m = 2$kg is represented w.r.t. its x-coordinate. The particle moves under the effect of this conservative force along the x-axis. Which of the following statements is incorrect about the particle:



(A) If it is released at the origin it will move in negative x-axis.

(B) If it is released at $x = 2 + \Delta$ where $\Delta \to 0$ then its maximum speed will be 5 m/s and it will perform oscillatory motion

(C) If initially $x = -10$ and $\vec{u} = \sqrt{6}\,\hat{i}$ then it will cross $x = 10$

(D) $x = -5$ and $x = +5$ are unstable equilibrium positions of the particle

Solution If the particle is released at the origin, it will try to go in the direction of force. Here $\frac{du}{dx}$ is positive and hence force is negative, as a result it will move towards –ve x-axis.

When the particle is released at $x = 2 + \Delta$; it will reach the point of least possible potential energy (–15 *J*) where it will have maximum kinetic energy.

$$\therefore \quad \frac{1}{2}mv_{max}^2 = 25 \quad \Rightarrow \quad v_{max} = 5 \text{ m/s}$$

The particle will now perform oscillatory motion between $-15 \le U \le 15$, because reaching $U = +15$ J, the kinetic energy and hence speed becomes zero.

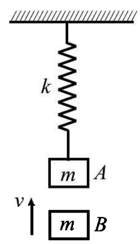
In (C); $E_i = U_i + k_i = 15 + 6 = 21\,J$

At $x = 10$; $U_f = 20 \Rightarrow k_f = 1 \neq 0$

$\Rightarrow$ The particle cross $x = 10$.

$\therefore$ (D) is the right option.

Example 100. Block '*A*' is hanging from a vertical spring and is at rest. Block '*B*' strikes the block '*A*' with velocity '*v*' and sticks to it. Then the value of '*v*' for which the spring just attains natural length is:



(A) $\sqrt{\frac{60\,m\,g^2}{k}}$ (B) $\sqrt{\frac{6\,m\,g^2}{k}}$ (C) $\sqrt{\frac{10\,m\,g^2}{k}}$

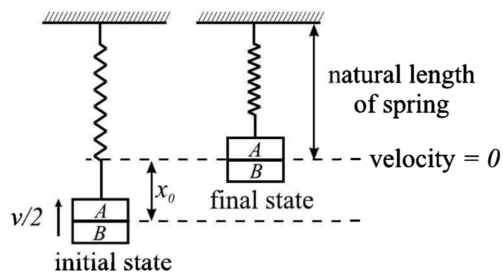
(D) none of these

Solution The initial extension in spring is $x_0 = \frac{mg}{k}$

Just after collision of B with A the speed of combined mass is $\frac{v}{2}$.

For the spring to just attain natural length the combined mass must rise up by $x_0 = \frac{mg}{k}$ (sec fig.) and comes to rest.

Applying conservation of energy between initial and final state



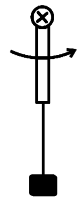
$$\frac{1}{2}\,2m\left(\frac{v}{2}\right)^2 + \frac{1}{2}\,k\left(\frac{mg}{k}\right)^2 = 2mg\left(\frac{mg}{k}\right)$$

Solving we get $v = \sqrt{\frac{6mg^2}{k}}$

Alternative solution by SHM

$$\frac{v}{2} = \sqrt{\frac{k}{2m}}\sqrt{\left(\frac{2mg}{k}\right)^2 - \left(\frac{mg}{k}\right)^2}\ ; \qquad v = \sqrt{\frac{2k}{m}}\sqrt{\frac{3m^2g^2}{k^2}} = \sqrt{\frac{6mg^2}{k}}\ ]$$

Example 101. One end of a light rod of length 1 m is attached with a string of length 1m. Other end of the rod is attached at point O such that rod can move in a vertical circle. Other end of the string is attached with a block of mass 2kg. The minimum velocity that must be given to the block in horizontal direction so that it can complete the vertical circle is (g = 10 m/s²).

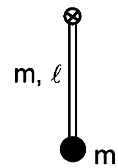


(A) $4\sqrt{5}$ (B) $5\sqrt{5}$ (C) 10 (D) $3\sqrt{5}$

Solution $V_{min} = \sqrt{5gR} = \sqrt{5\times10\times2} = 10$ m/s

Example 102. A particle is attached to the lower end of a uniform rod which is hinged at its other end as shown in the figure. The minimum speed given to the particle so that the rod performs circular motion in a vertical plane will be:

[Length of the rod is ℓ, consider masses of both rod and particle to be same.]



(A) $\sqrt{5g\ell}$ (B) $\sqrt{4g\ell}$ (C) $\sqrt{4.5g\ell}$ (D) none of these

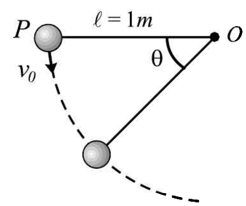
Solution For (rod + particle) system:

$$\frac{1}{2}\left(\frac{m\ell^2}{3}\right)\left(\frac{v^2}{\ell^2}\right) + \frac{1}{2}mv^2 = 2\,mg\left(\frac{3\ell}{2}\right)$$

[Since, com will finally reach a height $2\left(\frac{3\ell}{4}\right)$] $\Rightarrow v = \sqrt{4.5g\ell}$

$\therefore$ (C) is the right answer.

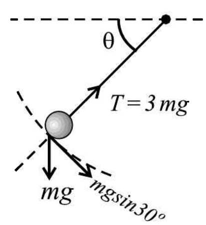
Example 103. The sphere at P is given a downward velocity v_0 and swings in a vertical plane at the end of a rope of ℓ = 1m attached to a support at O. The rope breaks at angle 30° from horizontal, knowing that it can withstand a maximum tension equal to three times the weight of the sphere. Then the value of v_0 will be: ($g = \pi^2$ m/s²)



(A) $\frac{\pi}{2}$ m/s (B) $\frac{2\pi}{3}$ m/s

(C) $\sqrt{\frac{3}{2}}\pi$ m/s (D) $\frac{\pi}{3}$ m/s

Solution $T - mg\sin\theta = \dfrac{mv^2}{R}$

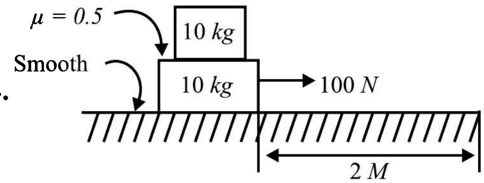


$\Rightarrow$ $3\,mg - mg\sin 30r = \dfrac{m.(u_0^2 + 2g\ell\sin 30°)}{\ell}$

$\therefore$ $u_0 = \sqrt{3g/2}$

$\therefore$ (C) is the right option.

Example 104.



(i) Find out w.d. by applied force during displacement 2m.

Solution

$100 \times 2 \times \cos 0° = 200$ J

(ii) Find out w.d. by frictional force on B by A during the displacement.

Solution

$f_{smax} = \mu mg = 0.5 \times 10\ g = 5$ N

Assuming they move together.

$100 = 2a \Rightarrow a = 5$ m/s^2

Check Friction on A

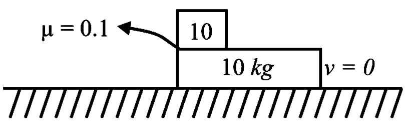
$f = 10 \times 5 = 50$ N

$f_{reqd} = r_{available}$ ∴ They move together

Hence, $\left.\begin{matrix}(W_f)_{on\ B} = 100J \\ (W_f)_{on\ A} = 100J\end{matrix}\right\}$ Net Zero

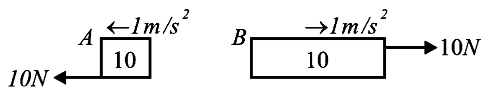
i.e. w.d by internal static friction is zero.

Example 105.



(i) Find out the velocity of two blocks when frictional force stops acting.

Solution



$V_A = 10 - 1\,t$

$V_B = 1t$

$V_A = V_B$

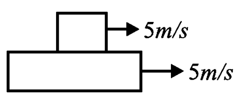
$10 - t = t$

$10 = 2t$

$t = 25$ sec.

$V_B = V_A = 5$ m/s

Situation becomes



(ii) Find out displacement of A and B till velocity becomes equal.

$S_A = 10 \times 5 - \frac{1}{2} \times 1 \times 5^2$

$$S_B = \frac{1}{2} \times 1 \times 5^2 = 12.5 \text{ m}$$

Example 106. In the above question find work done by kinetic friction on A and B.

Solution $W_{KF})_{on}$ A = 10 × 37.5 cos 180ţ = – 375 J

$W_{KF})_{on}$ B = 10 × 12.5 cos 0ţ = 125 J

Work done by KF on system of A and B

= – 375 + 125 = – 250 J

Workdone by KF on a system is always negative.

Heat generated = – (W_{KF}) on a system

(W_{KF}) on system = – ($f_k \times S_{relative}$)

= – 10 × 25 = – 250 J

True and False:

Example 107. Workdone by KF on a body in never zero.

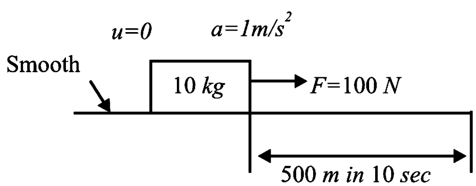
Solution False

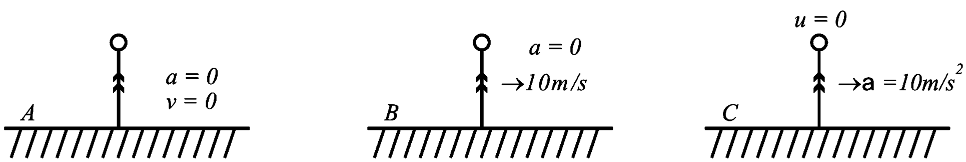
Example 108. Work done by KF on a system is always negative.

Solution True

Kinetic Energy of a body frame dependents as velocity is as frame dependent quantity. Therefore pseudo force work has to be considered.

Work Done by Pseudo Force





Find out work done by the force F in 10 seconds as observed by A, B and C.

Solution $(W_F)_{\text{on block w.r.t. A}}$ = 100 × 500 J = 50,000 J

$(W_F)_{\text{on block w.r.t. B}}$ = 100 [500 – 10 × 10] = 40,000 J

$(W_F)_{\text{on block w.r.t. C}}$ = 100 [500 – 500] = 0

Work Done by Internal Force

$F_{AB} = F_{BA}$ i.e. sum of internal force is zero.

But is not necessary that work done by internal force is zero. There must be some determation or reformation between the system to do internal work. Incase of a rigid body work done by internal force is zero.

Assertion Reason Questions

Example 1.

STATEMENT–1: The work done by all forces on a system equals to the change in kinetic energy of that system. This statement is true even if nonconservative forces act on the system.

STATEMENT–2: The total work done by internal forces may be positive.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

[**Ans.** (B)]

Solution Both the statements are true. The work done by all forces on a system is equal to change in its kinetic energy, irrespective of fact whether work done by internal forces is positive, is zero or is negative.

Example 2.

STATEMENT–1: Work done by a force on a body whose centre of mass does not move may be non-zero.

STATEMENT–2: Work done by a force depends on the displacement of the centre of mass.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

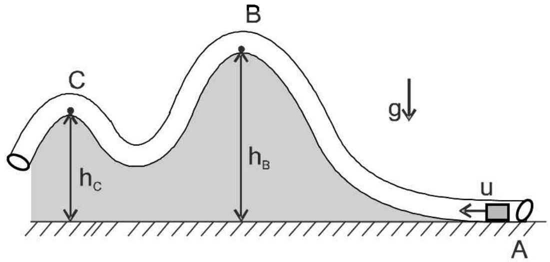
(D) Statement-1 is False, Statement-2 is True.

Solution Work done depends on displacement of point of application of force and not on displacement of centre of mass. Hence statement-2 is false.

[**Ans.** (C)]

Example 3.

STATEMENT-1: A small block of mass m is projected with some speed from point A on smooth vertical tube–track of small diameter as shown. The vertical height of points B and C from point of projection are h_B and h_C such that $h_B = 2h$ and $h_C = h$. Then minimum possible speed of block at point C is $\sqrt{2gh}$ (where g is acceleration due to gravity).



STATEMENT-2: The minimum speed of block at point C in situation given in statement-1 depends on $h_B - h_C$,

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Solution For speed to be minimum at C, the speed at B should be just zero.

Hence statement-2 is correct explanation of statement-1.

[**Ans.** (A)]

Example **4.**

STATEMENT-1: The sum of potential and kinetic energy for a system of moving objects is conserved only when no net external force acts on the objects.

STATEMENT-2: If no nonconservative force acts on a system of objects, the work done by external forces on a system of objects is equal to change in potential energy plus change in kinetic energy of the system.

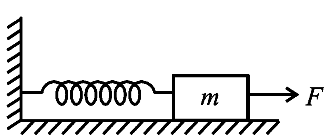
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Solution Even if net external force on system is zero, work done by all external forces may not be zero. Hence statement -1 is false.

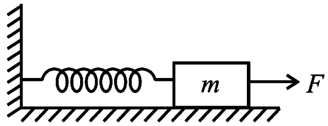
[**Ans.** (D)]

Example **5.**

STATEMENT-1: One end of ideal massless spring is connected to fixed vertical wall and other end to a block of mass m initially at rest on smooth horizontal surface. The spring is initially in natural length. Now a horizontal force F acts on block as shown. Then the maximum extension in spring is equal to maximum compression in spring.



STATEMENT-2: To compress and to expand an ideal unstretched spring by equal amount, same work is to be done on spring.



(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

Solution The maximum extension is non-zero, while the spring never undergoes compression.

Hence statement-1 is false.

[Ans. (D)]

Match the following

***Example* 1.** Motion of particle is described in column-I. In column-II, some statements about work done by forces on the particle from ground frame is given. Match the particle's motion given in column-I with corresponding possible work done on the particle in certain time interval given in column-II.

Column I	Column II
(A) A particle is moving in horizontal circle	(p) work done by all the forces may be positive
(B) A particle is moving in vertical circle with uniform speed	(q) work done by all the forces may be negative
(C) A particle is moving in air (projectile motion without any air resistance) under gravity	(r) work done by all the forces must be zero
(D) A particle is attached to roof of moving train on inclined surface.	(s) work done by gravity may be positive.

[Ans. (A) p,q (B) r, s (C) p,q,s (D) p,q,s]

Solution

(A) If motion is uniform circular motion (constant speed), change in kinetic energy of particle is zero

$$W_{all} = KE_2 - KE_1$$
$$W_{all} = 0$$

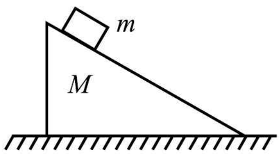
If motion is non uniform circular motion then kinetic energy of particle may decrease or increase. So work done by all the forces may be positive or negative.

(B) The particle's speed is constant, so work done by all the force is zero. For vertical downward displacement, work done by gravity is positive.

(C) In projectile motion, for upward vertical displacement, speed particle decreases, so work done by all the forces will be negative. For vertical downward displacement, speed of particle increases, so work done by all the force will be positive.

(D) If the speed of train is increasing, then work done by all the forces is positive and vice versa. If train is moving downward the incline, work done by gravity on the particle is positive.

Example 2. A block of mass m lies on wedge of mass M. The wedge in turn lies on smooth horizontal surface. Friction is absent everywhere. The wedge block system is released from rest. All situation given in column-I are to be estimated in duration the block undergoes a vertical displacement 'h' starting from rest (assume the block to be still on the wedge). Match the statement in column-I with the results in column-II. (g is acceleration due to gravity).



	Column I		Column II
(A)	Work done by normal reaction acting on the block is	(p)	positive
(B)	Work done by normal reaction (exerted by block) acting on wedge is	(q)	negative
(C)	The sum of work done by normal reaction on block and work done by normal reaction (exerted by block) on wedge is	(r)	zero
(D)	Net work done by all forces on block is	(s)	less than mgh in magnitude

[**Ans.** (A) q, s (B) p, s (C) r, s (D) p, s]

Solution

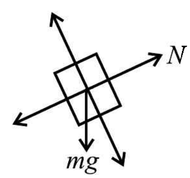
(A) The *FBD* of block is angle between velocity of block and normal reaction on block is obtuse

$\therefore$ work by normal reaction on block is negative.

As the block fall by vertical distance h, from work energy Theorem

Work done by mg + work done by $N = KE$ of block

$\therefore$ $|\text{work done by } N| = mgh - \frac{1}{2}mv^2$



$\because$ $\frac{1}{2}mv^2 < mgh$

$\therefore$ $|\text{work done by } N| < mgh$

(B) Work done by normal reaction on wedge is positive

Since loss in PE of block = $K.E.$ of wedge + $K.E.$ of block

Work done by normal reaction on wedge = KE of wedge.

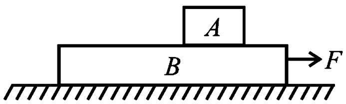
∴ Work done by $N < mgh$.

(C) Net work done by normal reaction on block and wedge is zero.

(D) Net work done by at force on block is positive, because its kinetic energy has increased. Also KE of block $< mgh$

∴ Net work done on block = final KE of block $< mgh$.

Example 3. A block A of mass m kg lies on block B of mass m kg. B in turn lies on smooth horizontal plane. The coefficient of friction between A and B is μ. Both the blocks are initially at rest. A horizontal force F is applied to lower block B at $t = 0$ such that there is relative motion between A and B. In the duration from $t = 0$ second till the lower block B undergoes a displacement of magnitude L, match the statements in column-I with results in column-II.



Column I	Column II
(A) Work done by friction force on block A is	(p) positive
(B) Work done by friction force on block B is	(q) negative
(C) Work done by friction on block A plus work done by friction on block B is	(r) less than μ mgL in magnitude
(D) Work done by force F on block B is	(s) equal to μ mgL in magnitude

[**Ans.** (A) *p*, *r* (B) *q*, *s* (C) *q*, *r* (D) *p*]

Solution The displacement of A shall be less than displacement L of block B.

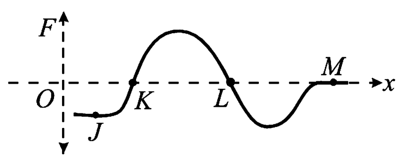
Hence work done by friction on block A is positive and its magnitude is less than μmgL.

And the work done by friction on block B is negative and its magnitude is equal to μmgL.

Therefore workdone by friction on block A plus on block B is negative its magnitude is less than μmgL.

Work done by F is positive. Since $F > 2\mu mg$, magnitude of work done by F shall be more than $2\mu mgL$.

Example 4. A particle moving along x-axis is being acted upon by one dimensional conservative force F. In the F–x curve shown, four points J, K, L, M are marked on the curve. Column II gives different type of equilibrium for the particle at different positions. Column I gives certain positions on the force position graphs. Match the positions in Column-I with the corresponding nature of equilibrium at these positions.



Column I	Column II
(A) Point J is position of	(p) Neutral equilibrium
(B) Point K is position of	(q) Unstable equilibrium

(C) Point L is position of (r) Stable equilibrium

(D) Point M is position of (s) No equilibrium

Ans. (A) s (B) q (C) r (D) p

Solution Point

$J \longrightarrow$ No equilibrium

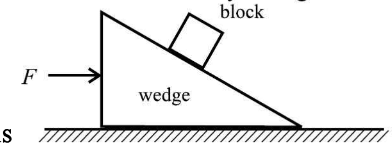
$K \longrightarrow$ Unstable equilibrium

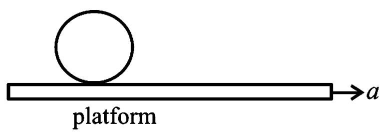
$L \longrightarrow$ Stable equilibrium

$M \longrightarrow$ Neutral equilibrium

***Example* 5.** Match the statements in Column I with the results in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

Column I	Column II
(a) Work done by ideal gas during free expansion	(P) zero
(b) A wedge block system is as shown in the fig. The wedge lying on horizontal surface is accelerated to right by a horizontal force F. All surfaces are smooth. Work done by normal reaction exerted by wedge on block in any time interval is	(Q) non zero
(c) Two identical conducting spheres of radius 'a' are separated by a distance 'b' (b>>a). Both spheres carry equal and opposite charge. Net electrostatic potential energy of system of both spheres is	(R) negative
(d) A uniform cylinder lies over a rough horizontal platform. The platform is accelerated horizontally as shown with acceleration a. The cylinder does not slip over the platform. The work done by the force of friction on the cylinder w.r.t ground in any time interval is	(S) positive





[**Ans.** (A) p (B) q,s (C) q,s (D) q,s]

Solution

(A) Work done by an ideal gas during free expansion is zero.

(B) The angle between normal reaction on block and velocity of block is acute (whether the block moves up or down the incline). Hence work done by this force is non-zero and positive.

(C) Net electrostatic potential energy

$$= S_1 + S_2 + M_{12} = \frac{Q^2}{8\pi \in_0 a} + \frac{Q^2}{8\pi \in_0 a} - \frac{Q^2}{4\pi \in_0 b}$$

$$= \frac{Q^2}{4\pi \in_0 a} - \frac{Q^2}{4\pi \in_0 b} = \text{non-zero and positive. } (\because b >> a)$$

(D) The kinetic energy of cylinder is increasing and work is done on cylinder by only force of friction.

Therefore work done by force of friction on cylinder is non-zero and positive.

Example 6. Net force on a system of particles in ground frame is zero. In each situation of column-I a statement is given regarding this system. Match the statements in column-I with the results in column-II.

	Column I		Column II
(A)	Acceleration of centre of mass of system from ground frame	(p)	is constant
(B)	Net momentum of system from ground frame.	(q)	is zero
(C)	Net momentum of system from frame of centre of mass of system	(r)	may be zero
(D)	*K.E.* of system from frame of centre of mass of system	(s)	may be constant

[**Ans.** (A) *p*,*q* (B) *p*,*r* (C) *p*,*q* (D) *r*,*s*]

Solution

(A) If net force = 0, then a_{cm} from ground frame is zero and constant.

(B) If $a_{cm} = 0$, then v_{cm} = constant and constant may be zero.

(C) Net momentum from centre of mass frame is always zero.

(D) *KE* (of system with $a_{cm} = 0$) is least from centre of mass frame. This may be constant or may be zero.

True/False

Example 1. Select True/False

Whenever a block is placed on the rough horizontal surface of another block and lower block is moved horizontally, work done by friction on upper block is always positive.

Solution The friction on '*A*' is in the direction of its motion

∴ w.d. is positive

Example 2.

S_1: Path of a particle moving along a straight line with respect to the observer moving along another straight line must be straight line.

S_2: A man is standing inside a lift which is moving upwards. He shall feel more weight as compared to when lift was at rest

S_3: A block moves down the inclined surface of a wedge. The wedge lies on smooth horizontal surface. The work done by normal reaction (exerted by wedge on the block) on the block must be zero.

S_4: Whenever a block is placed on top of the rough horizontal surface of another block and lower block is moved horizontally, work done by friction on upper block is always positive.

(A) *F F T T* (B) *F F F T* (C) *F T F T* (D) *T T F T*

Solution If the initial relative velocity and relative acceleration are neither parallel nor anti parallel, the particle shall move in any not straight line curve. Hence S_1 is false

If the lift is retarding upwards , he shall feel lighter. Hence S_2 is false

For moving wedge, the work done by normal reaction on block (exerted by wedge) is non zero. Hence S_3 is false.

The friction on 'upper block is in the direction of its motion. ∴ w.d. is positive. Hence S_4 is true.

[Ans. (B)]

Example 3. If the internal forces within a system are conservative, then the work done by the external forces on the system is equal to the change in mechanical energy of the system.

Solution True

Example 4. The work kinetic theorem ($W_{all} = \Delta K$) is not valid in non-inertial frame.

Solution (False)

Example 5.

S_1: If the internal forces within a system are conservative, then the work done by the external forces on the system is equal to the change in mechanical energy of the system.

S_2: The potential energy of a particle moving along x-axis in a conservative force field is $U = 2x^2 - 5x + 1$ in S.I. units. No other forces are acting on it. It has a stable equilibrium position at one point on x-axis.

S_3: Internal forces can perform net work on a rigid body.

S_4: Internal forces can perform net work on a non–rigid body.

(A) $TTFT$ (B) $TFFT$ (C) $FFTT$ (D) $FTFT$

Solution S_1: The statement is true from Work Energy Theorem

S_2: $F = -\dfrac{dU}{dx} = -4x + 5$ ∴ SHM

S_3 & S_4: A rigid body by definition cannot be expanded or compressed, thus it cannot store mechanical potential energy. Hence internal forces can do no work on rigid body, but can do work on non-rigid body. Hence S_3 is false and S_4 is true.

[Ans. (A)]

Example 6. The potential energy of a particle moving along x-axis in a conservative force field is $U = 2x^2 - 5x + 1$ in S.I. units. No other forces are acting on it. It performs SHM.

Solution $F = -\dfrac{dU}{dx} = -4x + 5$ ∴ SHM **(True)**

Example 7. Internal forces can perform net work on a rigid body.

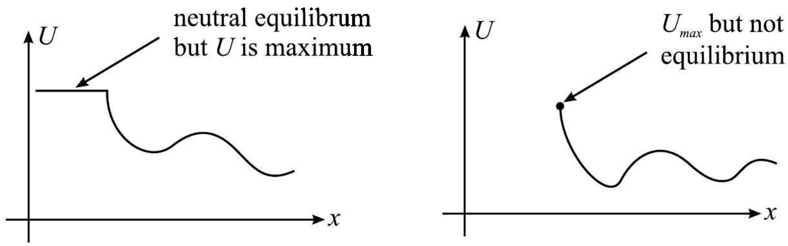
Solution Net work done by internal forces is equal to change is $P.E.$ of system.

Example 8. Internal forces can perform net work on a non–rigid body.

Solution A rigid body by definition cannot be expanded or compressed, thus it cannot store mechanical potential energy. Hence internal forces can do no work on rigid body, but can do work on non-rigid body.

Example 9. "The position of maximum potential energy is the position of unstable equilibrium". State whether the statement is true or false, with short reason.

Solution False, it can be neutral but not equilibrium at all

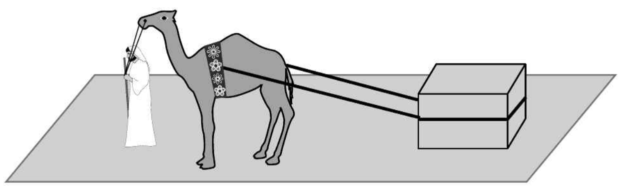


Comprehension Questions

Comprehension - 1

Ram and Ali are two fast friends since childhood. Ali neglected studies and now has no means to earn money other than a camel whereas Ram becomes an engineer. Now both are working in the same factory. Ali uses camel to transport the load within the factory.

Due to low salary & degradation in health of camel, Ali becomes worried and meet his friend Ram and discusses his problem. Ram collected some data and with some assumptions concluded the following.



(i) The load used in each trip is 1000 kg and has friction coefficient $\mu_k = 0.1$ and $\mu_s = 0.2$.

(ii) Mass of camel is 500 kg.

(iii) Load is accelerated for first 50 m with constant acceleration, then it is pulled at a constant speed of 5m/s for 2 km and at last stopped with constant retardation in 50 m.

(iv) From biological data, the rate of consumption of energy of camel can be expressed as $P = 18 \times 10^3 V + 10^4$ J/s where P is the power and V is the velocity of the camel.

After calculations on different issues Ram suggested proper food, speed of camel etc. to his friend. For the welfare of Ali, Ram wrote a letter to the management to increase his salary.

(Assuming that the camel exerts a horizontal force on the load)

Example 6. Sign of work done by the camel on the load during parts of motion: accelerated motion, uniform motion and retarted motion respectively are:

(A) +ve, + ve, +ve (B) +ve, +ve, – ve
(C) +ve, zero, – ve (D) +ve, zero, +ve

Solution (A) $W_{CL} + W_f = \Delta KE$ $\therefore W_{CL} = \Delta KE - W_f$

(a) During accelerated motion negative work is done against friction and there is also change is kinetic energy. Hence net work needed is +ve.

(b) During uniform motion work is done against friction only and that is +ve.

(c) During retarded motion, the load has to be stopped in exactly 50 metres. If only friction is considered then the load stops in 12.5 metres which is less than where it has to stop.

Hence the camel has to apply some force so that the load stops in 50m (>12.5 m). Therefore the work done in this case is also +ve. **[Ans.** (A)]

Example 7. The ratio of magnitude of work done by camel on the load during accelerated motion to retarded motion is:

(A) 3: 5 (B) 2.2: 1 (C) 1: 1 (D) 5: 3

Solution $W_{CL}|_{\text{accelerated motion}} = \Delta KE - W_{\text{friction}}$ where W_{CL} is work done by camel on load.

$$= \left[\frac{1}{2}mv^2 - 0\right] - [-\mu_k mg.50]$$

$$= \frac{1}{2}\times 1000\times 5^2 + 0.1\times 10\times 1000\times 50 = 1000\left[\frac{125}{2}\right]$$

similarly, $W_{CL}|_{\text{retardation}} = \Delta KE - W_{\text{friction}}$

$$\left[0 - \frac{1}{2}mv^2\right] - [-\mu_k mg.50] = 1000\left[\frac{75}{2}\right]$$

$$\therefore \quad \frac{W_{CL}|_{accelerated\ \text{motion}}}{W_{CL}|_{retarded\ \text{motion}}}$$

$$= \frac{125}{75} = \frac{5}{3}$$

$\Rightarrow$ 5: 3

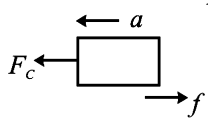
[Ans. (D)]

Example 8. Maximum power transmitted by the camel to load is:

(A) 6250 *J/s* (B) 5000 *J/s* (C) 10^5 *J/s* (D) 1250 *J/s*

Solution Maximum power = $F_{\max} \times V$

Maximum force applied by camel is during the accelerated motion.



We have $V^2 - U^2 = 2as$

$$25 = 0^2 + 2\,.\,a\,.\,50$$

$a = 0.25$ m/s^2 ; for accelerated motion

$\therefore \quad F_C - f = ma$

$\therefore \quad F_C = \mu mg + ma = 0.1 \times 1000 \times 10 + 1000 \times 2.5$

$= 1000 + 250 = 1250\ N$

This is the critical point just before the point where it attains maximum velocity of almost 5 m/s.

Hence maximum power at this point is = 1250 × 5 = 6250 J/s.

[Ans. (A)]

Example 9. The ratio of the energy consumed of the camel during uniform motion for the two cases when it moves with speed 5 m/s to the case when it moves with 10 m/s:

(A) $\frac{19}{20}$ (B) $\frac{19}{10}$ (C) $\frac{10}{19}$ (D) $\frac{20}{19}$

Solution We have

$W = P\Delta T$

$P = 18 \times 10^3 V + 10^4 J/s$

$\therefore \quad P_5 = 18 \times 10^3 \times 5 + 10^4 J/s$ and $\Delta T_5 = \frac{2000m}{5\ m/s} = 400$ s

$P_{10} = 18 \times 10^3 \times 10^4 J/s$ and $\Delta T_{10} = \frac{2000m}{10\ m/s} = 200$ s

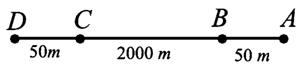
$\therefore \quad W_5/W_{10} = \frac{10^4(9+1) \times 400}{10^4(18+1) \times 200} = \frac{20}{19}$

[Ans. (D)]

Example 5. The total energy consumed of the camel during the trip of 2100 m is:

(A) $2.1 \times 10^6 J$ (B) $4.22 \times 10^7 J$ (C) $2.22 \times 10^4 J$ (D) $4.22 \times 10^6 J$

Solution The time of travel in accelerated motion = time of travel in retarded motion.



$T_{AB} = T_{CD} = \frac{V}{a} = \frac{5}{0.25} = 20$ sec.

Now time for uniform motion = $T_{BC} = \frac{2000}{5} = 400$ sec.

$\therefore$ Total energy consumed = $\int_0^{440} Pdt$

$= \int_0^{20}[18.10^3 V + 10^4]dt + \int_{20}^{420}[18.10^3.5 + 10^4]dt + \int_{420}^{440}[18.10^3 V + 10^4]dt$

$= \int_0^{20} 18.10^3 V\, dt + \int_0^{20} 10^4\, dt + \left[10^5 t\right]_{20}^{420} + \int_{420}^{440} 18.10^3 Vdt + \int_{420}^{440} 10^4 dt$

Putting $Vdt = dx$ and changing limits appropriately.

it becomes $\int_0^{50} 18.10^3 dx + \left[10^4 t\right]_0^{20} + 10^5[420 - 20] + \int_{2050}^{2100} 18.10^3 dx + \left[10^4\right]_{420}^{440}$

$= 18.10^3.50 + 10^4[20] + 10^5.400 + 18.10^3[50] + 10^4[20]$ Joules

$= 90 \times 10^4 + 20 \times 10^4 + 400 \times 10^5 + 90 \times 10^4 + 20 \times 10^4$ Joules

$= 4.22 \times 10^7$ Joules

[Ans. (B)]

Alternate:

$E = \int Pdt = \int (18 \times 10^3 V + 10^4)\, dt$

$= 18 \times 10^3 \boxed{\int Vdt} \rightarrow$ [Total distance travelled]

$+ 10^4 \int dt$

$$E = 18 \times 10^3 \times 2100 + 10^4\,[T]$$

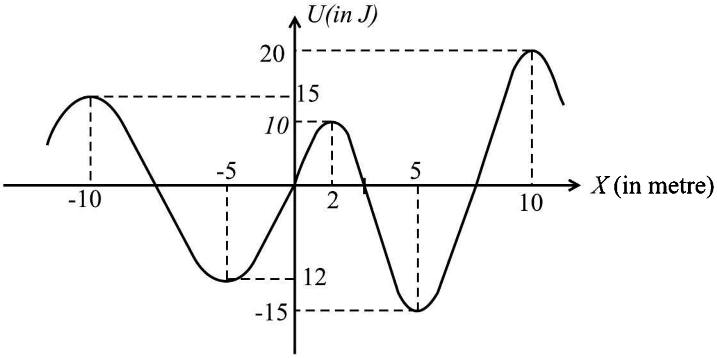
$$T = \frac{50}{\left(0+\frac{5}{2}\right)} \times 2 + \frac{2000}{5}$$

$$T = \frac{440}{5}$$

$$E = 18 \times 10^3 \times 2100 + \frac{10^4 \times 440}{5}$$

Comprehension – 2

In the figure the variation of potential energy of a particle of mass m = 2kg is represented w.r.t. its x-coordinate. The particle moves under the effect of this conservative force along the x-axis.



Example If the particle is released at the origin then:

(A) it will move towards positive x-axis.
(B) it will move towards negative x-axis.
(C) it will remain stationary at the origin.
(D) its subsequent motion cannot be decided due to lack of information.

Solution If the particle is released at the origin, it will try to go in the direction of force. Here $\frac{du}{dx}$ is positive and hence force is negative, as a result it will move towards – ve x-axis.

[Ans. (B)]

Example If the particle is released at $x = 2 + \Delta$ where $\Delta \to 0$ (it is positive) then its maximum speed in subsequent motion will be:

(A) $\sqrt{10}$ m/s (B) 5 m/s (C) $5\sqrt{2}$ (D) 7.5 m/s

Solution When the particle is released at $x = 2 + \Delta$; it will reach the point of least possible potential energy (–15 J) where it will have maximum kinetic energy.

$$\therefore \quad \frac{1}{2} m v_{\max}^2 = 25 \quad \Rightarrow \quad v_{\max} = 5 \text{ m/s}$$

[Ans. (B)]

Example $x = -5$ m and $x = 10$ m positions of the particle are respectively of

(A) neutral and stable equilibrium. (B) neutral and unstable equilibrium.
(C) unstable and stable equilibrium. (D) stable and unstable equilibrium.

Solution **[Ans.** (D)]

Comprehension – 3

A body of mass m is moving along x-axis under the influence of conservative force with a potential energy given by

$$U(x) = \frac{-cx}{x^2 + a^2}$$

Where c and a are positive constants. When displaced slightly from stable equilibrium position $x = x_0$, it will experience restoring force proportional to its displacement, the force constant being

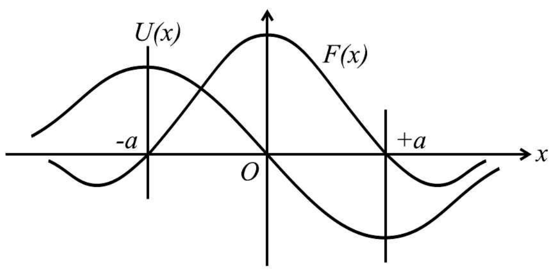
$$\left[\frac{d^2U}{dx^2}\right]_{x=x_0}$$

Example 13. The magnitude of force is maximum at:

(A) $x = 0$ (B) $x = +a$ (C) $x = -a$ (D) no value of x

Solution $F = -\frac{dU}{dx} = \frac{c(x^2 + a^2) - 2x.cx}{(x^2 + a^2)^2} = \frac{(a^2 - x^2)}{(x^2 + a^2)^2}$

[Ans. (A)]



Example 14. The body is in stable equilibrium at

(A) $x = 0$ (B) $x = +a$ (C) $x = -a$ (D) both $x = \pm a$

Solution At $x = \pm a$, $F = 0$ but the stable equilibrium is at $x = +a$ only.

[Ans. (B)]

Example 15. If body is at $x = x_0$ where (i) $x_0 = 2a$ (ii) $x_0 = +a$ (iii) $x_0 = -a$.

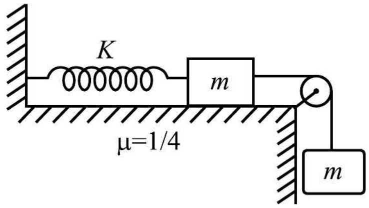
If it is displaced slightly towards right, it will experience restoring force in

(A) all the three cases (B) case (ii) only
(C) case (iii) only (D) cases (i) and (ii) only.

Solution For $x_0 = 2a$ and $x_0 = +a$ for will be restoring when displaced towards right.

Comprehension – 4

Consider the system shown below, with two equal masses m and a spring with spring constant K. The coefficient of friction between the left mass and horizontal table is $\mu = 1/4$, and the pulley is frictionless. The string connecting both the blocks is massless and inelastic. The system is held with the spring at its unstretched length and then released.



Example 17. The extension in spring when the masses come to momentary rest for the first time is

(A) $\frac{3mg}{2K}$ (B) $\frac{mg}{2K}$ (C) $\frac{mg}{K}$ (D) $\frac{2mg}{K}$

Solution From work energy theorem, the masses stop when-total work done on them is zero.

$$W = mgx - \frac{1}{2}kx^2 - \mu mgx = 0$$

$\therefore \quad \frac{2mg}{k}(1-\mu) = \frac{3mg}{2k}$

[Ans. (A)**]**

Example 18. The minimum value of μ for which the system remains at rest once it has stopped for the first time is

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

Solution When the masses are stopped at this value of x, the forces on left mass for it to remain at rest is zero

$\Rightarrow \quad kx = mg + f$

$\Rightarrow \quad k\frac{2mg}{k}(1-\mu) < mg + \mu mg$

$\therefore \quad \mu > 1/3 \quad \therefore$ least value of μ is 1/3.

[Ans. (B)**]**

Example 19. If the string connecting both the masses is cut just at the instant both masses came to momentary rest for the first time in question 17, then maximum compression of spring during resulting motion is (Take $\mu = 1/4$)

(A) $\frac{2mg}{3K}$ (B) $\frac{mg}{2K}$ (C) $\frac{mg}{K}$ (D) $\frac{1mg}{3K}$

Solution At the instant string is cut, let the extension in spring be x_0. The maximum compression x will occur for spring when left block comes to rest first time after the string is cut

$\therefore$ From work energy Theorem $\Delta W = 0$

$$\frac{1}{2}kx_0^2 - \frac{1}{2}kx^2 - \mu mg(x + x_0) = 0$$

$x_0 = \frac{3mg}{2k}$ and $\mu = \frac{1}{4}$

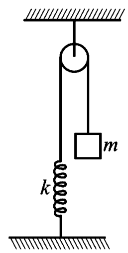
solving we get, $x = \frac{mg}{k}$

[Ans. (C)**]**

EXERCISE

Exercise–1: Subjective Problems

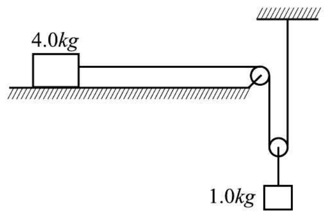
1. Calculate the work done by a coolie in carrying a load of mass 10 kg on his head when the walks uniformly a distance of 5 m in the (i) horizontal direction (ii) vertical direction. (Take $g = 10\ m/s^2$)
2. A cluster of clouds at a height of 1000 metre above the Earth burst and enough rain fell to cover an area of 10^6 square metre with a depth of 2 cm. How much work would have been done in raising water to the height of clouds? Given: g = 980 cm s^{-2} and density of water = 1 g cm^{-3}.
3. A particle moves along the x-axis from $x = 0$ to $x = 5$ m under the influence of a force F (in N) given by $F = 3x^2 - 2x + 7$. Calculate the work done by this force.
4. A flexible chain of length ℓ and mass m is slowly pulled at constant speed up over the edge of a table by a force F parallel to the surface of the table. Assuming that there is no friction between the table and chain, calculate the work done by force F till the chain reaches to the horizontal surface of the table.
5. In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 m s^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet ?
6. It is well known that a raindrop or a small pebble falls under the influence of the downward gravitational force and the opposing resistive force. The latter is known to be proportional to the speed of the drop but is otherwise undetermined. Consider a drop or small pebble of 1 g falling from a cliff of height 1.00 km. It hits the ground with a speed of 50.0 ms^{-1}. What is the work done by the unknown resistive force?
7. A force of 1000 N acts on a particle parallel to its direction of motion which is horizontal. Its velocity increases from 1 ms^{-1} to 10 ms^{-1}, when the force acts through a distance of 4 metre. Calculate the mass of the particle. Given: a force of 10 newton is necessary for overcoming friction.
8. A block of mass m moving at a speed v compresses a spring through a distance x before its speed is halved. Find the spring constant of the spring.
9. Consider the situation shown in figure. Initially the spring is unstretched when the system is released from rest. Assuming no friction in the pulley, find the maximum elongation of the spring.



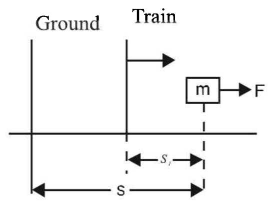
10. A rigid body of mass 0.3 kg is taken slowly up an inclined plane of length 10 m and height 5 m, and then allowed to slide down to the bottom again. The co-efficient of friction between the body and the plane is 0.15. Using $g = 9.8\ m/s^2$ find the

(a) work done by the gravitational force over the round trip.
(b) work done by the applied force (assuming it to be parallel to the inclined plane) over the upward journey
(c) work done by frictional force over the round trip.
(d) kinetic energy of the body at the end of the trip?

11. Consider the situation shown in figure. The system is released from rest and the block of mass 1.0 kg is found to have a speed 0.3 m/s after it has descended through a distance of 1m. Find the coefficient of kinetic friction between the block and the table.

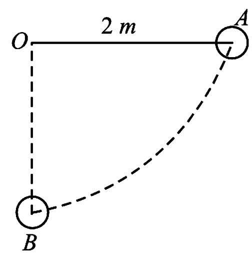


12. A block of mass m sits at rest on a frictionless table in a rail car that is moving with speed v_c along a straight horizontal track (fig.) A person riding in the car pushes on the block with a net horizontal force F for a time t in the direction of the car's motion.



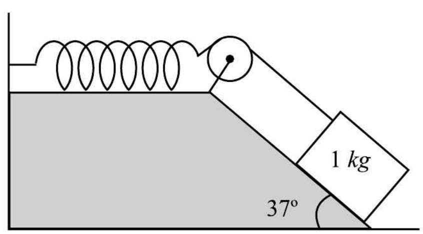
(a) What is the final speed of the block according to a person in the car?
(b) According to a person standing on the ground outside the train?
(c) How much did K of the block change according to the person in the car?
(d) According to the person on the ground?
(e) In terms of F, m, & t, how far did the force displace the object according to the person in car?
(f) According to the person on the ground?
(g) How much work does each say the force did?
(h) Compare the work done to the K gain according to each person.
(i) What can you conclude from this computation?

13. The bob of a pendulum is released from a horizontal position A as shown in figure. If the length of the pendulum is 2 m, what is the speed with which the bob arrives at the lowermost point B, given that it dissipated 10% of its initial energy against air resistance?



14. A person trying to lose weight (dieter) lifts a 10 kg mass through 0.5 m, 1000 times. Assume that the potential energy lost each time she lowers the mass is dissipated. (a) How much work does she do against the gravitational force? (b) Fat supplies $4 \times 10^7 J$ of energy per kilogram which is converted to mechanical energy with a 20% efficiency rate. How much fat will the dieter use up? (use $g = 10$ m/s^2)

15. A 1 kg block situated on a rough inclined plane is connected to a spring of spring constant 100 N m^{-1} as shown. The block is released from rest with the spring in the unstretched position. The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline assume that the spring has negligible mass and the pulley is frictionless.
Take $g = 10$ ms^{-2}.



16. The potential energy function of a particle in a region of space is given as:

$U = (2x^2 + 3y^3 + 2z)\ J$

Here x, y and z are in metres. Find the force acting on the particle at point P($1m$, $2m$, $3m$).

17. The potential energy function of a particle in a region of space is given as

$U = (2xy + yz)J$

Here x, y and z are in metre. Find the force acting on the particle at a general point $P(x, y, z)$.

18. Force acting on a particle in a conservative force field is:

(i) $\vec{F} = (2\hat{i} + 3\hat{j})$ (ii) $\vec{F} = (2x\hat{i} + 3y\hat{j})$

(iii) $\vec{F} = (y\hat{i} + x\hat{j})$

Find the potential energy function, if it is zero at origin.

19. An elevator weighing 500 kg is to be lifted up at a constant velocity of 0.4 m s^{-1}. What should be the minimum horse power of the motor to be used? (Take g = 10 m s^{-2} and 1 hp = 750 watts).

20. A lift is designed to carry a load of 4000 kg in 10 seconds through 10 floors of a building averaging 6 metre per floor . Calculate the hourse power of the lift. (Take g = 10 m s^{-2} and 1 hp = 750 watts).

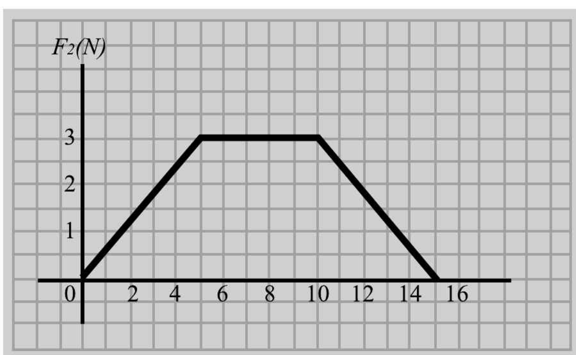
21. A labourer lifts 100 stones to a height of 6 metre in two minute. If mass of each stone be one kilogram, calculate the average power. Given: g = 10 m s^{-2}.

22. An engine lifts 90 metric ton of coal per hour from a mine whose depth is 200 metre. Calculate the power of the engine (use g = 9.8 m/s^2)

23. A motor is capable of rasing 400 kg of water in 5 minute from a well 120 m deep. What is the power developed by the man?

24. A man weighing 70 kg climbs up a vertical staircase at the rate of 1 ms^{-1}. What is the power developed by the man?

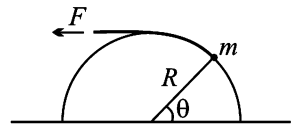
25. A particle is subject to a force F_x that varies with position as in figure. Find the work done by the force on the body as it moves (a) from x = 0 to x = 5.00 m, (b) from x = 5.00 m to x = 10.0 m, and (c) from x = 10.0 m to x = 15.0 m. (d) What is the total work done by the force over the distance x = 0 to x = 15.0 m?



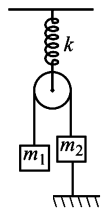
26. A spring, which is initially in its unstretched condition, is first stretched by a length x and then again by a further length x. The work done in the first case is W_1 and in the second case is W_2. Find $\frac{W_2}{W_1}$.

27. A 4 kg particle moves along the X-axis. It's position x varies with time according to $x(t) = t + 2t^3$, where x is in m and t is in seconds. Compute:
    (a) The kinetic energy at time t.
    (b) The force acting on the particle at time t.
    (c) The power delivered to the particle at time t.
    (d) The work done on the particle from $t = 0$ to $t = 2$ seconds.

28. A small object of mass m is pulled to the top of a frictionless half-cylinder (of radius R) by a cord that passes over the top of the cylinder, as illustrated in figure. If a the object moves at a constant speed, show that $F = mg \cos \theta$. Find the work done in moving the object at constant speed from the bottom to the top of the half cylinder.



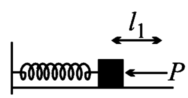
29. In the figure shown, pulley and spring are ideal. Find the potential energy stored in the spring ($m_1 > m_2$).



30. The $P.E.$ of a particle oscillating on x-axis is given as $U = 20 + (x - 2)^2$ here U is in Joules & x is in meters. Total mechanical energy of particle is 36 J
    (a) Find the mean position
    (b) Find the max. $K.E.$ of the particle

31. The potential function for a conservative force is given by $U = k(x + y)$. Find the work done by the conservative force in moving a particle from the point $A(1, 1)$ to point B (2, 3).

32. A block of mass m placed on a smooth horizontal surface is attached to a spring and is held at rest by a force P as shown. Suddenly the force P changes its direction opposite to the previous one. How many times is the maximum extension l_2 of the spring longer compared to its initial compression l_1?

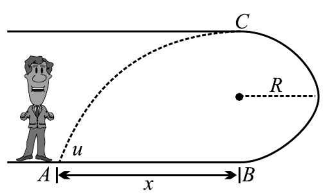


33. Power applied to a particle varies with time as $P = (3t^2 - 2t + 1)$ watt, where t is in second. Find the change in its kinetic energy between time $t = 2$ s and $t = 4$ s.

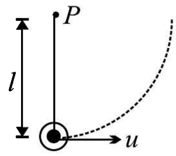
34. A 650-kg elevator starts from rest. It moves upward for 3.00 s with constant acceleration until it reaches its cruising speed of 1.75 m/s.
    (a) What is the average power of the elevator motor during this period?
    (b) How does this power compare with its power when it moves at its cruising speed?

35. Water is pumped from a depth of 10 m and delivered through a pipe of cross section 10^{-2} m^2 upto a height of 10 m. If it is needed to deliver a volume 0.2 m^3 per second, find the power required. [Use $g = 10$ m/s^2]

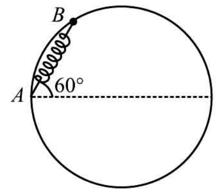
36. A person rolls a small ball with speed u along the floor from point A. If $x = 3R$, determine the required speed u so that the ball returns to A after rolling on the circular surface in the vertical plane from B to C and becoming a projectile at C. What is the minimum value of x for which the game could be played if contact must be maintained to point C? Neglect friction.



37. Consider the shown arrangement when a is bob of mass 'm' is suspended by means of a string connected to peg P. If the bob is given a horizontal velocity $\vec{u}$ having magnitude $\sqrt{3gl}$, find the minimum speed of the bob in subsequent motion.

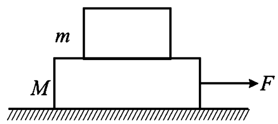


38. A bead of mass m is attached to one end of a spring of natural length $\sqrt{3}R$ and spring constant $k = \frac{(\sqrt{3}+1)mg}{R}$. The other end of the spring is fixed at point A on a smooth fixed vertical ring of radius R as shown in the figure. What is the normal reaction at B just after the bead is released?



39. A body of mass 2 kg is moving under the influence of a central force whose potential energy is given by $U(r) = 2r^3$ Joule. If the body is moving in a circular orbit of $5m$,then find its energy.

40. A block of mass m is kept over another block of mass M and the system rests on a horizontal surface (figure). A constant horizontal force F acting on the lower block produces an acceleration $\frac{F}{2(m+M)}$ in the system, the two blocks always move together.



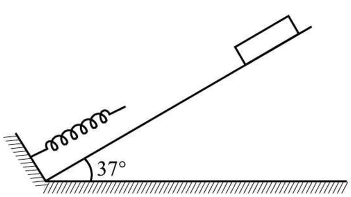
(a) Find the coefficient of kinetic friction between the bigger block and the horizontal surface. (b) Find the frictional force acting on the smaller block. (c) Find the work done by the force of friction on the smaller block by the bigger block during a displacement d of the system.

41. A box weighing 2000 N is to be slowly slid through 20 m on a straight track having friction coefficient 0.2 with the box. (a) Find the work done by the person pulling the box with a chain at an angle θ with the horizontal. (b) Find the work when the person has chosen a value of θ which ensures him the minimum magnitude of the force.

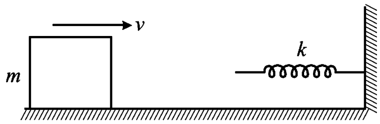
42. A particle of mass m moves on a straight line with its velocity varying with the distance travelled according to the equation $v = a\sqrt{x}$, where a is a constant. Find total work done by all the forces during a displacement from $x = 0$ to $x = d$.

43. The US athlete Florence Griffith-Joyner won the 100 m sprint gold medal at Seol Olympic 1988 setting a new Olympic record of 10.54 s. Assume that she achieved her maximum speed in a very short time and then ran the race with that speed till she crossed the line. Take her mass to be 50 kg. (a) Calculate the kinetic energy of Griffith-Joyner at her full speed. (b) Assuming that the track, the wind etc. offered an average resistance of one tenth of her weight, calculate the work done by the resistance during the run. (c) What power Griffith-Joyner had to exert to maintain uniform speed ?

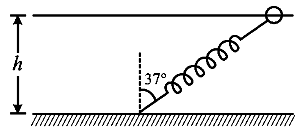
44. Figure shows a spring fixed at the bottom end of an incline of inclination 37°. A small block of mass 2 kg starts slipping down the incline from a point 4.8 m away from the spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance of 1 m up the incline. Find (a) The friction coefficient between the plane and the block and (b) the spring constant of the spring. Take g=10 m/s^2.



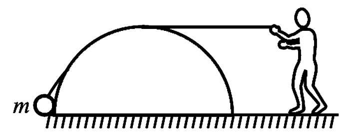
45. A block of mass m sliding on a smooth horizontal surface with a velocity $\vec{v}$ meets a long horizontal spring fixed at one end and having spring constant k as shown in figure. Find the maximum compression of the spring. Will the velocity of the block be the same as $\vec{v}$ when it comes back to the original position shown?



46. One end of a spring of natural length h and spring constant k is fixed at the ground and the other is fitted with a smooth ring of mass m which is allowed to slide on a horizontal rod fixed at a height h (figure). Initially, the spring makes an angle of 37° with the vertical when the system is released from rest. Find the speed of the ring when the spring becomes vertical.



47. As shown in the figure a person is pulling a mass 'm' from ground on a fixed rough hemispherical surface upto the top of the hemisphere with the help of a light inextensible string. Find the work done by tension in the string if radius of hemisphere is R and friction coefficient is μ. Assume that the block is pulled with negligible velocity.



48. Two blocks of masses m_1 and m_2 are connected by a spring of stiffness k. The coefficient of friction between the blocks and the surface is μ. Find the minimum constant force F to be applied to m_1 in order to slide the mass m_2.

49. A particle of mass m approaches a region of force starting from $r = +\infty$. The potential energy function in terms of distance r from the origin is given by,

$$U(r) = \frac{K}{2a^3}(3a^2 - r^2) \qquad \text{for,} \qquad 0 \le r \le a$$

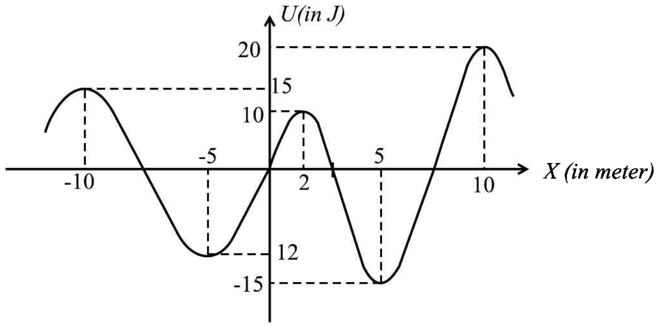
$$= K/r \qquad \text{for,} \qquad r \ge a$$

(a) Derive the force $F(r)$ and determine whether it is repulsive or attractive.

(b) With what velocity should the particle start at $r = \infty$ to cross over to $r =$ on the other side of the origin.

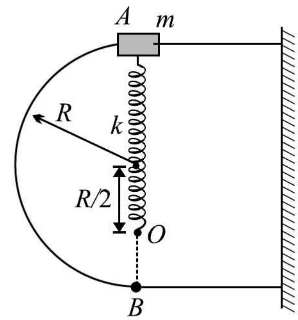
(c) If the velocity of the particle at $r = \infty$ is $\sqrt{\frac{2K}{am}}$, towards the origin describe the motion.

50. In the figure the variation of potential energy of a particle of mass m = 2kg is represented w.r.t. its x-coordinate. The particle moves under the effect of this conservative force along the x-axis. Which of the following statements is incorrect about the particle is true:



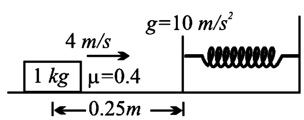
1. If it is released at the origin it will move in negative x-axis.
2. If it is released at $x = 2 + \Delta$ where $\Delta \to 0$ then its maximum speed will be 5 m/s and it will perform oscillatory motion
3. If initially $x = -10$ and $\vec{u} = \sqrt{6}\,\hat{i}$ then it will cross $x = 10$
4. $x = -5$ and $x = +5$ are unstable equilibrium positions of the particle

51. A bead of mass m is tied at one end of a spring of spring constant $\frac{mg}{R}$ and unstretched length $\frac{R}{2}$ and other end to fixed point O. The smooth semicircular wire frame is fixed in vertical plane. Find the normal reaction between bead and wire just before it reaches the lowest point.



52. A particle of mass m is hanging with the help of an elastic string of unstretched length a and force constant $\frac{mg}{a}$. The other end is fixed to a peg on vertical wall. String is given an additional extension of $2a$ in vertical downward direction by pulling the mass and released from rest. Find the maximum height reached by it during its subsequent motion above point of release. (Neglect interaction with peg if any.)

53. A particle of mass 1 kg is given a horizontal velocity of 4 m/s along a horizontal surface, with which it has a coefficient of friction (both static and kinetic) of 0.4. The particle strikes a fixed ideal spring of force constant 6 N/m after travelling a distance of 0.25 m. Assume acceleration due to gravity is 10 m/s^2. Find the final displacement of the particle from its starting point.



54. A point moves along a circle having a radius 20 cm with a constant tangential acceleration 5 cm/s^2. How much time is needed after motion begins for the normal acceleration of the point to be equal to tangential acceleration ?

55. A single conservative force F(x) acts on a 1.0 kg particle that moves along the x-axis. The potential energy $U(x)$ is given by: $U(x) = 20 + (x - 2)^2$

where x is in meters. At $x = 5.0$ m the particle has a kinetic energy of 20 J.

(a) What is the mechanical energy of the system ?

(b) Make a plot of $U(x)$ as a function of x for $-10\text{ m} < x < 10\text{m}$, and on the same graph draw the line that represents the mechanical energy of the system. Use part (b) to determine

(c) The least value of x and

(d) The greatest value of x between which the particle can move.

(e) The maximum kinetic energy of the particle and

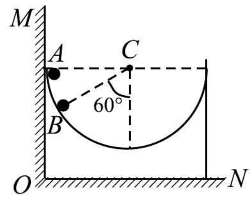
(f) The value of x at which it occurs.

(g) Determine the equation for $F(x)$ as a function of x.

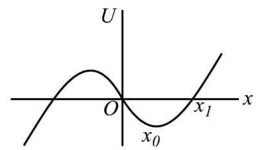
(h) For what (finite) value of x does $F(x) = 0$?

Exercise–2

1. A ball of mass 1 kg is released from position A inside a wedge with a hemispherical cut of radius 0.5 m as shown in the figure. Find the force exerted by the vertical wall OM on wedge, when the ball is in position B. (neglect friction everywhere).
Take ($g = 10\text{ m/s}^2$)



2. A particle is confined to move along the $+x$ axis under the action of a force $F(x)$ that is derivable from the potential $U(x) = ax^3 - bx$.

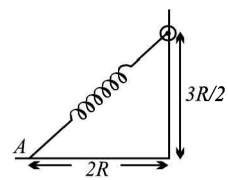


(a) Find the expression for $F(x)$

(b) When the total energy of the particle is zero, the particle can be trapped with in the interval $x = 0$ to $x = x_1$. For this case find the values of x_1.

(c) Determine the maximum kinetic energy that the trapped particle has in its motion. Express all answers in terms a and b. At what value of x will the kinetic energy be maximum ?

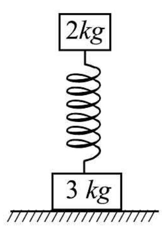
6. A ring of mass m can slide over a smooth vertical rod. The ring is connected to a spring of force constant $K = \dfrac{4mg}{R}$ where $2R$ is the natural length of the spring. The other end of the spring is fixed to the ground at a horizontal distance $2R$ from the base of the rod. The mass is released at a height of $1.5R$ from ground



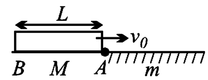
(a) calculate the work done by the spring.

(b) calculate the velocity of the ring as it reaches the ground.

7. The ends of spring are attached to blocks of mass 3*kg* and 2*kg*. The 3*kg* block rests on a horizontal surface and the 2*kg* block which is vertically above it is in equilibrium producing a compression of 1cm of the spring. The 2*kg* mass must be compressed further by at least _______, so that when it is released, the 3 kg block may be lifted off the ground.



8. A uniform rod of mass m length L is sliding along its length on a horizontal table whose top is partly smooth & rest rough with friction coefficient μ. If the rod after moving through smooth part, enters the rough with velocity v_0.

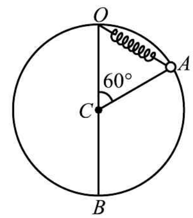


   (a) What will be the magnitude of the friction force when its x length ($< L$) lies in the rough part during sliding.
   (b) Determine the minimum velocity v_0 with which it must enter so that it lies completely in rough region before coming to rest.
   (c) If the velocity is double the minimum velocity as calculated in part (a) then what distance does its front end A would have travelled in rough region before rod comes to rest.

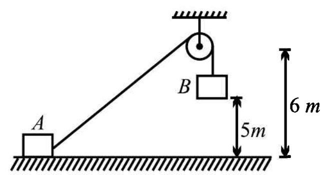
9. A car's engine can deliver 90kW of power. The car's mass is 1000kg. Assume the total resistive force is proportional to the velocity: $|F_{friction}| = \alpha v$. The drag coefficient α is α = 100Ns/m. Car can maintain on a level road a maximum speed of _______ m/s?

10. Two trains of equal masses are drawn along smooth level lines by engines; one of then X exerts a constant force while the other Y works at a constant rate. Both start from rest & after a time t both again have the same velocity v. Find the ratio of travelled distance during the interval.

12. A particle of mass 5 kg is free to slide on a smooth ring of radius r = 20 cm fixed in a vertical plane. The particle is attached to one end of a spring whose other end is fixed to the top point O of the ring. Initially the particle is at rest at a point A of the ring such that $\angle OCA = 60°$, C being the centre of the ring. The natural length of the spring is also equal to r = 20cm. After the particle is released and slides down the ring the contact force between the particle and the ring becomes zero when it reaches the lowest position B. Determine the force constant of the spring.



13. A block of mass m is held at rest on a smooth horizontal floor. A light frictionless, small pulley is fixed at a height of 6 m from the floor. A light inextensible string of length 16 m, connected with A passes over the pulley and another identical block B is hung from the string. Initial height of B is 5m from the floor as shown in Fig. When the system is released from rest, B starts to move vertically downwards and A slides on the floor towards right.

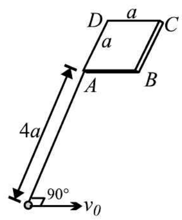


   (i) If at an instant string makes an angle θ with horizontal, calculate relation between velocity u of A and v of B.
   (ii) Calculate v when B strikes the floor.

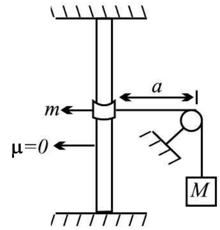
14. A particle of mass 2kg is subjected to a two dimensional conservative force given by $Fx = -2x + 2y$, $Fy = 2x - y^2$. (x, y in m and F in N) If the particle has kinetic energy of (8/3) J at point (2,3), find the speed of the particle when it reaches (1,2).

15. A square plate is firmly atached to a frictionless horizontal plane. One end of a taut cord is attached to point A of the plate and the other end is attached to a sphere of mass m. In the

process, the cord gets wrapped around the plate. The sphere is given an initial velocity v_0 on the horizontal plane perpendicular to the cord which causes it to make a complete circuit of the plate and return to point A. Find the velocity of the sphere when it hits point A again after moving in a circuit on the horizontal plane. Also find the time taken by the sphere to complete the circuit.

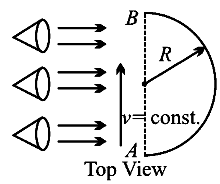


16. A ring of mass m slides on a smooth vertical rod. A light string is attached to the ring and is passing over a smooth peg distant a from the rod, and at the other end of the string is a mass M (> m). The ring is held on a level with the peg and released:
Show that it first comes to rest after falling a distance:

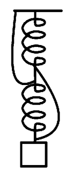


$$\frac{2mMa}{M^2 - m^2}$$

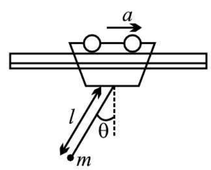
17. A small block can move in a straight horizontal linea along AB. Flash lights from one side projects its shadow on a vertical wall which has horizontal cross section as a circle. Find tangential & normal acceleration of shadow of the block on the wall as a function of time if the velocity of the block is constant (v).



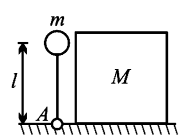
18. In figure two identical springs, each with a relaxed length of 50cm and a spring constant of 500N/m, are connected by a short cord of length 10cm. The upper string is attached to the ceiling, a box that weighs 100N hangs from the lower spring. Two additional cords, each 85 cm long, are also tied to the assembly; they are limp (i.e. slack).
(a) If the short cord is cut, so that the box then hangs from the springs and the two longer cords, does the box move up or down?
(b) How far does the box move before coming to rest again?

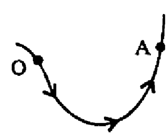


19. The small pendulum of mass m is suspended from a trolley that runs on a horizontal rail. The trolley and pendulum are initially at rest with $\theta = 0$. If the trolley is given a constant acceleration $a = g$ determine the maximum angle θ_{max} through which the pendulum swings. Also find the tension T in the cord in terms of θ.



20. A weightless rod of length l with a small load of mass m at the end is hinged at point A as shown in the figure and occupies a strictly vertical position, touching a body of mass M. A light jerk sets the system in motion. For what mass ratio M/m will the rod form an angle $\alpha = \pi/6$ with the horizontal at the moment of the separation from the body? What will be the velocity u of the body at this moment? Friction should be neglected.

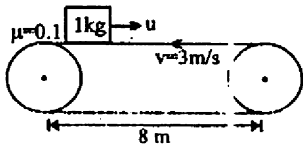




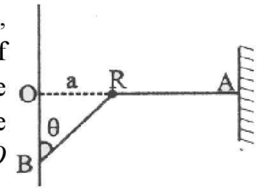
21. A particle which moves along the curved path shown passes point with a speed of 12m/s and slows down to 5m/s at point A in a distance of 18 m

measured along the curve from O. The deceleration of the particle is 10 m/s^2 on it passes A. Find the radius of curvature of A.

22. Find the velocity with which a block of mass 1 kg must be horizontally projected on a conveyer belt moving uniformly at a velocity of 3 m/s so that maximum heat is liberated. Take coefficient of friction of 0.1. Also find the corresponding amount of heat liberated. What happens when belt velocity is 5m/s ?

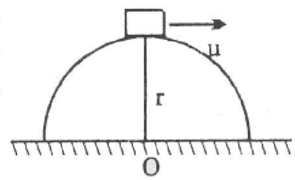


23. A small bead of mass m is free to slide on a fixed smooth vertical wire, as indicated in the diagram. One end of of a light elastic string, of unstretched length a and force constant 2 mg/a is attahced to B. The string passes through a smooth fixed ring R and the other end of the string is attached to the fixed point A, AR being horizontal. The point O on the wire is at same horizontal level as R, and $AR = PO = a$.



    (i) In the equilibrium position, find OB.
    (ii) The bead B is raised to a point C of the wire above C, where $OC = a$, and is released from rest. Find the speed of the bead as it passes O, and find the greatest depth below O of the bead in the subsequent motion.

24. A small block of mass m is projected horizontally from the top of the smooth hemisphere of radius r with speed u as shown. For values of $u > u_0$, it does not slide on the hemisphere (i.e. leaves the surface at the top itself).



    (a) For $u = 2u_0$, it lands at point P on ground Find OP.
    (b) For $u = u_0/3$, find the height from the ground at which it leaves the hemisphere.
    (c) Find its net acceleration at the instant it leaves the hemisphere.

Exercise–3

(JEE/REE Questions of Previous Years)

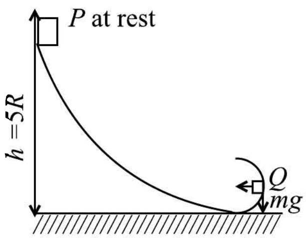
1. A hemispherical bowl of radius $r = 0.1$m is rotating about its axis (which is vertical) with an angular velocity ω. A particle of mass 10^{-2}kg on the frictionless inner surface of the bowl is also rotating with the same ω. the particle is at a height h from the bottom of the bowl. (a) Obtain the relation between h and ω. What is the minimum value of ω needed in order to have a nonzero value of h. (b) It is desired to measure 'g' using this setup by measuring h accurately. Assuming that r and ω are known precisely and that the least count in the measurement of h is 10^{-4}m. What is minimum error Δg in the measured value of g.[$g = 9.8$m/s^2] **[JEE 1993]**

2. A particle of mass m is moving in a circular path of constant radius r such that is centripetal acceleration a_c is varying with time t as $a_c = k^2 rt^2$ where k is a constant. The power delivered to the particle by the force acting on it is- **[JEE 1994]**

    (A) $2\pi mk^2 r^2$ (B) $mk^2 r^2 t$ (C) $\dfrac{(mk^4r^2t^5)}{3}$ (D) Zero

3. A smooth semicircular wire track of radius R is fixed in a vertical plane (figure). On end of a massless spring of natural length $3R/4$ is attached to the lowest point O of the wire track. A small ring of mass m which can slide on the track is attached to the other end of the spring. The ring is held stationary at point P such that the spring make an angle 60° with the vertical. The spring constant $K = mg/R$. Consider the instant when the ring is released **[JEE 1996]**
   (i) Draw the free body diagram of the ring.
   (ii) Determine the tangential acceleration of the ring and the normal reaction.

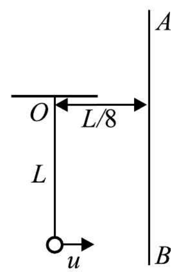
4. A small block of mass m slides along a smooth frictional track as shown in figure. (i) If it starts from rest at P, when is the resultant force acting on it at Q ? (ii) At what height above the bottom of the loop should the block be released so that the force it exerts against the track at the top of the loop equals its weight ? **[REE 1997]**



5. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time the stone is at its lowest position and has a speed u. The magnitude of the change in its velocity as it reaches a position, where the string is horizontal, is **[JEE 1998]**
   (A) $\sqrt{u^2 - 2gL}$ (B) $\sqrt{2gL}$ (C) $\sqrt{u^2 - gL}$ (D) $\sqrt{2(u^2 - gL)}$

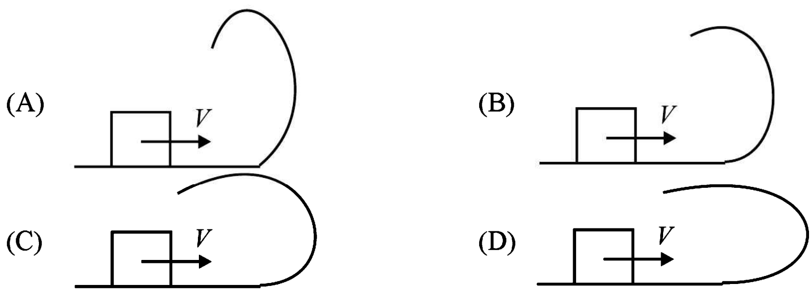
6. A particle at rest starts rolling from the top of a large frictionless sphere of radius R. The sphere is fixed on the ground. Calculate that height from the ground at which the particle leaves the surface of the sphere.
   [REE 1998]

7. A particle is suspended vertically from a point O by an inextensible massless string of length L. A vertical line AB is at a distance $L/8$ from O as shown. The object given a horizontal velocity u. At some point, its motion ceases to be circular and eventually the object passes through the line AB. At the instant of crossing AB, its velocity is horizontal. Find u. **[JEE 1999]**

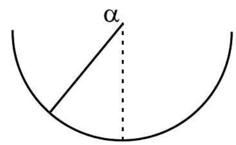


8. A long horizontal rod has a bead which can slide along its length and is initially placed at a distance L from one end A of the rod. The rod is set in angular motion about A with a constant angular acceleration, α . If the coefficient of friction between the rod and the bead is μ, and gravity is neglected, then the time after which the bead starts slipping is - **[JEE 2000]**
   (A) $\sqrt{\frac{\mu}{\alpha}}$ (B) $\frac{1}{\sqrt{\alpha}}$ (C) $\frac{1}{\sqrt{\mu\alpha}}$ (D) Infinitesimal

9. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum is- **[JEE 2001]**



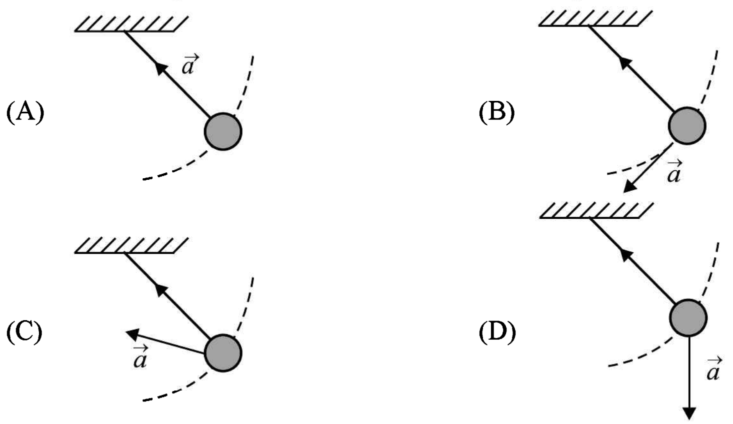
10. An insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the surface and the insect is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given **[JEE 2001]**



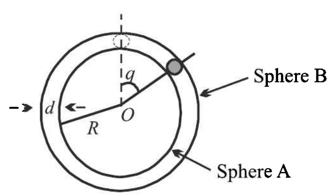
(A) $\cot \alpha = 3$ (B) $\tan \alpha = 3$ (C) $\sec \alpha = 3$ (D) $\operatorname{cosec} \alpha = 3$

11. A block is placed inside a horizontal hollow cylinder. The cylinder starts rotating with one revolution per second about its axis. The angular position of the block at which it begins to slide is 30° below the horizontal level passing through the center. Find the radius of the cylinder if the coefficient of friction is 0.6. What should be the minimum angular speed of the cylinder so that the block reach the highest point of the cylinder? **[REE 2001]**

12. A simple pendulum is oscillating without damping. When the displacements of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in **[JEE 2002]**



13. A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is depended by θ (shown in figure)



(a) Express the total normal reaction force exerted by the spheres on the ball as a function of angle θ.

(b) Let N_A and N_B denote the magnitudes of the normal reaction force on the ball extorted by the spheres *A* and *B*, respectively. Sketch the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two separate graphs in your answer book, taking $\cos\theta$ on the horizontal axis **[JEE 2002]**

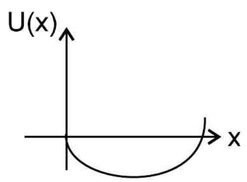
14. A double star system consists of two stars *A* and *B* which have time period T_A and T_B. Radius R_A and R_B and mass M_A and M_B. Choose the correct option. **[IIT 2006]**
(A) If $T_A > T_B$ then $R_A > R_B$ (B) If $T_A > T_B$ then $M_A > M_B$
(C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$ (D) $T_A = T_B$

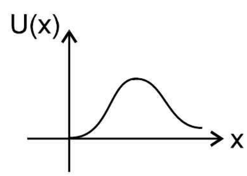
15. A force $\vec{F} = -K(y\hat{i} + x\hat{j})$ where *K* is a positive constant, acts on a particle moving in the *x-y* plane. Starting from the origin, the particle is taken along the positive *x*-axis to the point (a,0) and then parallel to the *y*-axis to the point (*a, a*). The total work done by the force $\vec{F}$ on the particle is **[JEE 1998]**
(A) $-2Ka^2$ (B) $2Ka^2$ (C) $-Ka^2$ (D) Ka^2

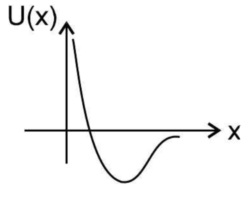
16. A pump motor is used to deliver water at a certain rate from a given pipe. To obtain "*n*" times water from the same pipe in the same time, the amount by which the power of the motor should be increased is **[REE 1998]**
(A) n^2 (B) n^3 (C) n^4 (D) $n^{1/2}$

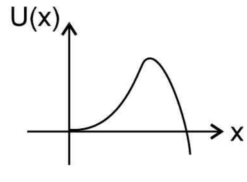
17. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to **[JEE 2000]**
(A) v (B) v^2 (C) v^3 (D) v^4

18. A particle, which is constrained to move along the *x*-axis, is subjected to a force in the same direction which varies with the distance *x* of the particle from the origin as $F(x) = -kx + ax^3$. Here *k* and *a* are positive constants. For $x \ge 0$, the functional form of the potential energy $U(x)$ of the particle is **[JEE 2002]**

(A) 

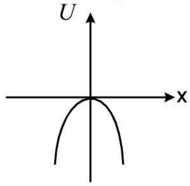
(B) 

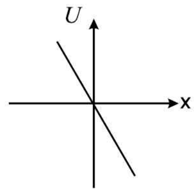
(C) 

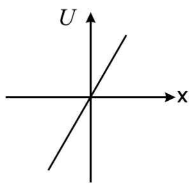
(D) 

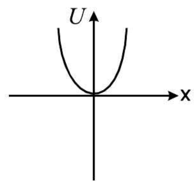
19. An ideal spring with spring-constant *k* is hung from the ceiling and a block of mass *M* is attached to its lower end. The mass is released with the spring initially unstreched. Then the maximum extension in the spring is **[JEE 2002]**
(A) $4Mg/k$ (B) $2Mg/k$ (C) Mg/k (D) $Mg/2k$

20. A particle moves under the influence of a force $F = kx$ in one dimensions (k is a positive constant and x is the distance of the particle from the origin). Assume that the potential energy of the particle at the origin is zero, the schematic diagram of the potential energy U as a function of x is given by **[JEE 2004]**

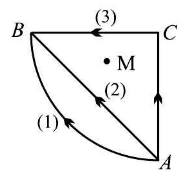
(A) 

(B) 

(C) 

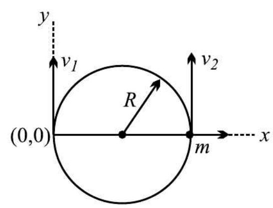
(D) 

21. In a region of only gravitational field of mass 'M' a particle is shifted from A to B via three different paths in the figure. The work done in different paths are W_1, W_2, W_3 respectively then **[JEE 2003]**



(A) $W_1 = W_2 = W_3$
(B) $W_1 = W_2 > W_3$
(C) $W_1 > W_2 > W_3$
(D) $W_1 < W_2 < W_3$

22. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point $(2R, 0)$ at time $t = 0$ and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time $t = 0$. Calculate the linear momentum of the particle w.r.t. the man as a function of time. **[JEE 2003]**



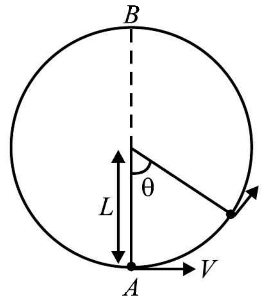
23. **STATEMENT-1**: A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation. **because**

STATEMENT-2: The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True **[JEE 2007]**

24. A bob of mass M is suspended by a massless string of length L. The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies

[JEE 2008]



(A) $\theta = \frac{\pi}{4}$ (B) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

(C) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (D) $\frac{3\pi}{4} < \theta < \pi$

25. **STATEMENT-1:** For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary. **[JEE 2008]**

and

STATEMENT-2: If the observer and the object are moving at velocities $\vec{V}_1$ and $\vec{V}_2$ respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is $\vec{V}_2 - \vec{V}_1$.

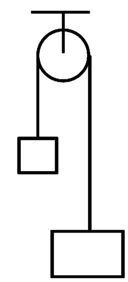
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

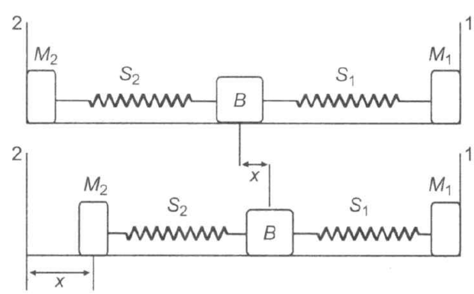
(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

26. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10\ \text{m/s}^2$, find the work done (in **joules**) by the string on the block of mass 0.36 kg during the first second after the system is released from rest. **[JEE 2009]**

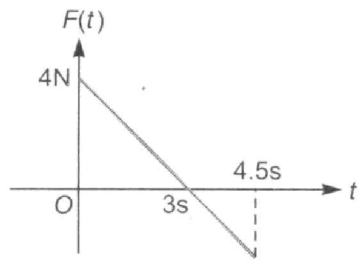


27. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$, respectively. The other ends are attached to two supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x and released. The block returns Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio $\frac{y}{x}$ is **[JEE 2008]**



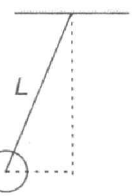
(A) 4 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

28. A block of mass 2 kg is free to move along the x-axis. It is at rest and from $t = 0$ onwards it is subjected to a time-dependent force $F(t)$ in the x-direction. The force $F(t)$ varies with t as shown in the figure. The kinetic energy of the block after 4.5 s is **[JEE 2010]**



(A) 4.50 J (B) 7.50 J
(C) 5.06 J (D) 14.06 J

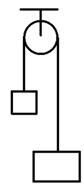
29. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in rad/s) is **[JEE 2011]**



(A) 9 (B) 18
(C) 27 (D) 36

30. A block of mass 0.18 kg is attached to a spring of force constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is $v = \frac{N}{10}$. Then N is **[JEE 2011]**

31. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10$ m/s², find the work done (in **joules**) by the string on the block of mass 0.36 kg during the first second after the system is released from rest. **[JEE-2010]**

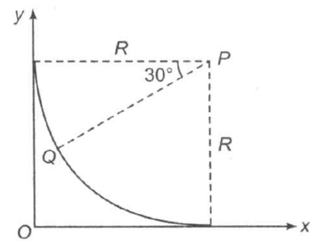


32. The work done on a particle of mass m by a force, $K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is **[JEE ADVANCE 2013]**

(A) $\frac{2K\pi}{a}$ (B) $\frac{K\pi}{a}$ (C) $\frac{K\pi}{2a}$ (D) 0

33. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5s is **[JEE ADVANCE 2013]**

Paragraph (34 – 35)



A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 *m*. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point *Q*, as shown in the figure, is 150 *J*. (Take the acceleration due to gravity, $g = 10 ms^{-2}$) **[JEE 2013]**

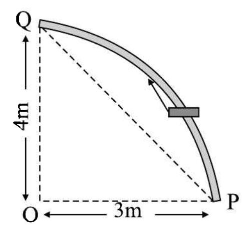
34. The speed of the block when it reaches the point *Q* is

(A) $5\ ms^{-1}$ (B) $10\ ms^{-1}$ (C) $10\sqrt{3}\ ms^{-1}$ (D) $20\ ms^{-1}$

35. The magnitude of the normal reaction that acts on the block at the point Q is

(A) 7.5 *N* (B) 8.6 *N* (C) 11.5 *N* (D) 22.5 *N*

36. Consider an elliptically shaped rail *PQ* in the vertical plane with *OP* = 3m and *OQ* = 4m. A block of mass 1 kg is pulled along the rail from *P* to *Q* with a force of 18*N*, which is always parallel to line *PQ* (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches *Q* is ($n \times 10$) Joules. The value of n is (take acceleration due to gravity = 10 m/s^2) **[JEE Advanced (Integer Type) 2014]**
[Ans. (5)**]**



Exercise-4: Previous years' AIEEE questions

1. If a body loses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest? **[AIEEE 2002]**

(A) 1 cm (B) 2 cm (C) 3 cm (D) 4 cm

2. A spring of force constant 800 N/m has an extension of 5cm. The work done in extending it from 5cm to 15cm is 800 N/m: **[AIEEE 2002]**

(A) 16 *J* (B) 8 *J* (C) 32 *J* (D) 24 *J*

3. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then the work required to stretch it further by another 5 cm is: **[AIEEE 2003]**

(A) 12.50 N-m (B) 18.75 N-m (C) 25.00 N-m (D) 6.25 N-m

4. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time *t* is proportional to: **[AIEEE 2003]**

(A) $t^{3/4}$ (B) $t^{3/2}$ (C) $t^{1/4}$ (D) $t^{1/2}$

5. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table? **[AIEEE 2004]**

(A) 7.2 *J* (B) 3.6 *J* (C) 120 *J* (D) 1200 *J*

6. A force $\vec{F} = (5\hat{i} + 3\hat{j} + 2\hat{k})$ *N* is applied over a particle which displaces it from origin to the point $\vec{r} = (2\hat{i} - \hat{j})m$. The work done on the particle in joules is: **[AIEEE 2004]**

(A) – 7 (B) + 7 (C) + 10 (D) + 13

7. A body of mass *m* accelerates uniformly from rest to v_1 in time t_1. The instantaneous power delivered to the body as a function of time *t* is: **[AIEEE 2004]**

(A) $\dfrac{mv_1 t}{t_1}$ (B) $\dfrac{mv_1^2 t}{t_1^2}$ (C) $\dfrac{mv_1 t}{t_1}$ (D) $\dfrac{mv_1^2 t}{t_1}$

8. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time, is given by: **[AIEEE 2005]**
(A) $\frac{mv^2}{T^2}.t$ (B) $\frac{mv^2}{T^2}.t^2$ (C) $\frac{1}{2}\frac{mv^2}{T^2}.t$ (D) $\frac{1}{2}\frac{mv^2}{T^2}.t^2$

9. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. the work done by the force of gravity during the time the particle goes up is **[AIEEE 2006]**
(A) $-0.5\,J$ (B) $-1.25\,J$ (C) $+1.25\,J$ (D) $0.5\,J$

10. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[AIEEE 2007]**
(A) K (B) zero (C) $K/4$ (D) $K/2$

11. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range **[AIEEE 2008]**
(A) $2 \times 10^5\,J - 3 \times 10^5\,J$ (B) $20{,}000\,J - 50{,}000\,J$
(C) $2{,}000\,J - 5{,}000\,J$ (D) $200\,J - 500\,J$

12. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = \left[U(x=\infty) - U_{at\ equilibrium}\right]$, D is **[AIEEE 2010]**
(A) $\frac{b^2}{2a}$ (B) $\frac{b^2}{12a}$ (C) $\frac{b^2}{4a}$ (D) $\frac{b^2}{6a}$

13. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done is stretching the unstretched rubber-band by L is **[JEE Main 2014]**
(A) $\frac{1}{2}(aL^2 + bL^3)$ (B) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (C) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$ (D) $aL^2 + bL^3$

[Ans. **(B)**]

Exercise-4: Objective Problems

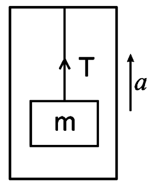
Only One Option Correct

Work Done by a Variable Force and Constant Force

1. A rigid body moves a distance of 10 m along a straight line under the action of a force of 5 N. If the work done by this force on the body is 25 joules, the angle which the force makes with the direction of motion of the body is
(A) 0ţ (B) 30ţ (C) 60ţ (D) 90ţ

2. A rigid body of mass 6 kg is under a force which causes displacement in it given by $S = \frac{t^2}{4}$ metres where t is time. The work done by the force in 2 seconds is
(A) 12 J (B) 9 J (C) 6 J (D) 3 J

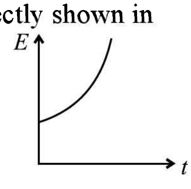
3. A ball is released from the top of a tower. The ratio of work done by force of gravity in first, second and third second of the motion of the ball is
   (A) 1: 2: 3 (B) 1: 4: 9 (C) 1: 3: 5 (D) 1: 5: 3
4. When a rigid body of mass M slides down an inclined plane of inclination θ, having coefficient of friction μ through a distance s, the work done against friction is:
   (A) $\mu\,(Mg\cos\theta)s$ (B) $\mu\,(Mg\sin\theta)\,s$
   (C) $Mg\,(\mu\cos\theta - \sin\theta)s$ (D) None of the above
5. A block of mass m is suspended by a light thread from an elevator. The elevator is accelerating upward with uniform acceleration a. The work done by tension on the block during t seconds is

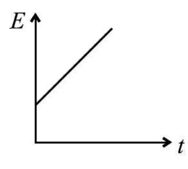


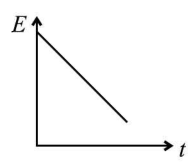
   (A) $\frac{m}{2}(g+a)\,at^2$ (B) $\frac{m}{2}(g-a)at^2$ (C) $\frac{m}{2}gat^2$ (D) 0
6 A particle moves under the effect of a force $F = Cx$ from $x = 0$ to $x = x_1$. The work done in the process is
   (A) $Cx_1^{\,2}$ (B) $\frac{1}{2}Cx_1^2$ (C) Cx_1 (D) Zero
7. Two springs have their force constant as k_1 and $k_2 (k_1 > k_2)$. When they are stretched by the same force
   (A) No work is done by this force in case of both the springs
   (B) Equal work is done by this force in case of both the springs
   (C) More work is done by this force in case of second spring
   (D) More work is done by this force in case of first spring
8. Two equal masses are attached to the two ends of a spring of spring constant k. The masses are pulled out symmetrically to stretch the spring by a length x over its natural length. The work done by the spring on each mass during the above pulling is
   (A) $\frac{1}{2}kx^2$ (B) $-\frac{1}{2}kx^2$ (C) $\frac{1}{4}kx^2$ (D) $-\frac{1}{4}kx^2$
9. The work done by the frictional force on a surface in drawing a circle of radius r on the surface by a pencil of negligible mass with a normal pressing force N (coefficient of friction μ_k) is
   (A) $4\pi r^2 \mu_K N$ (B) $-2\pi r^2 \mu_K \mathrm{N}$ (C) $-3\pi r^2 \mu_K N$ (D) $-2\pi r \mu_K N$
10. A uniform chain of length 2m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done by a force parallel to the horizontal surface in pulling the entire chain slowly on the table?
   (A) 7.2 J (B) 3.6 J (C) 120 J (D) 1200 J

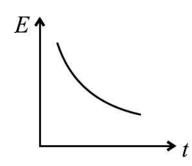
Work Energy Theorem

11. A particle is dropped from a height h. A constant horizontal velocity is given to the particle. Taking g to be constant every where, kinetic energy E of the particle with respect to time t is correctly shown in

(A)


(B)


(C)


(D)


12. If v, p and E denote the velocity, momentum and kinetic energy of the particle, then:
(A) $p = dE/dv$ (B) $p = dE/dt$ (C) $p = dv/dt$ (D) None of these

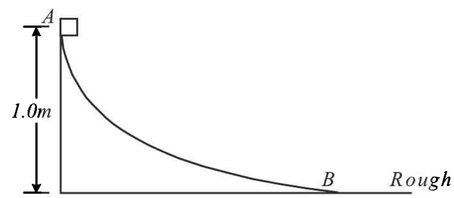
13. A heavy stone is thrown from a cliff of height h with a speed v. The stone will hit the ground with maximum speed if it is thrown
(A) Vertically downward
(B) Vertically upward
(C) Horizontally
(D) The speed does not depend on the initial direction.

14. A body moving at 2 m/s can be stopped over a distance x. If its kinetic energy is doubled, how long will it go before coming to rest, if the retarding force remains unchanged ?
(A) x (B) $2x$ (C) $4x$ (D) $8x$

15. A rod of length 1m and mass 0.5 kg hinged at one end, is initially hanging vertical. The other end is now raised slowly until it makes an angle 60ş with the vertical. The required work is:(use g=9.8 m/s^2)
(A) 1.522 J (B) 1.225 J
(C) 2.125 J (D) 3.125 K

16. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to
(A) x^2 (B) e^x (C) x (D) $\log_e x$

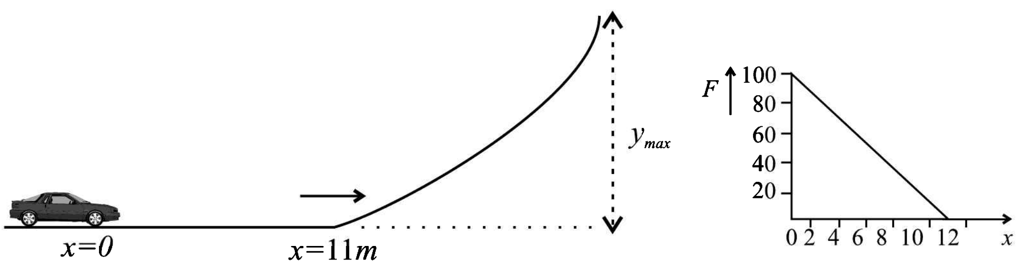
17. A block weighing 10 N travles down a smooth curved track AB joined to a rough horizontal surface (figure). The rough surface has a friction coefficient of 0.20 with the block. If the block starts slipping on the track from a point 1.0 m above the horizontal surface, the distance it will move on the rough surface is:



(A) 5.0 m (B) 10.0 m (C) 15.0 m (D) 20.0 m

Mechanical Energy Conservation

18 A toy car of mass 5 kg moves up a ramp under the influence of force F plotted against displacement x. The maximum height attained is given by



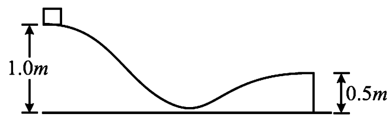
(A) y_{max} = 20 m (B) y_{max} = 15 m (C) y_{max} = 11 m (D) y_{max} = 5 m

19. The negative of the work done by the conservative internal forces on a system equals the change in
(A) Total energy (B) Kinetic energy (C) Potential energy (D) None of these

20. ________ of a two particle system depends only on the separation between the two particles. The most appropriate choice for the blank space in the above sentence is
(A) Kinetic energy (B) Total mechanical energy
(C) Potential energy (D) Total energy

21 A stone projected up with a velocity u reaches a maximum height h. When it is at a height of 3h/4 from the ground, the ratio of KE and PE at that point is: (consider $PE = 0$ at the point of projectory)
(A) 1: 1 (B) 1: 2 (C) 1: 3 (D) 3: 1

22. Figure shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground ?

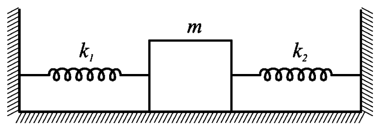


(A) At a horizontal distance of 1 m from the end of the track.
(B) At a horizontal distance of 2 m from the end of the track.
(C) At a horizontal distance of 3 m from the end of the track.
(D) Insufficient information

23 Two springs A and B ($k_A = 2k_B$) are strettched by applying forces of equal magnitudes at the four ends. If the energy stored in A is E, that in B is
(A) $E/2$ (B) $2E$ (C) E (D) $E/4$

24. When a spring is stretched by 2 cm, it stores 100 J of energy. If it is stretched further by 2 cm, the stored energy will be increased by
(A) $100\,J$ (B) $200\,J$ (C) $300\,J$ (D) $400\,J$

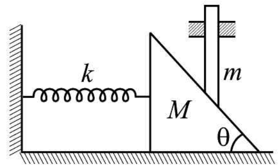
25. A block of mass m is attached to two unstretched springs of spring constants k_1 and k_2 as shown in figure. The block is displaced towards right through a distance x and is released. Find the speed of the block as it passes through the mean position shown.



(A) $\sqrt{\frac{k_1 + k_2}{m}}x$ (B) $\sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}x$ (C) $\sqrt{\frac{k_1^2 k_2^2}{m(k_1^2 + k_2^2)}}x$ (D) $\sqrt{\frac{k_1^3 k_2^3}{m(k_1^3 + k_2^3)}}$

26. A spring of spring constant k placed horizontally on a rough horizontal surface is compressed against a block of mass m placed on the surface so as to store maximum energy in the spring. If the coefficient of friction between the block and the surface is μ, the potential energy stored the spring is
(A) $\frac{\mu^2 m^2 g^2}{k}$ (B) $\frac{2\mu m^2 g^2}{k}$
(C) $\frac{\mu^2 m^2 g^2}{2k}$ (D) $\frac{3\mu^2 m g^2}{k}$

27. A wedge of mass M fitted with a spring of stiffness 'k' is kept on a smooth horizontal surface. A rod of mass m is kept on the wedge as shown in the figure. System is in equilibrium.



Assuming that all surfaces are smooth, the potential energy stored in the spring is:

(A) $\frac{mg^2\tan^2\theta}{2K}$ (B) $\frac{m^2g\tan^2\theta}{2K}$

(C) $\frac{m^2g^2\tan^2\theta}{2K}$ (D) $\frac{m^2g^2\tan^2\theta}{K}$

Power

28. A car of mass 'm' is driven with acceleration 'a' along a straight level road against a constant external resistive force 'R'. When the velocity of the car is 'V', the rate at which the engine of the car is doing work will be

(A) RV (B) maV (C) $(R + ma)V$ (D) $(ma - R)V$

29. A particle moves with a velocity $\vec{v} = (5\hat{i} - 3\hat{j} + 6\hat{k})$m/s under the influence of a constant force $\vec{F} = (10\hat{i} + 10\hat{j} + 20\hat{k})N$. The instantaneous power applied to the particle is

(A) 200 J/s (B) 40 J/s (C) 140 J/s (D) 170 J/s

30. An electric motor creates a tension of 4500 N in hoisting cable and reels it at the rate of 2 m/s. What is the power of electric motor ?

(A) 9 W (B) 9 KW (C) 225 W (D) 9000 $H.P.$

31. A man M_1 of mass 80 kg runs up a staircase in 15 s. Another man M_2 also of mass 80 kg runs up the stair case in 20 s. The ratio of the power developed by them (P_1 / P_2) will be

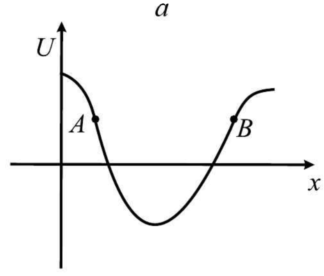
(A) 1 (B) 4/3 (C) 16/9 (D) None of the above

Conservative & Non-Conservative Forces and Equilibrium

32. The potential energy of a particle in a field is $U = \frac{a}{r^2} - \frac{b}{r}$, where a and b are constant. The value of r in terms of a and b where force on the particle is zero will be

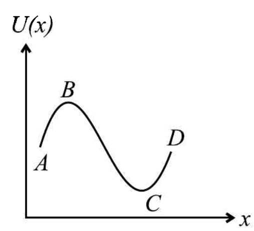
(A) $\frac{a}{b}$ (B) $\frac{b}{a}$ (C) $\frac{2a}{b}$ (D) $\frac{2b}{a}$

33. Potential energy v/s displacement curve for one dimensional conservative field is shown. Force at A and B is respectively.



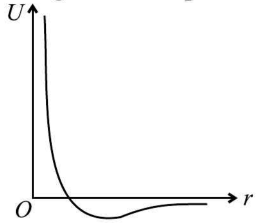
(A) Positive, Positive
(B) Positive, Negative
(C) Negative, Positive
(D) Negative, Negative

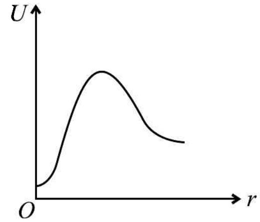
34. The potential energy of a particle varies with distance x as shown in the graph. The force acting on the particle is zero at

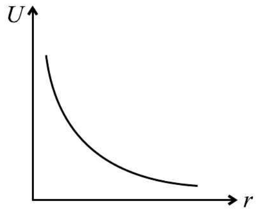


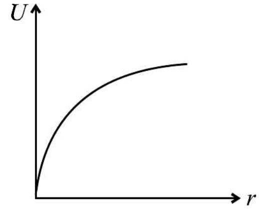
(A) C (B) B
(C) B and C (D) A and D.

35. The diagrams represent the potential energy U of a function of the inter-atomic distance r. Which diagram corresponds to stable molecules found in nature?

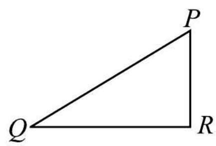
(A) 

(B) 

(C) 

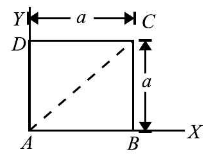
(D) 

36. For the path PQR in a conservative force field (fig.), the amounts work done in carrying a body from P to Q & from Q to R are 5 J & 2 J respectively . The work done in carrying the body from P to R will be



(A) 7 J (B) 3 J
(C) $\sqrt{21}J$ (D) zero

37. A force $F = x^2y^2i + x^2y^2j$ (N) acts on a particle which moves in the XY plane.
(a) Determine if F is conservative and
(b) find the work done by $\boldsymbol{F}$ as it moves the particle from A to C (fig.) along each of the paths ABC, ADC, and AC.



38. Calculate the forces $F(y)$ associated with the following one-dimensional potential energies
(a) $U = -\omega y$ (b) $U = ay^3 - by$ (c) $U = U_0 \sin \beta y$

39. The potential energy for a force field $\vec{F}$ is given by $U(x, y) = \sin(x + y)$. The force acting on the particle of mass m at $\left(0, \frac{\pi}{4}\right)$ is

(A) 1 (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) 0

40. A particle is taken from point A to point B under the influence of a force field. Now it is taken back from B to A and it is observed that the work done in taking the particle from A to B is not equal to the work done in taking it from B to A. If W_{nc} and W_c is the work done by non-conservative forces and conservative forces present in the system respectively, ΔU is the change in potential energy, Δk is the change kinetic energy, then
(A) $W_{nc} - \Delta U = \Delta k$ (B) $W_c = -\Delta U$ (C) $W_{nc} + W_c = \Delta k$ (D) $W_{nc} - \Delta U = -\Delta k$

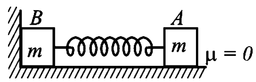
41. A body of mass m accelerates uniformly from rest to a speed v_0 in time t_0. The work done on the body till any time t is
(A) $\frac{1}{2} m v_0^2 \left(\frac{t^2}{t_0^2}\right)$ (B) $\frac{1}{2} m v_0^2 \left(\frac{t_0}{t}\right)$ (C) $m v_0^2 \left(\frac{t}{t_0}\right)$ (D) $m v_0^2 \left(\frac{t}{t_0}\right)^3$

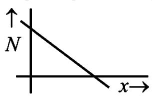
42. A man who is running has half the kinetic energy of the boy of half his mass. The man speeds up by 1 m/s and then has the same kinetic energy as the boy. The original speed of the man was
(A) $\sqrt{2}$ m/s (B) $(\sqrt{2} - 1)$ m/s (C) 2 m/s (D) $(\sqrt{2} + 1)$ m/s

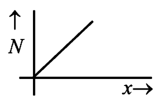
43. $F = 2x^2 - 3x - 2$. Choose correct option
(A) $x = -1/2$ is position of stable equilibrium
(B) $x = 2$ is position of stable equilibrium
(C) $x = -1/2$ is position of unstable equilibrium
(D) $x = 2$ is position of neutral equilibrium

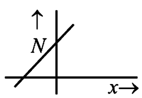
44. A block of mass m is hung vertically from an elastic thread of force constant mg/a. Initially the thread was at its natural length and the block is allowed to fall freely. The kinetic energy of the block when it passes through the equilibrium position will be:
(A) mga (B) $mga/2$ (C) zero (D) $2mga$

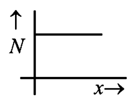
45. The block A is pushed towards the wall by a distance and released. The normal reaction by vertical wall on the block B v/s compression in spring is given by



(A) 

(B) 

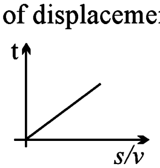
(C) 

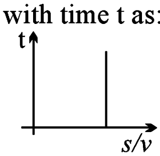
(D) 

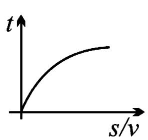
46. Force acting on a body of mass 1 kg is related to its position x as $F = x^3 - 3x$ N. It is at rest at $x = 1$. Its velocity at $x = 3$ can be
(A) 4 m/s (B) 3 m/s (C) 2 m/s (D) 5 m/s

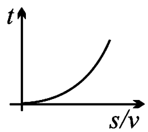
47. Assume the aerodynamic drag force on a car is proportional to its speed. If the power output from the engine is doubled, then the maximum speed of the car.
(A) Is unchanged (B) Increases by a factor of $\sqrt{2}$
(C) Is also doubled (D) Increases by a factor of four.

48. A body is moved from rest along a straight line by a machine delivering constant power. The ratio of displacement and velocity (s/v) varies with time t as:

(A) 

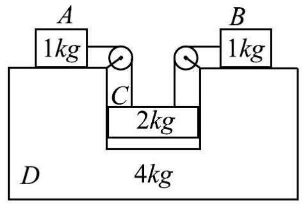
(B) 

(C) 

(D) 

49. A particle is released from rest at origin. It moves under influence of potential field $U = x^2 - 3x$, kinetic energy at $x = 2$ is
(A) $2J$ (B) $1\,J$ (C) $1.5\,J$ (D) $0\,J$

50. In the system shown in the figure there is no friction anywhere. The block C goes down by a distance $x_0 = 10$ cm with respect to wedge D when system is released from rest. The velocity of A with respect to B will be ($g = 10$ m/s^2)



(A) Zero (B) 1 m/s
(C) 2 m/s (D) None of these

51. Potential energy of a particle is related to x coordinate by equation $x^2 - 2x$. Particle will be in stable equilibrium at
(A) $x = 0.5$ (B) $x = 1$ (C) $x = 2$ (D) $x = 4$

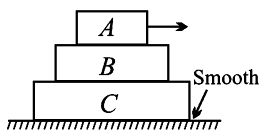
52. A force $\vec{F} = k[y\hat{i} + x\hat{j}]$ where k is a positive constant acts on a particle moving in x-y plane starting from the point (3,5), the particle is taken along a straight line to (5, 7). The work done by the force is:
(A) Zero (B) 35 K (C) 20 K (D) 15 K

53. Water is pumped from a depth of 10 m and delivered through a pipe of cross section 10^{-2}m^2. If it is needed to deliver a volume of 10^{-1} m^3 per second the power required will be
(A) 10 kW (B) 9.8 kW (C) 15 kW (D) 4.9 kW

54. A light spring of length 20 cm and force constant 2 kg/cm is placed vertically on a table.A small block of mass 1 kg. falls on it. The length h from the surface of the table at which the ball will have the maximum velocity is
(A) 20 cm (B) 15 cm (C) 10 cm (D) 5 cm

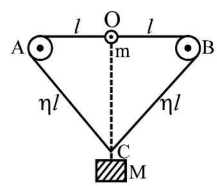
55. The work done in joules in increasing the extension of a spring of stiffness 10 N/cm from 4 cm to 6 cm is:
(A) 1 (B) 10 (C) 50 (D) 100

56. Three blocks A, B and C are kept as shown in the figure. The coefficient of friction between A and B is 0.2, B and C is 0.1, C and ground is 0.0. The mass of A, B and C are 3 kg, 2 kg and 1 kg respectively. A is given a horizontal velocity 10 m/s. A, B and C always remain in contact i.e. lies as in figure. The total work done by friction will be



(A) $-75\,J$ (B) $75\,J$ (C) $-150\,J$ (D) $-100\,J$

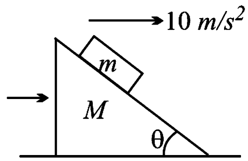
57. A loop of light inextensible string passes over smooth small pulleys A and B. Two masses m and M are attached to the points O and C respectively. Then the condition that m and M will cross each other.
[Take AB = 2l and $AC = AB = \eta$l] will be



(A) $\frac{m}{M} > 2\sqrt{\frac{\eta+1}{\eta+3}} - 1$ (B) $\frac{m}{M} > 2\sqrt{\frac{\eta+3}{\eta+1}} - 1$

(C) $\frac{m}{M} > \sqrt{\frac{\eta+1}{\eta+3}} + 1$ (D) none of these

58. In the figure shown all the surfaces are frictionless, and mass of the block, $m = 1$ kg. The block and wedge are held initially at rest. Now wedge is given a horizontal acceleration of 10 m/s^2 by applying a force on the wedge, so that the block does not slip on the wedge. Then work done by the normal force in ground frame on the block in $\sqrt{3}$ seconds is

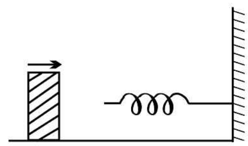


(A) 30 *J* (B) 60 *J* (C) 150 *J* (D) $100\sqrt{3}$ *J*

59. When a conservative force does positive work on a body
(A) The potential energy increases (B) The potential energy decreases
(C) Total energy increases (D) Total energy decreases

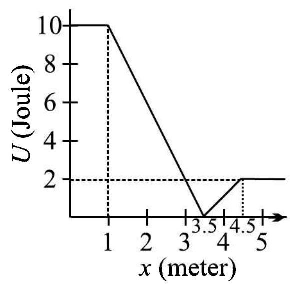
60. The *P.E.* of a certain spring when stretched from natural length through a distance 0.3 m is 10 *J*. The amount of work in joule that must be done on this spring to stretch it through an additional distance 0.15 m will be
(A) 10 *J* (B) 20 *J* (C) 7.5 *J* (D) 12.5 *J*

61. A 1.0 kg block collides with a horizontal weightless spring of force constant 2.75 Nm^{-1} as shown in figure. The block compresses the spring 4.0 m from the rest position. If the coefficient of kinetic friction between the block and horizontal surface is 0.25, the speed of the block at the instant of collision is



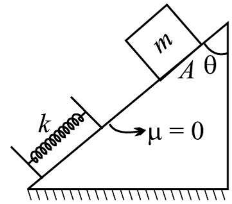
(A) 0.4 ms^{-1} (B) 4 ms^{-1}
(C) 0.8 ms^{-1} (D) 8 ms^{-1}

62. A body with mass 2 kg moves in one direction in the presence of a force which is described by the potential energy graph. If the body is released from rest at $x = 2$m, then its speed when it crosses $x = 5$m is



(A) zero (B) 1 ms^{-1}
(C) 2 ms^{-1} (D) 3 ms^{-1}

63. A block of mass '*m*' is released from rest at point A. The compression in spring, when the speed of block is maximum

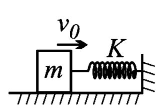


(A) $\dfrac{mg\sin\theta}{k}$ (B) $\dfrac{2mg\sin\theta}{k}$

(C) $\dfrac{mg\cos\theta}{k}$ (D) $\dfrac{mg}{k}$

Question No. 64 to 69 (6 questions)

A block of mass *m* moving with a velocity v_0 on a smooth horizontal surface strikes and compresses a spring of stiffness *k* till mass comes to rest as shown in the figure. This phenomenon is observed by two observers:

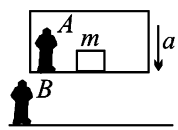


A: standing on the horizontal surface

B: standing on the block

64. To an observer A, the work done by spring force is
    (A) negative but nothing can be said about its magnitude
    (B) $-\frac{1}{2}mv_0^2$
    (C) positive but nothing can be said about its magnitude
    (D) $+\frac{1}{2}mv_0^2$
65. To an observer A, the work done by the normal reaction N between the block and the spring on the block is
    (A) zero (B) $-\frac{1}{2}mv_0^2$ (C) $+\frac{1}{2}mv_0^2$ (D) none of these
66. To an observer A, the net work done on the block is
    (A) $-mv_0^2$ (B) $+mv_0^2$ (C) $-\frac{1}{2}mv_0^2$ (D) zero
67. According to the observer A
    (A) The kinetic energy of the block is converted into the potential energy of the spring
    (B) The mechanical energy of the spring-mass system is conserved
    (C) The block loses its kinetic energy because of the negative work done by the conservative force of spring
    (D) All the above
68. To an observer B, when the block is compressing the spring
    (A) Velocity of the block is decreasing (B) Retardation of the block is increasing
    (C) Kinetic energy of the block is zero (D) All the above
69. According to observer B, the potential energy of the spring increases
    (A) Due to the positive work done by pseudo force
    (B) Due to the positive work done by normal reaction between spring & wall
    (C) Due to the decrease in the kinetic energy of the block
    (D) All the above

Question No. 70 to 73 (4 questions)

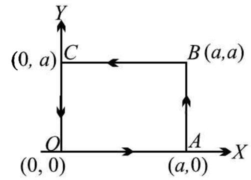


A block of mass m is kept in an elevation which starts moving downward with an acceleration a as shown in figure. The block is observed by two observers A and B for a time interval t_0.

70. The observer B finds that the work done by gravity on the block is
    (A) $\frac{1}{2}mg^2t_0^2$ (B) $-\frac{1}{2}mg^2t_0^2$ (C) $\frac{1}{2}mgat_0^2$ (D) $-\frac{1}{2}mgat_0^2$
71. The observer B finds that the work done by pseudo force on the block is
    (A) zero (B) $-ma^2t_0$ (C) $+ma^2t_0$ (D) $-mgat_0$
72. According to observer B, the net work done on the block is
    (A) $-\frac{1}{2}ma^2t_0^2$ (B) $\frac{1}{2}ma^2t_0^2$ (C) $\frac{1}{2}mgat_0^2$ (D) $-\frac{1}{2}mgat_0^2$

73. According to the observer A
(A) The work done by gravity is zero
(B) The work done by normal reaction is zero
(C) The work done by pseudo force is zero
(D) All the above

74. The work done by the force $\vec{F} = x^2\hat{i} + y^2\hat{j}$ around the path shown in the figure is

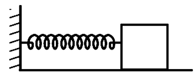


(A) $\frac{2}{3}a^3$ (B) zero
(C) a^3 (D) $\frac{4}{3}a^3$

Question No. 75 to 76 (2 questions)

A spring block system is placed on a rough horizontal floor. The block is pulled towards right to give spring an elongation less than $\frac{2\mu mg}{K}$ but more than $\frac{\mu mg}{K}$ and released.

75. Which of the following laws/principles of physics can be applied on the spring block system



(A) Conservation of mechanical energy
(B) Conservation of momentum
(C) Work energy principle
(D) None

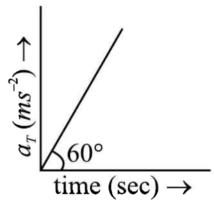
76. The correct statement is
(A) The block will cross the mean position.
(B) The block will come to rest when the forces acting on it are exactly balanced
(C) The block will come to rest when the work done by friction becomes equal to the change in energy stored in spring.
(D) None

77. A particle is rotated in a vertical circle by connecting it to a light rod of length l and keeping the other end of the rod fixed. The minimum speed of the particle when the light rod is horizontal for which the particle will complete the circle is
(A) $\sqrt{gl}$ (B) $\sqrt{2gl}$ (C) $\sqrt{3gl}$ (D) none

78. A body is moving uni-directionally under the influence of a source of constant power. Its displacement in time t is proportional to
(A) $t^{1/2}$ (B) t (C) $t^{3/2}$ (D) t^2

79. If angular velocity of a disc depends an angle rotated θ as $\omega = \theta^2 + 2\theta$, then its angular acceleration α at $\theta = 1$ rad is:
(A) 8 rad/sec^2 (B) 10 rad/sec^2 (C) 12 rad/sec^2 (D) None

80. Tangential acceleration of a particle moving in a circle of radius 1 m varies with time t as (initial velocity of particle is zero). Time after which total acceleration of particle makes and angle of 30° with radial acceleration is



(A) 4 sec (B) 4/3 sec
(C) $2^{2/3}$ sec (D) $\sqrt{2}$ sec

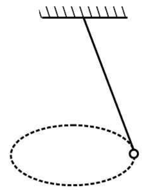
81. The magnitude of displacement of a particle moving in a circle of radius a with constant angular speed ω varies with time t as

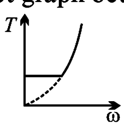
(A) $2\,a\sin\omega t$ (B) $2a\sin\frac{\omega t}{2}$ (C) $2a\cos\omega t$ (D) $2a\cos\frac{\omega t}{2}$

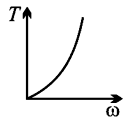
82. A particle originally at rest at the highest point of a smooth vertical circle is slightly displaced. It will leave the circle at a vertical distance h below the highest point, such that

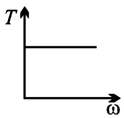
(A) $h = R$ (B) $h = R/3$ (C) $h = R/2$ (D) $h = 2R$

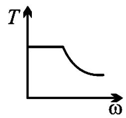
83. In a conical pendulum, the bob is rotated with different angular velocities and tension in the string is calculated for different values of ω. Which of them is correct graph between T & ω?



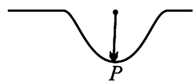
(A) 

(B) 

(C) 

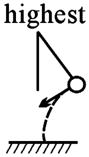
(D) 

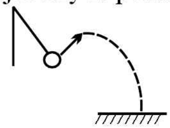
84. A car travelling on a smooth road passes through a curved portion of the road in form of an arc of circle of radius 10 m. If the mass of car is 500 kg, the reaction on car at lowest point P where its speed is 20 m/s is

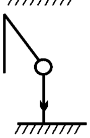


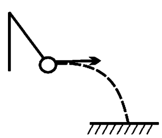
(A) 35 kN (B) 30 kN
(C) 25 kN (D) 20 kN

85. A pendulum bob is swinging in a vertical plane such that its angular amplitude is less than 90^0. At its highest point, the string is cut. Which trajectory is possible for the bob afterwards.

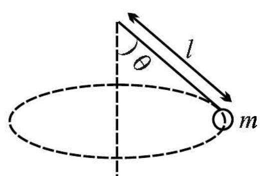
(A) 

(B) 

(C) 

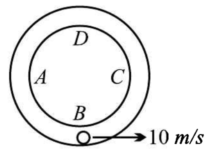
(D) 

86. A conical pendulum is moving in a circle with angular velocity ω as shown. If tension in the string is T, which of following equations are correct ?



(A) $T = m\omega^2 l$ (B) $T\sin\theta = m\omega^2 l$
(C) $T = mg\cos\theta$ (D) $T = m\omega^2 l\sin\theta$

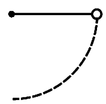
87. A ball whose size is slightly smaller than width of the tube of radius 2.5 m is projected from bottommost point of a smooth tube fixed in a vertical plane with velocity of 10 m/s. If N_1 and N_2 are the normal reactions exerted by inner side and outer side of the tube on the ball

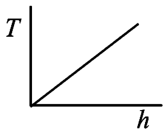


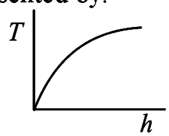
(A) $N_1 > 0$ for motion in *ABC*, $N_2 > 0$ for motion in *CDA*
(B) $N_1 > 0$ for motion in *CDA*, $N_2 > 0$ for motion in *ABC*
(C) $N_2 > 0$ for motion in *ABC* & part of *CDA*
(D) N_1 is always zero.

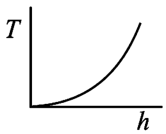
88. A road is banked at an angle of 30° to the horizontal for negotiating a curve of radius $10\sqrt{3}$ m. At what velocity will a car experience no friction while negotiating the curve?
(A) 54 km/hr (B) 72 km/hr (C) 36 km/hr (D) 18 km/hr

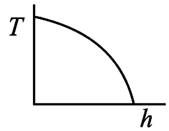
89. A bob attached to a string is held horizontal and released. The tension and vertical distance from point of suspension can be represented by.



(A)


(B)


(C)


(D)


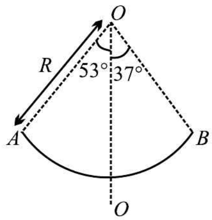
90. A particle of mass *m* is tied to one end of a string of length l. The particle is held horizontal with the string taut. It is then projected upward with a velocity *u*. The tension in the string is $\frac{mg}{2}$ when it is inclined at an angle 30° to the horizontal. The value of *u* is
(A) $\sqrt{lg}$ (B) $\sqrt{2lg}$ (C) $\sqrt{\frac{lg}{2}}$ (D) $2\sqrt{lg}$

91. The ratio of period of oscillation of the conical pendulum to that of the simple pendulum is: (Assume the strings are of the same length in the two cases and θ is the angle made by the string with the verticla in case of conical pendulum)
(A) cos θ (B) $\sqrt{\cos\theta}$ (C) 1 (D) none of these

92. A particle is moving in a circle:
(A) The resultant force on the particle must be towards the centre.
(B) The cross product of the tangential acceleration and the angular velocity will be zero.
(C) The direction of the angular acceleration and the angular velocity must be the same.
(D) The resultant force may be towards the centre.

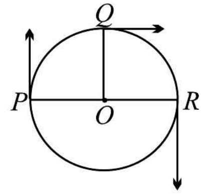
93. A particle is moving along the circle $x^2 + y^2 = a^2$ in anti clock wise direction. The *x–y* plane is a rough horizontal stationary surface. At the point (a cosθ, a sinθ), the unit vector in the direction of friction on the particle is:
(A) $\cos\theta\hat{i} + \sin\theta\hat{j}$ (B) $-(\cos\theta\hat{i} + \sin\theta\hat{j})$
(C) $\sin\theta\hat{i} - \cos\theta\hat{j}$ (D) $\cos\theta\hat{i} - \sin\theta\hat{j}$

94. A section of fixed smooth circular track of radius R in vertical plane is shown in the figure. A block is released from position A and leaves the track at B. The radius of curvature of its trajectory when it just leaves the track at B is:



(A) R (B) $R/4$
(C) $R/2$ (D) None of these

95. Three point particles P, Q, R move in circle of radius 'r' with different but constant speeds. They start moving at $t = 0$ from their initial positions as shown in the figure. The angular velocities (in rad/ sec) of P, Q and R are 5π, 2π & 3π respectively, in the same sense. The time interval after which they are at sameangular position.



(A) 2/3 sec (B) 1/6 sec
(C) 1/2 sec (D) 3/2 sec

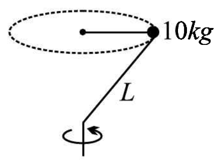
96. In the above question, the number of times P and Q meet in that time interval is
(A) 4 (B) 1 (C) 3 (D) 9

97. A particle inside the rough surface of a rotating cone about its axis is at rest relative to it at a height of 1m above its vertex. Friction coefficient is $\mu = 0.5$, if half angle of cone is 45°, the maximum angular velocity of revolution of cone can be
(A) $\sqrt{10}$ rad/s (B) $\sqrt{30}$ rad/s (C) $\frac{\sqrt{40}}{3}$ rad/s (D) $\sqrt{50}$ rad/s

98. A body of mass 1 kg starts moving from rest at $t = 0$, in a circular path of radius 8 m. Its kinetic energy varies as a function of time as: $K.E. = 2t^2$ Joules, where t is in seconds. Then
(A) tangential acceleration = 4 m/s^2
(B) power of all forces at $t = 2$ sec is 8 watt
(C) first round is completed in 2 sec.
(D) tangential force at $t = 2$ sec is 4 newton.

99. A particle is moving along a circular path of radius R in such a way that at any instant magnitude of radial acceleration & tangential acceleration are equal. If at $t = 0$ velocity of particle is V_0, the time period of first revolution of the particle is
(A) $\frac{R}{V_0} e^{-2\pi}$ (B) $\frac{R}{V_0}(e^{2\pi}-1)$ (C) $\frac{R}{V_0}$ (D) $\frac{R}{V_0}(1 - e^{-2\pi})$

100. A 10 kg ball attached to the end of a rigid massless rod of length 1 m rotates at constant speed in a horizontal circle of radius 0.5 m and period 1.57 sec as in fig. The force exerted by rod on the ball is

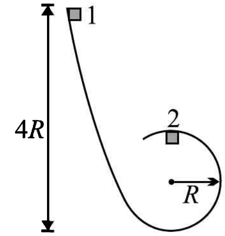


(A) 1.28 N (B) 128 N
(C) 10 N (D) 12.8 N

101. Two bodies A & B rotate about an axis, such that angle θ_A (in radians) covered by first body is proportional to square of time, and θ_B (in radians) covered by second body varies linearly. At $t = 0$, $\theta_A = \theta_B = 0$. If A completes its first revolution in $\sqrt{\pi}$ sec. and B needs 4π sec. to complete half revolution then; angular velocity $\omega_A : \omega_B$ at $t = 5$ sec. are in the ratio

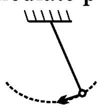
(A) 4: 1 (B) 20: 1
(C) 80: 1 (D) 20: 4

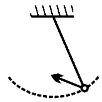
102. A small cube with mass M starts at rest at point 1 at a height $4R$, where R is the radius of the circular part of the track. The cube slides down the frictionless track and around the loop. The force that the track exerts on the cube at point 2 is nearly _____ times the cube's weight Mg.

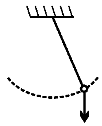


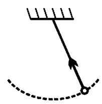
(A) 1 (B) 2
(C) 3 (D) 4

103. Which vector in the figures best represents the acceleration of a pendulum mass at the intermediate point in its swing?

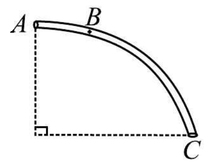
(A) 

(B) 

(C) 

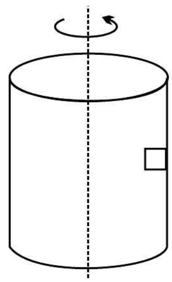
(D) 

104. The tube AC forms a quarter circle in a vertical plane. The ball B has an area of cross–section slightly smaller than that of the tube, and can move without friction through it. B is placed at A and displaced slightly. It will

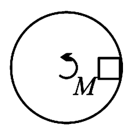


(A) Always be in contact with the inner wall of the tube
(B) Always be in contact with the outer wall of the tube
(C) Initially be in contact with the inner wall and later with the outer wall
(D) Initially be in contact with the outer wall and later with the inner wall

105. A hollow vertical cylinder of radius R is rotated with angular velocity ω about an axis through its center. What is the minimum coefficient of static friction necessary to keep the mass M suspended on the inside of the cylinder as it rotates?



Side view



Top view

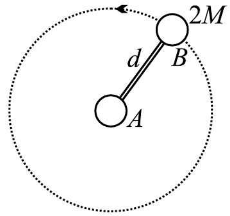
(A) $\mu = \dfrac{gR}{\omega^2}$ (B) $\mu = \dfrac{\omega^2 g}{R}$

(C) $\mu = \dfrac{\omega^2 R}{g}$ (D) $\mu = \dfrac{g}{\omega^2 R}$

106. A horizontal curve on a racing track is banked at a 45° angle. When a vehicle goes around this curve at the curve's safe speed (no friction needed to stay on the track), what is its centripetal acceleration?

(A) g (B) 2g (C) 0.5g (D) none

107. The dumbell is placed on a frictionless horizontal table. Sphere A is attached to a frictionless pivot so that B can be made to rotate about A with constant angular velocity. If B makes one revolution in period P, the tension in the rod is



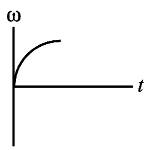
(A) $\frac{4\pi^2 Md}{P^2}$ (B) $\frac{8\pi^2 Md}{P^2}$

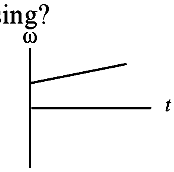
(C) $\frac{4\pi^2 Md}{P}$ (D) $\frac{2Md}{P}$

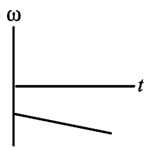
108. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time t. The ratio of the angular speeds of the first to the second car is

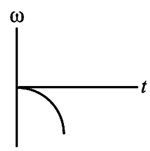
(A) 1: 1 (B) $m_1 : m_2$ (C) $r_1 : r_2$ (D) $m_1 m_2 : r_1 r_2$

109. The graphs below show angular velocity as a function of time. In which one is the magnitude of the angular acceleration constantly decreasing?

(A) 

(B) 

(C) 

(D) 

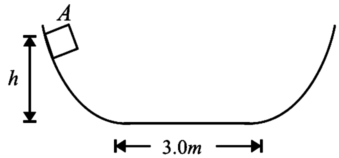
Exercise–6: Objective Problems

One or More than One Option Correct

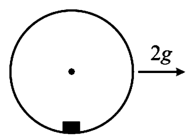
1. The work done by the external forces on a system equals the change in
   (A) Total energy (B) Kinetic energy
   (C) Potential energy (D) None of these
2. A small block of mass m is kept on a rough inclined surface of inclination θ fixed in a elevator. The elevator goes up with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on the block in time t will be
   (A) Zero (B) $mgvt \cos^2\theta$ (C) $mgvt \sin^2\theta$ (D) $mgvt \sin 2\theta$
3. A heavy stone is thrown from a cliff of height h in a given direction. The speed with which it hits the ground
   (A) Must depend on the speed of projection
   (B) Must be larger than the speed of projection
   (C) Must be independent of the speed of projection
   (D) May be smaller than the speed of projection
4. The total work done on a particle is equal to the change in its kinetic energy
   (A) Always
   (B) Only if the forces acting on it are conservative
   (C) Only if gravitational force alone acts on it
   (D) Only if elastic force alone acts on it.

5. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. If follows that
   (A) Its velocity is constant (B) Its acceleration is constant
   (C) Its kinetic energy is constant (D) It moves in a circular path
6. Consider two observers moving with respect to each other at a speed v along a straight line. They observe a block of mass m moving a distance ℓ on a rough surface. The following quantities will be same as observed by the two observers
   (A) Kinetic energy of the block at time t (B) Work done by friction
   (C) Total work done on the block (D) Acceleration of the block
7. You lift a suitcase from the floor and keep it on a table. The work done by you on the suitcase does not depend on
   (A) The path taken by the suitcase (B) The time taken by you in doing so
   (C) The weight of the suitcase (D) Your weight
8. No work is done by a force on an object if
   (A) The force is always perpendicular to its velocity
   (B) The force is always perpendicular to its acceleration
   (C) The object is stationary but the point of application of the force moves on the object
   (D) The object moves in such a way that the point of application of the force remains fixed.
9. The kinetic energy of a particle continuously increases with time
   (A) The resultant force on the particle must be parallel to the velocity at all instants.
   (B) The resultant force on the particle must be at an angle less than 90° all the time
   (C) Its height above the ground level must continuously decrease
   (D) The magnitude of its linear momentum is increasing continuously
10. One end of a light spring of spring constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. In a displacement , the work done by the spring is $\frac{1}{2}kx^2$. The possible cases are
   (A) The spring was initially compressed by a distance x and was finally in its natural length
   (B) It was initially stretched by a distance x and finally was in its natural length
   (C) It was initially in its natural length and finally in a compressed position
   (D) It was initially in its natural length and finally in a stretched position
11. A block of mass M is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force F. The kinetic energy of the block increases by 20 J in 1s.
   (A) The tension in the string is Mg
   (B) The tension in the string is F
   (C) The work done by the tension on the block is 20 J in the above 1 s.
   (D) The work done by the force of gravity is – 20 J in the above 1s.
12. If force is always perpendicular to motion
   (A) KE remains constant (B) Work done = 0
   (C) Speed is constant (D) Velocity is constant
13. Work done by force of friction
   (A) Can be zero (B) Can be positive
   (C) Can be negative (D) Information insufficient

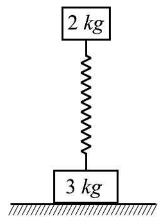
14. When work done by force of gravity is negative (Assume only gravitational force to be acting)
    (A) *KE* increases (B) *KE* decreases
    (C) *PE* increases (D) *PE* stays constant
15. When total work done on a particle is positive
    (A) *KE* remains constant (B) Momentum increases
    (C) *KE* decreases (D) *KE* increases
16. When a man walks on a horizontal surface with constant velocity, work done by
    (A) Friction is zero (B) Contact force is zero
    (C) Gravity is zero (D) Man is zero
17. A small particle slides along a track with elevated ends and a flat central part, as shown in figure. The flat part has a length 3m. the curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is $\mu = 0.2$. The particle is released at point A, which is at a height $h = 1.5$ m above the flat part of the track. The position where the particle finally come to rest is:



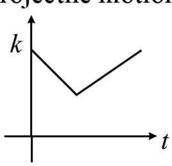
    (A) Left to mid point of the flat part (B) Right to the mid point of the flat part
    (C) Mid point of the flat part (D) None of these
18. A block of mass m is placed inside a smooth hollow cylinder of radius *R* kept horizontally. Initially system was at rest. Now cylinder is given constant acceleration $2g$ in the horizontal direction by external agent. The maximum angular displacement of the block with the vertical is:

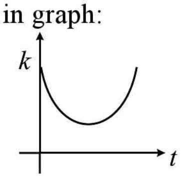


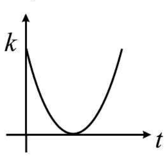
    (A) $2\tan^{-1}2$ (B) $\tan^{-1}2$
    (C) $\tan^{-1}1$ (D) $\tan^{-1}\left(\frac{1}{2}\right)$
19. The ends of a spring are attached to blocks of masses 3 kg and 2 kg. The 3 kg block rests on a horizontal surface and the 2 kg block which is vertically above it is in equilibrium producing a compression of 1 cm of the spring. The value of the length to which the 2 kg mass must be compressed, so that when it is released, the 3 kg block may be lifted off the ground is:

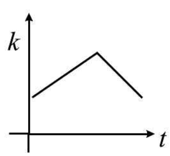


    (A) 5 cm (B) 2.5 cm
    (C) 8 cm (D) 10 cm
20. In a projectile motion, *KE* varies with time as in graph:

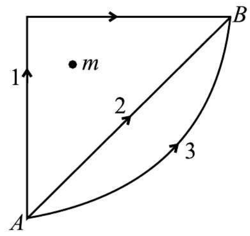
(A)


(B)


(C)


(D)


21. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1, 2, 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3



(A) $W_1 > W_2 > W_3$ (B) $W_1 = W_2 = W_3$

(C) $W_1 < W_2 < W_3$ (D) $W_2 > W_1 > W_3$

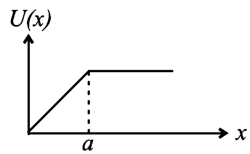
22. A lorry and a car moving with the same $K.E.$ are brought to rest by applying the same retarding force, then
(A) Lorry will come to rest in a shortest distance
(B) Car will come to rest in a shorter distance
(C) Both come to rest in a same distance
(D) None of the above

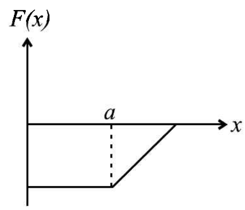
23. An open knife edge of mass 'm' is dropped from a height 'h' on a wooden floor. If the blade penetrates upto depth 'd' into the wood, the average resistance offered by the wood to the knife edge is

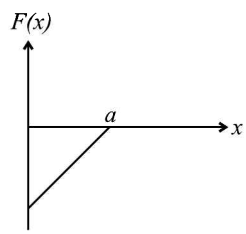
(A) mg (B) $mg\left(1-\frac{h}{d}\right)$ (C) $mg\left(1+\frac{h}{d}\right)$ (D) $mg\left(1+\frac{h}{d}\right)^2$

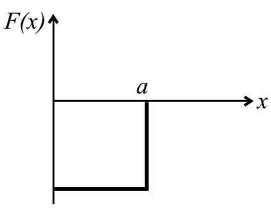
24. A body is moved along a straight line by a machine delivering constant power. The distance moved by the body in time t is proportional to
(A) $t^{1/2}$ (B) $t^{3/4}$ (C) $t^{3/2}$ (D) t^2

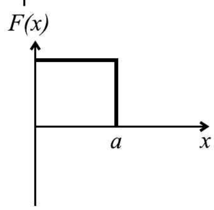
25. The potential energy of a system is represented in the first figure, the force acting on the system will be represented by



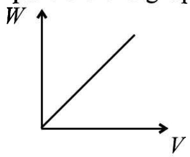
(A)


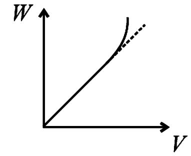
(B)


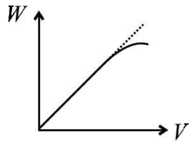
(C)


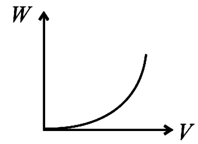
(D)


26. A particle, initially at rest on a frictionless horizontal surface, is acted upon by a horizontal force which is constant in size and direction. A graph is plotted between the work done (W) on the particle, against the speed of the particle, (v). If there are no other horizontal forces acting on the particle the graph would look like

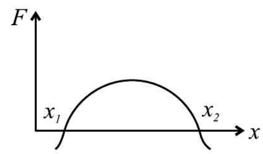
(A) 

(B) 

(C) 

(D) 

27. The force acting on a body moving along x-axis varies with the position of the particle as shown in the figure. The body is in stable equilibrium at



(A) $x = x_1$ (B) $x = x_2$
(C) Both x_1 and x_2 (D) Neither x_1 nor x_2

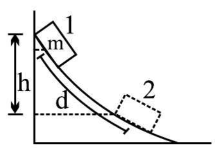
28. A particle with constant total energy E moves in one dimension in a region where the potential energy is $U(x)$. The speed of the particle is zero where

(A) $U(x) = E$ (B) $U(x) = 0$ (C) $\frac{dU(x)}{dx} = 0$ (D) $\frac{d^2U(x)}{dx^2} = 0$

29. A block of mass m slides down a plane inclined at an angle θ. Which of the following will NOT increase the energy lost by the block due to friction?
(A) Increasing the angle of inclination
(B) Increasing the distance that the block travels
(C) Increasing the accelertion due to gravity
(D) Increasing the mass of the block

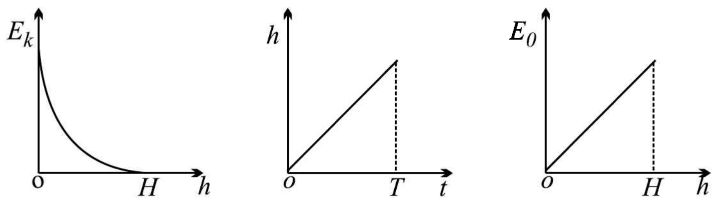
30. The potential energy in joules of a particle of mass 1 kg moving in a plane is given by $U = 3x + 4y$, the position coordinates of the point being x and y, measured in metres. If the particle is initially at rest at (6,4), then
(A) Its acceleration is of magnitude 5 m/s^2
(B) Its speed when it crosses the y-axis is 10 m/s
(C) It crosses the y-axis ($x = 0$) at $y = -4$
(D) It moves in a straight line passing through the origin (0,0)

31. The potential energy of a particle of mass 5kg moving in the XY plane is given by $V = -7x + 24y$ joules, x and y being in metres. Initially at $t = 0$ the particle is at the origin (0,0) moving with a velocity of $6[\hat{i}\,(2.4) + \hat{j}\,(0.7)]$ m/s. Then

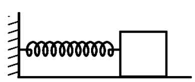


(A) The magnitude of velocity of the particle at $t = 4$ sec is 25m/s
(B) The magnitude of acceleration of the particle is 5m/s^2
(C) The direction of motion of the particle initially at $t = 0$ is at right angles to the direction of acceleration
(D) The path of the particle is a circle.

32. A box of mass m is released from rest at position 1 on the frictionless curved track shown. It slides a distance d along the track in time t to reach position 2, dropping a vertical distance h. Let v and a be the instantaneous speed and instantaneous acceleration, respectively, of the box at position 2. Which of the following equations is valid for this situation?
(A) $h = vt$
(B) $h = (1/2)gt^2$
(C) $d = (1/2)at^2$
(D) $mgh = (1/2)mv^2$
33. A ball of mass m is attached to the lower end of light vertical spring of force constant k. The upper end of the spring is fixed. The ball is released from rest with the spring at its normal (unstretched) length, comes to rest again after descending through a distance x.
(A) $x = mg/k$
(B) $x = 2\ mg/k$
(C) The ball will have no acceleration at the position where it has descended through $x/2$.
(D) The ball will have an upward acceleration equal to g at its lowermost position.
34. A ball is projected vertically upwards. Air resistance & variation in g may be neglected. The ball rises to its maximum height H in a time T, the height being h after a time t
1. The graph of kinetic energy E_k of the ball against height h is shown in figure 1
2. The graph of height h against time t is shown in figure 2
3. The graph of gravitational energy E_g of the ball against height h is shown in figure 3



Which of A, B, C, D, E shows the correct answers?
(A) 3 only
(B) 1, 2
(C) 2, 3
(D) 1 only
35. A spring block system is placed on a rough horizontal floor. The block is pulled towards right to give spring some elongation and released.

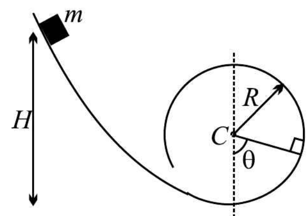


(A) The block may stop before the spring attains its mean position.
(B) The block must stop with spring having some compression.
(C) The block may stop with spring having some compression.
(D) It is not possible that the block stops at mean position.
36. In the above situation the block will have maximum velocity when
(A) The spring force becomes zero
(B) The frictional force becomes zero
(C) The net force becomes zero
(D) The acceleration of block becomes zero
37. A particle of mass m is at rest in a train moving with constant velocity with respect to ground. Now the particle is accelerated by a constant force F_0 acting along the direction of motion of train for time t_0. A girl in the train and a boy on the ground measure the work done by this force. Which of the following are **INCORRECT**?
(A) Both will measure the same work
(B) Boy will measure higher value than the girl
(C) Girl will measure higher value than the boy
(D) Data are insufficient for the measurement of work done by the force F_0

38. Two particles move on a circular path (one just inside and the other just outside) with angular velocities ω and 5ω starting from the same point. Then
    (A) They cross each other at regular intervals of time $\frac{2\pi}{4\omega}$ when their angular velocities are oppositely directed.
    (B) They cross each other at points on the path subtending an angle of 60° at the centre if their angular velocities are oppositely directed.
    (C) They cross at intervals of time $\frac{\pi}{3\omega}$ if their angular velocities are oppositely directed.
    (D) They cross each other at points on the path subtending 90° at the centre if their angular velocities are in the same sense.
39. A cart moves with a constant speed along a horizontal circular path. From the cart, a particle is thrown up vertically with respect to the cart
    (A) The particle will land somewhere on the circular path
    (B) The particle will land outside the circular path
    (C) The particle will follow an elliptical path
    (D) The particle will follow a parabolic path

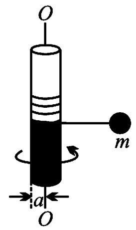
Question No. 40 to 42 (3 questions)

A particle of mass m is released from a height H on a smooth curved surface which ends into a vertical loop of radius R, as shown



40. Choose the correct alternative(s) if $H = 2R$
    (A) The particles reaches the top of the loop with zero velocity
    (B) The particle cannot reach the top of the loop
    (C) The particle breaks off at a height $H = R$ from the base of the loop
    (D) The particle break off at a height $R < H < 2R$
41. If θ is instantaneous angle which the line joining the particle and the centre of the loop makes with the vertical, then identify the correct statement(s) related to the normal reaction N between the block and the surface
    (A) The maximum value N occurs at $\theta = 0$
    (B) The minimum value of N occurs at $N = \pi$ for $H > 5R/2$
    (C) The value of N becomes negative for $\pi/2 < \theta < 3\pi/2$
    (D) The value of N becomes zero only when $\theta \geq \pi/2$
42. The minimum value of H required so that the particle makes a complete vertical circle is given by
    (A) $5R$ (B) $4R$ (C) $2.5R$ (D) $2R$

43. A small particle of mass m is given an initial high velocity in the horizontal plane and winds its cord around the fixed vertical shaft of radius a. All motion occurs essentially in horizontal plane. If the angular velocity of the cord is ω_0 when the distance from the particle to the tangency point is r_0, then the angular velocity of the cord ω after it has turned through an angle θ is



(A) $\omega = \omega_0$ (B) $\omega = \dfrac{a\omega_0}{r_0}$

(C) $\omega = \dfrac{\omega_0}{1 - \dfrac{a\theta}{r_0}}$ (D) $\omega = \omega_0\theta$

ANSWER KEY

Exercise-1

1. (i) Zero (ii) 500J
2. $1.96 \times 10^{11}\, J$
3. 135 J.
4. $\dfrac{mg\ell}{2}$
5. $v_f = 63.2\ \text{ms}^{-1}$.
6. – 8.75 J
7. 80 kg
8. $\dfrac{3mv^2}{4x^2}$
9. $2mg/k$
10. (a) Since the gravitational force is a conservative force therefore the work done in round trip is zero.
    (b) 18.5 J
    (c) –7.6 J (d) 10.9 J.
11. 0.12
12. (a) $a_1 = F/m$, so $v_1 = a_1 t = Ft/m$.
    (b) Since velocities add, $v = v_c + v_1 = v_c + Ft/m$
    (c) $\Delta K_1 = m(v_1)^2/2 = F^2t^2/2m$
    (d) $\Delta K = m\,(v_c + v_1)^2/2 - mv_c^{\,2}/2$.
    (e) s_1 is $a_1t^2/2 = Ft^2/2m$
    (f) $s_1 + v_c t$
    (h) Compare W and W_1 and ΔK and ΔK_1, they are respectively equal.
    (i) The work - energy theorem holds for moving observers.
13. $6\ \text{m s}^{-1}$.
14. (a) 5×10^4 (b) 6.25×10^{-3} kg
15. 0.13.
16. $\vec{F} = -(4\hat{i} + 36\hat{j} + 2\hat{k})N$
17. $\vec{F} = -[2y\hat{i} + (2x + z)\hat{j} + y\hat{k}]$

18. (i) $U(x, y, z) = (-2x - 3y)$ (ii) $U(x, y, z) = -(x^2 + y^2)$ (iii) $U(x, y, z) = -xy$.

19. $\frac{8}{3} hp$

20. 320 hp

21. 50 W

22. 49 kW

23. 1568 W

24. 686 W

25. (a) 7.5 J (b) 15 J (c) 7.5 J (d) 30 J

26. $W_2 = 3W_1$

27. (a) $2 + 24t^2 + 72t^4 J$, (b) $48\,t\,N$, (c) $48t + 288t^3\ W$, (d) 1248 J

28. mgR

29. $\frac{2m_1^2 g^2}{k}$

30. (a) $x=2$, (b) 16 J

31. $-3k$

32. 3.

33. 46 J

34. (a) $6 \times 10^3\ W$ (b) $1 \times 10^4\ W$

35. 80 kW

36. $5/2\sqrt{gR}$, $x_{min}=2R$

37. $\frac{1}{3}\sqrt{\frac{gl}{3}}$

38. $(1-\sqrt{3}/2)mg$

39. 625 J

40. (a) $\frac{F}{2(M+m)g}$ (b) $\frac{mF}{2(M+m)}$ (c) $\frac{mFd}{2(M+m)}$

41. (a) $\frac{40000}{5+\tan\theta} J$ (b) 7690 J

42. $ma^2d/2$

43. (a) 2250 J (b) – 4900 J (c) 465W

44. (a) 0.5 (b) 1000 N/m

45. $v\sqrt{m/k}$, No

46. $\frac{h}{4}\sqrt{k/m}$

47. $W = (\mu + 1)\ mgR$

48. $\mu m_1 g + \frac{\mu m_2 g}{2}$

49. (a) repulsive (b) $\sqrt{\frac{3k}{am}}$

50. 1, 2, 3.

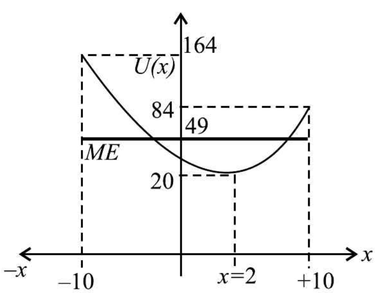
51. $6mg$

52. $9a/2$.

53. $\frac{7}{12}$ m

54. 2 sec

55. (a) 49 J (b)



(c) –3.38 m (d) 7.38 m (e) 29 J (*f*) x = 2m (*g*) $F = 2(2 - x)$

Exercise-2

1. $\frac{15\sqrt{3}}{2}N$

2. $F = -3ax^2 + b,\ x = \sqrt{\frac{b}{a}},\ KE_{max} = \frac{2b}{3\sqrt{3}}\sqrt{\frac{b}{a}},\ x = \sqrt{\frac{b}{3a}}$

6. $mgR/2,\ 2\sqrt{gR}$

7. 2.5cm

8. (a) $f = -\frac{\mu m}{\ell}xg$; (b) $\sqrt{\mu g \ell}$; (c) $\frac{5\ell}{2}$

9. 30

10. 3/4.

12. 500N/m

13. $u = v \sec\theta,\ v = \frac{40}{\sqrt{41}}$ m/s

14. 2 m/s

15. $v = v_0,\ 5\pi a/v_0$.

16. $a_N = \frac{vR}{(2Rt - vt^2)},\ a_t = \frac{R(vt - R)v^{1/2}}{(2Rt - vt^2)^{3/2}}$

17. up, 10cm

18. $\theta_{max} = \pi/2,\quad T = mg(3\sin\theta + 3\cos\theta - 2)$

19. 4, $\sqrt{gl/8}$

20. 3.3 m

21. 4m/s, 24.5*J*, 40 *J*

22. (i) $\frac{a}{2}$, (ii) $2\sqrt{ag}$, $2a$

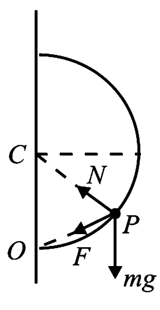
23. (a) $2\sqrt{2r}$, (b) $h = \frac{19}{27}r$, (c) g

Exercise-3

1. (a) $7\sqrt{2}\ rad/s$ (b) -9.8×10^{-3} m/s^2.

2. *B*

3. (i)



(ii) $a_T = \frac{5\sqrt{3}}{8}g,\ N = \frac{3mg}{8}$

4. $(\sqrt{65})\,mg$, $3R$

5. *D*

6. $H = (5/3)R$

7. $u = \sqrt{gL\left(\frac{3\sqrt{3}}{2} + 2\right)}$

8. *A*

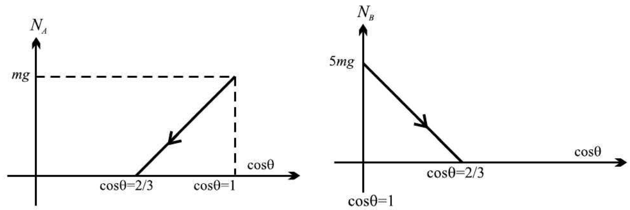
9. *A*

10. *A*

11. 0.24m, 6.4rad/sec

12. *C*

13. (a) $N = 3mg\cos\theta - 2mg$, (b)



14. D
15. C
16. B
17. C
18. D
19. B
20. A
21. A
22. $\vec{P}_{PM} = m\overline{v}_{PM} = -mv_2 \sin\omega t\,\hat{i} + m(v_2\cos\omega t - v_1)\,\hat{j}$
23. *C*
24. *D*
25. *B*
26. 8

Exercise-4

27.	(C)	28.	(C)	29.	(D)	30.	(4)
31.	(8)	32.	(D)	33.	(5)	34.	(B)
35.	(A)						

Previous Year's AIEEE Questions

1.	(A)	2.	(B)	3.	(B)	4.	(B)
5.	(B)	6.	(B)	7.	(B)	8.	(A)
9.	(C)	10.	(C)	11.	(C)	12.	(C)

Exercise-5

1.	C	2.	D	3.	C	4.	A
5.	A	6.	B	7.	C	8.	D
9.	D	10.	B	11.	A	12.	A
13.	D	14.	B	15.	B	16.	A
17.	A	18.	C	19.	C	20.	C
21.	C	22.	A	23.	B	24.	C
25.	A	26.	C	27.	C	28.	C
29.	C	30.	B	31.	B	32.	C
33.	B	34.	C	35.	A	36.	A

37. (b) $W_{ABC} = W_{ADC} = \frac{a^5}{3}$ (*J*), $W_{AC} = \frac{2a^5}{5}$ (*J*)]

38. (a) $F = -\frac{dU}{dy} = \omega$ (b) $F = -\frac{dU}{dy} = -3ay^2 + 2by$ (c) $F = -\frac{dU}{dy} = -\beta U_0\cos\beta y$]

39.	A	40.	ABC	41.	A	42.	D
43.	A	44.	B	45.	B	46.	A
47.	B	48.	A	49.	A	50.	C
51.	B	52.	C	53.	C	54.	B
55.	A	56.	A	57.	A	58.	C
59.	B	60.	D	61.	D	62.	C
63.	C	64.	B	65.	B	66.	C
67.	D	68.	C	69.	B	70.	C
71.	A	72.	B	73.	D	74.	B
75.	C	76.	C	77.	B	78.	C
79.	C	80.	C	81.	B	82.	B
83.	A	84.	C	85.	C	86.	A
87.	C	88.	C	89.	A	90.	B
91.	B	92.	D	93.	C	94.	C
95.	D	96.	C	97.	B	98.	B
99.	D	100.	B	101.	C	102.	C
103.	B	104.	C	105.	D	106.	A
107.	B	108.	A	109.	A		

Exercise-6

1.	A	2.	C	3.	AB	4.	A
5.	CD	6.	D	7.	ABD	8.	ACD
9.	BD	10.	AB	11.	B	12.	ABC
13.	ABC	14.	BC	15.	BD	16.	ABC
17.	C	18.	A	19.	B	20.	B
21.	B	22.	C	23.	C	24.	C
25.	C	26.	D	27.	B	28.	A
29.	A	30.	A, B, C	31.	A, B, C	32.	D
33.	B, C, D	34.	A	35.	A, C	36.	C, D
37.	A, C	38.	B, C, D	39.	B, D	40.	B, D
41.	A, B, D	42.	C	43.	C		

Chapter

7

Motion in a Vertical Circle

INTRODUCTION

Suppose a particle of mass m is attached to an inextensible light string of length R. The particle is moving in a vertical circle of radius R about a fixed point O. It is imparted a velocity u in horizontal direction at lowest point A. Let v be its velocity at point B of the circle as shown in the figure here.

$$h = R(1 - \cos\theta) \quad \text{... (i)}$$

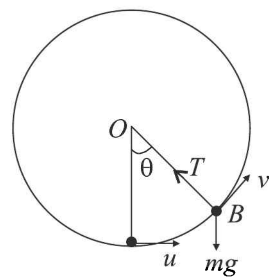
From conservation of mechanical energy

$$\frac{1}{2}M(u^2 - v^2) = mgh$$

or $$v^2 = u^2 - 2gh \quad \text{... (ii)}$$

The necessary centripetal force is provided by the resultant of tension T and $mg\cos\theta$

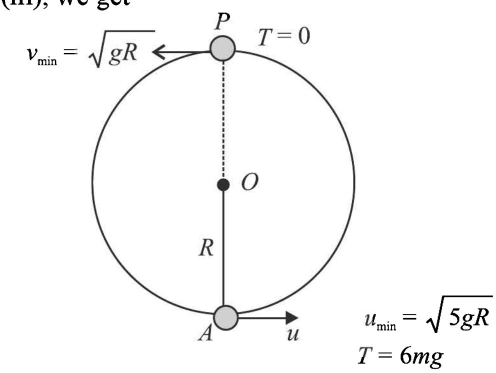
$$\therefore \quad T - mg\cos\theta = \frac{mv^2}{R} \quad \text{... (iii)}$$



Now, following three conditions arise depending on the value of u.

Case 1: ($u \geq \sqrt{5gR}$)

The particle will complete the circle if the string does not slack even at the highest point ($\theta = \pi$). Thus, tension in the string should be greater than or equal to zero ($T \geq 0$) at $\theta = \pi$. In critical case, substituting $T = 0$ and $\theta = \pi$ in equation (iii), we get



$$mg = \frac{mv^2_{min}}{R} \quad \text{or} \quad v^2_{min} = gr$$

or $$v_{min} = \sqrt{gR} \quad \text{(at highest point)}$$

Substituting $\theta = \pi$ in equation (i) $h = 2R$

Therefore, from equation (ii) $u^2_{min} = v^2_{min} + 2gh$

or $u^2_{min} = gR + 2g(2R) = 5gR$ or $u_{min} = \sqrt{5gR}$

Thus, if $u \geq \sqrt{5gR}$, the particle will complete the circle.

At $u = \sqrt{5gR}$, velocity at highest point is $v = \sqrt{gR}$, and tension in the string is zero.

Substituting $\theta = 0^0$ and $v = u = \sqrt{5gR}$ in equation (iii),

we get $T = 6\ mg$ or in the critical condition tension in the string at lowest position is 6 mg. This is shown in figure.

If $u < \sqrt{5gR}$, following two cases are possible.

Case 2: $(\sqrt{2gR} < u < \sqrt{5gR})$

If $u < \sqrt{5gR}$, the tension in the string will become zero before reaching the highest point. From equation (iii), tension in the string becomes zero ($T = 0$)

where $\cos\theta = \dfrac{-v^2}{Rg}$ or $\cos\theta = \dfrac{2gh - u^2}{Rg}$

Substituting, this value of cosθ in equation (i), we get

$$\frac{2gh - u^2}{Rg} = 1 - \frac{h}{R}$$

$$h = h_1 = \frac{1}{3}\left(\frac{u^2 + Rg}{g}\right) \qquad \text{...(iv)}$$

or we can say that at height h_1 tension in the string becomes zero. Further, if $u < \sqrt{5gR}$, velocity of the particle becomes zero when

$$0 = u^2 - 2gh$$

or

$$h = \frac{u^2}{2g} = h_2 \quad \text{(say)} \qquad \text{... (v)}$$

i.e. at height h_2 velocity of particle becomes zero.

Now, the particle will leave the circle if tension in the string becomes zero but velocity is not zero or $T = 0$ but $v \neq 0$. This is possible only when

$$h_1 < h_2$$

or

$$\frac{u^2 + Rg}{3g} < \frac{u^2}{2g}$$

or

$$2u^2 + 2Rg < 3u^2$$

or

$$u^2 > 2Rg$$

or

$$u > \sqrt{2Rg}$$

Therefore, if $\sqrt{2gR} < u < \sqrt{5gR}$, the particle leaves the circle.

From equation (v), we can see that $h > R$ if $u^2 > 2gR$. Thus, the particle will leave the circle when $h > R$ or $90^0 < \theta < 180^0$. This situation is shown in the figure 7.2.

$$\sqrt{2gR} < u < \sqrt{5gR} \quad \text{or} \quad 90^0 < \theta < 180^0$$

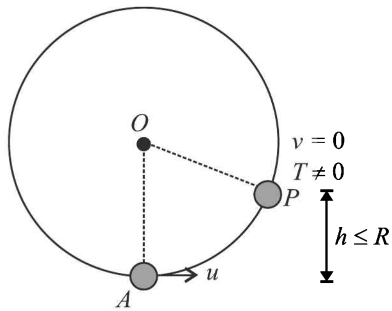
Case 3: ($0 < u \le \sqrt{2Rg}$)

The particle will oscillate if velocity of the particle becomes zero but tension in the string is not zero. or $v = 0$, but $T \neq 0$. This is possible when

$$h_2 < h_1$$

or $$\frac{u^2}{2g} < \frac{u^2 + Rg}{3g} \quad \text{or} \quad 3u^2 < 2u^2 + 2Rg$$

or $$u^2 < 2Rg \text{ or } u < \sqrt{2Rg}$$



Further, if $h_1 = h_2$, $u = \sqrt{2Rg}$ and tension and velocity both becomes zero simultaneously.

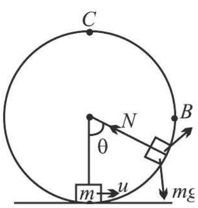
Further, from equation (v), we can say that

$h \le R$ if $u \le \sqrt{2Rg}$.

Thus, for $0 < u \le \sqrt{2gR}$ particle oscillates in lower half of the circle ($0^0 < \theta \le 90^0$). This situation is shown in the figure given alongside.

$$0 < u \le \sqrt{2gR} \quad \text{or} \quad 0^0 < \theta \le 90^0$$

Example 1. What is the minimum speed to reach B and C?



(B) $\frac{1}{2} mu^2 = mg$ (R)

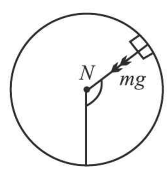
$u = \sqrt{2gR}$

Solve for (C) like this

(C) $\frac{1}{2} mu^2 = mg\ (R)$

$u = \sqrt{4gR}$. This is wrong. Why?

At any θ with vertical.



$$N = mg \cos\theta = \frac{mV^2}{R}$$

$$N = mg \cos\theta + \frac{mV^2}{R}$$

This equation is valid through out for $\theta > \frac{\pi}{2}$ as $\cos\theta$ will go negative and comp. of mg will act in the direction of normal. For $0 < \theta < \frac{\pi}{2}$ N will never be zero as both $mg \cos\theta$ and $\frac{mV^2}{R}$ are positive. Hence, it will be contact and will have circular motion.

Using Work Energy

$$-mg\,(R(1-\cos\theta)) = \frac{mv^2}{2} - \frac{mu^2}{2}$$

$$V^2 = \frac{2}{m}\left(\frac{mu^2}{2} - mgR(1-\cos\theta)\right)$$

$$V^2 = u^2 - 2gR\,(1-\cos\theta)$$

$$V^2 = u^2 - 2gR + 2g\,R\cos\theta$$

$$N = mg\cos\theta + \frac{m(u^2 - 2gR + 2gR\cos\theta)}{R}$$

$$= mg\cos\theta + \frac{mu^2}{R} - 2\,mg + 2\,mg\ \cos\theta$$

$$N = \frac{\mathrm{m}}{\mathrm{R}}\,[u^2 - 2gR + 3g\,R\cos\theta]$$

$$0 < \theta < \frac{\pi}{2}$$

Normal will not become zero.

If we want to find minimum value to reach B there is no need to see the equation of normal all that matters is speed.

at $\theta = \frac{\pi}{2}$

$$0 = u^2 - 2gR + 2gR\,(0)$$

$$u = \sqrt{2gR}$$

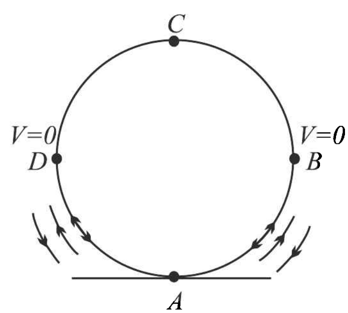
Case I: u = $\sqrt{2gR}$ it will just reach B.

Motion : A → B → A → D → A → B

At B, N = 0 but it wil not loose constant.

Case II: $u < \sqrt{2gR}$

The body will not reach B but its velocity will become zero before B.



e.g., Let $u = \sqrt{gR}$

$$0 = gR - 2gR + 2gR\cos\theta$$

$$\cos\theta = \frac{1}{2}$$

∴ at 60° the body will stop. The body will not remain stationary as its tangential acceleration will not be zero.

what if $\theta > \frac{\pi}{2}$

Here the normal will become zero before velocity. This is why $\sqrt{4gR}$ was wrong as we were considering speed and not normal where as to reach C it is necessary that '*N*' does not become zero.

Find minimum speed to reach C.

$$0 = u^2 - 5gR \qquad [\theta = \pi]$$

$$u = \sqrt{5gR}$$

Case III:

$$u = \sqrt{5gR}$$

$$v^2 = 5gR - 2gR - 2gR$$

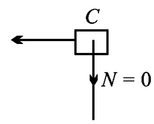
$$= gR$$

$$v = \sqrt{gR}$$

Minimum possible value of '*N*' and '*v*' is at 'C'.

$$v = \sqrt{gR}$$

As $mg = \dfrac{mv^2}{R}$ is valid



so the body will continue moving in circular motion.

$u = \sqrt{5gR}$ implies the body has just completed circular motion.

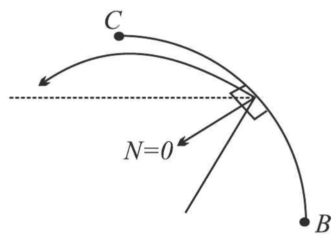
Note

We check for 'π' as cos π has maximum negative value. If *N* is not 0 at this point then for all $\theta < \pi$ the normal will never be zero.

Case IV: $u > \sqrt{5gR}$

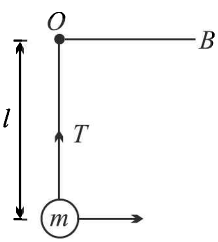
The body will freely move in a circle and '*N*' will never be zero.

Case V: $\sqrt{5gR} > u > \sqrt{2gR}$.



The normal will become zero some where between B and C. At this point $v \neq 0$. It will leave circular motion and will become projectile because symmetry will no more be there as in the next instant velocity will decrease further for which *N* should be negative which is not possible and so it will leave circular motion and will have projectile motion.

For a mass tied by a string about O.

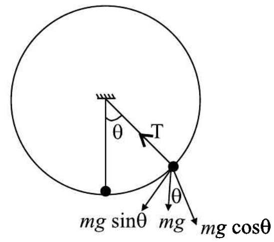


Here instead of normal 'Tension' is the worrying factor.

$T = 0 \rightarrow$ String is slack and 'm' wil leave circular motion.

MOTION IN A VERTICAL CIRCLE

To understand this consider the motion of a small body (say stone) tied to a string and whirled in a vertical circle. If at any time the body is at angular position θ, as shown in the figure, the forces acting on it are tension T in the string along the radius towards the center and the weight of the body mg acting vertically down wards.



Applying Newton's law towards centre

$$T - mg\cos\theta = \frac{mv^2}{r} \quad \text{or} \quad T = \frac{mv^2}{r} + mg\cos\theta$$

The body will move on the circular path only and only if $T_{min} > 0$ (as if $T_{min} \leq 0$, the string will slack and the body will fall down instead of moving on the circle. So, for completing the circle, i.e., 'looping the loop'

$$\frac{mv_H^2}{r} - mg \geq 0 \quad \text{i.e.} \quad v_H = \sqrt{gr} \quad ...(1)$$

Now applying conservation of mechanical energy between highest point H and lowest point L

we get $v_L = \sqrt{5gr}$

i.e., for looping the loop, velocity at lowest point must be $\geq \sqrt{5gr}$.

In case of motion in a vertical plane tension is maximum at lowest position and in case of looping the loop

$$T_{min} \geq 6 \text{ mg}.$$

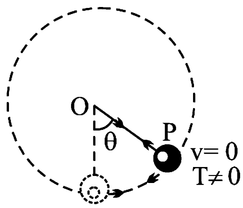
CONDITION FOR OSCILLATION OR LEAVING THE CIRCLE

In case of non uniform circular motion in a vertical plane if velocity of body at lowest point is lesser than $\sqrt{5gr}$, the particle will not complete the circle in vertical plane. Now it can either oscillate about the lowest point or after reaching a certain height may loose contact with the path.

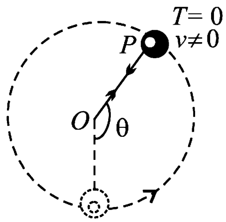
From the theory of looping the loop we know that if $v_L \geq \sqrt{5gr}$, the body will loop the loop. So, if the velocity of a body at lowest point is such that

$$\sqrt{2gr} < v_L < \sqrt{5gr}$$

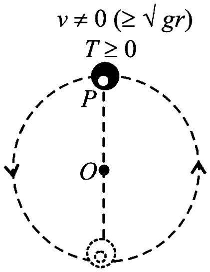
the body will move along the circle for $\theta > 90°$ and will not reach upto highest point but will leave the circle somewhere between $90° < \theta < 180°$. Here, it is worth noting that at the point of leaving the circle $T = 0$ but $v_0 \neq 0$. This all is shown in the three figures on the next page.



(A)
For Oscillation
$0 < v_L \le \sqrt{2gr}$
$0 < \theta \le 90°$



(B)
For Leaving the circle
$\sqrt{2gr} < v_L < \sqrt{5gr}$
$90° < \theta < 180°$



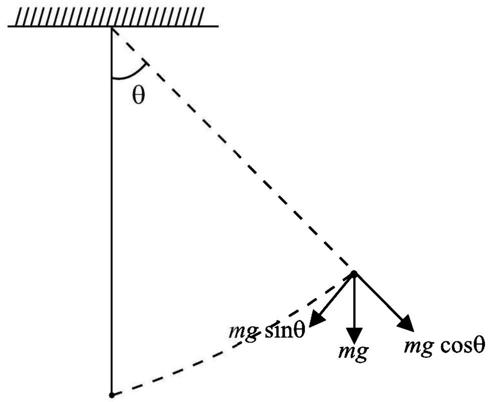
(C)
For Looping the loop
$V_L \ge \sqrt{5gr}$

Example 2. A simple pendulum is constructed by attaching a bob of mass *m* to a string of length L fixed at its upper end. The bob oscillates in a vertical circle. It is found that the speed of the bob is *v* when the string makes an angle θ with the vertical. Find the tension in the string at this instant.

Solution The forces acting on the bob are (figure)

(a) the tension *T*

(b) the weight mg.



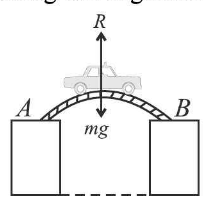
As the bob moves in a vertical circle with centre at O, the radial acceleration is v^2 / L towards O. Taking the components along this radius and applying Newton's second law , we get

$$T - mg\cos\theta = mv^2 / L$$

or, $$T = m(g\cos + v^2 / L).$$

Example 3. Prove that a motor car moving over a convex bridge is lighter than the same car resting on the same bridge.

Solution The motion of the motor car over a convex bridge *AB* is the motion along the segment of a circle *AB* (Figure)



The centripetal force is provided by the difference of weight *mg* of the car and the normal reaction *R* of the bridge.

$$\therefore \quad mg - R = \frac{mv^2}{r} \quad \text{or} \quad R = mg - \frac{mv^2}{r}$$

Clearly $R < mg$, i.e., the weight of the moving car is less than the weight of the stationary car.

Example 4. A body weighing 0.4 kg is whirled in a vertical circle with a string making 2 revolutions per second. If the radius of the circle is 1.2 m. Find the tension (a) at the top of the circle, (b) at the bottom of the circle. **Given: $g = 10$ m s^{-2} and $\pi = 3.14$.**

Solution Mass, $m = 0.4$ kg;

time period $= \frac{1}{2}$ second, radius, $r = 1.2$ m

Angular velocity, $\omega = \frac{2\pi}{1/2} = 4\pi$ rad s^{-1} = 12.56 rad s^{-1} .

(a) At the top of the circle,.

$$T = \frac{mv^2}{r} - mg = mr\omega^2 - mg = m(r\omega^2 - g)$$

$$= 0.4\,(1.2 \times 12.56 \times 12.56 - 9.8)\,N = 71.2\,N$$

(b) At the lowest point, $T = m(r\omega^2 + g) = 80\,N$

Example 5. You may have seen in a circus a motorcyclist driving in vertical loops inside a 'death wall' (a hollow spherical chamber with holes, so that the cyclist does not drop down when he is at the uppermost point, with no support from below. What is the minimum speed required at the uppermost position to perform a vertical loop if the radius of the chamber is 25 m?

Solution When the motorcyclist is at the highest point of the death-well, the normal reaction R on the motorcyclist by the ceiling of the chamber acts downwards. His weight mg also act downwards. These two forces are balanced by the outward centrifugal force acting on him

$$\therefore \quad R + mg = \frac{mv^2}{r}$$

Here v is the speed of the motorcyclist and m is the mass of the motorcyclist (including the mass of the motor cycle). Because of the balancing of the forces, the motorcyclist does not fall down.

The minimum speed required to perform a vertical loop is given by equation (1) when $R = 0$.

$$\therefore \quad mg = \frac{mv_{min}^2}{r} \quad \text{or } v^2_{min} = gr$$

$$\text{or} \quad v_{min} = \sqrt{gr} = \sqrt{9.8 \times 25} \text{ m s}^{-1} = 15.65 \text{ ms}^{-1} .$$

So, the minimum speed, at the top, required to perform a vertical loop is 15.65ms^{-1}.

Example 6. A heavy particle hanging from a fixed point by a light inextensible string of length l is projected horizontally with speed $\sqrt{(gl)}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle.

Solution Let tension in the string becomes equal to the weight of the particle when particle reaches the point B and deflection of the string from vertical is θ. Resolving mg along the string and perpendicular to the string, we get net radial force on the particle at B i.e.

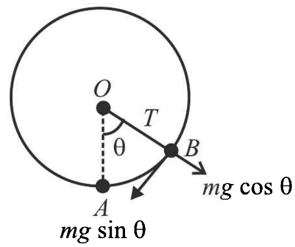
$$F_R = T - mg\cos\theta \qquad \text{...(i)}$$

If v be the speed of the particle at B, then

$$F_R = \frac{mv^2}{l} \quad \text{... (ii)}$$

From (i) and (ii), we get

$$T - mg\cos\theta = \frac{mv^2}{l} \quad \text{... (iii)}$$



Since at B, $T = mg$

$$\Rightarrow \quad mg(1 - \cos\theta) = \frac{mv^2}{l}$$

$$\Rightarrow \quad v^2 = gl(1 - \cos\theta) \quad \text{... (iv)}$$

Conserving the energy of the particle at point A and B, we have

$$\frac{1}{2}mv_0^2 = mgl(1-\cos\theta) + \frac{1}{2}mv^2$$

where $v_0 = \sqrt{gl}$ and $v = \sqrt{gl(1-\cos\theta)}$

$$\Rightarrow \quad gl = 2gl(1 - \cos\theta) + gl(1 - \cos\theta)$$

$$\Rightarrow \quad \cos\theta = 2/3 \quad \text{... (v)}$$

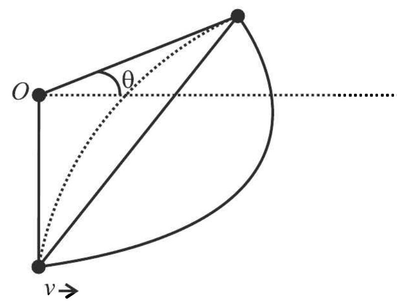
Putting the value of $\cos\theta$ in equation (iv) we get

$$v = \sqrt{\frac{gl}{3}}$$

Example 7. A particle is suspended by a string of length 'l'. It is projected with such a velocity v along the horizontal such that after the string becomes slack it flies through its initial position. Find v.

Solution Let the velocity be v' at B where the string become slack and the string makes angle θ with horizontal by the law of conservation of energy

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + mg\ell(1+\sin\theta) \quad \text{...(i)}$$



$$\text{or,} \quad v'^2 = v^2 - 2g\ell\,(1 + \sin\theta) \quad \text{...(ii)}$$

By the dynamics of circular motion

$$mg\sin\theta = \frac{mv'^2}{\ell}$$

$$\Rightarrow \quad v'^2 = g\ell\sin\theta \quad \text{...(iii)}$$

from equation (ii) and (iii) we get

$$\therefore \quad g\,\ell\sin\theta = v^2 - 2g\ell\,(1 + \sin\theta) \quad \text{...(iv)}$$

At B the particle becomes a projectile of velocity v' at 90 - θ with the horizontal.

Here, $u_x = v'\sin\theta$ and $u_y = v'\cos\theta$

$a_x = 0$ and $a_y = -g$

$$\therefore \quad \ell\cos\theta = v'\sin\theta\, t \quad \text{...(v)}$$

$$\therefore \quad t = \frac{\ell\cos\theta}{v'\sin\theta} \;\&\; -\ell\,(1+\sin\theta)$$

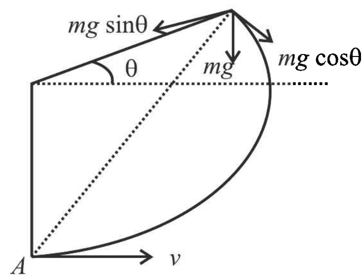
$$= v'\cos\theta\,\frac{l\cos\theta}{v'\sin\theta} - \frac{1}{2}g\frac{l^2\cos^2\theta}{v'^2\sin^2\theta}$$

$$\Rightarrow \quad 2\sin^3\theta + 3\sin^2\theta - 1 = 0$$

$$\therefore \quad \sin\theta = \frac{1}{2} \text{ is the acceptable solution}$$

$$\therefore \quad v^2 = 2g\ell + 3g\ell \times \frac{1}{2} = \frac{7g\ell}{2} \Rightarrow v = \sqrt{\frac{7g\ell}{2}}$$

(from equation (iv))



A BODY MOVING INSIDE A HOLLOW TUBE

The same discussion holds good for this case, but instead of tension in the string we have the normal reaction of the surface. If N is the normal reaction at the lowest point, then

$$N - mg = \frac{mv_1^2}{r}; \quad N = m\left(\frac{v_1^2}{r} + g\right)$$

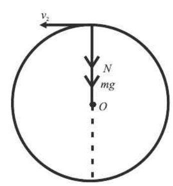
At the highest point of the circle,

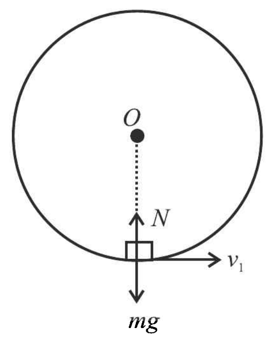
$$N + mg = \frac{mv_2^2}{r}$$

$$N = m\left(\frac{v_2^2}{r} - g\right)$$

The condition $v_1 \geq \sqrt{5rg}$

All other equations (can be) similarly obtained by replacing tension T by reaction N.





BODY MOVING ON A SPHERICAL SURFACE

The small body of mass m is placed on the top of a smooth sphere of radius r.

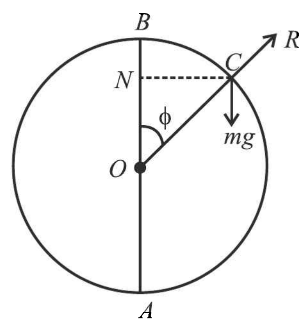
If the body slides down the surface, at what point does it fly off the surface?

Consider the point C where the mass is, at a certain instant. The forces are the normal reaction R and the weight mg. The radial component of the weight is mg cosϕ acting towards the centre.

The centripetal force is

$$mg\cos\phi - R = \frac{mv^2}{r}$$

where v is the velocity of the body at O.



$$R = m\left(g\cos\varphi - \frac{v^2}{r}\right) \qquad \text{... (i)}$$

The body flies off the surface at the point where R becomes zero.

i.e., $g\cos\phi = \frac{v^2}{r}$; $\cos\phi = \frac{v^2}{rg}$ ... (ii)

To find v, we use conservation of energy

i.e., $\frac{1}{2}mv^2 = mg(BN) = mg(OB - ON) = mgr(1 - \cos\phi)$

$$v^2 = 2rg(1 - \cos\phi)$$

$$2(1 - \cos\phi) = \frac{v^2}{rg} \qquad \text{... (iii)}$$

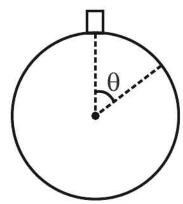
From equation (ii) and (iii) we get

$$\cos\phi = 2 - 2\cos\phi;\quad 3\cos\phi = 2$$

$$\cos\phi = \frac{2}{3};\ \phi = \cos^{-1}\left(\frac{2}{3}\right) \qquad \text{... (iv)}$$

This gives the angle at which the body goes off the surface. The height from the ground of that point $= AN = r(1 + \cos\phi)$

$$= r\left(1 + \frac{2}{3}\right) = \frac{5}{3}r$$

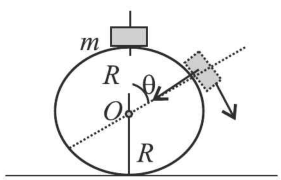


Example 8. A block is kept on a fixed smooth sphere.

(a) Find θ at which block will break off.

(b) Initial velocity, so that block breaks off in initial position itself.

Example 9. A point mass '*m*' starts from rest and slides down the surface of a frictionless solid sphere of radius '*R*' as shown in the figure. At what angle will this body break off the surface of sphere? Find the velocity with which it will break off.



Solution Applying *COE*, we have at the point *A* and *B*,

we have $mgR(1 - \cos\theta) = \frac{1}{2}mv^2$ ...(i)

Force equation gives, $mg\cos\theta - N = mv^2/R$ ...(ii)

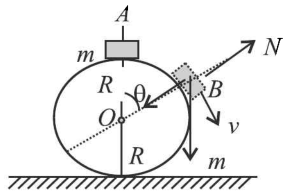
$N = 0$ for break off.

$\therefore \quad v = \sqrt{gR\cos\theta}$ ... (iii)

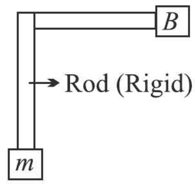
Putting it in (i)

We get cos è = 2/3

Putting this in (iii) we get $v = \sqrt{\frac{2}{3}gR}$.



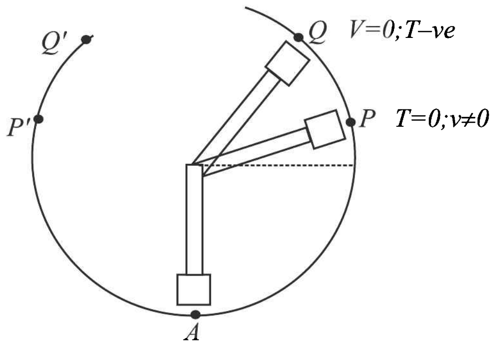
CONCEPT OF MOTION OF LIGHT RIGID ROD



Case 1: $u < \sqrt{2gR}$ – Pendulum

Case 2: $u = \sqrt{2gR}$ will reach B and come back.

Case 3: $\sqrt{4gR} > u > \sqrt{2gR}$. The body will continue moving in circular motion as tension of a rod can go negative which is allowed as then the rod instead of pulling the body will push it.



$$A \rightarrow P \rightarrow Q \rightarrow P \rightarrow A \rightarrow P' \rightarrow Q'$$

Case 4: $u = \sqrt{4gR}$

$v = 0, T \rightarrow -ve$

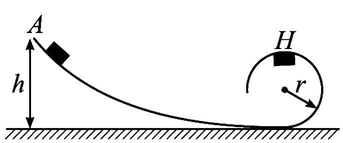
The body will stop at the top.

Case 5: $u > \sqrt{4gR}$ Forever will do circular motion.

EXERCISE

Exercise–1: Subjective Problems

1. A small body of mass m is allowed to slide on an inclined frictionless track from rest position as shown in the figure.



(i) Find the minimum height h, so that body may successfully complete the loop of radius 'r'.

(ii) If h is double of that minimum height, find the resultant force on the block at position H

2. A nail is located at a certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from the position where the string makes an angle of 60° from the vertical. Calculate the distance of the nail from the point of suspension such that the bob will just perform revolution with the nail as centre. Assume the length of pendulum to be 1 m.
3. A bucket tied at the end of a 1.6 m long string is whirled in a vertical circle with constant speed. What should be the minimum speed so that the water from the bucket does not spill, when the bucket is at the highest position (Take $g = 10$ m/sec^2)

Exercise–2: Objective Problems

Only One Option Correct

1. The driver of a car travelling at speed suddenly sees a wall at a distance r directly in front of him. To avoid collision,
   (A) He should apply brakes sharply
   (B) He should turn the car sharply
   (C) He should apply brakes and then sharply turn
   (D) None of these
2. A curved section of a road is banked for a speed v. If there is no friction between road and tyres of the car, then:
   (A) Car is more likely to slip at speeds higher than v than speeds lower than v
   (B) Car cannot remain in static equilibrium on the curved section
   (C) Car will not slip when moving with speed v
   (D) None of the above
3. A particle of mass m is observed from an inertial frame of reference and is found to move in a circle of radius r with a unifrom speed v. The centrifugal force on it is
   (A) $\frac{mv^2}{r}$ towards the centre
   (B) $\frac{mv^2}{r}$ away from the centre
   (C) $\frac{mv^2}{r}$ along the tangent through the particle
   (D) Zero
4. A train A runs from east to west and another train B of the same mass runs from west to east at the same speed along the equator. A presses the track with a force F_1 and B presses the track with a force F_2.
   (A) $F_1 > F_2$
   (B) $F_1 < F_2$
   (C) $F_1 = F_2$
   (D) The information is insufficient to find the relation between F_1 and F_2.
5. A car of mass M is moving horizontaly on a circular path of radius r. At an instant its speed is v and is increasing at a rate a.

(A) The acceleration of the car is towards the centre of the path

(B) The magnitude of the frictional force on the car is greater than $\frac{mv^2}{r}$

(C) The friction coefficient between the ground and the car is not less than a/g.

(D) The friction coefficient between the ground and the car is $\mu = \tan^{-1}\frac{v^2}{rg}$

6. A circular road of radius r is banked for a speed v = 40 km/hr. A car of mass attempts to go on the circular road. The friction coefficient between the tyre and the road is negligible.
   (A) The car cannot make a turn without skidding.
   (B) If the car turns at a speed less than 40 km/hr, it will slip down
   (C) If the car turns at the current speed of 40 km/hr, the force by the road on the car is equal to $\frac{mv^2}{r}$
   (D) If the car turns at the correct speed of 40 km/hr, the force by the road on the car is greater than mg as well as greater than $\frac{mv^2}{r}$

7. A person applies a constant force $\vec{F}$ on a particle of mass m and finds that the particle moves in a circle of radius r with a uniform speed v as seen from an inertial frame of reference.
   (A) This is not possible.
   (B) There are other forces on the particle
   (C) The resultant of the other forces is $\frac{mv^2}{r}$ towards the centre.
   (D) The resultant of the other forces varies in magnitude as well as in direction.

ANSWER KEY

Exercise-1

1. (i) $h_{min} = \frac{5}{2}r$, (ii) $F = 6\,mg$
2. 0.8 m
3. 4 m/sec

Exercise-2

1. A	2. C	3. D
4. A	5. BC	6. BD
7. BD		

Chapter

8

Centre of Mass

INTRODUCTION

Every physical system is associated with certain points where motion characterises the motion of the whole system. When the system moves under some external forces, then this point moves as if the entire mass of the system is concentrated at this point and also the external force is applied at this point for translational motion. This point is called the centre of mass of the system.

Centre of mass of system of N point masses is that point at which moment of mass of the system is zero. It means that if at a particular origin the moment of mass of system of N point masses is zero then that particular origin is the centre of mass of the system.

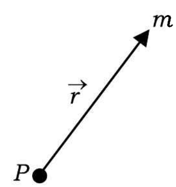
In a system of particles, there is one special point with some interesting and simple properties, no matter how complicated the system is. This special point is called the centre of mass.

- The centre of mass of a system is an imaginary point where the whole mass of the system is supposed to be acting.
- The centre of mass of a body may lie inside or outside the body.
- The centre of mass always lie on the axis of symmetry of the body if it exists. For a body in which there are two or more axes of symmetry, then the centre of mass lies at their point of intersection.
- In a system of n particles, the centre of mass of the system may or may not coincide with any of the particles.
- In a system of n coplaner particles the centre or at the system must lie within or at the edge of at least one of the polygons formed by joining $(n - 1)$ particles.
- To locate the centre of mass of different system, we define a vector quantity associated with all the particles of system this is **Mass moment** of a particle.

Moment of Point Mass 'M' About an Origin 'O'

Mass Moment

It is defined as the product of mass of the particle and distance of the particle from the point about which mass moment is taken. It is a vector quantity and its direction is directed from the point about which it is taken to the particle



Let P be the point where mass 'm' is located. Take position vector of point P with respect to origin O. The moment of point mass m about origin O is defined as

$$\vec{Z} = m\vec{r}$$

Here, $\vec{Z} =$ mass moment of particle about point P

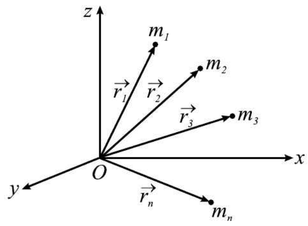
$\vec{r} =$ Position vector of particle of mass 'm' about point P

The direction of mass moment along the vector $(\vec{r})$

The physical significance of moment of mass is that when differentiated with respect to time it gives momentum of the particle.

It is worth noting that moment of point mass depends on choice of origin.

Moment of System of *N* Point Masses about an Origin



Consider a system of N point masses m_1, m_2, m_3 .... m_n whose position vectors from origin O are given by $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$, ... $\vec{r}_n$ respectively.

The moment of system of point masses about origin O is the sum of individual moment of each point mass about origin O.

$$\vec{M} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \ldots\ldots\ldots\ldots + m_n\vec{r}_n$$

There is an important property of centre of mass associated with the mass moments of each particle of the system which helps to determination of centre of mass of a system. The property is — *"The summation of mass moments of all the particles of the system about its centre of mass is always equal to zero."*

This statement is an experimentally verified property which does not require any analytical proof. It can be used as a universal property in all type of particles of system.

Centre of mass of a system of '*N*' discrete particles :

$$\vec{r}_{\text{com}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \ldots. + m_n\vec{r}_n}{m_1 + m_2 + \ldots\ldots + m_n}$$

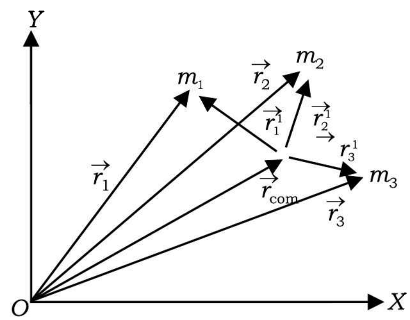
$$\vec{r}_{\text{com}} = \frac{\sum m_i\vec{r}_i}{\sum m_i}$$

Proof of position of centre of mass by the help of concept of mass moment.

$\vec{r}_1$ = Positive vector of mass (m_1) w.r.t. origin

$\vec{r}_2$ = Positive vector of mass (m_2) w.r.t. origin

$\vec{r}_3$ = Positive vector of mass (m_3) w.r.t. origin



$\vec{r}_1^{\,1}$ = Positive vector of (m_1) w.r.t. centre of mass

$\vec{r}_2^{\,1}$ = Positive vector of (m_2) w.r.t. centre of mass

$\vec{r}_3^{\,1}$ = Positive vector of (m_3) w.r.t. centre of mass

$\vec{Z}_1 = m_1\ \vec{r}_1^{\,1}$ [mass moment of (m_1) about centre of mass]

$\vec{Z}_2 = m_2\ \vec{r}_1^{\,1}$ [mass moment of (m_2) about centre of mass]

$\vec{Z}_3^{\,1} = m_3\ \vec{r}_1^{\,1}$ [mass moment of (m_3) about centre of mass]

By applying the concept

$\vec{Z}_1+\vec{Z}_2+\vec{Z}_3=0 \quad \Rightarrow \quad m_1\vec{r}_1^{\,1}+m_2\vec{r}_2^{\,1}+m_3\vec{r}_3^{\,1}=0$ **...(A)**

By applying triangle law of vector addition

$$\vec{r}_{com}+\vec{r}_1^{\,1}=\vec{r}_1 \Rightarrow \vec{r}_1^{\,1}+\vec{r}_1=\vec{r}_{com}$$

Similarly : $\vec{r}_2^{\,1}=\vec{r}_2-\vec{r}_{com}$ and $\vec{r}_3^{\,1}=\vec{r}_3-\vec{r}_{com}$

By putting these above values in the equation (*A*), we get

$$m_1(\vec{r}_1-\vec{r}_{com})+m_2(\vec{r}_2-\vec{r}_{com})+m_3(\vec{r}_3-\vec{r}_{com})=0$$

$$\vec{r}_{com}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+m_3\vec{r}_3}{m_1+m_2+m_3}$$

This relation can also be generalized for '*n*' mass system also.

Hence, we can write the *X*-cordinate of centre of mass of a system of '*N*' discrete particle :

$$X_{com}=\frac{m_1x_1+m_2x_2+.....m_nx_n}{m_1+m_2+.....+m_n}$$

$$X_{com}=\frac{\sum m_ix_i}{\sum m_i}$$

Similarly, $Y_{com}=\dfrac{m_1y_1+m_2y_2+.....m_ny_n}{m_1+m_2+.....+m_n}$

$$Y_{com}=\frac{\sum m_iy_i}{\sum m_i}$$

and $Z_{com}=\dfrac{m_1z_1+m_2z_2+.....m_nz_n}{m_1+m_2+.....+m_n}$

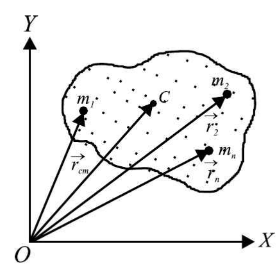
$$Z_{com}=\frac{\sum m_iz_i}{\sum m_i}$$

Centre of Mass of a System of '*N*' Discrete Particles

Consider a system of *N* point masses m_1, m_2, m_3, ... m_n whose position vectors from origin *O* are given by $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$,............... $\vec{r}_n$ respectively. Then the position vector of the centre of mass *C* of the system is given by.

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+........+m_n\vec{r}_n}{m_1+m_2+........+m_n} \ ; \ \vec{r}_{cm}=\frac{\sum_{i=1}^{n}m_i\vec{r}_i}{\sum_{i=1}^{n}m_i}$$

$$\vec{r}_{cm}=\frac{1}{M}\sum_{i=1}^{n}m_i\vec{r}_i$$



where M $\left(=\sum_{i=1}^{n} m_i\right)$ is the total mass of the system.

Position of Centre of Mass

(a) System of two particles : Consider first a system of two particles m_1 and m_2 at distances x_1 and x_2 respectively, from some origin O. We define a point C, the centre of mass of the system, as a distance x_{cm} from the origin O, where x_{cm} is defined by

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \quad ...(1)$$

x_{cm} can be regarded as mass -weighted mean of x_1 and x_2

(b) System of many particles :

(i) If $m_1, m_2 \ldots\ldots, m_n$, are along a straight line, by definition,

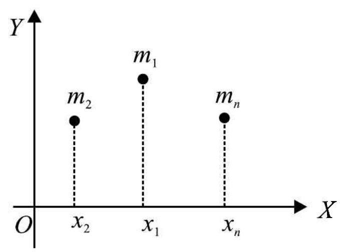
$$x_{cm} = \frac{m_1x_1 + m_2x_2 + \ldots\ldots + m_nx_n}{m_1 + m_2 + \ldots\ldots + m_n} = \frac{\sum m_ix_i}{\sum m_i} = \frac{\sum m_ix_i}{M} \quad ...(2)$$

where M is total mass of the system.

(ii) If particles do not lie in a straight line but lie in a plane, (suppose x-y plane) the centre of mass C is defined and located by the coordinates x_{cm} and y_{cm},

where $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + \ldots\ldots + m_nx_n}{m_1 + m_2 + \ldots\ldots + m_n} = \dfrac{\sum m_ix_i}{\sum m_i} = \dfrac{\sum m_ix_i}{M}$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + \ldots\ldots + m_ny_n}{m_1 + m_2 + \ldots\ldots + m_n} = \frac{\sum m_iy_i}{\sum m_i} = \frac{\sum m_iy_i}{M} \quad ...(3)$$



(iii) If the particles are distributed in space,

$$x_{cm} = \frac{\sum m_ix_i}{M}, \; y_{cm} = \frac{\sum m_iy_i}{M}, \; z_{cm} = \frac{\sum m_iz_i}{M} \quad ...(4)$$

So, position vector of C is given by

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k} = \frac{\sum m_ir_i}{\sum m_i} = \frac{\sum m_ir_i}{M} \quad ...(5)$$

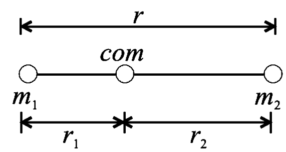
(c) Continuous bodies : $x_{cm} = \underset{\Delta m_i \to 0}{Lt} \dfrac{\sum \Delta m_i x_i}{\sum \Delta m_i} = \dfrac{\int x\,dm}{\int dm} = \dfrac{1}{M}\int x\,dm$

Similarly $y_{cm} = \dfrac{\int y dm}{\int dm} = \dfrac{1}{M}\int y\,dm$ and $z_{cm} = \dfrac{\int z dm}{\int dm} = \dfrac{1}{M}\int z\,dm$

$$\therefore \quad \vec{r}_{cm} = \frac{\int \vec{r} dm}{\int dm} = \frac{1}{M}\int \vec{r} dm \qquad \text{...(6)}$$

Position of Com of Two Particles

Centre of mass of two particles of mass m_1 and m_2 separated by a distance r lies in between the two particles. The distance of centre of mass from any of the particle (r) is inversely proportional to the mass of the particle (m)



i.e. $r \propto \dfrac{1}{m}$

or $\dfrac{r_1}{r_2} = \dfrac{m_2}{m_1}$

or $m_1 r_1 = m_2 r_2$

or $r_1 = \left(\dfrac{m_2}{m_2 + m_1}\right) r$ and $r_2 = \left(\dfrac{m_1}{m_1 + m_2}\right) r$

Here, r_1 = distance or COM from m_1

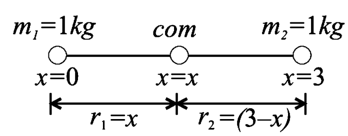
and r_2 = distance or COM from m_2

From the above discussion, we see that

$r_1 = r_2 = \dfrac{1}{2}$ if $m_1 = m_2$, i.e., COM lies midway between the two particles of equal masses.

Similarly, $r_1 \geq r_2$ if $m_1 < m_2$ and $r_1 < r_2$ if $m_2 < m_1$, i.e., *COM* is nearer to the particle having larger mass.

Example 1. Two particles of mass 1 kg and 2 kg are located at $x = 0$ and $x = 3$ m. Find the position of their centre of mass.



Solution Since, both the particles lies on x-axis, the COM will also lie on x-axis. Let the COM is located at $x = x$, then

r_1 = distance of COM from the particle of mass 1 kg = x

and r_2 = distance of COM from the particle of mass 2 kg = $(3 - x)$

Using $\dfrac{r_1}{r_2} = \dfrac{m_2}{m_1}$ or $\dfrac{x}{3-x} = \dfrac{2}{1}$ or $x = 2$ m

Thus, the COM of the two particles is located at $x = 2$ m.

Example 2. The position vector of three particles of mass m_1 = 1 kg, m_2 = 2 kg and m_3 = 3 kg are $\vec{r}_1 = (\hat{i} + 4\hat{j} + \hat{k})m$, $\vec{r}_2 = (\hat{i} + \hat{j} + \hat{k})m$ and $\vec{r}_3 = (2\hat{i} - \hat{j} - 2\hat{k})m$ respectively. Find the position vector of their centre of mass.

Solution The position vector of COM of the three particles will be given by

$$\vec{r}_{\text{COM}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3}$$

Substituting the values, we get

$$\vec{r}_{\text{COM}} = \frac{(1)(\hat{i} + 4\hat{j} + \hat{k}) + (2)(\hat{i} + \hat{j} + \hat{k}) + (3)(2\hat{i} - \hat{j} - 2\hat{k})}{1 + 2 + 3}$$

$$= \frac{9\hat{i} + 3\hat{j} - 3\hat{k}}{6}$$

$$\vec{r}_{\text{COM}} = \frac{1}{2}(3\hat{i} + \hat{j} - \hat{k})\text{m}$$

Example 3. Four particles of mass 1 kg, 2 kg, 3 kg and 4 kg are placed at the four vertices A, B, C and D of a square of side 1 m. Find the position of centre of mass of the particles.

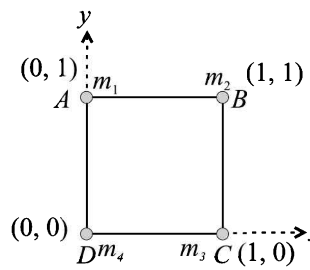
Solution Assuming D as the origin, DC as x -axis and DA as y-axis, we have

$m_1 = 1$ kg, $(x_1, y_1) = (0, 1m)$

$m_2 = 2$ kg, $(x_2, y_2) = (1m, 1m)$

$m_3 = 3$ kg, $(x_3, y_3) = (1m, 0)$

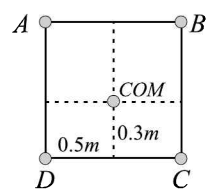
and $m_4 = 4$ kg, $(x_4, y_4) = (0, 0)$



Co-ordinates of their COM are

$$x_{\text{COM}} = \frac{m_1x_1 + m_2x_2 + m_3m_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{(1)(0) + 2(1) + 3(1) + 4(0)}{1 + 2 + 3 + 4} = \frac{5}{10} = \frac{1}{2}m = 0.5 \text{ m}$$

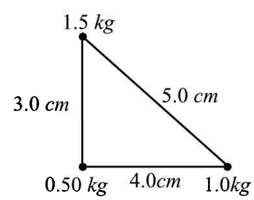


$$\text{Similarly, } y_{\text{COM}} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4}{m_1 + m_2 + m_3 + m_4}$$

$$= \frac{(1)(1) + 2(1) + 3(0) + 4(0)}{1 + 2 + 3 + 4} = \frac{3}{10}m = 0.3 \text{ m}$$

$\therefore$ $(x_{\text{COM}}, y_{\text{COM}}) = (0.5\ m, 0.3\ m)$

Thus, position of COM of the four particles is as shown in figure.



Example 4. Three particles of masses 0.5 kg, 1.0 kg and 1.5 kg are placed at the three corners of a right angled triangle of sides 3.0 cm, 4.0 cm and 5.0 cm as shown in figure. Locate the centre of mass of the system.

Solution The centre of mass is 1.3 cm to the right and 1.5 cm above the 0.5 kg particle.

Example 5. Consider a two-particle system with the particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved so as to keep the centre of mass at the same position?

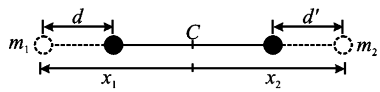
Solution Consider figure. Suppose the distance of m_1 from the centre of mass C is x_1 and that of m_2 from C is x_2. Suppose the mass m_2 is moved through a distance d' towards C so as to keep the centre of mass at C.

Then, $$m_1x_2 = m_2x_2 \quad ...(i)$$

and $$m_1(x_1 - d) = m_2(x_2 - d') \quad ...(ii)$$

Subtracting (ii) from (i) $$m_1d = m_2d'$$

or, $$d' = \frac{m_1}{m_2}d,$$



Thus $$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + ...}{m_1 + m_2 + m_3 + ...}$$

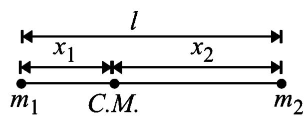
E.g. $m_1 = 3$ kg at (1,0,2)

$m_2 = 4$ kg at (1,2,3)

$m_3 = 3$ kg at (4,3,–1) Find $P.V.$ of this system.

Example 6. Distance between m_1 and m_2 is 1. Find distance between m_1 and $C.M.$

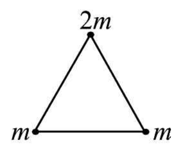
Solution $$x_1 = \frac{m_2 1}{m_1 + m_2} \qquad x_2 = \frac{m_1 1}{m_1 + m_2}$$



Note

C.M. divides two point masses in inverse ratio of their masses $\frac{x_1}{x_2} = \frac{m_2}{m_1}$

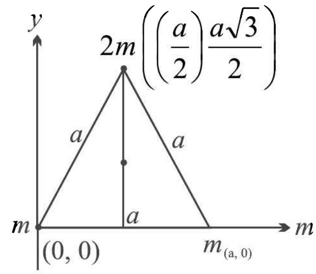
Example 7. Find $C.M.$ (equilateral triangle)



Solution $$x_{cm} = \frac{m \times 0 + ma + 2m\frac{a}{2}}{4m} = \frac{2a}{4} = \frac{a}{2}$$

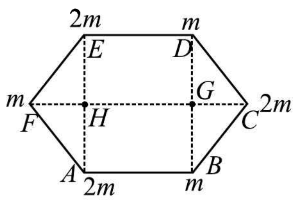
$$y_{cm} = \frac{m \times 0 + 2m \times a\frac{\sqrt{3}}{2}}{4m} = \frac{a\sqrt{3}}{4}$$

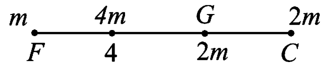
$$C.M.\left(\left(\frac{a}{2}\right)\frac{a\sqrt{3}}{4}\right)$$



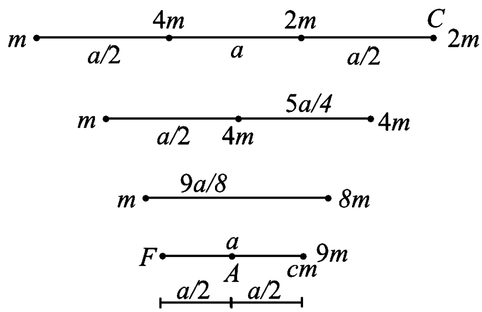
Example 8. **Regular hexagon →**

Masses at A & E can be placed at centre of AE, similarly masses at B & D can be placed at centre of BD.





Hence $y_{cm} = \frac{a}{2}\sqrt{3} = AH$ $\quad x_{cm}$ from $A = \frac{a}{2}$



Example **9.** Two particles of equal mass have velocities $\vec{v}_1 = 4\hat{i}$ m/s and $\vec{v}_2 = 4\hat{j}$ m/s. First particle has an acceleration $\vec{a}_1 = (\hat{i} + \hat{j})$ m/s² while the acceleration of the other particle is zero. The centre of mass of the two particles moves on a :

(a) circle (b) parabola (c) straight line (d) ellipse.

[Ans. (c)]

Solution

$$\vec{v}_{COM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2} = \frac{\vec{v}_1 + \vec{v}_2}{2} = 2(\hat{i} + \hat{j}) \text{ m/s}$$

Similarly, $\vec{a}_{COM} = \frac{\vec{a}_1 + \vec{a}_2}{2} = \frac{1}{2}(\hat{i} + \hat{j}) \text{ m/s}^2$

$\Rightarrow$ $\vec{v}_{COM}$ is parallel to $\vec{a}_{COM}$ the path will be a straight line.

Centre of Mass of A Continuous Mass Distribution

Centre of Mass of a Continuous Mass Distribution

For continuous mass distribution the centre of mass can be located by replacing summation sign with an integral sign. Proper limits for the integral are chosen according to the situation

$$x_{cm} = \frac{\int x\,dm}{\int dm},\ y_{cm} = \frac{\int y\,dm}{\int dm},\ z_{cm} = \frac{\int z\,dm}{\int dm}$$

$$\int \text{dm} = M \text{ (mass of the body)}$$

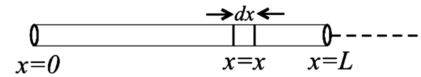
$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}dm.$$

Note

If an object has symmetric uniform mass distribution about x-axis than y coordinate of COM is zero and vice-versa

Centre of Mass of a Uniform ROD

Suppose a rod of mass M and length L is lying along the x-axis with its one end at $x = 0$ and the other at $x = L$.



Mass per unit length of the rod = $\frac{M}{L}$

Hence, dm, (the mass of the element dx situated at $x = x$ is) = $\frac{M}{L}dx$

The coordinates of the element PQ are $(x, 0, 0)$. Therefore, x-coordinate of COM of the rod will be

$$x_{\text{COM}} = \frac{\int_0^L x\, dm}{\int dm}$$

$$= \frac{\int_0^L (x)\left(\frac{M}{L}dx\right)}{M} = \frac{1}{L}\int_0^L x\, dx = \frac{L}{2}$$

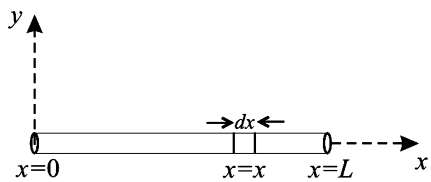
The y-coordinate of COM is

$$y_{\text{COM}} = \frac{\int y\, dm}{\int dm} = 0$$

Similarly, $z_{\text{COM}} = 0$

i.e., the coordinates of COM of the rod are $\left(\frac{L}{2}, 0, 0\right)$. Or it lies at the centre of the rod.

***Example* 10.** A rod of length L is placed along the x-axis between $x = 0$ and $x = L$. The linear density (mass/length) λ of the rod varies with the distance x from the origin as $\lambda = Rx$. Here, R is a positive constant. Find the position of centre of mass of this rod.



Solution Mass of element dx situated at $x = x$ is

$$dm = \lambda\, dx = Rx\, dx$$

The COM of the element has coordinates $(x, 0, 0)$.

Therefore, x-coordinate of COM of the rod will be

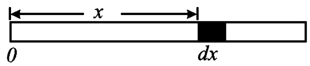
$$x_{\text{COM}} = \frac{\int_0^L x\, dm}{\int_0^L dm} = \frac{\int_0^L (x)(Rx)\, dx}{\int_0^L (Rx)\, dx} = \frac{R\int_0^L x^2 dx}{R\int_0^L x\, dx} = \frac{\left[\frac{x^3}{3}\right]_0^L}{\left[\frac{x^2}{2}\right]_0^L} = \frac{2L}{3}$$

The y-coordinate of COM of the rod is $y_{\text{COM}} = \dfrac{\int y\,dm}{\int dm} = 0$ (as $y = 0$)

Similarly, $z_{\text{COM}} = 0$

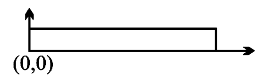
Hence, the centre of mass of the rod lies at $\left[\dfrac{2L}{3}, 0, 0\right]$

Example 11. The density of a straight rod of length L varies as $\rho = A + Bx$ where x is the distance from the left end. Locate the centre of mass.



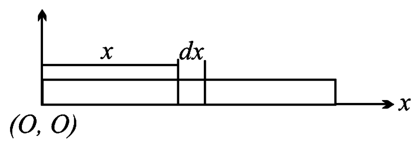
Solution $\dfrac{2AL + 2BL^2}{3(2A + BL)}$

Example 12. mass / length = kx^2



Solution $\dfrac{Mass}{Length} = \lambda = kx^2$

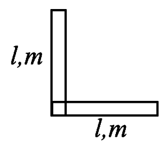
$$dm = \lambda\, dx = kx^2\, dx$$

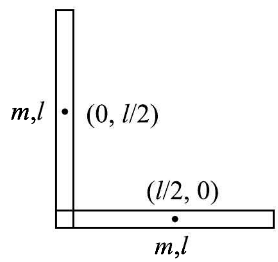


$$x_{\text{cm}} = \frac{\int x\,dm}{m} = \frac{K\int_0^L x^3\,dx}{K\frac{\ell^3}{3}} = \frac{3L}{4}$$

$$m = K\int_0^L x^2\,dx = \frac{KL^3}{3}$$

Example 13. Calculate *C.M.* of the system





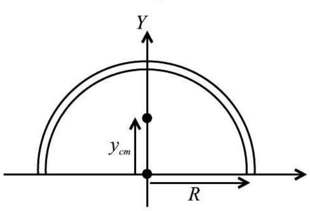
Solution

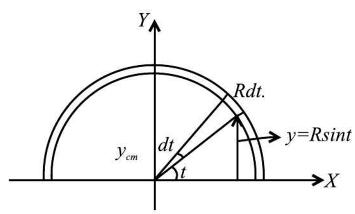
$$x_{\text{cm}} = \frac{ml/2 + 0}{2m} = \frac{1}{4}$$

$$y_{\text{cm}} = \frac{0 + m\frac{1}{2}}{2m} = \frac{1}{4}$$

Centre of Mass of a Semicircular Ring

The figure below shows the object (semi circular ring). By observation we can say that the x-coordinate of the centre of mass of the ring is zero as the half ring is symmetrical on both sides of the origin. Only we are required to find the y-coordinate of the centre of mass.





To find y_{cm} we use $\quad y_{cm} = \dfrac{1}{M}\displaystyle\int dm\, y$ ...(i)

Here for *dm* we consider an elemental arc of the ring at an angle θ from the *x*-direction of angular width dθ. If radius of the ring is *R* then its *y* coordinate will be *R* sinθ, here dm is given as

$$dm = \frac{M}{\pi R} \times R\, d\theta$$

So from equation (i), we have

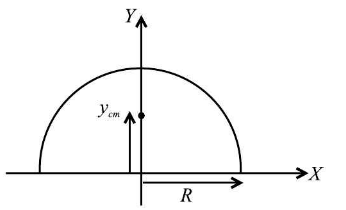
$$y_{cm} = \frac{1}{M}\int_0^{\pi} \frac{M}{\pi R} R d\theta \qquad (R \sin\theta) = \frac{R}{\pi}\int_0^{\pi} \sin\theta\, d\theta$$

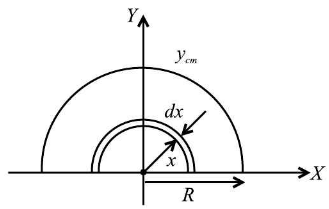
$$y_{cm} = \frac{2R}{\pi} \qquad \text{...(ii)}$$

Centre of Mass of Semicircular Disc

The figure here shows the half disc of mass *M* and radius *R*. Here, we are only required to find the *y*-coordinate of the centre of mass of this disc as centre of mass will be located on its half vertical diameter. Here to find y_{cm}, we consider a small elemental ring of mass dm of radius *x* on the disc (disc can be considered to be made up such thin rings of increasing radii) which will be integrated from 0 to *R*. Here dm is given as

$$dm = \frac{2M}{\pi R^2}(\pi\, x)dx$$



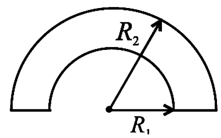


Now the *y*-coordinate of the element is taken as $\dfrac{2x}{\pi}$, as in previous section, we have derived that the centre of mass of a semi circular ring is concentrated at $\dfrac{2R}{\pi}$

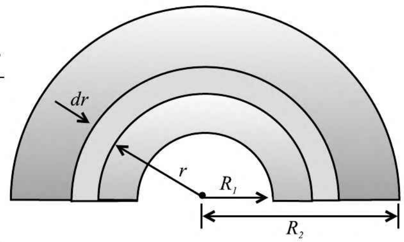
Here y_{cm} is given as $\quad y_{cm} = \dfrac{1}{M}\displaystyle\int_0^R dm\ \dfrac{2x}{\pi} = \dfrac{1}{M}\int_0^R \dfrac{4M}{\pi R^2} x^2\, dx$

$$y_{cm} = \frac{4R}{3\pi}$$

***Example* 14.** Find the centre of mass of an annular half disc shown in figure.



Solution Let ρ be the mass per unit area of the object. To find its centre of mass we consider an element as a half ring of mass dm as shown in figure of radius *r* and width *dr* and there we have



Now, $dm = \rho\pi r\, dr$

Centre of mass of this half ring will be at height $\frac{2r}{\pi}$

$$y_{cm} = \frac{1}{M}\int_{R_1}^{R_2} (\rho.\pi r\, dr).\frac{2r}{\pi}$$

$$y_{cm} = \frac{2\rho}{\rho\frac{\pi}{2}(R_2^2 - R_1^2)}\int_{R_1}^{R_2} r^2 dr = \frac{4(R_2^3 - R_1^3)}{3\pi(R_2^2 - R_1^2)}$$

Alternate Solution

We can also find the centre of mass of this object by considering it to be complete half disc of radius R_2 and a smaller half disc of radius R_1 cut from it. If y_{cm} be the centre of mass of this disc we have from the mass moments.

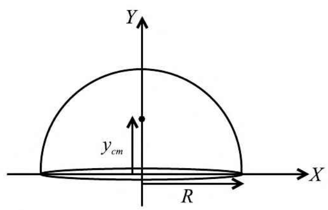
$$\left(\rho.\frac{\pi R_1^2}{2}\right)\times\left(\frac{4R_1}{3\pi}\right)+\left(\rho.\frac{\pi}{2}(R_2^2 - R_1^2)\right)(y_{cm}) = \left(\rho\cdot\frac{\pi R_2^2}{2}\right)\times\left(\frac{4R_2}{3\pi}\right)$$

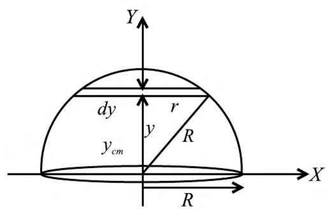
$$y_{cm} = \frac{4(R_2^3 - R_1^3)}{3\pi(R_2^2 - R_1^2)}$$

Centre of Mass of a Solid Hemisphere

The hemisphere is of mass M and radius R. To find its centre of mass (only y-coordinate), we consider an element disc of width dy, mass dm at a distance y from the centre of the hemisphere. The radius of this elemental disc will be given as

$$r = \sqrt{R^2 - y^2}$$





The mass dm of this disc can be given as

$$dm = \frac{3M}{2\pi R^3}\times\pi r^2\, dy = \frac{3M}{2R^3}(R^2 - y^2)\, dy$$

y_{cm} of the hemisphere is given as

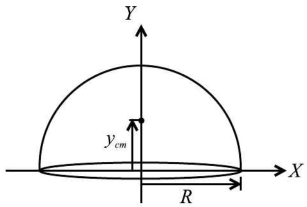
$$y_{cm} = \frac{1}{M}\int_0^R dm\, y = \frac{1}{M}\int_0^R \frac{3M}{2R^3}(R^2 - y^2)\, dy \quad y = \frac{3}{2R^3}\int_0^R (R^2 - y^2)\, y\, dy$$

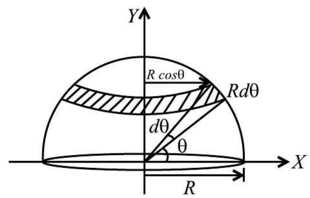
$$y_{cm} = \frac{3R}{8}$$

Centre of Mass of a Hollow Hemisphere

A hollow hemisphere of mass M and radius R. Now we consider an elemental circular strip of angular width $d\theta$ at an angular distance θ from the base of the hemisphere. This strip will have an area.

$$dS = 2\pi R \cos\theta \, R d\theta$$





Its mass dm is given as $dm = \dfrac{M}{2\pi R^2} 2\pi R \cos\theta \, R d\theta$

Here y-coordinate of this strip of mass dm can be taken as $R \sin\theta$. Now we can obtain the centre of mass of the system as.

$$y_{cm} = \frac{1}{M}\int_0^{\frac{\pi}{2}} dm \, R\sin\theta$$

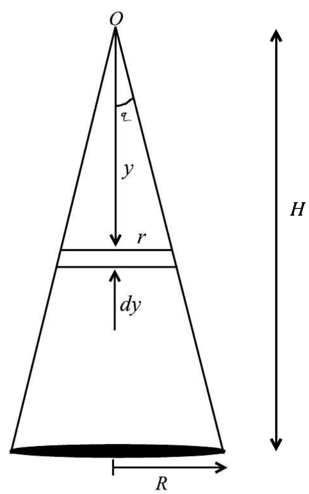
$$= \frac{1}{M}\int_0^{\frac{\pi}{2}} \left(\frac{M}{2\pi R^2} 2\pi R^2 \cos\theta \, d\theta\right)$$

$$R \sin\theta = R\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta \, d\theta$$

$$y_{cm} = \frac{R}{2}$$

Centre of Masss of A Solid Cone

A solid cone has mass M, height H and base radius R. Obviously the centre of mass of this cone will lie somewhere on its axis, at a height less than $H/2$. To locate the centre of mass we consider an elemental disc of width dy and radius r, at a distance y from the apex of the cone. Let the mass of this disc be dm, which can be given as



$$dm = \frac{3M}{\pi R^2 H} \times \pi r^2 \, dy$$

here y_{cm} can be given as

$$y_{cm} = \frac{1}{M}\int_0^H y \, dm$$

$$= \frac{1}{M}\int_0^R \left(\frac{3M}{\pi R^2 H}\pi\left(\frac{Ry}{H}\right)^2 dy\right) y$$

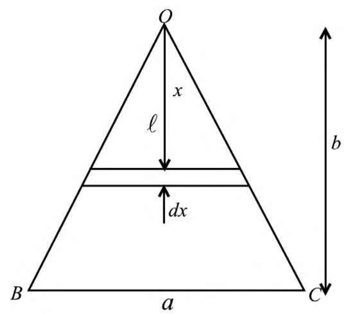
$$= \frac{3}{H^3}\int_0^H y^3\, dy$$

$$= \frac{3H}{4}$$

Example 15. Find out the centre of mass of an isosceles triangle of base length a and altitude b. Assume that the mass of the triangle is uniformly distributed over its area.

Solution To locate the centre of mass of the triangle, we take a strip of width dx at a distance x from the vertex of the triangle. Length of this strip can be evaluated by similar triangles as

$\ell = x.\ (a/b)$



Mass of the strip is $$dm = \frac{2M}{ab}\ell dx$$

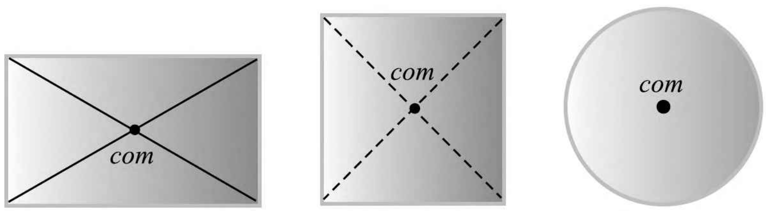
Distance of centre of mass from the vertex of the triangle is

$$x_{\text{CM}} = \frac{1}{M}\int x\, dm$$

$$= \int_0^b \frac{2x^2}{b^2}dx = \frac{2}{3}b$$

Proceeding in the similar manner, we can find the COM of certain rigid bodies. Centre of mass of some well known rigid bodies are given below :

1. Centre of mass of a uniform rectangular, square or circular plate lies at its centre. Axis of symmetry plane of symmetry.



2. For a laminar type (2-dimensional) body with uniform negligible thickness the formulae for finding the position of centre of mass are as follows :

$$\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...}{m_1 + m_2 + ...} = \frac{\rho A_1 t\vec{r}_1 + \rho A_2 t\vec{r}_2 + ...}{\rho A_1 t + \rho A_2 t + ...} \quad (\because m = \rho At)$$

or $$\vec{r}_{COM} = \frac{A_1\vec{r}_1 + A_2\vec{r}_2 + ...}{A_1 + A_2 + ...}$$

Here, A stands for the area,

3. If some mass of area is removed from a rigid body, then the position of centre of mass of the remaining portion is obtained from the following formulae:

(i) $$\vec{r}_{COM} = \frac{m_1\vec{r}_1 - m_2\vec{r}_2}{m_1 - m_2} \text{ or } \vec{r}_{COM} = \frac{A_1\vec{r}_1 - A_2\vec{r}_2}{A_1 - A_2}$$

(ii) $x_{COM} = \dfrac{m_1x_1 - m_2x_2}{m_1 - m_2}$ or $x_{COM} = \dfrac{A_1x_1 - A_2x_2}{A_1 - A_2}$

$y_{COM} = \dfrac{m_1y_1 - m_2y_2}{m_1 - m_2}$ or $y_{COM} = \dfrac{A_1y_1 - A_2y_2}{A_1 - A_2}$

and $z_{COM} = \dfrac{m_1z_1 - m_2z_2}{m_1 - m_2}$ or $z_{COM} = \dfrac{A_1z_1 - A_2z_2}{A_1 - A_2}$

Here, m_1, A_1, $\vec{r}_1$, x_1, y_1 and z_1 are the values for the whole mass while m_2, A_2, $\vec{r}_2$, $\vec{x}_2$, y_2 and z_2 are the values for the mass which has been removed. Let us see two examples in support of the above theory.

Example 16. Find the position of centre of mass of the uniform lamina shown in figure.

Solution Here, A_1 = area of complete circle = πa^2

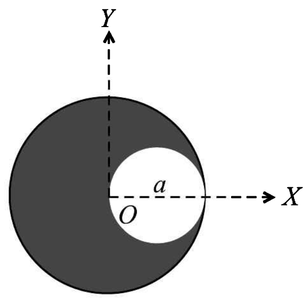
A_2 = area of small circle = $\pi\left(\dfrac{a}{2}\right)^2 = \dfrac{\pi a^2}{4}$

(x_1, y_1) = coordinates of centre of mass of large circle = (0, 0)

and (x_2, y_2) = coordinates of centre of mass of small circle = $\left(\dfrac{a}{2}, 0\right)$

Using $x_{\text{COM}} = \dfrac{A_1x_1 - A_2x_2}{A_1 - A_2}$

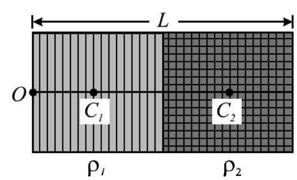
we get $x_{\text{COM}} = \dfrac{-\dfrac{\pi a^2}{4}\left(\dfrac{a}{2}\right)}{\pi a^2 - \dfrac{\pi a^2}{4}} = \dfrac{-\left(\dfrac{1}{8}\right)}{\left(\dfrac{3}{4}\right)} a = -\dfrac{a}{6}$



and $y_{\text{COM}} = 0$ as y_1 and y_2 both are zero.

Therefore, coordinates of COM of the lamina shown in figure are $\left(-\dfrac{a}{6}, 0\right)$

Example 17. Half of the rectangular plate shown in figure is made of a material of density ρ_1 and the other half of density ρ_2. The length of the plate is L. Locate the centre of mass of the plate.



Solution $X = \dfrac{(\rho_1 + 3\rho_2)}{4(\rho_1 + \rho_2)} L$

Example 18. The centre of mass of rigid body always lie inside the body. Is this statement true or false?

Solution False

Example 19. The centre of mass always lie on the axis of symmetry if it exists. Is this statement true of false?

Solution True

Example 20. If all the particles of a system lie in y-z plane, the x-coordinate of the centre of mass will be zero. Is this statement true or not?

Solution True

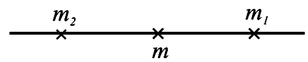
Example 21. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42 cm is removed from one edge of the plate as shown in figure. Find the centre of mass of the remaining portion.

Solution Let O be the centre of circular plate and O_1, the centre of circular portion removed from the plate. Let O_2 be the centre of mass of the remaining part

$$\text{Area of original plate} = \pi R^2 = \pi\left(\frac{56}{2}\right)^2 = 28^2\pi \text{ cm}^2$$

$$\text{Area removed from circular part} = \pi r^2$$

$$= \pi\left(\frac{42}{2}\right)^2 = (21)^2\pi\ cm^2$$



Let σ be the mass per cm^2.

Then mass of original plate, $m = (28)^2\sigma\pi$

mass of the removed part, $m_1 = (21)^2\ \sigma\pi$

mass of remaining part, $m_2 = (28)^2\sigma\pi - (21)^2\sigma\pi = 343\sigma\pi$

Now the masses m_1 and m_2 may be supposed to be concentrated at O_1 and O_2 respectively. Their combined centre of mass is at O. Taking O as origin we have from definition of centre of mass,

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$x_1 = OO_1 = OA - O_1A = 28 - 21 = 7 \text{ cm}$$

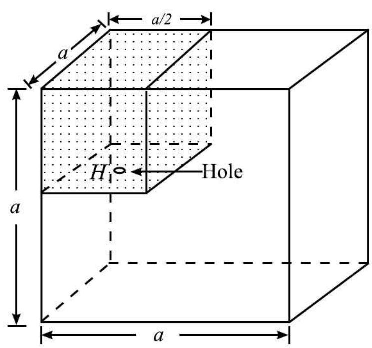
$$x_2 = OO_2 = ?,\ x_{cm} = 0$$

$$\therefore \quad 0 = \frac{(21)^2\pi\sigma \times 7 + 343\pi\sigma \times x_2}{(m_1 + m_2)}$$

$$x_2 = \frac{(21)^2\pi\sigma \times 7\pi\sigma}{343\pi\sigma} = \frac{441 \times 7}{343} = -9\ cm$$

This means that centre of mass of the remaining plate is at a distance 9 cm from the centre of given circular plate opposite to the removed portion.

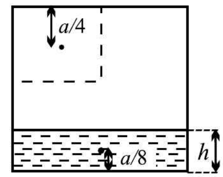
Example 22. The figure shows a hollow cube of side 'a' of volume V. There is a small chamber of volume $\frac{V}{4}$ in the cube as shown. This chamber is completely filled by m kg of water. Water leaks through a hole H and spreads in the whole cube. Then the work done by gravity in this process assuming that the complete water finally lies at the bottom of the cube is :



(A) $\frac{1}{2}\ mg\ a$ (B) $\frac{3}{8}\ mg\ a$

(C) $\frac{5}{8}\ mga$ (D) $\frac{1}{8}\ mga$

Solution Let h be the height of water surface, finally



$$a^2h = a\ .\ \frac{a}{2}\cdot\frac{a}{2}\ ;\ h = \frac{a}{4}$$

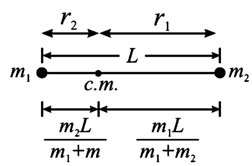
∴ $C.M.$ gets lowered by $a - \left(\frac{a}{4}+\frac{a}{8}\right) = a - \frac{3a}{8} = \frac{5a}{8}$

∴ Work done by gravity $= mg\ \frac{5a}{8}$

∴ (C) is the right answer.

Centre of Mass of Some Common Systems

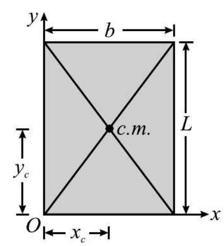
⇒ A system of two point masses $m_1 r_1 = m_2 r_2$



The centre of mass lies closer to the heavier mass.

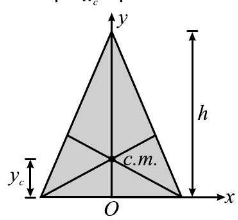
⇒ Rectangular plate (By symmetry)

$x_c = \frac{b}{2}\quad y_c = \frac{L}{2}$



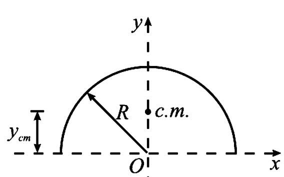
⇒ A triangular plate (By qualitative argument)

at the centroid : $y_c = \frac{h}{3}$



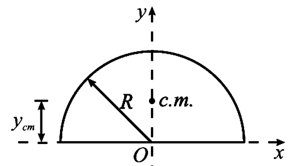
⇒ A semi-circular ring

$y_c = \frac{2R}{\pi}\qquad x_c = O$



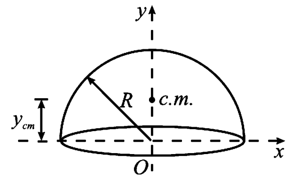
⇒ A semi-circular disc

$y_c = \dfrac{4R}{3\pi} \qquad x_c = O$

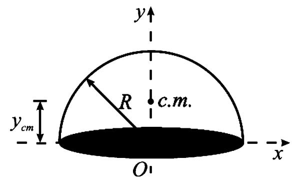


⇒ A hemispherical shell

$y_c = \dfrac{R}{2} \qquad x_c = O$

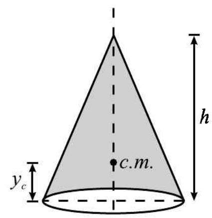


⇒ A solid hemisphere $\quad y_c = \dfrac{3R}{8} \qquad x_c = O$



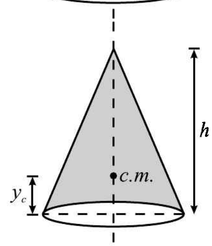
⇒ A circular cone (solid)

$y_c = \dfrac{h}{4}$



⇒ A circular cone (hollow)

$y_c = \dfrac{h}{3}$



DISTINCTION BETWEEN CENTRE OF MASS AND CENTRE OF GRAVITY

The position of the centre of mass of a system depends only upon the mass and position of each constituent particles,

i.e., $$\vec{r}_{CM} = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i} \quad \text{...(a)}$$

The location of G, the centre of gravity of the system, depends however upon the moment of the gravitational force acting on each particle in the system (about any point, the sum of the moments for all the constituent particles is equal to the moment for the whole system concentrated at G).

Hence, if g_i is the acceleration vector due to gravity of a particle P, the position vector r_G of the centre of gravity of the system is given by

$$\vec{r}_G \times \Sigma m_i \vec{g}_i = \Sigma(\vec{r}_i \times m_i \vec{g}_i) \quad \text{...(b)}$$

It is only when the system is in a uniform gravitational field, where the acceleration due to gravity (g) is the same for all particles, then equation (b)

Becomes $\vec{r}_G = \dfrac{\Sigma m_i \vec{r}_i}{m_i} = \vec{r}_{CM}$

In this case, therefore the centre of gravity and the centre of mass coincide.

If, however the gravitational field is not uniform and g_i is not constant then, in general equation (b) cannot be simplified and $\vec{r}_G = \vec{r}_{CM}$.

Velocity of Centre of Mass of System

$$\vec{v}_{cm} = \frac{m_1 \dfrac{\overrightarrow{dr_1}}{dt} + m_2 \dfrac{\overrightarrow{dr_2}}{dt} + m_3 \dfrac{\overrightarrow{dr_3}}{dt} \ldots\ldots\ldots\ldots + m_n \dfrac{\overrightarrow{dr_n}}{dt}}{M}$$

$$= \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 \ldots\ldots\ldots + m_n \vec{v}_n}{M}$$

Here numerator of the right hand side term is the total momentum of the system i.e., summation of momentum of the individual component (particle) of the system

Hence velocity of centre of mass of the system is the ratio of momentum of the system per unit mass of the system.

Acceleration of Centre of Mass of System

$$\vec{a}_{cm} = \frac{m_1 \dfrac{\overrightarrow{dv_1}}{dt} + m_2 \dfrac{\overrightarrow{dv_2}}{dt} + m_3 \dfrac{\overrightarrow{dv_3}}{dt} \ldots\ldots\ldots\ldots + m_n \dfrac{\overrightarrow{dv_n}}{dt}}{M}$$

$$= \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3 \ldots\ldots\ldots + m_n \vec{a}_n}{M}$$

$$= \frac{\text{Net force on system}}{\text{M}} = \frac{\text{Net External Force} + \text{Net internal Force}}{\text{M}}$$

$$= \frac{\text{Net External Force}}{\text{M}}$$

($\because$ action and reaction both of an internal force must be within the system. Vector summation will cancel all internal forces and hence net internal force on system is zero)

$\therefore \quad \vec{F}_{ext} = M\vec{a}_{cm}$

where $\vec{F}_{ext}$ is the sum of the 'external' forces acting on the system. The internal forces which the particles exert on one another play absolutely no role in the motion of the centre of mass.

If no external force is acting on a system of particles, the acceleration of centre of mass of the system will be zero. If $a_c = 0$, it implies that v_c must be a constant and if v_{cm} is a constant, it implies that the total momentum of the system must remain constant. It leads to the principle of conservation of momentum in absence of external forces.

If $\vec{F}_{ext} = 0$ then $\vec{v}_{cm}$ = constant

"If no external force is acting on the system, net momentum of the system must remain constant".

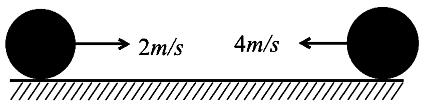
Motion of Com in a Moving System of Particles

(1) COM at rest : If $F_{ext} = 0$ and $V_{cm} = 0$, then COM remains at rest. Individual components of the system may move and have non-zero momentum due to mutual forces (internal), but the net momentum of the system remains zero.

(i) All the particles of the system are at rest.

(ii) Particles are moving such that their net momentum is zero.

Example :



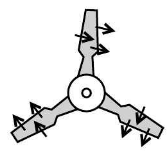
(iii) A bomb at rest suddenly explodes into various smaller fragments, all moving in different directions then, since the explosive forces are internal and there is no external force on the system for explosion therefore, the COM of the bomb will remain at the original position and the fragment fly such that their net momentum remains zero.

(iv) Two men standing on a frictionless platform, push each other, then also their net momentum remains zero because the push forces are internal for the two men system.

(v) A boat floating in a lake, also has net momentum zero if the people on it changes their position, because the friction force required to move the people is internal of the boat system.

(vi) Objects initially at rest, if moving under mutual forces (electrostatic or gravitation) also have net momentum zero.

(vii) A light spring of spring constant k kept copressed between two blocks of masses m_1 and m_2 on a smooth horizontal surface. When released, the blocks acquire velocities in opposite directions, such that the net momentum is zero.

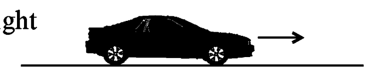


(viii) In a fan, all particles are moving but com is at rest

(2) COM moving with uniform velocity : If $F_{ext} = 0$, then V_{cm} remains constant therefore, net momentum of the system also remains conserved. Individual components of the system may have variable velocity and momentum due to mutual forces (internal), but the net momentum of the system remains constant and COM continues to move with the initial velocity.

(i) All the particles of the system are moving with same velocity.

Example : A car moving with uniform speed on a straight road, has its COM moving with a constant velocity.



(ii) Internal explosions / breaking does not change the motion of COM and net momentum remains conserved. A bomb moving in a straight line suddenly explodes into various smaller fragments, all moving in different directions then, since the explosive forces are internal & there is no external force on the system for explosion therefore, the COM of the bomb will continue the original motion and the fragment fly such that their net momentum remains conserved.

(iii) Man jumping from cart or buggy also exert internal forces therefore net momentum of the system and hence, Motion of COM remains conserved.

(iv) Two moving blocks connected by a light spring of spring constant on a smooth horizontal surface. If the acting forces is only due to spring then COM will remain in its motion and momentum will remain conserved.

(v) Particles colliding in absence of external impulsive forces also have their momentum conserved.

(3) COM moving with acceleration : If an external force is present then COM continues its original motion as if the external force is acting on it, irrespective of internal forces.

Example

Projectile Motion : An axe is thrown in air at an angle θ with the horizontal will perform a complicated motion of rotation as well as parabolic motion under the effect of gravitation

$$H_{\text{com}} = \frac{u^2 \sin^2\theta}{2g}$$

$$R_{\text{com}} = \frac{u^2 \sin^2\theta}{g}; \qquad T = \frac{2u\sin\theta}{g}$$

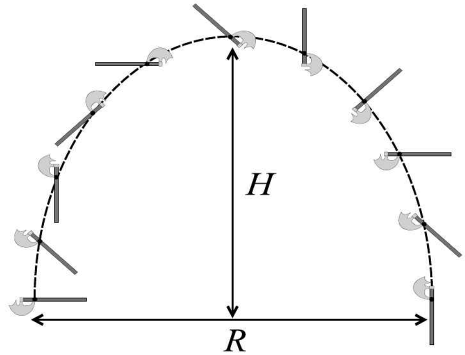
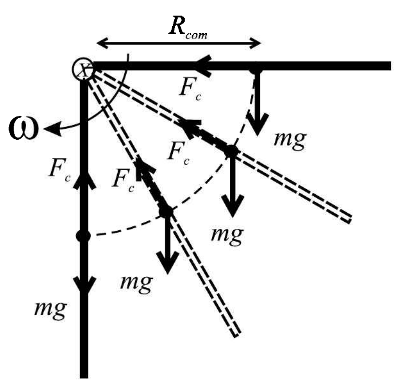


Fig 8.3: The motion of axe is complicated but the COM is moving in a parabolic motion

Example

Circular Motion: A rod hinged at an end, rotates, than its COM performs circular motion. The centripetal force (F_c) required in the circular motion is assumed to be acting on the com.

$$F_C = \frac{m\omega^2}{R_{com}}$$



Concept

Whatever may be the rearrangement of the bodies in a system, due to internal forces (such as one part moving away from the other or an internal explosion taking place, breaking a body into pieces).

(a) If the body was originally at rest, the *C.M.* will continue to be at rest.

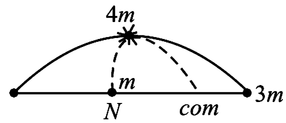
(b) If before the change, the body had been moving with a constant velocity, it will continue to move with a constant velocity and

In presence of external force: if body had been moving with constant acceleration in a particular trajectory, the *C.M.* will continue to move in the same trajectory, with the same acceleration as if it had never experienced any explosion only if there is no change in external force.

Example 23. A projectile is fired at a speed of 100 m/s at an angle of 37ŗ above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1 : 3, the smaller coming to rest. Find the distance from the launching point to the point where the heavier piece lands.

Solution Internal force do not effect the motion of the centre of mass, the centre of mass hits the ground at the position where the original projectile would have landed. The range of the original projectile is,

$$x_{\text{COM}} = \frac{2u^2 \sin\theta\cos\theta}{g} = \frac{2\times10^4 \times \frac{3}{5}\times\frac{4}{5}}{10}\ m = 960\ \text{m}$$



The centre of mass will hit the ground at this position. As the smaller block comes to rest after breaking, it falls down vertically and hits the ground at half of the range, i.e., at $x = 480$ m. If the heavier block hits the ground at x_2, then

$$x_{\text{COM}} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$960 = \frac{(m)(480) + (3m)(x_2)}{(m + 3m)}$$

$$x_2 = 1120\ \text{m}$$

Example 24. A shell flying with a velocity u = 500 m/s bursts into three identical fragments so that the kinetic energy of the system increases k times. What maximum velocity can one of the fragments obtain if k = 1.5 ?

Solution Let the mass of the shell be 3m. The mass of each fragment is m. θ

The particle with maximum velocity must be in the forward direction.

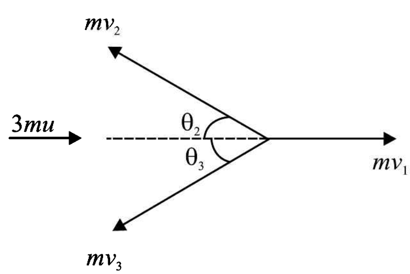
By law of conservation of momentum,

$$3mu = mv_1 - mv_2\cos\theta_2 - mv_3\cos\theta_3$$

$$3u = v_1 - v_2\cos\theta_2 - v_3\cos\theta_3$$

$$v_1 = 3u + v_2\cos\theta_2 + v_3\cos\theta_3 \quad ...(i)$$

Also $mv_2\sin\theta_2 = mv_3\sin\theta_3$ ...(ii)



If v_1 is to be maximum

$$\theta_2 = \theta_3 = 0$$

From (ii), if $\theta_2 = \theta_3$

$$v_2 = v_3 = v \text{ (say)}$$

Equation (i) becomes

$$v_1 = 3u + 2v$$

$$v = (v_1 - 3u)/2 \quad \text{...(iii)}$$

from question,

$$k\frac{1}{2}(3m)u^2 = \left(\frac{1}{2}mv_1^2 + 2\times\frac{1}{2}mv^2\right)$$

$$3ku^2 = v_1^2 + 2v^2 \quad \text{...(iv)}$$

Substituting for v from (iii) $\quad 3ku^2 = v_1^2 + \frac{1}{2}(v_1^2 + 9u^2 - 6v_1u)$

Solving for v_1 $\quad v_1 = u\left[1+\sqrt{2(k-1)}\right]$

For $u = 500$ m/s and $k = 1.5$ $\quad v_1 = 500\left[1+\sqrt{2(1.5-1)}\right] = 1000$ m/s.

Example 25. A stationary body explodes into four identical fragments such that three of them fly off mutually perpendicular to each other, each with same *K.E*. Find the energy of explosion.

Solution Let the three fragments move along *X*, *Y* and *Z* axes. Therefore their velocities can be given as $\vec{V_1} = V\hat{i}$, $\vec{V_2} = V\hat{j}$, and $\vec{V_3} = V\hat{k}$,

where V = speed of each of the three fragments. Let the velocity of the fourth fragment be $\vec{V_4}$. Since, in explosion no net external force is involved, the net momentum of the system remains conserved just before and after the explosion.

$$\Rightarrow \quad (\vec{P})_f = (\vec{P})_i \quad \Rightarrow \quad m\vec{V}_1 + m\vec{V}_2 + m\vec{V}_3 + m\vec{V}_4 = 0$$

($P_i = 0$ because the body was stationary), putting the values of $\vec{V}_1, \vec{V}_2$ and $\vec{V}_3$, we obtain,

$$\vec{V}_4 = -V(\hat{i} + \hat{j} + \hat{k})$$

Therefore, $V_4 = \sqrt{3}V$

The energy of explosion

$$\therefore \quad E = KE_f - KE_i = \left(\frac{1}{2}mV_1^2 + \frac{1}{2}mV_2^2 + \frac{1}{2}mV_3^2 + \frac{1}{2}mV_4^2\right) - (0)$$

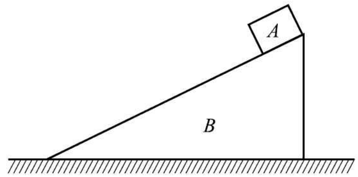
Putting $V_1 = V_2 = V_3 = V$ & setting $\frac{1}{2}mV^2 = E_0$,

we obtain, $E = 6E_0$.

Example 26. In a boat of mass 4 *M* and length ℓ on a frictionless water surface. Two men *A* (mass = *M*) and *B* (mass 2*M*) are standing on the two opposite ends. Now A travels a distance $\ell/4$ relative to boat towards its centre and B moves a distance $3\ell/4$ relative to boat and meet A. Find the distance travelled by the boat on water till A and B meet.

Solution [**Ans.** $5\ell/28$]

Example 27. A block A (mass = $4M$) is placed on the top of a wedge B of base length ℓ (mass = $20M$) as shown in figure. When the system is released from rest. Find the distance moved by the wedge B till the block A reaches ground. Assume all surfaces are frictionless.



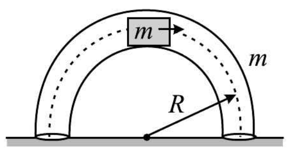
Solution [**Ans.** $\ell/6$]

Example 28. A balloon having mass 'm' is filled with gas and is held in hands of a boy. Then suddenly it get released and gas starts coming out of it with a constant rate. The velocities of the ejected gases is also constant 2 m/s with respect to the balloon. Find out the velocity of the balloon when the mass of gas is reduced to half.

(A) $\ell n\, 2$ (B) $2\, \ell n\, 4$ (C) $2\, \ell n\, 2$ (D) none of these

[Ans. (C)]

Example 29. In a vertical plane inside a smooth hollow thin tube a block of same mass as that of tube is released as shown in figure. When it is slightly disturbed it moves towards right. By the time the block reaches the right end of the tube then the displacement of the tube will be (where 'R' is mean radius of tube). Assume that the tube remains in vertical plane.



(A) $\dfrac{2R}{\pi}$ (B) $\dfrac{4R}{\pi}$

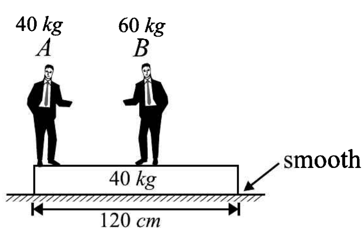
(C) $\dfrac{R}{2}$ (D) R

Solution Let the tube displaced by x towards left,

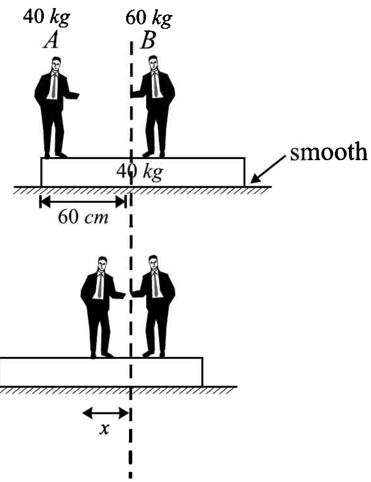
then $mx = m(R - x) \Rightarrow x = \dfrac{R}{2}$

$\therefore$ (C) is right answer.

Example 30. Two men 'A' and 'B' are standing on a plank. 'B' is at the middle of the plank and 'A' is the left end of the plank. Surface of the plank is smooth. System is initially at rest and masses are as shown in figure. 'A' and 'B' starts moving such that the position of 'B' remains fixed with respect to ground then 'A' meets 'B'. Then the point where A meets B is located at :



(A) the middle of the plank
(B) 30 cm from the left end of the plank
(C) the right end of the plank
(D) None of these



Solution Taking the origin at the centre of the plank.

$m_1\Delta x_1 + m_2\,\Delta x_2 + m_3\Delta x_3 = 0$

(Assuming the centres of the two men are exactly at the axis shown.)

$$60(0) + 40(60) + 40\,(-x) = 0,$$

x is the displacement of the block.

$\Rightarrow$ $x = 60$ cm i.e. A & B meet at the right end of the plank.

Hence, (C) is correct.

Example 31. Which of the following is incorrect ?

(A) If centre of mass of three particles is at rest, and it is known that two of them are moving along different lines then the third particle must also be moving.
(B) If centre of mass remains at rest, then net work done by the forces acting on the system must be zero.
(C) If centre of mass remains at rest then net external force must be zero
(D) None of these statement is incorrect

Solution When the centre of mass remains at rest, it is possible that different individual forces do individual works though the net resultant force is zero. As work is a scalar quantity, they gets added up.

Also, $\Sigma F_{ext} = 0 \quad \Rightarrow \quad a_{CM} = 0.$

$\therefore$ (B) is correct.

Example 32. The centre of mass of two masses m & m' move by distance $\frac{x}{5}$ when mass m is moved by distance x and m' is kept fixed. The ratio $\frac{m'}{m}$ is:

(A) 2 (B) 4 (C) $\frac{1}{4}$ (D) None of these

Solution $(m + m')\left(\frac{x}{5}\right) = mx + m'(0) \quad \Rightarrow \quad \frac{m'}{m} = 4$

$\therefore$ (B) is correct answer.

Example 33. An isolated particle of mass m is moving in a horizontal xy plane, along x-axis, at a certain height above ground. It suddenly explodes into two fragments of masses $m/4$ and $3m/4$. An instant later, the smaller fragment is at $y = +\,15$ cm. Find the position of heaver fragment at this instant.

Solution [**Ans.** $y = -\,5$ cm]

MOMENTUM

Momentum of a single particle $\vec{P} = m\vec{v}$

Note

Momentum depends on reference frame as velocity is frame dependent.

Momentum of system : For a system consisting of particles of mass $m_1, m_2, m_3....$

$$\vec{P}_{sys} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ...$$

Note

1. *While adding momentum we should remember it is vector sum.*

   Example: Momentum of rotating fan as a system is zero. Since all three equidistant pts. of blades will be having momentum at 120° and will add up to zero.

2. *We can take consider particle as part of system irrespective of its location and association with other particles.*

$$\vec{r}_{C.M.} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...}$$

   if we differentiate above equation w.r.t. time.

$$\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ...}{m_1 + m_2 + m_3 + ...}$$

   If we multiply both sides by total mass

$$\vec{P}_{sys} = M\vec{V}_{cm}$$

   Thus we can see the importance of $C.M.$ As momentum of system can be interpreted simply by the motion of $C.M.$

   For eg. in previous case of rotating fan, we can now very easily conclude that momentum is zero, since $C.M.$ will be at axis and have zero velocity.

Momentum Conservation

The total linear momentum of a system of particles is equal to the product of the total mass of the system and the velocity of its centre of mass. $\vec{P} = M\,\vec{v}_{cm}$

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt} \quad \text{If} \quad \vec{F}_{ext} = 0 \quad \Rightarrow \quad \frac{d\vec{P}}{dt} = 0; \ \vec{P} = \text{constant}$$

When the vector sum of the external forces acting on a system is zero, the total linear momentum of the system remains constant.

$$\vec{P}_1 + \vec{P}_1 + \vec{P} + ...+ \vec{P}_n = \text{constant.}$$

So, when the resultant external force acting on the system is zero, the total vector momentum of the system remains constant. This is called as principle of the conservation of linear momentum. The momentum of the individual particles may change, but their sum remains constant if there is no net external force.

Kinetic Energy of a System

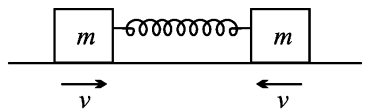
$$KE_{sys} = \sum \frac{1}{2} m_i v_i^2 \quad 0 = \sum \frac{1}{2} m_i \left(\vec{v}_{i/c} + \vec{v}_c\right)^2$$

$$= \sum \frac{1}{2} m_i v_{i/c}^2 + \sum \frac{1}{2} m_i v_c^2 + \sum \frac{1}{2} m_i \, 2\vec{v}_{i/c}.\vec{v}_c$$

$$= \sum \frac{1}{2} m_i v_{i/c}^2 + \frac{1}{2}\left(\sum m_i\right) v_c^2 + \left(\sum m_i \vec{v}_{i/c}\right).\vec{v}_c = KE_{sys/cm} + \frac{1}{2} M_{sys.} v_c^2 + 0$$

$$KE_{sys} = KE_{sys/cm} + \frac{1}{2} M_{sys.} v_c^2$$

Say that $K_{sys} \neq \frac{1}{2} MV_c^2$ e.g.



$K_{sys} \neq 0$

but $\frac{1}{2} MV_c^2 = 0$

Force on a System

We consider a system consisting of N particles of masses $m_1, m_2, ..., m_N$. The total mass is

$$M = m_1 + m_2 + ... + m_N = \Sigma m_n.$$

Each particle in the system can be represented by its mass m_n (where n = 1, 2, ..., N), its location at the coordinate $\vec{r}_n$ (whose components are x_n, y_n, and z_n), its velocity $\vec{v}_n$ (whose components are v_{nx}, v_{ny}, and v_{nz}), and its acceleration $\vec{a}_n$. The net force on particle m_n is $\Sigma \vec{F}_n$, which in general differs from one particle to another. This force may arise partly from the other $N - 1$ particles and partly from an external agent.

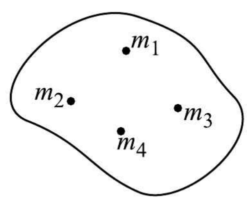
$$\vec{F}_1 = m_1 \vec{a}_1$$

$$\vec{F}_2 = m_2 \vec{a}_2 \qquad \left[\begin{array}{l}\text{Where } \vec{F}_1 \text{ force is} \\ \text{sum of forces applied by} \\ \text{parts of system and ext. agent}\end{array}\right]$$

$$\vec{F}_3 = m_3 \vec{a}_3$$

| |

$$\sum \vec{F}_k = \sum m_k \vec{a}_k$$



$$\sum \vec{F}_{ext} + \sum \vec{F}_{\text{int}} = M\,\vec{a}_{cm} = \frac{d}{dt}\left(\sum m_k \vec{v}_k\right)$$

But $\qquad \sum \vec{F}_{\text{int}} = 0$

because $\qquad \sum \vec{F}_{1\text{int}} = \vec{F}_{1/2} + \vec{F}_{1/3} + \vec{F}_{1/4} + \vec{F}_{1/5} + ...$

$\left[b/c\, \vec{F}_{1/2} = -\vec{F}_{2/1} \text{ by Newton's III}^{rd} \text{ law} \right]$

$$\sum \vec{F}_{2\text{int}} = \vec{F}_{2/1} + \vec{F}_{2/3} + \vec{F}_{2/4} + ...$$

$$\sum \vec{F}_{3\text{int}} = \vec{F}_{3/1} + \vec{F}_{3/2} + \vec{F}_{3/4} + ...$$

| | | |

$$\sum \vec{F}_{\text{int}} = 0$$

Hence $\qquad \sum \vec{F}_{ext} = \dfrac{d(\vec{P}_{sys})}{dt} = m\,\vec{a}_{cm}$

$$M\vec{a}_{cm} = \Sigma\vec{F}_1 + \Sigma\vec{F}_2 + ... + \Sigma\vec{F}_N$$

where the last result follows from applying Newton's second law, $\Sigma\vec{F}_n = m_n\vec{a}_n$, to each individual particle. The total force acting on a system of particles is thus equal to the total mass of the system time the acceleration of the centre of mass. Equation $M\vec{a}_{cm} = \Sigma\vec{F}_1 + \Sigma\vec{F}_2 + ... + \Sigma\vec{F}_N$ is just Newton's second law for the system of *N* particles treated as a single particle of mass *M* located at the centre of mass, moving with velocity $\vec{v}_{cm}$ and experiencing acceleration $\vec{a}_{cm}$.

It is helpful to simplify $M\vec{a}_{cm} = \Sigma\vec{F}_1 + \Sigma\vec{F}_2 + ... + \Sigma\vec{F}_N$ even bit mor

Any given particle m_n may experience force exerted on it by particle m_k, which we write as $\vec{F}_{nk}$. This particular force is one among the many that make up $\Sigma\vec{F}_n$, the total force on m_n. Similarly, the total force on particle m_k includes a term $\vec{F}_{kn}$ due to the interaction with paritcle m_n. By Newton's third law, $\vec{F}_{nk} = -\vec{F}_{kn}$, and thus there two particular forces cancel when we carry out the sum of all the forces in $M\vec{a}_{cm} = \Sigma\vec{F}_1 + \Sigma\vec{F}_2 + ... + \Sigma\vec{F}_N$. In fact, all such internal fcorces are part of action-reaction pairs and cancel. (In NLM we cautioned that the action and reaction forces must apply to different particles and thus, do not oppose one another on a given particle. We are not violtaing that caution here, because we are applying the action to one particle and the reaction to another. The distinction here is that we are addtion to get the net force on the two particles, in which case the action and reaction components, which still apply to different particles, do indeed cancel).

All that remains in equation $M\vec{a}_{cm} = \Sigma\vec{F}_1 + \Sigma\vec{F}_2 + ... + \Sigma\vec{F}_N$ is the total of all the external forces, and equation reduces to

$$\Sigma\vec{F}_{ext} = M\vec{a}_{cm}$$

which can be written in terms of its components as

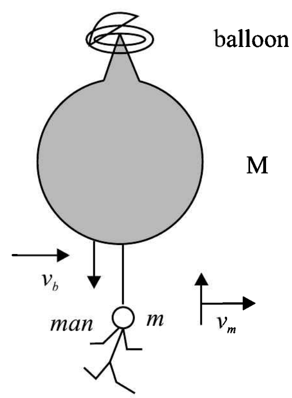
$$\Sigma F_{ext,x} = Ma_{cm,x} \qquad \Sigma F_{ext,y} = Ma_{cm,y} \text{ and } \Sigma F_{ext,z} = Ma_{cm,z}$$

We can summarize this important result as follows:

The overall translational motion of a system of particles can be analyzed using Newton's laws as if all the mass and the total external force were applied at that point.

Example 34. A man of mass m climbs a rope of length L suspended below a balloon of mass M. The balloon is stationary with respect to ground. If the man begins to climb up the rope at a speed v (relative to rope) in upward direction then with what speed (relative to ground) will the balloon move?

Solution Balloon is stationary



$\Rightarrow$ No net external force acts on it.

$\Rightarrow$ The conservation of linear momentum of the system (balloon + man) is valid

$\Rightarrow$ $M\vec{v}_b + m\vec{v}_m = 0$, where $\vec{v}_m = \vec{v}_{mb} + \vec{v}_b$

$\Rightarrow$ $M\vec{v}_b + m[\vec{v}_{mb} + \vec{v}_b] = 0$

where v_{mb} = velocity of man relative to the balloon (rope)

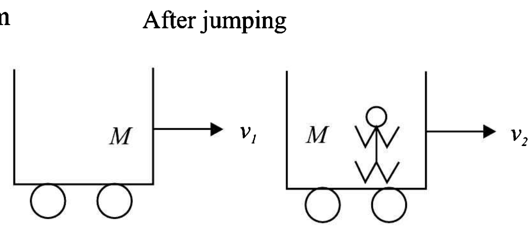
$$\Rightarrow \quad \vec{v}_b = \frac{m\vec{v}_{mb}}{M+m}$$

where $v_{mb} = v \Rightarrow v_b = \dfrac{mv}{M+m}$ and directed opposite to that of the motion of the man.

Example 35. Two identical buggies move one after the other due to inertia (without friction) with the same velocity v_0. A man of mass m rides the rear buggy. At a certain moment the man jumps into the front buggy with a velocity u relative to his buggy. The mass of each buggy is M. Find the velocities with which the buggies will move afterwards.

Solution Initial momentum of rear buggy = $(M + m)v_0$. The momentum of man when he jumps = $m(v_1 + u)$, where v_1 is the velocity of buggy as he jumps.

By the conservation of linear momentum



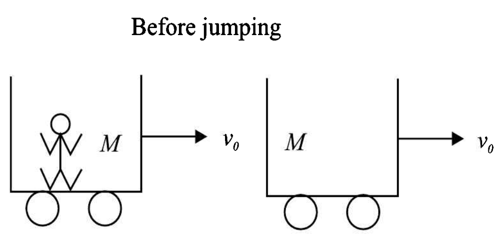
$$(M + m)v_0 = Mv_1 + m(v_1 + u)$$

$$\Rightarrow \quad v_1(M + m) = (M + m)v_0 - mu$$

$$v_1 = v_0 - \frac{m}{M+m}u$$

Initial momentum of front buggy $= Mv_0$

$$Mv_0 + m(v_1 + u) = (M + m)v_2$$



$$Mv_0 + m\left(v_0 - \frac{mu}{M+m}u + u\right) = (M+m)v_2$$

$$\Rightarrow \quad Mv_0 + m\left(v_0 - \frac{mu}{M+m}u + u\right) = (M+m)v_2$$

$$\Rightarrow \quad (M+m)v_0 + \frac{mMu}{M+m} = (M+m)v_2$$

$$\Rightarrow \quad v_2 = v_0 + \frac{mMu}{(M+m)^2}$$

Example 36. Two particles A and B start moving due to their mutual interaction only. If at any time 't', $\vec{a}_A$ and $\vec{a}_B$ are their respective accelerations, $\vec{v}_A$ and $\vec{v}_B$ are their respective velocities, and upto that time w_A and w_B are the work done on A and B respectively by the mutual force, m_A and m_B are their masses respectively, then which of the following is always correct.

(A) $\vec{v}_A + \vec{v}_B = 0$ (B) $m_A\vec{v}_A + m_B\vec{v}_B = 0$

(C) $w_A + w_B = 0$ (D) $\vec{a}_A + \vec{a}_B = 0$

Solution (B) is correct, since $\Sigma\, \vec{F}_{ext} = \vec{0}$ $\therefore$ Moment of system will remain conserved, equal to zero.

Example 37. A train of mass M is moving on a circular track of radius 'R' with constant speed V. The length of the train is half of the perimeter of the track. The linear momentum of the train will be

(A) zero (B) $\frac{2MV}{\pi}$ (C) MVR (D) MV

Solution If we treat the train as a ring of mass 'M' then its COM will be at a distance $\frac{2R}{\pi}$ from the centre of the circle. Velocity of centre of mass is :

$$V_{CM} = R_{CM}.\omega = \frac{2R}{\pi}.\omega = \frac{2R}{\pi}.\left(\frac{V}{R}\right) \qquad (\because \omega = \frac{V}{R})$$

$$\Rightarrow \quad V_{CM} = \frac{2V}{\pi} \Rightarrow MV_{CM} = \frac{2MV}{\pi}$$

As the linear momentum of any system $= MV_{CM}$

$\therefore$ The linear momentum of the train $= \frac{2MV}{\pi}$

Hence, (B) is correct.

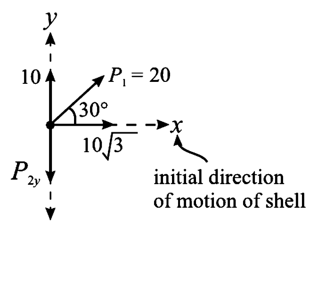
Example 38. A canon shell moving along a straight line bursts in to two parts. Just after the burst one part moves with momentum 20 Ns making an angle 30ŗ with the original line of motion. The minimum momentum of the other part of shell just after the burst is:

(A) 0 Ns (B) 5 Ns

(C) 10 Ns (D) 17.32 Ns

Solution As shown in figure the component of momentum of one shell along initial direction and perpendicular to initial direction are

$P_{1X} = 10\sqrt{3}\ Ns$ and $P_{1y} = 10Ns$.



For momentum of the system to be zero in y-direction P_{2y} must be 10 Ns. 2nd part of shell may or may not have momentum in x-direction

$\therefore \quad P_{2min} = 10\ Ns.$

$\therefore$ (C) is correct.

Example 39. A shell is fired from a cannon with a speed of 100 m/s at an angle 60° with the horizontal (positive x-direction). At the highest point of its trajectory, the shell explodes into two equal fragments. One of the fragments moves along the negative x-direction with a speed of 50 m/s. What is the speed of the other fragment at the time of explosion.

Solution As we know in absence of external force the motion of centre of mass of a body remains unaffected. Thus, here the centre of mass of the two fragment will continue to follow the original projectile path. The velocity of the shell at the highest point of trajectory is

$$v_M = u\cos\theta = 100 \times \cos 60° = 50 \text{ m/s}.$$

Let v_1 be the speed of the fragment which moves along the negative x-direction and the other fragment has speed v_2, which must be along + ve x-direction. Now from momentum conservation, we have

$$mv = \frac{-m}{2}v_1 + \frac{m}{2}v_2 \qquad \text{or } 2v = v_2 - v_1$$

or $\quad v_2 = 2v + v_1 = (2 \times 50) + 50 = 150$ m/s

Example 40. A shell is fired from a cannon with a speed of 100 m/s at an angle 30° with the vertical (y-direction). At the highest point of its trajectory, the shell explodes into two fragments of masses in the ratio 1 : 2. The lighter fragments moves vertically upwards with an initial speed of 200 m/s. What is the speed of the heavier fragment at the time of explosion.

Solution [**Ans.** 125 m/sec]

Example 41. A shell at rest at origin explodes into three fragments of masses 1 kg, 2 kg and *mkg*. The fragments of masses 1 kg and 2 kg fly off with speeds 12 m/s along x-axis and 8 m/s along y-axis respectively. If mkg files off with speed 40 m/s then find the total mass of the shell.

Solution [**Ans.** 3.5 kg]

Example 42. A block moving horizontally on a smooth surface with a speed of 20 m/s bursts into two equal parts continuing in the same direction. If one of the parts moves at 30 m/s, with what speed does the second part move and what is the fractional change in the kinetic energy?

Solution [**Ans.** v = 10 m/s, $\frac{1}{4}$]

Example 43. A block at rest explodes into three equal parts. Two parts starts moving along X and Y axis respectively with equal speeds of 10 m/s. Find the initial velocity of the third part.

Solution [**Ans.** $10\sqrt{2}$ m/s 135° below the X-axis]

Example 44. A boy of mass 25 kg stands on a board of mass 10 kg which in turn is kept on a frictionless horizontal ice surface. The boy makes a jump with a velocity component 5 m/s in horizontal direction with respect to the ice. With what velocity does the board recoil? With what rate are the boy and the board separating from each other?

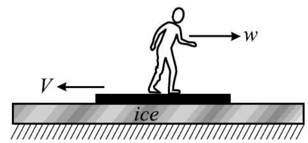
Solution [**Ans.** v = 12.5 m/s; 17.5 m/s]

Example 45. A man of mass m is standing on a platform of mass M kept on smooth ice. If the man starts moving on the platform with a speed v relative to the platform, with what velocity relative to the ice does the platform recoil ?

Solution Consider the situation shown in figure. Suppose the man moves at a speed w towards right and the platform recoils at a speed V towards left, both relative to the ice. Hence, the speed of the man relative to the platform is $V + w$. By the question,

$$V + w = v, \text{ or } w = v - V \quad \text{...(i)}$$

Taking the platform and the man to be the system, there is no external horizontal force on the system. The linear momentum of the system remains constant. Initially both the man and the platform were at rest. Thus,

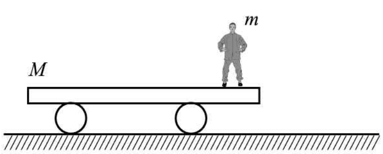


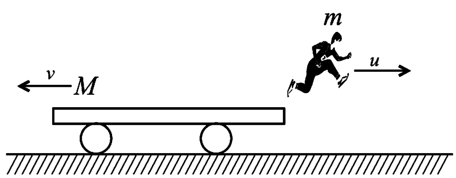
$$0 = MV - mw \quad \text{or,} \quad MV = m(v - V) \quad \text{[Using (i)]}$$

or, $$V = \frac{mv}{M+m}.$$

Example 46. A flat car of mass M is at rest on a frictionless floor with a child of mass m standing at its edge. If child jumps off from the car towards right with an initial velocity u, with respect to the car, find the velocity of the car after its jump.

Solution Let car attains a velocity v, and the net velocity of the child with respect to earth will be $u - v$, as u is its velocity with respect to car.





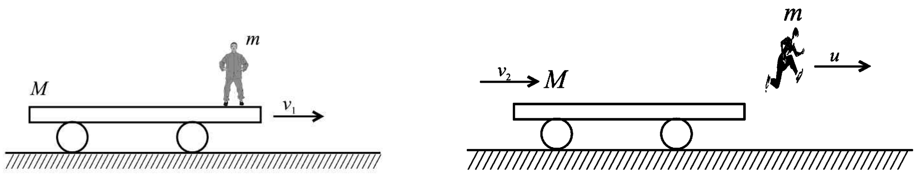
Initially, the system was at rest, thus according to momentum conservation, momentum after jump must be zero, as

$$m(u - v) = Mv$$

$$v = \frac{mu}{m+M}$$

Example 47. A flat car of mass M with a child of mass m is moving with a velocity v_1. The child jumps in the direction of motion of car with a velocity u with respect to car. Find the final velocities of the child and that of the car after jump.

Solution This case is similar to the previous example, except now the car is moving before jump. Here also no external force is acting on the system in horizontal direction, hence momentum remains conserved in this direction. After jump car attains a velocity v_2 in the same direction, which is less than v_1, due to backward push of the child for jumping. After jump child attains a velocity $u + v_2$ in the direction of motion of car, with respect to ground.

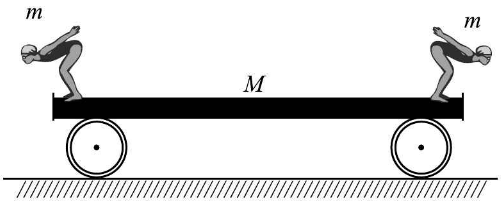


According to momentum conservation $(M+m)v_1 = Mv_2 + m\,(u+v_2)$

Velocity of car after jump is $\quad v_2 = \dfrac{(M+m)v_1 - mu}{M+m}$

Velocity of child after jump is $\quad u + v_2 = \dfrac{(M+m)v_1 + (M)u}{M+m}$

Example 48. Two persons *A* and *B*, each of mass *m* are standing at the two ends of rail-road car of mass *M*. The person *A* jumps to the left with a horizontal speed *u* with respect to the car. Thereafter, the person *B* jumps to the right, again with the same horizontal speed *u* with respect to the car. Find the velocity of the car after both the persons have jumped off.



Solution $\quad \dfrac{m^2u}{(M+2m)(M+m)}$

Example 49. Two identical buggies move one after the other due to inertia (without friction) with the same velocity v_0. A man of mass *m* jumps into the front buggy from the rear buggy with a velocity *u* relative to his buggy. Knowing that the mass of each huggy is equal to *M*, find the velocities with which the buggies will move after that.

Solution $\quad v_F = v_0 + \dfrac{Mmu}{(M+m)^2}; \qquad v_A = v_0 - \dfrac{mu}{(M+m)}$

Example 50. A man of mass 100 kg. is standing on a platform of mass 200 kg. which is kept on a smooth ice surface. If the man starts moving on the platform with a speed 30 m/sec relative to the platform then calculate with what velocity relative to the ice the platform will recoil?

(A) 5 m/sec (B) 10 m/sec (C) 15 m/sec (D) 20 m/sec.

Solution Let us suppose that the platform recoils with speed *V* towards right, when the man moves with speed W towards left, both velocities are taken relative to ice.

If V_1 is the speed of the man relative to the platform, then

$V + W = V_1. \qquad \therefore \quad W = V_1 - V$

Since, initially both man and platform are at rest, therefore

$MV = mW$

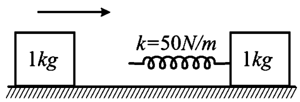
where, M = mass of platform and m = mass of the man.

or, $\quad MV = m(V_1 - V)$

$$V = \frac{mV_1}{M+m} = \frac{100 \times 30}{(200+100)} = 10 \text{ m/sec.}$$

$\therefore$ (B) is correct.

Example 51. Each of the blocks shown in figure has mass 1 kg. The rear block moves with a speed of 2 m/s towards the front block kept at rest. The spring attached to the front block is light and has a spring constant 50 N/m. Find the maximum compression of the spring.



Solution Maximum compression will take place when the blocks move with equal velocity. As no net external horizontal force acts on the system of the two blocks, the total linear momentum will remain constant. If V is the common speed at maximum compression, we have,

$$(1 \text{ kg})(2 \text{ m/s}) = (1 \text{ kg})V + (1 \text{ kg})V \quad \text{or,} \quad V = 1 \text{ m/s.}$$

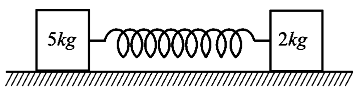
$$\text{Initial kinetic energy} = \frac{1}{2}(1 \text{ kg})(2 \text{ m/s})^2 = 2\,J.$$

$$\text{Final kinetic energy} = \frac{1}{2}(1 \text{ kg})(1 \text{m/s})^2 + \frac{1}{2}(1 \text{ kg})(1 \text{ m/s})^2 = 1\,J$$

The kinetic energy lost is stored as the elastic energy in the spring.

$$\text{Hence,} \quad \frac{1}{2}(50 \text{ N/m})\,x^2 = 2J - 1J = 1\,J \quad \text{or,} \quad x = 0.2 \text{ m.}$$

Example 52. Figure shows two blocks of masses 5 kg and 2 kg placed on a frictionless surface and connected with a spring. An external kick gives a velocity 14 m/s to the heavier block in the direction of lighter one.



Deduce (a) velocity gained by the centre of mass and (b) the separate velocities of the two blocks with respect to centre of mass just after the kick.

Solution

(a) Velocity of centre of mass is

$$v_{cm} = \frac{5 \times 14 + 2 \times 0}{5+2} = 10 \text{ m/s}$$

(b) Due to kick on 5 kg block, it starts moving with a velocity 14 m/s immediately, but due to inertia 2 kg block remains at rest, at that moment. Thus

Velocity of 5 kg block with respect to the centre of mass is $v_1 = 14 - 10 = 4$ m/s and the velocity of 2 kg block w.r.t. to centre of mass is $v_2 = 0 - 10 = -10$ m/s

Example 53. A light spring of spring constant k is kept compressed between two blocks of masses m and M on a smooth horizontal surface. When released, the blocks acquire velocities in opposite directions.

The spring loses contact with the blocks when it acquires natural length. If the spring was initially compressed through a distance x, find the final speeds of the two blocks.

Solution Consider the two blocks plus the spring to be the system. No external force acts on this system in horizontal direction. Hence, the linear momentum will remain constant. Suppose, the block of mass M moves with a speed V and the other block with a speed v after losing contact with the spring. From conservation of linear momentum in horizontal direction we have

$$MV - mv = 0 \quad \text{or} \quad V = \frac{m}{M}v, \qquad \text{...(i)}$$

Initially, the energy of the system $= \frac{1}{2}kx^2$

Finally, the energy of the system $= \frac{1}{2}mv^2 + \frac{1}{2}MV^2$

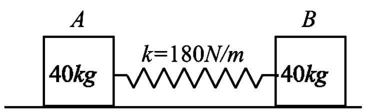
As there is no friction, mechanical energy will remain conserved.

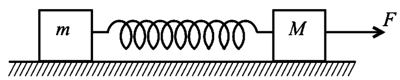
Therefore, $\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}kx^2$ ...(ii)

Solving Eqs. (i) and (ii), we get

or, $$V = \left[\frac{kM}{m(M+m)}\right]^{1/2} x \quad \text{and} \quad V = \left[\frac{km}{M(M+m)}\right]^{1/2} x$$

Example 54. Blocks A and B have masses 40 kg and 60 kg respectively. They are placed on a smooth surface and the spring connected between them is stretched by 2m. If they are released from rest, determine the speeds of both blocks at the instant the spring becomes unstretched.

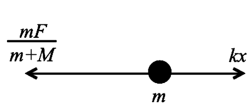


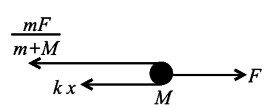


Solution **[Ans.** 3.2 m/s, 2.19 m/s]

Example 55. A block of mass m is connected to another block of mass M by a massless spring of spring constant k. The blocks are kept on a smooth horizontal plane and are at rest. The spring is unstretched when a constant force F starts acting on the block of mass M to pull it. Find the maximum extension of the spring.

Solution We solve the situation in the reference frame of centre of mass. As only F is the external force acting on the system, due to this force, the acceleration of the centre of mass is $F/(M' + m)$. Thus with respect to centre of mass there is a Pseudo force on the two masses in opposite direction, the free body diagram of m and M with respect to centre of mass (taking centre of mass at rest) is shown in figure.





Taking centre of mass at rest, if m moves maximum by a distance x_1 and M moves maximum by a distance x_2, then the work done by external forces (including Pseudo force) will be

$$W = \frac{mF}{m+M} \cdot x_1 + \left(F - \frac{MF}{m+M}\right) \cdot x_2 = \frac{mF}{m+M} \cdot (x_1 + x_2)$$

This work is stored in the form of potential energy of the spring as

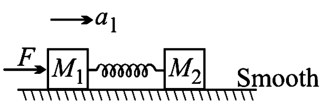
$$U = \frac{1}{2}k(x_1 + x_2)^2$$

Thus on equating we get the maximum extension in the spring, as after this instant the spring starts contracting.

$$\frac{1}{2}k(x_1 + x_2)^2 = \frac{mF}{m+M}\cdot(x_1 + x_2)$$

$$x_{max} = x_1 + x_2 = \frac{2mF}{k(m+M)}$$

Example 56. Two blocks of equal mass m are connected by an unstretched spring and the system is kept at rest on a frictionless horizontal surface. A constant force F is applied on one of the blocks pulling it away from the other as shown in figure



(a) Find the displacement of the centre of mass at time t

(b) if the extension of the spring is x_0 at time t, find the displacement of the two blocks at this instant.

Solution

(a) The acceleration of the centre of mass is

$$a_{COM} = \frac{F}{2m}$$

The displacement of the centre of mass at time t will be

$$x = \frac{1}{2}a_{COM}t^2 = \frac{Ft^2}{4m}$$

(b) Suppose the displacement of the first block is x_1 and that of the second is x_2. Then,

$$x = \frac{mx_1 + mx_2}{2m} \quad \text{or,} \quad \frac{Ft^2}{4m} = \frac{x_1 + x_2}{2}$$

$$\text{or,} \quad x_1 + x_2 = \frac{Ft^2}{2m} \qquad ...(i)$$

Further, the extension of the spring is $x_1 - x_2$. Therefore,

$$x_1 - x_2 = x_0 \qquad ...(ii)$$

From Eqs. (i) and (ii), $x_1 = \frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right)$ and $x_2 = \frac{1}{2}\left(\frac{Ft^2}{2m} - x_0\right)$

Example 57. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time $t = 0$, They collide at time t_0. Their velocities become $\vec{v}_1'$ and $\vec{v}_2'$ at time $2t_0$ while still moving in air. The value of $|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)|$ is :

(A) 0 (B) $(m_1 + m_2)gt_0$

(C) $2(m_1 + m_2)gt_0$ (D) $\frac{1}{2}(m_1 + m_2)gt_0$.

Solution Before collision, the velocities of m_1 and m_2 at t_0 are given as

$\vec{v}_1'' = \vec{v}_1 + \vec{g}t_0$ and $\vec{v}_2'' = \vec{v}_2 + \vec{g}t_0$

At $t = 2t_0$, the velocities of m_1 and m_2 are expresed as

$\vec{v}_1' = \vec{v}_1'' + 2\vec{g}t_0$ and $\vec{v}_2' = \vec{v}_2'' + 2\vec{g}t_0$

so, $(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2) = 2(m_1 + m_2)\vec{g}t_0$.

∴ (C) is correct.

Example 58. An electron of mass m moving with a velocity v collides head on with an atom of mass M. As a result of the collision a certain fixed amount of energy ΔE is stored internally in the atom. The minimum initial velocity possessed by the electron is:

(A) $\sqrt{\frac{2(M-m)\Delta E}{Mm}}$ (C) $\sqrt{\frac{2M\Delta E}{(M+m)m}}$

(B) $\sqrt{\frac{2(M+m)\Delta E}{Mm}}$ (D) none of the above is correct.

Solution $\frac{1}{2}\left(\frac{mM}{M+m}\right)v^2 = \Delta E \Rightarrow v = \sqrt{\frac{2(M+m)\Delta E}{Mm}}$.

∴ (B) is correct.

Example 59. A bullet of mass 0.01 kg, travelling at a speed of 500 ms^{-1}, strikes a block of mass 2 kg, which is suspended by a string of length 5 m, and emerges out. The block rises by a vertical distance of 0.1 m. The speed of the bullet after it emerges from the block is :

(A) 55 ms^{-1} (B) 110 ms^{-1} (C) 220 ms^{-1} (D) 440 ms^{-1}.

Solution By law of Conservation of Linear momentum $mu = mv + MV$ ...(1)

where m = mass of bullet

M = mass of block

u = velocity of bullet before collision

v = velocity of bullet after collision

V = velocity of block after collision

By law of Conservation of Energy

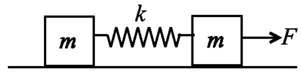
$$Mgh = \frac{1}{2}MV^2 \Rightarrow V = \sqrt{2\times 9.8\times 0.1} \Rightarrow V = 1.4\ ms^{-1}$$

Put in (1), we get

$$5 = 0.01v + 2(1.4) \qquad v = \frac{2.2}{0.01}ms^{-1} \Rightarrow v = 220\ ms^{-1}.$$

∴ (C) is correct.

Example 60. Two blocks of masses M_1 = 1kg and M_2 = 2kg kept on smooth surface, are connected to each other through a light spring (k = 100 N/m) as shown in the figure. When we push mass M_1 with a force F = 10N and M_1 is seen to move with a acceleration a_1 = 2 m/s^2, what will be the acceleration of M_2?



[Ans. 0004; $\frac{F - M_1 a_1}{M_2}$]

Solution From FBD of M_1

$F - kx = M_1 a_1$ and from FBD of M_2, $kx = M_2 a_2$

$$\therefore \quad a_2 = \frac{F - M_1 a_1}{M_2}$$

Note

To use $\vec{F}_{ext} = M_1\vec{a}_1 + M_2\vec{a}_2$

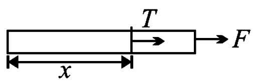
A corollary follows immediately in the case $\Sigma\vec{F}_{ext} = 0. \Rightarrow \vec{a}_{CM} = 0$

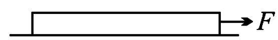
If the net external force on a system of particles is zero, then the centre of mass of the system moves with constant velocity.

Although mathematically internal forces cancel out each other but physically they are present.

Example 61. If F pull this rod from one end then at any point on rod there will be tension (internal force) driving the remaining body.

Solution





$$T = \left(\frac{F}{M}\right) \times \left(\left(\frac{m}{l}\right) x\right)$$

Note

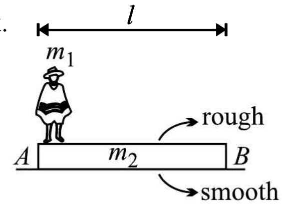
From the above discussion we can understand that momentum of a system depends only on external forces and internal forces can not effect net momentum of system. But internal forces can still contribute in changing energy of system and momentum of parts of system.

For example *in Chin–up exercise on horizontal bar force due to bar is required to generate momentum in vertical direction but it is not doing work. Increase in PE during rising up is due to work done by internal forces of body. But internal forces in themselves could not have changed momentum of the body.*

If a system is chosen such that net external force on it is zero.

$$\text{then } \vec{F}_{ext} = 0 \quad \Rightarrow \quad \frac{d\vec{P}_{sys}}{dt} = 0 \quad \text{also} \quad \Rightarrow \vec{a}_{cm} = 0$$

$$\Rightarrow \quad \vec{P}_{sys} \text{ is constant} \quad \Rightarrow v_{cm} \text{ is constant}$$



Example 62. If man walks from A to B, find displacement of man and plank.

Hint. Initial momentum of system man and plank is zero.

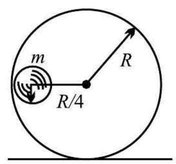
Net ext. force on this system is zero.

Thus $\vec{P}_{sys} = 0 \Rightarrow \vec{v}_{cm} = 0 \Rightarrow \vec{s}_{cm} = 0 \Rightarrow m_1\vec{s}_1 + m_2\vec{s}_2 = 0$

Solution $S_2 = \frac{-m_1}{m_1 + m_2}\ell$

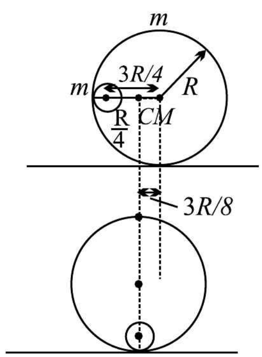
CM will not change its horizontal position since there is no external force.

Example 63. Inside a smooth spherical shell of the radius R a ball of the same mass is released from the shown position (Fig.). Find the distance travelled by the shell on the horizontal floor when the ball comes to the lowest point of the shell.



[Ans. $3R/8$**]**

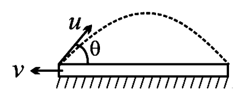
Solution CM will not change it's horizontal position since there is no external force.



Example 64. A frog sits on the end of a long board of length L. The board rests on a frictionless horizontal table. The frog wants to jump to the opposite end of the board. What is the minimum take-off speed i.e. relative to ground v that allows the frog to do the trick? The board and the frog have equal masses.

[Ans. $\sqrt{\frac{gL}{2}}$ **]**

Solution Taking v for the plank in ground frame and conserving linear momentum in horizontal direction



$$mv = m(u\cos\theta)$$

$$v = u\cos\theta$$

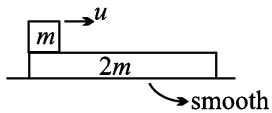
$$t = \frac{2u\sin\theta}{g} \quad \therefore\ L = \frac{2u(u\cos\theta + u\cos\theta)\sin\theta}{g} = \frac{2u^2\sin 2\theta}{g}$$

$$u = \sqrt{\frac{gL}{2\sin 2\theta}} \quad \therefore \text{ Minimum } u = \sqrt{\frac{gL}{2}}\ ]$$

Concept

Internal force are moving the two objects in opposite direction, so that *C.M.* do not have any disp.

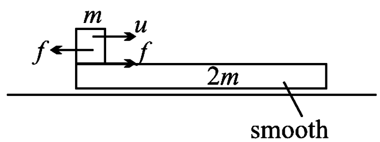
Example 65. Find total *W.D.* by friction assuming plank is sufficiently long.



Solution Where slipping stops both moves with same speed by momentum conservation

$$mu = mv$$

$$v = \frac{u}{3}$$



work done by friction = $\Delta KE = K_f - K_i$

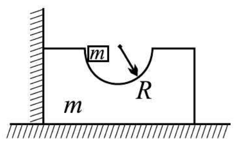
$$= \frac{1}{2}\, 2m \left(\frac{u}{3}\right)^2 + \frac{1}{2}\, m \left(\frac{u}{3}\right)^2 - \frac{1}{2}\, mu^2$$

$$= \frac{3mu^2}{18} - \frac{1}{2}\, mu^2 = -\frac{1}{3}\, mu^2 \text{ Joules}$$

Concept

P_{sys} = conserved if $f_{ext.} = 0$ although internal friction are doing work.

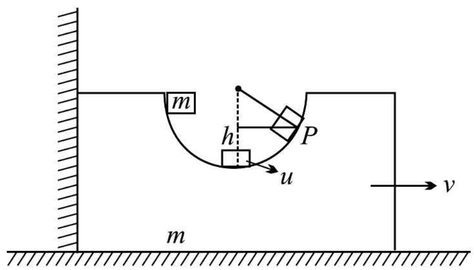
Example 66. Find maximum height reached by small mass *m* in fig assume frictionless.



Solution mass of both the block = *m*

bigger block remains at rest till smaller reaches at bottom of circular part.

Velocity of smaller block at lowest point $u = \sqrt{2gR}$.



Now bigger block also start moving let smaller block reaches up to height *h*.

By momentum conservation

$$mu = 2mv$$

$$v = \frac{u}{2}$$

By energy conservation increase in *PE* of smaller block = dec. in *KE* of smaller block + *KE* of bigger block

$$mgh + \frac{1}{2}\, mv^2 = \frac{1}{2}\, m\,(u^2 - v^2)$$

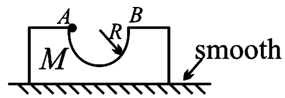
$$mgh = \frac{1}{2}mu^2 - \frac{1}{2}2mv^2 = \frac{1}{2}mu^2 - \frac{1}{2}2m \times \frac{u^2}{4} = -\frac{mu^2}{8}(4-2)$$

$$mgh = \frac{2mu^2}{8}; \qquad mgh = m\frac{2gR}{4}; \qquad h = \frac{R}{2}$$

Concept

Normal force by wall is F_{ext} and after losing contact, P_{system} is conserved.

Example 67. In the figure shown the wedge of mass M has a semicircular groove. A particle of mass $m = \frac{M}{2}$ is released from A. It slides on the smooth circular track and starts climbing on the right face.



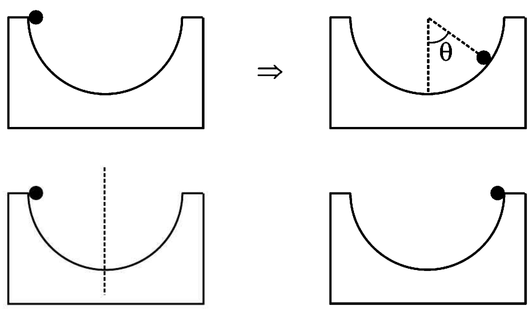
(i) Find the maximum value of θ which it can subtend with vertical and also find the distance displaced by wedge at this position.

(ii) Find the maximum velocity of wedge during process of motion.

Solution

(i) Initially no momentum along x-axis. So, final momentum will be zero also and relative velocity is also zero. So, no velocity of any object.

By energy conservation, initial potential energy = final potential energy.



Hence, $\theta = 90°$

$$\Delta x_{CM} = 0$$

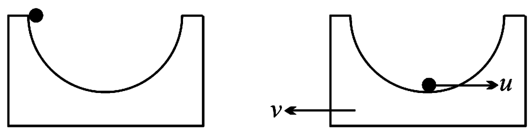
$$m(2R - x) = Mx$$

$$m(2R) = (M + m)x$$

$$x = \frac{2mR}{M+m} = \frac{2(M/2)R}{M + M/2}$$

$$x = \frac{2MR}{3M} = \frac{2R}{3}$$

(ii) Maximum velocity of wedge will be when the ball is at the lowest point in the wedge as till this point the horizontal component of normal on the wedge will be speeding the wedge.



$$p_i = 0; \qquad p_f = -MV + mu$$

$$p_i = p_f; \qquad u = \frac{Mv}{m} = 2v$$

$$U_i + K_i = U_f + K_f$$

$$mgR + 0 = 0 + \frac{1}{2}mu^2 + \frac{1}{2}mv^2; \qquad 2mgR = m(2v)^2 + Mv^2$$

$$2 \times \frac{M}{2} \times gR = 4mv^2 + Mv^2; \qquad MgR = 4 \times \frac{M}{2} \times v^2 + Mv^2$$

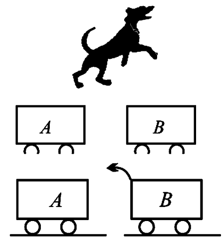
$MgR = 2Mv^2 + Mv^2$; $\qquad MgR = 3Mv^2$

$$v = \sqrt{\frac{gR}{3}}$$

Concept

Objective: P_{system} conservation, $Energy_{system}$ = conservation can be applied together.

Example 68 In a circus act, a 4kg dog is trained to jump from *B* cart to *A* and then immediately back to the *B* cart. The carts each have a mass of 20kg and they are initially at rest. In both cases the dog jumps at 6m/s relative to the cart. If the cart moves along the same line with negligible friction calculate the final velocity of each cart with respect to the floor.



[Ans: v_B = 55/36m/s, v_A = 11/6 m/s]

Solution Given $v_{D/C}$ = 6 m/sec

Case Ist : C.O.L.M.

$O = M_D \vec{v}_D + m_B \vec{v}_B$

$\vec{v}_D = -6i + \vec{v}_B \quad \therefore$ i.e. $v_D = -5\hat{i}$ toword A

Case IInd : In this condition when dog and trolly *A* will move with common speed again

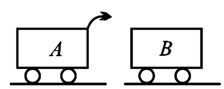
C.O.L.M. $-5i \times 4 = (20 + 4)\, v_{common}$

$v_{common} = -5/6\,\hat{i}$

so will jumping from $A \rightarrow B$

$\vec{v}_D = 6\hat{i} + \vec{v}_A$

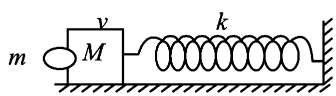
$-(24) \times \frac{5}{6}\hat{i} = \left(6\hat{i} + \vec{v}_A\right) \times 4 + \vec{v}_A \,.\, 20 \qquad \therefore v_A = -\frac{11}{6}\hat{i}$



Case IIIrd : $v_D = 6 - \frac{11}{6}\hat{i} = \frac{25}{6}\hat{i}$

$M_D \vec{v}_D + M_B \vec{v}_B = (M_B + M_D)\, v_{common} \qquad \therefore \quad v_{common} = \frac{11}{6}$ m/sec

Example 69. A bullet of mass *m* strikes a block of mass *M* connected to a light spring of stiffness *k*, with a speed v_0 and gets embedded into mass *M*. Find the loss of *K.E.* of the system just after impact.



Solution The process of impact of bullet and block is transient. Within a very short time of impact, the compression of the spring is negligible. Therefore the corresponding spring force is negligible. Even though it is external to the system ($M + m$), we can conserve its momentum just before and after the impact (impact force is internal). Conservation of linear momentum of bullet plus block just after and before impact yields

$$(M + m)V = mv_0 \Rightarrow V = \frac{mv_0}{M + m}$$

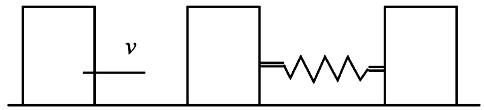
where V = common velocity of block and bullet.

Therefore the loss of $K.\,E.$ of the system

$$\Delta KE = \frac{1}{2}mv_0^2 - \frac{1}{2}(M+m)V^2$$

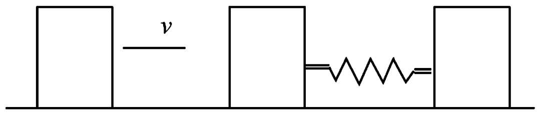
Putting $V = \dfrac{mv_0}{m+M}$ we obtain, $\Delta KE = \dfrac{Mmv_0^2}{2(M+m)}$.

Example 70. Two blocks B and C of mass m each connected by a spring of natural length 1 and spring constant k rest on an absolutely smooth horizontal surface as shown in figure. A third block A of same mass collides elastically block B with velocity v. Calculate the velocities of blocks, when the spring is compressed as much as possible and also the maximum compression.



A B C

m m m



A B C

Solution Let A be the moving block and B and C the stationary blocks. Since A and B are of equal mass, A is stopped dead and B takes off with its velocity. Now B and C move under their mutual action and reaction and so their momentum is conserved.

Let v_1 and v_2 be their instantaneous velocities when the compression of spring is x.

By the principle of conservation of momentum,

$$mv = m(v_1 + v_2)$$

$$v_1 + v_2 = v \text{ (a constant)}$$

By the principle of conservation of energy.

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \frac{1}{2}kx^2 \quad \Rightarrow \quad v^2 = v_1^2 + v_2^2 + \frac{k}{m}x^2$$

$$\Rightarrow \quad v^2 = (v_1 + v_2)^2 - 2v_1v_2 + \frac{k}{m}.x^2 \quad \Rightarrow \quad v^2 = v^2 - 2v_1v_2 + \frac{k}{m}.x^2$$

$$\Rightarrow \quad v_1v_2 = \frac{k}{2m}.x^2$$

Obviously compression (x) is maximum, when v_1v_2 is maximum under the condition that their sum $(v_1 + v_2)$ is constant.

We have, $(v_1 + v_2)^2 = (v_1 - v_2)^2 + 4v_1v_2 \quad \Rightarrow \quad v^2 = (v_1 - v_2)^2 + 4v_1v_2$

$$\Rightarrow \quad 4v_1v_2 = v^2 - (v_1 - v_2)^2$$

Obviously v_1v_2 is maximum when $(v_1 - v_2)^2$ is minimum. But it is a real positive quantity. Its minimum value is zero.

$$(v_1v_2)_{\max} = \frac{v^2}{4} \text{ when } v_1 = v_2\,; \quad x_{\max}^2 = \frac{2m}{k}(v_1v_2)_{\max} = \frac{2m}{k}.\frac{v^2}{4}$$

$$x_{max} = \sqrt{\frac{m}{2k}}.v$$

Example 71 A ball of mass m is projected with speed u into the barrel of spring gun of mass M initially at rest on a frictionless surface. The mass m sticks in the barrel at the point of maximum compression of the spring. What fraction of the initial kinetic energy of the ball is stored in the spring ? Neglect the friction.

Solution Let v be the velocity of system after the ball of mass m sticks in the barrel. Applying law of conservation of linear momentum, we have

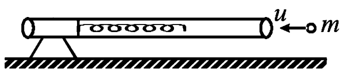
$$mu = (m+M)v \quad \text{...(i)}$$

The initial $K.\,E.= \frac{1}{2}mu^2$ of the ball is converted into elastic potential energy $= \frac{1}{2}kx^2$ of the spring and kinetic energy of the whole system $= \frac{1}{2}(m+M)v^2$. That is

$$\frac{1}{2}mu^2 = \frac{1}{2}kx^2 + \frac{1}{2}(m+M)v^2 \quad \text{...(ii)}$$

where k is the spring constant and x is its maximum compression.

Dividing equation (ii) by $\frac{1}{2}mu^2$



$$1 = \frac{\frac{1}{2}kx^2}{\frac{1}{2}mu^2} + \frac{\frac{1}{2}(m+M)v^2}{\frac{1}{2}mu^2} \quad \text{...(iii)}$$

$$1 = \frac{kx^2}{mu^2} + \frac{(m+M)v^2}{mu^2} \quad \text{...(iv)}$$

From equation (i), $\frac{v}{u} = \frac{m}{(M+m)}$

Substituting this value in equation (iv)

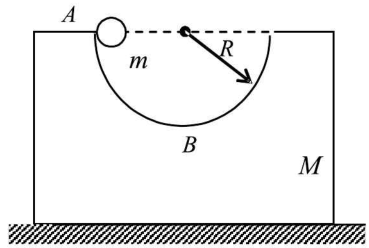
$$1 = \frac{kx^2}{mu^2} + \frac{(m+M)}{m}\frac{m^2}{(m+M)^2} \Rightarrow \frac{kx^2}{mu^2} + \frac{m}{m+M} \Rightarrow \frac{kx^2}{mu^2} = 1 - \frac{m}{m+M} = \frac{M}{(m+M)}$$

The energy stored in spring $= \frac{1}{2}kx^2$

Initial $K.E.$ of the ball $= \frac{1}{2}mu^2$

Hence, $\frac{kx^2}{mu^2}$ represents the fraction of initial energy, which is stored in the spring.

$\therefore$ fraction $= \frac{M}{m+M}$



Example 72. A block of mass M with a semicircular track of radius R rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see figure). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track? How fast is the block moving when the cylinder reaches the bottom of the track?

Solution The horizontal component of forces acting on M–m system is zero and the centre of mass of the system cannot have any horizontal displacement.

When the cylinder is at B its displacement relative to the block in the horizontal direction is $(R - r)$. Let the consequent displacement of the block to the left be x. The displacement of the cylinder relative to the ground is $(R - r - x)$

Since the centre of mass has no horizontal displacement

$$M.x = m(R - r - x)$$

$$x(M + m) = (R - r)m$$

$$x = \frac{(R - r)m}{(M + m)}$$

When the cylinder is at A, the total momentum of the system in the horizontal direction is zero. If v is the velocity of the cylinder at B and V, the velocity of the block at the same instant, then $mv + MV = 0$, by principle of conservation of momentum.

Potential energy of the system at $A = mg(R - r)$

Kinetic energy of the cylinder at $B = \frac{1}{2}mv^2$

The kinetic energy of the block at that instant $= \frac{1}{2}MV^2$

By principle of conservation of energy, $mg(R - r) = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$ since $v = -\frac{MV}{m}$

$$mg(R - r) = \frac{1}{2}m\left(-\frac{MV}{m}\right)^2 + \frac{1}{2}MV^2 = \frac{V^2}{2}\left(\frac{M^2}{m} + M\right)$$

$$mg(R - r) = \frac{V^2}{2m}(M^2 + Mm)$$

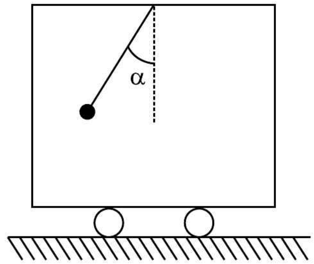
$$V^2 = \frac{2M^2g(R - r)}{(M^2 + Mm)}; \qquad V = \sqrt{\frac{2M^2g(R - r)}{M(M + r)}}.$$

Example 73. A wagon of mass M can move without friction along horizontal rails. A simple pendulum consisting of a bob of mass m is suspended from the ceiling by a string of length l. At the initial moment, the wagon and pendulum are at rest and the string is deflected through an angle α from the vertical. Find : (i) the velocity of wagon, when the string forms an angle β ($\beta < \alpha$) with vertical. (ii) the velocity of wagon, when the pendulum crosses its mean position.

Solution

(i) Let v be the leftward velocity of wagon (absolute that is relative to earth). Let u be the velocity of pendulum in a frame fixed to the wagon. Then $u\cos\beta$ is the relative horizontal velocity of the bob and $u\sin\beta$ is its vertical velocity. Let v_x and v_y be the absolute horizontal and vertical downward velocities of the bob.

$$\Rightarrow \quad v_x = u\cos\beta - v \quad \text{and} \quad u\sin\beta = v_y$$



Before releasing the bob

There is no external force on the system in the horizontal direction.

Therefore, by the principle of conservation of momentum to the right,

$$0 = m(u\cos\beta - v) - Mv$$

$$\Rightarrow \quad u\cos\beta - v = \frac{M}{m}\cdot v \quad \Rightarrow \quad u = \frac{(M+m)v}{m\cos\beta}$$

Kinetic energy of bob $= \frac{1}{2}m(v_x^2 + v_y^2)$

By the conservation of energy,

$$mgl(1-\cos\alpha) = mgl(1-\cos\beta) + \frac{1}{2}Mv^2 + \frac{1}{2}m[(u\cos\beta - v)^2 + u^2\sin^2\beta]$$

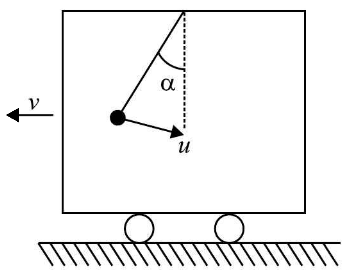
or, $$2mgl(\cos\beta - \cos\alpha) = Mv^2 + m\frac{M^2v^2}{m^2} + \frac{m\sin^2\beta(M+m)^2v^2}{m^2\cos^2\beta}$$

$$= Mv^2\left\{1 + \frac{M}{m}\right\} + \frac{(M+m)^2v^2}{m\cos^2\beta}\sin^2\beta = \frac{M(M+m)}{m}v^2 + \frac{(M+m)^2v^2\sin^2\beta}{m\cos^2\beta}$$

or $$2m^2gl(\cos\beta - \cos\alpha)\cos^2\beta = M(M+m)v^2\cos^2\beta + (M+m)^2v^2\sin^2\beta$$

$$= (M+m)v^2[M\cos^2\beta + (M+m)\sin^2\beta]$$

$$\Rightarrow \quad v^2 = \frac{2m^2gl}{M+m}\left[\frac{(\cos\beta - \cos\alpha)\cos^2\beta}{M + m\sin^2\beta}\right]$$



After releasing the bob

$$\therefore \qquad v = \sqrt{\frac{2m^2gl}{M+m}\left[\frac{(\cos\beta - \cos\alpha)\cos^2\beta}{M + m\sin^2\beta}\right]}$$

(ii) In this particular case when $\beta = 0$,

$$\sqrt{\frac{2m^2gl}{M+m}\frac{(1-\cos\alpha)}{M}} = \sqrt{\frac{2m^2gl}{M+m}\frac{2\sin^2\frac{\alpha}{2}}{M}}$$

$$v = 2\,m\,\sin\frac{\alpha}{2}\sqrt{\frac{gl}{(M+m)M}}.$$

Example 74. Two balls of masses m and 2m are suspended by two threads of same length l from the same point on the ceiling. The ball m is pulled aside through an angle α and released after giving a tangential velocity v_0 towards the other stationary ball is imparted to it. To what heights will the balls rise after collision, if the collision is perfectly elastic?

Solution The velocity acquired by m on reaching the lowest position is v (say).

Then, $\frac{1}{2}mv_0^2 + mgl(1-\cos\alpha) = \frac{1}{2}mv^2$

$$v^2 = v_0^2 + 2gl(1-\cos\alpha)$$

by conservation of momentum, $mv = mv_1 + 2mv_2$

$$v = v_1 + 2v_2 \qquad \text{or} \qquad v - v_1 = 2v_2 \qquad \text{...(i)}$$

By conservation of kinetic energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2; \qquad v^2 = v_1^2 + 2v_2^2$$

$$\left(v^2 - v_1^2\right) = 2v_2^2$$

$$(v - v_1)(v_1 + v) = 2v_2^2 \qquad \text{...(ii)}$$

Using (i) in (ii), $v_1 + v = v_2$ ...(iii)

Solving (i) and (iii), $v_2 = \frac{2}{3}v$ and $v_1 = -\frac{v}{3}$

Let m rise by h_1 and $2m$ by h_2, then

$$\frac{1}{2}mv_1^2 = mgh_1 \text{ or } gh_1 = \frac{1}{2}\times\frac{v^2}{9} = \frac{1}{18}\left[v_0^2 + 2gl(1-\cos\alpha)\right]$$

$$h_1 = \frac{1}{18g}\left[v_0^2 + 2gl(1-\cos\alpha)\right]; \qquad \frac{1}{2}\times 2mv_2^2 = 2mgh_2$$

$$gh_2 = \frac{1}{2}\times\frac{4v^2}{9} = \frac{4}{18}\left[v_0^2 + 2gl(1-\cos\alpha)\right]$$

$$h_2 = \frac{4}{18g}\left[v_0^2 + 2gl(1-\cos\alpha)\right]$$

Example 75. A gun is mounted on a gun carriage movable on a smooth horizontal plane and the gun is elevated at an angle 45ᵳ to the horizon. A shot is fired and leaves the gun inclined at an angle θ to the horizontal. If the mass of gun and carriage is n times that of the shot, find the value of θ.

Solution Let m be the mass of shot.

mn = mass of gun

w = velocity of shot relative to gun

v = velocity of recoil of gun

Since the gun is inclined at an angle α to horizontal, the direction of w makes an angle α with horizontal. The horizontal and vertical components are $w\cos\alpha$ and $w\sin\alpha$. When the shot leaves the muzzle the horizontal velocity relative to ground $(w\cos\alpha - v)$.

The vertical component of shot relative to ground is the same as relative to gun since the gun moves horizontal. If the shot leaves at an angle θ to horizontal,

$$\tan\theta = \frac{\text{Vertical component of velocity of shot}}{\text{Horizontal component of velocity of shot}} = \frac{w\sin\alpha}{w\cos\alpha - v} \quad ...(i)$$

by conservation of momentum in horizontal direction,

$$mnv = m(w\cos\alpha - v)$$

$$v = \frac{w\cos\alpha}{(n+1)}$$

Substituting in (i), $\tan\theta = \dfrac{w\sin\alpha}{w\cos\alpha - \dfrac{w\cos\alpha}{n+1}}$

$$\tan\theta = \frac{(n+1)\sin\alpha}{n\cos\alpha} = \left(1+\frac{1}{n}\right)\tan\alpha$$

$$\theta = \tan^{-1}\left(\frac{n+1}{n}\right) \quad (\because \tan 45^0 = 1)$$

IMPULSE

Impulse of a force F action on a body is defined as :-

$$\vec{J} = \int_{t_i}^{t_f} F dt$$

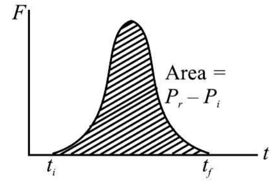
$$\vec{J} = \int F dt = \int m\frac{dv}{dt}dt = \int m\,dv$$

$$\vec{J} = m(v_2 - v_1)$$

It is also defined as change in momentum

$\vec{J} = \Delta\vec{P}$ **(impulse - momentum theorem)**

Instantaneous Impulse



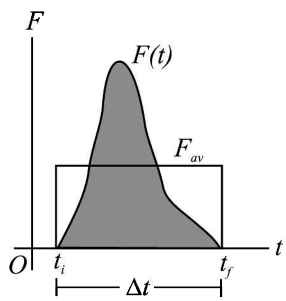
There are many occasions when a force acts for such a short time that the effect is instantaneous, e.g., a bat striking a ball. In such cases, although the magnitude of the force and the time for which it acts may each be unknown but the value of their product (i.e., impulse) can be known by measuring the initial and final momenta. Thus, we can write.

$$\vec{J} = \int \vec{F}\, dt = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$$

Regarding the impulse it is important to note that impulse applied to an object in a given time interval can also be calculated from the area under force time (F-t) graph in the same time interval.

Concept

1. It is a vector quantity.
2. Dimensions = $[MLT^{-1}]$
3. SI unit = kg m/s
4. Direction is along change in momentum.
5. Magnitude is equal to area under the F-t. graph.
6. $J = \int F dt = F_{av} \int dt = F_{av} \Delta t$
7. It is not a property of any particle, but it is a measure of the degree, to which an external force changes the momentum of the particle.



From Newton's second law in the form of $\Sigma\vec{F} = \frac{d\vec{p}}{dt}$ $\left(\Sigma\vec{F} = d\vec{p}/dt\right)$, we can write the change in momentum $d\vec{p}$ of a particle in a time interval dt during which a net force $\Sigma\vec{F}$ acts on it as

$$d\vec{p} = \Sigma\vec{F}\, dt$$

To find the total change in momentum during the entireevent, we integrate over the time, starting at time t_i (when the momentum is $\vec{p}_i$) and ending at time t_f (when the momentum is $\vec{p}_f$).

$$\int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}\, dt$$

The left side of equation is the change in momentum, $\Delta\vec{p} = \vec{p}_f - \vec{p}_i$. The right side defines a new quantity called as the impulse. For any arbitrary force $\vec{F}$, the impulse $\vec{J}$ is defined as

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}\, dt$$

The impulse depends on the strength of the force and on its duration. The impulse is a vector and, as equation $\int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}\, dt$ shows, the impulse has the same units and dimensions as momentum.

The right side of equation $\int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}\, dt$ is the impulse of the net force, $\vec{J}_{net} = \int \Sigma\vec{F} dt$. We can therefore write equation as

$$\vec{J}_{net} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

Equation is the mathematical statement of the impulse momentum theorem :

The impulse of the net force acting on a particle during a given time interval is equal to the change in momentum of the particle during that interval.

As a vector relationship, equation contains within it the three component equations :

$$J_{\text{net, x}} = \Delta p_x = p_{fx} - p_{ix}$$
$$J_{\text{net, y}} = \Delta p_y = p_{fy} - p_{iy}$$
$$J_{\text{net, z}} = \Delta p_z = p_{fz} - p_{iz}$$

Although we use $\vec{J}_{net} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$ mostly in situations involving impulsive forces (that is, those of short duration compared with the time of observation), no such limitation is built into that equation. Equation $\vec{J}_{net} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$ is just as general as Newton's second law, from which it was derived. We could, for example use eq. $\vec{J}_{net} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$ to find the momentum acquired by a body falling in the Earth's gravity.

We defined the impulse in terms of a single force, but the impulse-momentum theorem deals with the change in momentum due to the impulse of the net force-that is, the combined effect of all the force that act on the particle.

The magnitude of the impulse of this force is represented by the area under the $F(t)$ curve. We can represent that same area by the rectangle in figure of width Δt and height F_{av}, where F_{av} is the magnitude of the average force that acts during the interval Δt. Thus

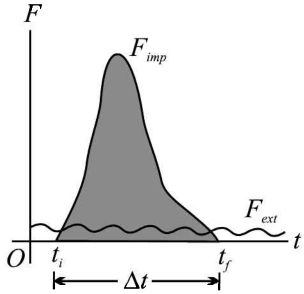
$$J = F_{\text{av}} \Delta t$$

Effect of small forces can be neglected during impulsive situations (collision, Jerk, Explosion, Jumping).

For all interactions that occurs in a time Δt that is negligible compared to the time during which we are observing the system, the small external forces that may act on the system during the time Δt produce negligible impulse compared to the impulsive forces. While a bat strikes a baseball, ablock is jerked into motion by pulling a rope sharply, or one billiard ball strikes another, external forces act on the system. Gravity or friction may exert forces on these bodies, for example; these external forces may not be the same on each colliding body nor are they necessarily canceled by other external forces. Even so, it is quite safe to neglect these external forces during the impulsive situations. As a result, the change in momentum of a particle arising from an external force during a impulsive situations is negligible compared to the change in momentum of that particle arising from the impulsive force (Refer to the figure that follows).

For example, when a bat strikes a baseball, the collision lasts only a few milliseconds. Because the change in momentum of the ball is large and the time of collision is small, it follows from

$$\Delta\vec{p} = \vec{F}_{av}\Delta t$$

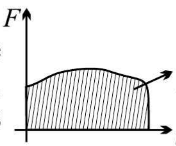


that the average impulsive force $\vec{F}_{av}$ is relatively large. Compared to this force, the external force of gravity is negligible.

During the collision we can safely ignore this external force in determining the change in motion of the ball ; the shorter the duration of the collision is, the more likely this is to be true.

$$\sum F_{ext} = \frac{dP_{sys}}{dt}$$

$$\text{Impulse} = \int_0^{\Delta t} \sum F_{ext} dt \text{ d t} = \int dP_{sys}$$



***Example* 76.** The hero of a stunt film fires 50 g bullets from a machine gun, each at a speed of 1.0 km/s. If he fires 20 bullets in 4 seconds, what average force does he exert against the machine gun during this period?

Solution The momentum of each bullet = (0.050 kg) (1000 m/s) = 50 kg-m/s.

The gun is imparted this much of momentum by each bullet fired. Thus, the rate of change of momentum of the gun

$$= \frac{(50kg - m/s) \times 20}{4s} = 250 \text{ N}.$$

In order to hold the gun, the hero must exert a force of 250 N against the gun.

Impulsive force

A force, of relatively higher magnitude and acting for relatively shorter time, is called as impulsive force.

An impulsive force can change the momentum of a body in a finite magnitude in a very short time interval. **Impulsive force** is a relative term. There is no clear boundary between an impulsive and Non-Impulsive force.

Concept

Usually colliding forces are impulsive in nature.

Since, the application time is very small, hence, very little motion of the particle takes place.

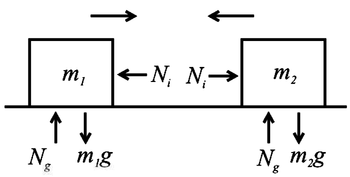
Important Points

1. Gravitational force and spring force are always non-impulsive.
2. Normal, tension and friction are case dependent.
3. An impulsive force can only be balanced by another impulsive force.

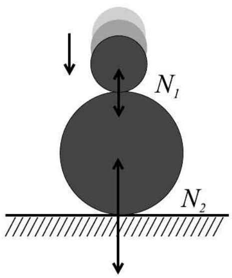
Impulsive Normal

In case of collision, normal forces at the surface of collision are always impulsive

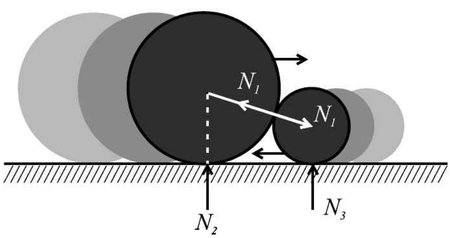
ex.



N_i = Impulsive; N_g = Non-impulsive

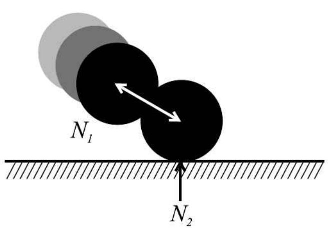


Both normals are Impulsive



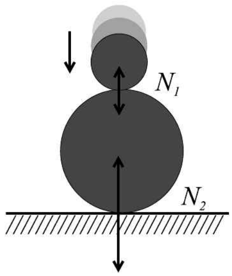
N_1, N_3 = Impulsive; $N_{2\text{ = non-impulsive}}$

Both normal are impulsive



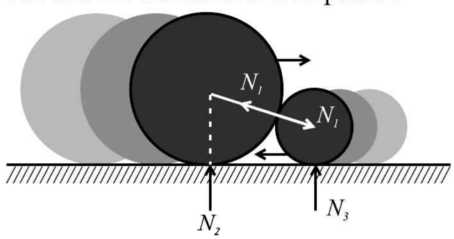
Impulsive Friction

If the normal between the two objects is impulsive, then the friction between the two will also be impulsive.



Both normals are Impulsive

Friction at both surfaces is impulsive



Friction due to N_2 is non-impulsive and due to N_3 is impulsive

Impulsive Tensions

When a string jerks, equal and opposite tension act suddenly at each end. Consequently, equal and opposite impulses act on the bodies attached with the string in the direction of the string. There are two cases to be considered.

(a) **One end of the string is fixed :** The impulse which acts at the fixed end of the string cannot change the momentum of the fixed object there. The object attached to the free end however will undergo a change in momentum in the direction of the string. The momentum remains unchanged in a direction perpendicular to the string where no impulsive forces act.

(b) **Both ends of the string attached to movable objects :** In this case equal and opposite impulses act on the two objects, producing equal and opposite changes in momentum. The total momentum of the system therefore remains constant, although the momentum of each individual object is changed in the direction of the string. Perpendicular to the string however, no impulse acts and the momentum of each particle in this direction is unchanged.

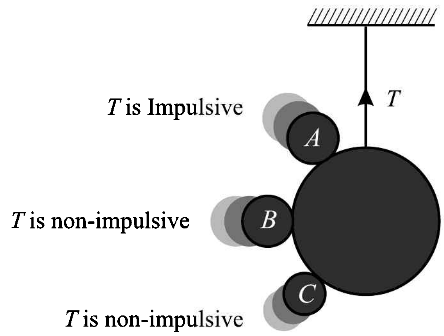


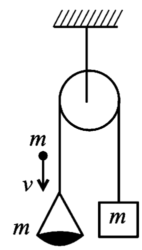
Fig 8.5: All normal are impulsive but tension T is impulsive only for the ball A

Concept

In case of rod Tension is always impulsive

In case of spring Tension is always non-impulsive.

Example 77. A block of mass m and a pan of equal mass are connected by a string going over a smooth light pulley. Initially the system is at rest when a particle of mass m falls on the pan and sticks to it. If the particle strikes the pan with a speed v, find the speed with which the system moves just after the collision.



Solution Let the required speed is V.

Further, let J_1 = impulse between particle and pan

and J_2 = impulse imparted to the block and the pan by the string

Using impulse = change in momentum

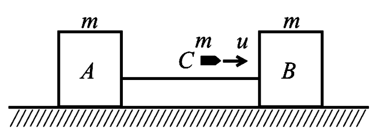
For particle $J_1 = mv - mV$ ...(i)

For pan $J_1 - J_2 = mV$ ...(ii)

For block $J_2 = mV$ ...(iii)

Solving, these three equation, we get $V = \frac{v}{3}$

Example 78. Two identical block A and B, connected by al massless string are placed on a frictionless horizontal plane. A bullet having same mass, moving with speed u strikes block B from behind as shown. If the bullet gets embedded into the block B then find :



(a) The velocity of A,B,C after collision.

(b) Impulse on A due to tension in the string

(c) Impulse on C due to normal force of collision.

(d) Impulse on B due to normal force of collision.

Solution

(a) By Conservation of linear momentum $v = \dfrac{u}{3}$

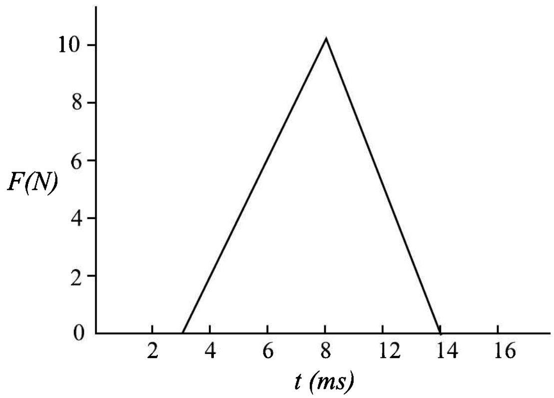
(b) $\int T\,dt = \dfrac{mu}{3}$

(c) $\int N\,dt = m\left(\dfrac{u}{3} - u\right) = \dfrac{-2mu}{3}$

(d) $\int (N - T)\,dt = \int N dt - \int T\,dt = \dfrac{mu}{3}$

$\Rightarrow \quad \int N\,dt = \dfrac{2mu}{3}$

Example 79. A cart of mass m_1 = 0.24 kg moves on a linear track without friction with an initial velocity of 0.17 m/s. It collides with another cart of mass m_2 = 0.68 kg that is initially at rest. The first cart carries a force probe that registers the magnitude of the force exerted by one cart on the other during the collision. The output of the force probe is shown in figure. Find the velocity of each cart after the collision.



Solution

$$v_{1fx} = \frac{p_{1fx}}{m_1} = \frac{-0.014\,kg.m/s}{0.24\,kg}$$

$$= -0.058 \text{ m/s} = -5.8 \text{ cm/s}.$$

$$v_{2fx} = \frac{p_{2fx}}{m_2} = \frac{+0.055\,kg.m/s}{0.68\,kg}$$

$$= +0.081 \text{ m/s} = +8.1 \text{ cm/s}.$$

Concept

Area under *f/t* graph gives impulse or ΔP

Note: *Impulsive fores are those forces which can have very large value in very small time. eg. Tension, Normal & friction. When impulsive forces act then momentum along the direction of force cannot be conserved.*

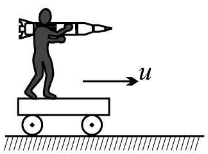
Example 80. In (a) momentum cannot be conserved in vertical direction just after collision while in (b) it can be conserved just after collision. *b/c.* in (a) Tension will reach a large value in small time and $\int_0^{\Delta t} T\,dt \neq 0$.

Note: *Problems involving impulsive forces: "Relative means after the event"*

***Example* 81.** Man + Rocket Launcher = M = 90

u = 5 m/s Rocket = m = 10

Muzzle velocity of Rocket = V_0= 30



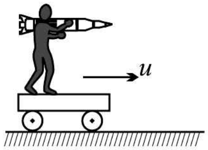
(a) What will be man's & rocket's velocity after firing.

(b) Find energy of explosion

Solution Man fire rocket with muzzle velocity = v_0

Let velocity of rocket is V_2 & man is V_1 momentum in horizontal direction remains constant

Initial momentum = final momentum



$$(m + M)\, u = MV_1 + mV_2$$

$$V_2 = V_0 + u$$

$$(m + M)\, u = MV_1 + m\,(V_0 + u)$$

$$V_1 = \frac{mu - MV_0}{m}$$

$$u = 5 \text{ m/s}$$

$$v_1 = \frac{90\times5-10\times30}{90} = \frac{450-300}{90} = \frac{150}{90} = \frac{5}{3}\text{ m/s}$$

$$\text{Energy of explosion} = KE_f - KE_i = \left(\frac{1}{2}MV_1^2 + \frac{1}{2}mV_2^2\right) - \frac{1}{2}\,(m + M)\,u^2$$

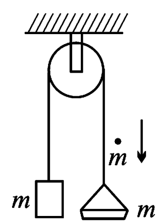
$$= \frac{1}{2}\,(m + M)\left[\frac{Mu^2 + mV_0^2}{M} - u^2\right]$$

$$= \frac{1}{2}\,(m + M)\left[\frac{mv_0^2}{m}\right] = \frac{1}{2}\,(100)\left[\frac{10}{90}\times 900\right] = 5000\,J$$

Concept

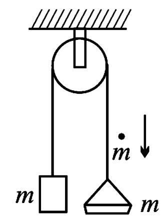
Muzzle velocity = velocity of bullet w.r.t. gun and internal forces can change KE but not P_{system}.

***Example* 82.** A block of mass m and a pan of equal mass are connected by a string going over a smooth light pulley as shown in figure. Initially the system is at rest when a particle of mass m falls on the pan and sticks to it. If the particle strikes the pan with a speed v find the speed with which the system moves just after the collision.



Solution Let the required speed be V.

As there is a sudden change in the speed of the block,



the tension must change by a larger amount during the collision

Mass m strikes the pan with speed v and stick to it.

Let speed of system just after collision is v_1. For pan and mass m.

Impulse = change in momentum

$$-\int T dt = 2mv_1 - mv$$

for block of mass m

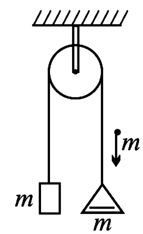
impulse = mv_1

$$\int T dt = mv_1$$

$$-mv = 2mv_1 - mv$$

$$mv = 3mv_1$$

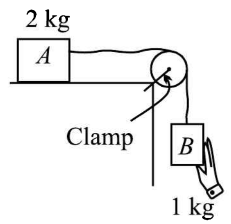
$$v_1 = v/3$$



Concept

Tension impulse on one side is equal on both sides so momentum can be conserved along the length of string.

Example 83. Two blocks A and B are joined by means of a slacked string passing over a massless pulley as shown in diagram. The system is released from rest and it becomes taut when B falls a distance 0.5 m, then



(a) Find the common velocity of two blocks just after string become taut.

(b) Find the magnitude of impulse on the pulley by the clamp during the small interval while string becomes taut.

[**Ans.** (a) $\frac{\sqrt{10}}{3}$ m/s, (b) $\frac{4}{3}\sqrt{5}$ N-m/s]

Solution Velocity of B just before the string is taut

$$v_B = \sqrt{2g\Delta h} = \sqrt{10} \text{ m/s}$$

(a) Common velocity = v

$$\therefore \quad (m_A + m_B)v = m_B v_B \Rightarrow v = v_B/3 = \frac{\sqrt{10}}{3} \text{ m/s}$$

(b) Magnitude of impulse on A = Magnitude of impulse on

$$B = 1\left(\sqrt{10} - \frac{\sqrt{10}}{3}\right) = \frac{2}{3}\sqrt{10} \ N\text{-m/s}$$

$\therefore$ Impulse on pulley = $\sqrt{2}$ Impulse on A = $\frac{4}{3}\sqrt{5}$ N-m/s.

Concept

Constraint motion can be applied only when string is tight.

C- FRAME

We may attach a frame of reference, designated $X_CY_CZ_C$, to the center of mass of a system. Relative to this frame, the center of mass is at rest (v_{CM} = 0). This is called the centre of mass or C-frame of reference. In view of equation $P = Mv_{cm}$, the total momentum of a system of particles referred to the C-frame of reference is always zero.

$$\vec{P} = \sum_i \vec{p}_i = 0 \text{ in the } C\text{-frame of reference.}$$

For that reason the C-frame is sometimes called the zero momentum frame. The C-frame is important because many problems can be more simply analyzed in the C-frame compared to ground frame . It is clear that the C-frame moves with a velocity v_{CM} relative to the ground frame. When no external forces act on a system, the C-frame can be considered as inertial.

Example 84. The velocities of two particles of masses m_1 and m_2 relative to an inertial observer are v_1 and v_2. Determine the velocity of the center of mass relative to the observer and the velocity of each particle relative to the center of mass.

Solution From equation $v_{cm} = \frac{dr_{cm}}{dt} = \frac{1}{M}\sum_i m_i \frac{dr_i}{dt} = \frac{\sum_i m_i v_i}{M}$ the velocity of the centre of mass relative to the observer is

$$v_{cm} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

The velocities of each particle relative to the centre of mass, using the Galilean transformation of velocities is

$$v_1' = v_1 - v_{cm} = v_1 - \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{m_2(v_1 - v_2)}{m_1 + m_2} = \frac{m_2v_{12}}{m_1 + m_2}$$

and

$$v_2' = v_2 - v_{cm} = \frac{m_1(v_2 - v_1)}{m_1 + m_2} = -\frac{m_1v_{12}}{m_1 + m_2}$$

where $v_{12} = v_1 - v_2$ is the relative velocity of the two particles. Thus, in the C-frame, the two particles appear to be moving in opposite directions, with velocities inversely proportional to their masses.

Let us find relation between kinetic energy of a system from ground frame and C-frame we have a system consisting of many particles, lets say speed of the i[th] particle is v_i. Then kinetic energy of system, K in ground frame will be summation of individual kinetic energy.

$$K_s = \sum\left(\frac{1}{2}m_iv_i^2\right) \qquad \text{now} \qquad \vec{v}_i = \vec{v}_{i/c} + \vec{v}_c$$

where $\vec{v}_i$ is velocity of the i[th] particle in ground, $\vec{v}_{i/c}$ is velocity of the i[th] particle in reference rame attached to the CM and $\vec{v}_c$ is velocity of CM in ground frame.

$$k_a = \frac{1}{2}\sum m_i\left(\vec{v}_{i/c} + \vec{v}_c\right)^2$$

$$k_a = \frac{1}{2}\sum m_i\vec{v}_{i/c}^{\,2} + \frac{1}{2}\sum m_i\vec{v}_c^{\,2} + \frac{1}{2}\cdot 2\left(\sum m_i \cdot \vec{v}_{i/c}\cdot \vec{v}_c\right)$$

$$k_a = \sum \frac{1}{2} m_i\vec{v}_{i/c}^{\,2} + \frac{1}{2}\left(\sum m_i\right)\vec{v}_c^{\,2} + \left(\sum m_i \cdot \vec{v}_{i/c}\right)\vec{v}_c$$

we can take $\vec{v}_c$ out of summation in second and third term as it is constant. Now third term becomes zero, as $\sum m_i\vec{v}_{i/c} = M\vec{v}_{c/c}$.

$\vec{v}_{c/c}$ is velocity of COM in frame of COM, which is zero. Also it represents momentum of system in C-frame which is zero

$$\left(\frac{1}{2}\sum m_i\vec{v}_{i/c}^{\,2}\right) = k_{s/c} \text{ Thus we get } \qquad k_a = k_{s/c} + \frac{1}{2}mv_c^2$$

where $k_{s/c}$ means kinetic energy of system in C-frame. This important conclusion will be again useful in rotational dynamics we can do little manipulation to write the equation as

$$k_a = k_{s/c} + \frac{P_c^2}{2M}$$

A system of two particles: Suppose the masses of the particles are equal to m_1 and m_2 and their velocities in the K reference frame to $\vec{v}_1$ and $\vec{v}_2$, respectively. Let us find the expressions defining their momenta and the total kinetic energy in the C-frame.

The momentum of the first particle in the C-system is

$$P_{1/c} = m_1\,\vec{v}_{1/c} = m_1(\vec{v}_1 - \vec{V}_c)$$

where $\vec{V}_c$ is the velocity of the centre of inertial (of the C system) in the K reference frame. Substituting in this formula expression $\vec{V}_c = \frac{1}{m}\sum m_i\vec{v}_i = \frac{1}{m}\sum \vec{p}_i$, we obtain

$$\vec{p}_i = \mu(\vec{v}_1 - \vec{v}_2)$$

where μ is the so called ***reduced mass*** of the system.

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

similarly, the momentum of the second particle in the C frame is

$$\vec{p}_{2/c} = \mu(\vec{v}_2 - \vec{v}_1)$$

This, the momenta of the two particles in the C-frame are equal in magnitude and opposite in direction; the modulus of the momentum of each particle is

$$\vec{p}_{1/c} = \mu v_{rel}$$

where $v_{rel} = |\vec{v}_1 - \vec{v}_2|$ is the velocity of one particle relative to another.

Finally, let us consider kinetic energy. The total kinetic energy of the two particles in the C-frame is

$$K_{s/c} = K_1 + K_2 = \frac{\vec{P}^2}{2m_1} + \frac{\vec{P}^2}{2m_2}$$

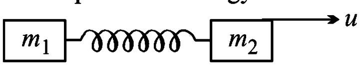
since in accordance with equation $\mu = \frac{m_1 m_2}{m_1 + m_2}$, $\frac{1}{m_2} + \frac{1}{m_2} = \frac{1}{\mu}$ then

$$K_{s/c} = \frac{p^2}{2u} = \frac{\mu v_{rel}^2}{2}$$

If the particles interace, then total mechanical energy in the C frame is $E = T + U$

where U is the potential energy of interation of the given particles.

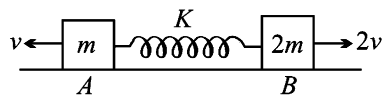
e.g.



Example 85. Two blocks A and B of masses m & $2m$ placed on smooth horizontal surface are connected with a light spring. The two blocks are given velocities as shown when spring is at natural length.

(a) Find velocity of centre of mass

(b) Maximum extension in the spring



Solution Velocity of $C.M.$ $v_{cm} = \frac{3mv - mv}{3m} = v$

In $C.O.M.$ frame. Initial momentum = 0

at the time of maximum elongation both the masses will be moving in same direction with same speed.

Initial relative velocity $v_{rel} = 3v$

Decline in KE = Increase in PE of spring

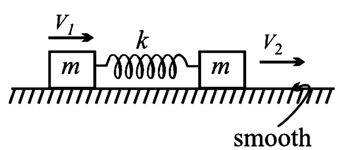
$$\frac{1}{2} m\, v_{rel}^2 = \frac{1}{2} kx^2 \qquad \frac{1}{2}\frac{m \times 2m}{3m}(3v)^2 = \frac{1}{2} kx^2$$

$$3mv^2 = \frac{1}{2} kx^2 \qquad x = v\sqrt{\frac{6m}{k}}$$

Concept

To learn how to apply C-frame.

Example 86. Two identical blocks of mass m, each are connected by a spring as shownin the figure. At any instant of time $t = 0$, one block is given a velocity v_1 and other is given a velocity v_2 ($v_1 \geq v_2$) in the same direction simultaneously as shown in the figure. Find the maximum energy stored in the spring.



[**Ans.** $\frac{1}{4} m (v_1 - v_2)^2$]

COLLISION OR IMPACT

Collision is a kind of interaction between two or more bodies which come in contact with each other for a very short time interal. The proper definition of collision between two bodies can be written as — *"When exchange of momentum takes place between two physical bodies only due to their mutual interaction force is defined as collision between two bodies."*

There are three distinct indentifable stages in a collision, namely, before, during and after. In the before and after stage the interaction force are zero. Between these two stages, the interaction forces are very large and often the dominating force governing the motion of bodies. The magnitude of the interacting force is often unknown, therefore, Newton's second law cannot be used. Hence, the law of conservation of momentum is useful in relating the initial and final velocities.

- In collision particle may or may not come in physical contact.
- The duration of collision is negligible as compared to the usual time intervals of observation of motion.

Line of Collision: Definition of line of collision. It is the common normal at the point of contact during collision is known as line of collision.

Common Normal (line of collision): Collision is an isolated event in which a strong force acts between two or more bodies for a short time, which results in change of their velocities.

Concept

(a) In collision particles may or may not come in physical contact.

(b) The duration of collision, Δt is negligible as compared to the usual time intervals of observation of motion.

(c) In a collision the effect of external non impulsive forces such as gravity are not taken into a account as due to small duration of collision (Δt) average impulsive force responsible for collision is much larger than external forces acting on the system.

The collision is in fact a redistribution of total momentum of the particles. Thus, law of conservation of linear momentum is indispensable in dealing with the phenomenon of collision between particles.

Line of Impact

The line passing through the common normal to the surfaces in contact during impact is called line of impact. The force during collision acts along this line on both the bodies.

Direction of Line of impact can be determined by:

(a) Geometry of colliding objects like spheres, discs, wedge etc.

(b) Direction of change of momentum.

If one particle is stationary before the collision then the line of impact will be along its motion after collision.

Classification of Collisions

(a) On the basis of line of impact

   (i) Head-on collision : If the velocities of the particles are along the same line before and after the collision.

   (ii) Oblique collision : If the velocities of the particles are along different lines before and after the collision.

(b) On the basis of energy :

   (i) Elastic collision : In an elastic collision, the particle regain their shape and size completely after collision i.e., no fraction of mechanical energy remains stored as deformation potential energy in the bodies. Thus, kinetic energy of system after collision is equal to kinetic energy of system before collision. Thus in addition to the linear momentum, kinetic energy also remains conserved before and after collision.

   (ii) Inelastic collision : In an inelastic collision, the particle do not regain their shape and size completely after collision. Some fraction of mechanical energy is retained by the colliding particles in the form of deformation potential energy. Thus, the kinetic energy of the particles no longer remains conserved. However, in the absence of external forces, law of conservation of linear momentum still holds good.

   (iii) Perfectly inelastic : If velocity of separation just after collision becomes zero then the collision is perfectly inelastic. Collision is said to be perfectly inelastic if both the particles stick together after collision and move with same velocity,

Concept

Actually collision between all real objects are neither perfectly elastic nor perfectly inelastic, its inelastic in nature.

Illustrations of line of impact and collisions based on line of impact

(i) Two balls A and B are approaching each other such that their centres are moving along line CD.

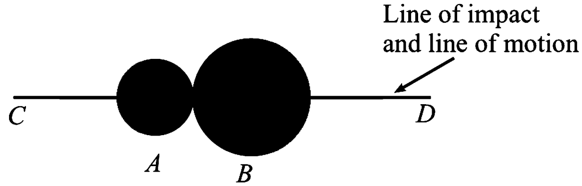


***Fig. 8.6:** Head on Collision*

(ii) Two balls A and B are approaching each other such that their centre are moving along dotted lines as shown in figure 8.7.

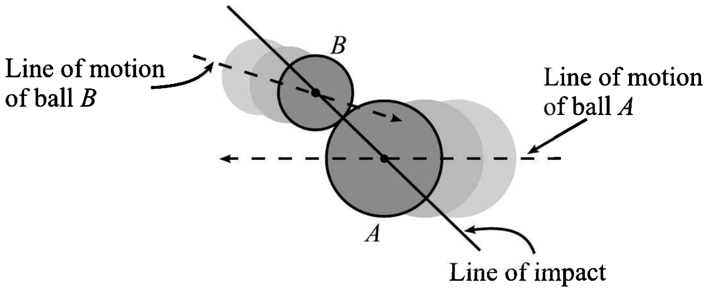


Fig. 8.7: *Oblique Collision*

(iii) Ball is falling on a stationary wedge.

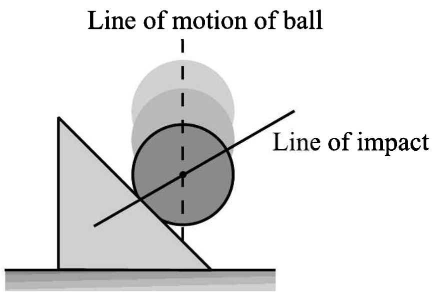


Fig. 8.8: *Oblique Collision*

When two bodies try to occupy same space at same time the event is called as collision.

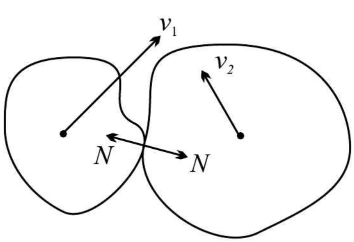
Assumptions :

(i) No external impulse is acting on the system

(ii) No friction is acting between colliding bodies.

(iii) Duration of collision is very-very small.

Thus when two bodies collide they apply only normal contact force on each other through point(s) of contact.



Line of Collision (L.O.C.)

Line along which normal contact force acts is called $L.O.C.$

Concept

L.O.C. is independent of direction of velocity of colliding bodies.

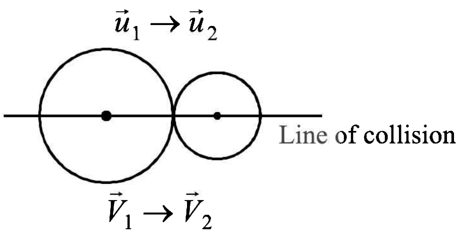
Collision is an event in which strong force acts between two or more bodies for a short time.

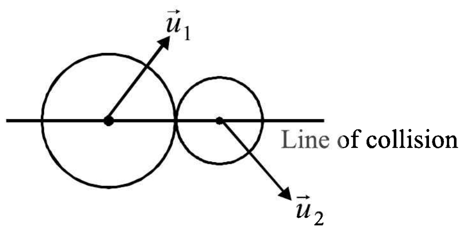
1. In a collision we consider situation before and after the collision. These terms refer to conditions when interactive force between particles effectively becomes zero. The duration of collision is negligible as compared to the time for which we observe the event.
2. In a collision effect of external non impulsive (e.g. gravity, friction) forces can be neglected because they are very small compared to i / a forces & time of collision is very small.
3. When two bodies collide, they exert force on each other through point of contact, perpendicular to the plane of contact. The direction of force of interaction is line of collision.
4. In case of collision it the external impulsive forces are not acting in a direction, the total momentum of system in that direction remains conserved

   i.e. $m_1\vec{u} + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$
5. After collision, only the components of velocity along line of collision changes, the $\perp$ components of velocity remain unaffected.
6. According to initial velocities and line of collision, collision is of two types :

   (A) head on

   (B) oblique





7. According to conservation of mechanical energy collision is of two types :

   (A) elastic $\rightarrow K.E._f = K.E._i$ (B) inelastic $\rightarrow K.E._f < K.E._i$

Laws of Collision

1. **Conservation of momentum**

   $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{u}_1 + m_2\vec{u}_2$ (if impulsive forces in this direction are zero)

   (This equation is generally written along line of collision)
2. **Conservation of energy :**

   (i) Elastic collision : $K.E._i = K.E._f$

   $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

   (ii) inelastic collision : ΔKE loss in $KE = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$

Coefficient of Restitution (e)

The coefficient of restitution is defined as the ratio of the impulses of recovery and deformation of either body.

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r\, dt}{\int F_d\, dt} = \frac{\text{Velocity of seperation along line of impact}}{\text{Velocity of approach along line of impact}}$$

The most general expression for coefficient of restitution is

$$e = \frac{\text{velocity of separation of points of contact along line of impact}}{\text{velocity of approach of point of contact along line of impact}}$$

3. Newton's law for collision :

(i) $\dfrac{v_2 - v_1}{u_2 - u_1} = -e$

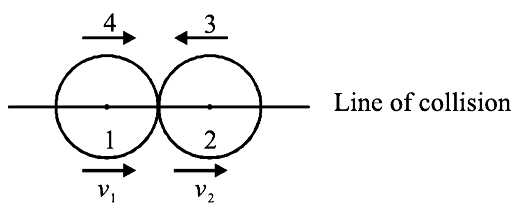
v_1 and v_2 : Components of velocities of masses colliding, along the line of contact, after collision (with sign).

u_1 and u_2 : Components of velocities of colliding masses, along the line of contact, before collision (with sign)

(ii) This Law is valid even when momentum is not conserved :

Eq. of sign :

$$-e = \frac{v_2 - v_1}{(-3) - (4)}$$

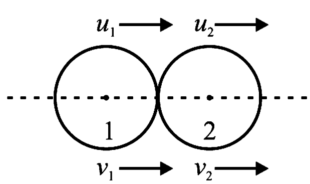


(iii) For $e = 1$ perfectly elastic collision

for $a < e < 1$ inelastic collision

for $e = 0$ perfectly inelastic collision (Bodies will move together)

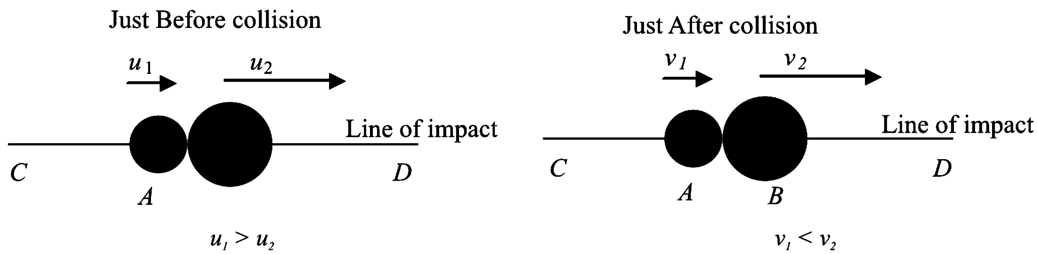
(iv) $v_2 - v_1 = e(u_1 - u_2)$

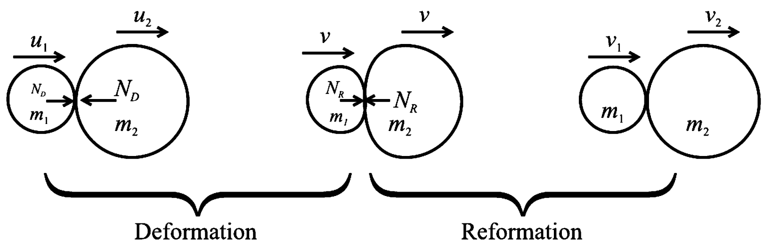


Relative velocity of receding = e [Relative velocity of approach before impact.]

Illustration for Calculation of e

Two smooth balls A and B approaching each other such that their centres are moving along line CD in absence of external impulsive force. The velocities of A and B just before collision be u_1 and u_2 respectively. The velocities of A and B just after collision be v_1 and v_2 respectively.





$\because$ $F_{ext} = 0$ momentum is conserved for the system.

$\Rightarrow$ $m_1u_1 + m_2u_2 = (m_1 + m_2)v = m_1v_1 + m_2v_2$

$$\Rightarrow v = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \quad \text{...(1)}$$

Impulse of Deformation

J_D = change in momentum of any one body during deformation.

$= m_2(v - u_2)$ for m_2

$= m_1(-v + u_1)$ for m_1

Impulse of Reformation

J_R = change in momentum of any one body during Reformation.

$= m_2(v_2 - v)$ for m_2

$= m_1(v - v_1)$ for m_1

$$e = \frac{\text{Impulse of Reformation } (\vec{J}_R)}{\text{Impulse of Deformation } (\vec{J}_D)} = \frac{v_2 - v}{v - v_1} = \frac{v_2 - v_1}{u_1 - u_2} \quad \text{(substituting v from (1))}$$

$$= \frac{\text{Velocity of separation along line of impact}}{\text{Velocity of approach along line of impact}}$$

Concept

e is independent of shape and mass of object but depends on the material.

The coefficient of restitution is constant for two particular objects.

(a) $e = 1 \Rightarrow$ Impulse of Reformation = Impulse of Deformation

$\Rightarrow$ Velocity of separation = Velocity of approach

$\Rightarrow$ Kinetic Energy may be conserved

$\Rightarrow$ Elastic collision.

(b) $e = 0 \Rightarrow$ Impulse of Reformation = 0

$\Rightarrow$ Velocity of separation = 0

$\Rightarrow$ Kinetic Energy is not conserved

$\Rightarrow$ Perfectly Inelastic collision.

(c) $0 < e < 1 \Rightarrow$ Impulse of Reformation < Impulse of Deformation

$\Rightarrow$ Velocity of separation < Velocity of approach

$\Rightarrow$ Kinetic Energy is not conserved

$\Rightarrow$ Inelastic collision.

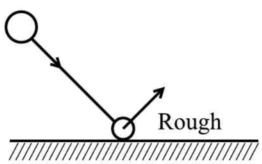
Concept

In case of contact collisions e is always less than unity.

$\therefore \quad 0 \le e \le 1$

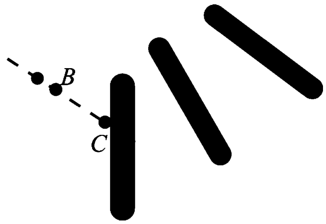
Concept

In case of elastic collision, if rough surface is present then $k_f < k_i$ (because friction is impulsive)

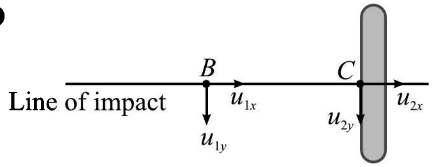


A particle 'B' moving along the dotted line collides with a rod also in state of motion as shown in the figure. The particle B comes in contact with point C on the rod.

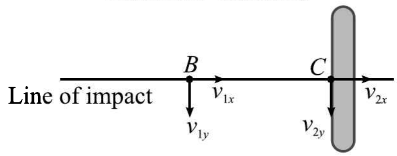
To write down the expression for coefficient of restitution e, we first draw the line of impact. Then we resolve the components of velocities of points of contact of both the bodies along line of impact just before and just after collision.



Just Before Collision

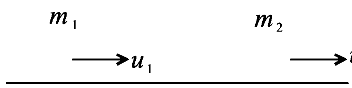


Just After Collision

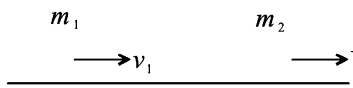


Then $e = \dfrac{v_{2x} - v_{1x}}{u_{1x} - u_{2x}}$

Collision in One Dimension (Head on)



(a)
Before Collision



(b)
After Collision

$u_1 \geq u_2$ $\qquad$ $v_2 \geq v_1$

$e = \dfrac{v_2 - v_1}{u_1 - u_2} \qquad \Rightarrow (u_1 - u_2)e = (v_2 - v_1)$

By momentum conservation,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$v_2 = v_1 + e(u_1 - u_2)$$

and

$$v_1 = \frac{m_1u_1 + m_2u_2 - m_2e(u_1 - u_2)}{m_1 + m_2} = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \frac{(1+e)m_2u_2}{m_1 + m_2}$$

$$v_2 = \frac{m_1u_1 + m_2u_2 + m_1e(u_1 - u_2)}{m_1 + m_2} = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 + \frac{(1+e)m_1u_1}{m_1 + m_2}$$

Special Case:

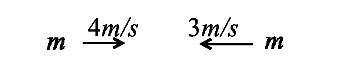
(1) $e = 0 \qquad \Rightarrow \quad v_1 = v_2$

$\Rightarrow$ for perfectly inelastic collision, both the bodies, move with same velocity after collision.

(2) $e = 1$ and $m_1 = m_2 = m$,

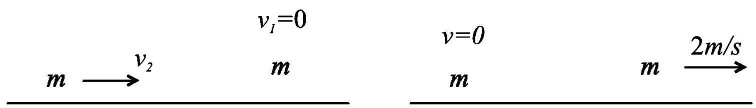
we get $\quad v_1 = u_2$ and $v_2 = u_1$

i.e., when two particles of equal mass collide elastically and the collision is head on, they exchange their velocities., e.g.



Before Collision

After Collision



Before Collision After Collision

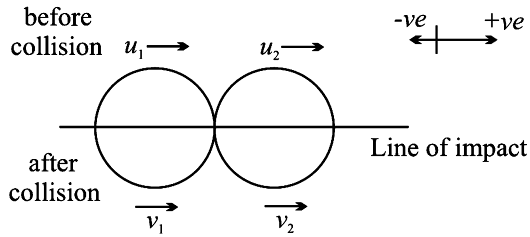
(3) $m_1 \geq\!\geq\!\geq\!\geq m_2$

$$m_1 + m_2 \approx m_1 \text{ and } \frac{m_2}{m_1} \approx 0$$

$\Rightarrow \quad v_1 = u_1$ No change and $\quad v_2 = u_1 + e(u_1 - u_2)$

Head-on Elastic Collision

Before collision



(i) Conserving momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

(ii) Since collision is perpectly elastic

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

(iii) Law of collision

$$\frac{v_2 - v_1}{u_2 - u_1} = -1 \qquad \text{(with sign)}$$

But actually only two equations are not trivial third depends on other equation.

Solving (i) & (iii)

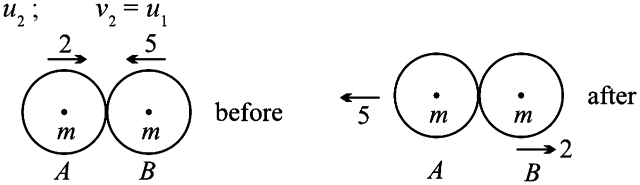
$$v_1 = \frac{2m_2u_2}{m_1 + m_2} + \frac{(m_1 - m_2)u_1}{m_1 + m_2}; \quad v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)u_2}{(m_1 + m_2)}$$

Cases:

(i) If $m_1 = m_2$ then

$v_1 = u_2$; $\quad v_2 = u_1$

e.g.



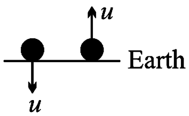
(ii) $m_1 = m_2$ and $u_2 = 0$

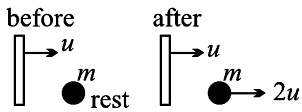
$v_1 = 0$; $v_2 = u_1$

(iii) If $m_1 \gg m_2$ then $v_1 \cong u_1$

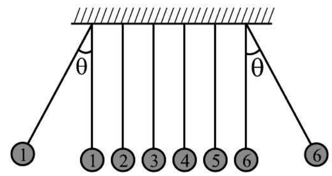
$u_2 = 2u_1 - u_2$

for $m_1 >> m_2$	for $m_1 >> m_2$
& $u_1 = 0$	if $u_2 = 0$
$v_1 = 0$	$v_1 = u_1$
$v_2 = -u_2$	$v_2 = 2u_2$





Example 87. Find which ball will move when ball 1 is released and collide with the ball 2. All balls are of same mass and all collisions are elastic.



Solution Ball 6 will move with same speed.

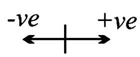
Concept

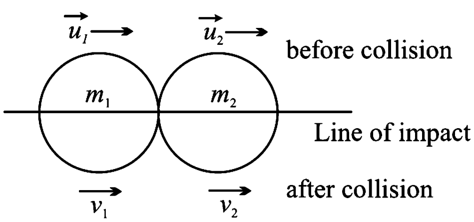
Identical masses under elastic head on collision, the velocity are interchanged.

Example 88. Repeat the previous problem if instead of one ball two balls are taken together.

Solution Balls 5 & 6 move with same speed.

Head-on Inelastic Collision





1. Conserving momentum $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ... (i)

2. $\dfrac{v_2 - v_1}{u_2 - u_1} = -e$ ... (ii)

From (i) & (ii)

$$v_1 = \frac{(m_1 - em_2)u_1}{m_1 + m_2} + \frac{(1+e)m_2u_2}{m_1+m_2}; \quad v_2 = \frac{(1+e)m_1u_1}{m_1+m_2} + \frac{(m_2 - em_1)u_2}{m_1+m_2}$$

$$\text{Loss in K.E.} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(1-e^2)\,|(\vec{u}_1 - \vec{u}_2)|^2$$

Result :

1. If $e = 0$ (perfectly inelastic collision)

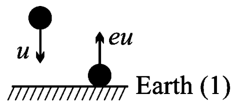
   then $v_1 = v_2 = \dfrac{m_1u_1 + m_2u_2}{m_1 + m_2}$; $\qquad \Delta E = \dfrac{1}{2}\left(\dfrac{m_1\, m_2}{m_1 + m_2}\right)(u_1 - u_2)^2$

2. if $m_1 \geq\geq m_2$ then $v_1 \cong u_1$

$$v_2 = (1 + e)\, u_1 - eu_2$$

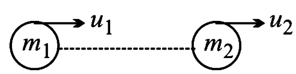
(i) if $m_1 >> m_2$
& $u_1 = 0$
then $v_1 = 0$
but $v_2 = -\, eu_2$

(ii) if $m_1 >> m_2$
& $u_2 = 0$
$v_1 = u_1$
but $v_2 = (1 + e)u_1$



Example 89. Two particles of mass m_1, m_2 moving with initial velocity u_1 and u_2 collied head-on. Find minimum Kinetic energy during collision. Thus prove that maximum kinetic energy is lost in perfectly inelastic collision

Solution In C-frame initial kinetic energy of system if $\frac{1}{2}\mu(v_2 - v_1)^2$ where $\mu = \dfrac{m_1m_2}{m_1 + m_2}$. During collision at the instant of maximum deformation we get minimum kinetic energy in C-frame as they attain same velocity thus no relative velocity. When system have minimum kinetic energy in C-frame it also has minimum kinetic energy in ground frame as velocity of CM is constant.



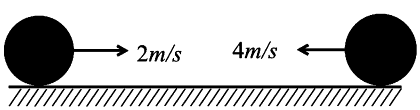
$K_G = \frac{1}{2}\mu v_{rel}^2 + \frac{1}{2}m_s v_c^2$ at maximum deformation. Thus minimum kinetic energy during collision is $\frac{1}{2}(m_1 + m_2)v_c^2$, where $v_c = \dfrac{(m_1u_1 + m_2u_2)}{m_1 + m_2}$

In inelastic collision final kinetic energy is $\frac{1}{2}m_s v_c^2$ of CM is constant.

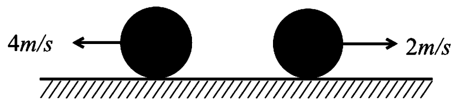
Concept

Internal froces have no effect on $C.M.$

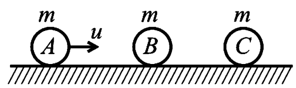
Example 90. Two identical balls are approaching towards each other on a straight line with velocity 2 m/s and 4 m/s respectively. Find the final velocities, after elastic collision between them.



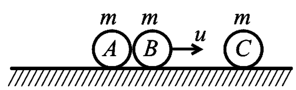
Solution The two velocities will be exchanged and the final motion is reverse of initial motion for both.



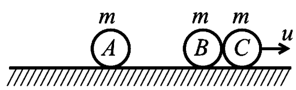
Example 91. Three balls A, B and C of same mass 'm' are placed on a frictionless horizontal plane in a straight line as shown. Ball A is moved with velocity u towards the middle ball B. If all the collisions are elastic then, find the final velocities of all the balls.



Solution A collides elastically with B and comes to rest but B starts moving with velocity u

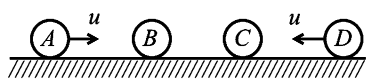


After a while B collides elastically with C and comes to rest but C starts moving with velocity u

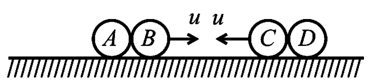


∴ Final velocities $V_A = 0$; $V_B = 0$ and $V_C = u$

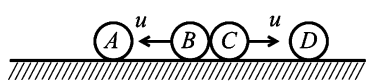
Example 92. Four identical balls A, B, C and D are placed in a line on a frictionless horizontal surface. A and D are moved with same speed 'u' towards the middle as shown. Assuming elastic collisions, find the final velocities.



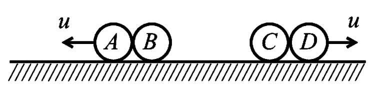
Solution A and D collides elastically with B and C respectively and come to rest but B and C starts moving with velocity u towards each other as shown



B and C collides elastically and exchange their velocities to move in opposite directions

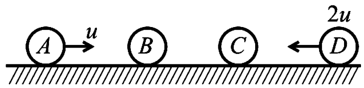


Now, B and C collides elastically with A and D respectively and come to rest but A and D starts moving with velocity u away from each other as shown

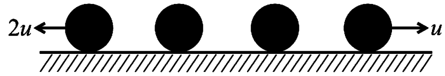


∴ Final velocities $V_A = u\ (\leftarrow)$; $V_B = 0$; $V_C = 0$ and $V_D = u\ (\rightarrow)$

Example 93. If A is moved with velocity u and D is moved with $2u$ as shown. What will be the final velocities now be?



Solution



Example 94. Two particles of mass m and $2m$ moving in opposite directions collide elastically with velocity v and $2v$ respectively. Find their velocities after collision.

Solution Let the final velocities of m and $2m$ be v_1 and v_2 respectively as shown in the figure:

By conservation of momentum:

$m(2v) + 2m(-v) = m(v_1) + 2m\ (v_2)$

or $0 = mv_1 + 2mv_2$ or $v_1 + 2v_2 = 0$ ...(1)

and since the collision is elastic:

$v_2 - v_1 = 2v - (-v)$ or $v_2 - v_1 = 3v$ ...(2)

Solving the above two equations, we get,

$v_2 = v$ and $v_1 = -2v$

i.e., the mass $2m$ returns with velocity v while the mass m returns with velocity $2v$ in the direction shown in figure:

Example 95. Find the fraction of kinetic energy lost by the colliding particles after collision in the above situation.

Solution The collision was elastic therefore, no kinetic energy is lost,

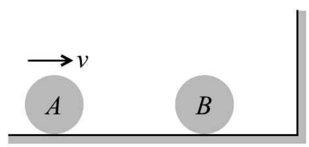
KE loss = KE_i - KE_f

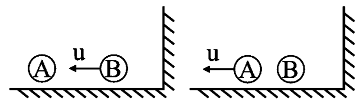
$$\left(\frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)(-v)^2\right) - \left(\frac{1}{2}m(-2v)^2 + \frac{1}{2}(2m)v^2\right) = 0$$

Example 96. Three balls A, B and C are placed on a smooth horizontal surface. Given that $m_A = m_C = 4m_B$. Ball B collides with ball C with an initial velocity v as shown in figure. Find the total number of collisions between the balls. All collisions are elastic.

Solution [**Ans.** 2 collisions]

Example 97. Two balls shown in figure are identical. Ball A is moving towards right with a speed v and the second ball is at rest. Assume all collisions to be elastic. Show that the speeds of the balls remains unchanged after all the collisions have taken place.



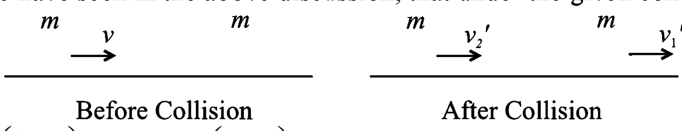


Solution Due to elastic collision if $m_1 = m_2$ then masses exchange their velocity with magnitude and driection. (A)(B)→ 4 (Just after the 1st collision)

Again (B) collides with the wall and return

Example 98. A ball of mass m moving at a speed v makes a head on collision with an identical ball at rest. The kinetic energy of the balls after the collision is 3/4th of the original. Find the coefficient of restitution.

Solution As we have seen in the above discussion, that under the given conditions :



$$v_1' = \left(\frac{1+e}{2}\right)v \text{ and } v_2' = \left(\frac{1-e}{2}\right)v$$

Given that $K_f = \frac{3}{4}K_i$ or $\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{4}\left(\frac{1}{2}mv^2\right)$

Substituting the value, we get

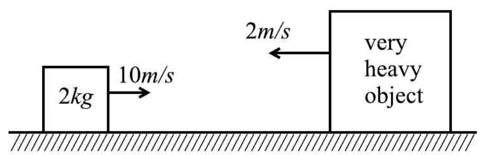
$$\left(\frac{1+e}{2}\right)^2 + \left(\frac{1-e}{2}\right)^2 = \frac{3}{4} \quad \text{or } (1+e)^2 + (1-e)^2 = 3$$

or $2 + 2e^2 = 3$ or $e^2 = \frac{1}{2}$ or $e = \frac{1}{\sqrt{2}}$

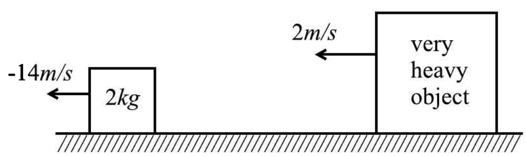
Example 99. A block of mass m moving at speed v collides with another block of mass 2 m at rest. The lighter block comes to rest after the collision. Find the coefficient of restitution.

Solution **[Ans.** $\frac{1}{2}$**]**

Example 100. A block of mass 2 kg is pushed towards a very heavy object moving with 2 m/s closer to the block (as shown). Assuming elastic collision and frictionless surfaces, find the final velocities of the blocks.



Solution Let v_1 and v_2 be the final velocities of 2kg block and heavy object respectively then,



$$v_1 = u_1 + 1\,(u_1 - u_2) = 2u_1 - u_2 = -14 \text{ m/s}$$

$$v_2 = -2\text{m/s}$$

Example 101. A ball of mass m moving with a certain velocity collides against a stationary ball of mass m. The two balls stick together during collision. If E be the initial kinetic energy, then the loss of kinetic energy in the collision is

(A) E (B) $\frac{E}{2}$ (C) $\frac{E}{3}$ (D) $\frac{E}{4}$

[Ans. (b)]

Solution From *COLM* $mu = (m+m)\,v$

$$v = \frac{u}{2}$$

$$\text{Final } E = \frac{1}{2}2mv^2 = \frac{1}{2}2m\frac{u^2}{4} = \frac{mu^2}{4}$$

$$\text{Initial } K.E. = \frac{1}{2}mu^2 = E$$

$$\text{Loss} = \text{Initial K.E.} - \text{Final K.E.} = \frac{mu^2}{2} - \frac{mu^2}{4} = \frac{mu^2}{4} = \frac{E}{2}$$

$\therefore$ (B) is correct.

Example 102. A 1 kg ball, moving at 12 ms^{-1}, collides head-on with a 2 kg ball moving in the opposite direction at 24 ms^{-1}. If the coefficient of restitution is $\frac{2}{3}$, then the energy lost in the collision is :

(A) 60 *J* (B) 120 *J* (C) 240 *J* (D) 480 *J*.

[Ans. (c)]

Solution Initial momentum = Final momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

(1) (12) + (2) (–24) = $1v_1 + 2v_2$

$\Rightarrow$ $v_1 + 2v_2 = -36$ ...(i)

Further $e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right)$ $\Rightarrow$ $\frac{2}{3} = -\left(\frac{v_2 - v_1}{-24-12}\right)$

$\Rightarrow$ $\frac{2}{3} = \frac{v_2 - v_1}{36}$ $\Rightarrow v_2 - v_1 = 24$ ...(ii)

Add (i) and (ii)

$3v_2 = -12$ $\Rightarrow$ $v_2 = -4$ ms^{-1} $\Rightarrow$ $v_1 = -28$ ms^{-1}

$\Rightarrow$ Loss = Total Initial *K.E.* – Total Final *K.E.* $\left(K.E. = \frac{1}{2}mv^2\right)$

Since Total Initial *K. E.* $= \frac{1}{2}(1)(144) + \frac{1}{2}(2)(576) \Rightarrow E_1 = 72 + 576 = 648\,J$

Similarly, total final *K.E.* $E_2 = \frac{1}{2}(1)(784) + \frac{1}{2}(2)(16)$

$\Rightarrow$ $E_2 = 392 + 16$

$\Rightarrow$ $E_2 = 408$ $J \Rightarrow$ Loss = 648 – 408 $\Rightarrow$ Loss = 240 *J*

Example 103. A ball of mass *m* is projected vertically up from a smooth horizontal floor with a speed *V*. Find the total momentum delivered by the ball to the surface, assuming *e* as the coefficient of restitution of impact.

Solution Referring the figure, the momentum delivered by the ball at first, second, third impact etc. can be given as the corresponding change, in its momenta (ΔP).

$$(\Delta \vec{P})_1 = \left|(mV_1\hat{j} - m(-V_0)\hat{j}\right| \Rightarrow \Delta P_1 = m(V_1 + V_0)$$

Similarly $\Delta P_2 = m(V_1 + V_2)$,

$\Rightarrow$ The total momentum transferred $\Delta P = \Delta P_1 + \Delta P_2 + \Delta P_3 + \ldots$

Putting the values of ΔP_1, ΔP_2 etc., we obtain,

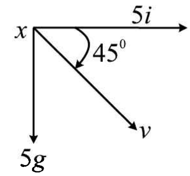
$$\Delta P = m[V_0 + 2(V_1 + V_2 + V_3 + \ldots)]$$

Putting $V_1 = eV_0$, $V_2 = e^2V_0$, $V_3 = e^3V_0$

we obtain, $\Delta P = mV_0\,[1 + 2(e + e^2 + e^3 + \ldots)]$

$$\Rightarrow \quad \Delta P = mV_0\left(1 + 2\frac{e}{1-e}\right) = mV_0\left(\frac{1+e}{1-e}\right)$$

Example 104. Body Q with linear momentum $\vec{P}_Q = (2\hat{i} - 3\hat{j})$ kg.m /s collides with and also sticks to body R, with linear momentum $\vec{P}_R = (3\hat{i} - 2\hat{j})$ kg.m /s. The bodies form a closed, isolated system. The direction in which they move after the collision makes an angle with positive x-axis that is equal to:



(A) 45° (B) 135° (C) 180° (D) 315°

Solution $m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}$

$(m_1 + m_2)\vec{v} = 5i - 5j\theta = 360 - 45 = 315°$

$\therefore$ (D) is correct.

Example 105. An object of mass 1.0 kg is projected vertically upward from the base of a 50 m tall building with an initial velocity of 40 m/s. At the same instant and directly overhead, a 2.0 kg object is dropped from rest from the top of the building. Maximum height above the ground attained by the centre of mass of two object system is (take $g = 10$ m/s^2).

(A) 8.9 m (B) 42 m (C) 33 m (D) 80 m

Solution Initial position of COM from base = $\dfrac{2}{3} \times 50 = \dfrac{100}{3}\,m$.

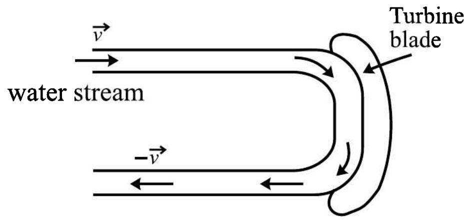
Initial velocity of COM = $\dfrac{40.1 + 2.0}{3} = \dfrac{40}{3}$ m/s upwards

It becomes zero after distance $\dfrac{v^2}{2g}$ from initial position $\Rightarrow \dfrac{v^2}{2g} = \dfrac{80}{9}m$

Total height of COM from the base = $\dfrac{80}{9} + \dfrac{100}{3} = 42\ m$.

Hence, (B) is correct.

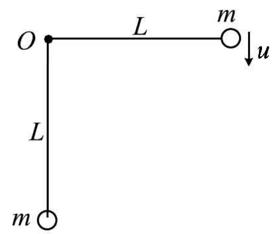
Example 106. A stream of water strikes a stationary turbine blade as shown. The speed of water v is 3 m/s, both before and after it strikes the curved surface of the blade and the constant rate at which the mass of water strikes is 5 kg/second. The magnitude of the force on the blade from the water is :



(A) 30 N (B) 15 N (C) 7.5 N (D) 5 N

Solution $f = \frac{dp}{dt} = 2v\frac{dm}{dt} = 30\ N.$

Hence, (A) is correct.



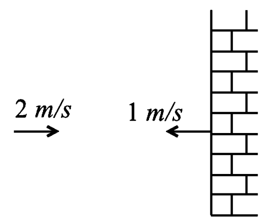
Example 107. Two same masses are tied with equal length of strings and are suspended at the same point. One mass is suspended freely whereas another is kept in a way that string is horizontal as shown. This is given initial velocity u in vertical downward direction. It strikes the freely suspended mass elastically that is just able to complete the circular motion after the collision about point of suspension O.

Magnitude of velocity *u* is:

(A) $\sqrt{gL}$ (B) $\sqrt{2.5gL}$ (C) $\sqrt{2gL}$ (D) $\sqrt{3gL}$

Solution As collision is elastic, freely suspended mass moves acquiring velocity of colliding mass after the collision. Hence, $mgL + \frac{1}{2}mu^2 = \frac{1}{2}m(5\ g\ell)\ u = \sqrt{3gL}$.

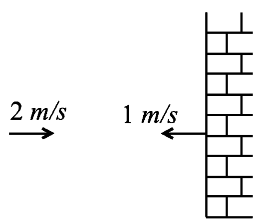
Hence, (C) is correct.



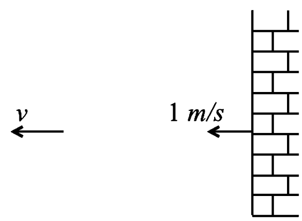
Example 108. A ball is moving with velocity 2 m/s towards a heavy wall moving towards the ball with speed 1m/s as shown in fig. Assuming collision to be elastic, find the velocity of the ball immediately after the collision.

Solution The speed of wall will not change after the collision.

So, let v be the velocity of the ball after collision in the direction shown in figure. Since collision is elastic ($e = 1$),



Before Collision

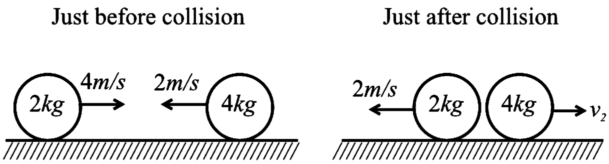


After Collision

separation speed = approach sped

or $v - 1 = 2 + 1$ or $v = 4$ m/s

Example 109. Two balls of masses 2 kg and 4 kg are moved towards each other with velocities 4 m/s and 2 m/s respectively on a frictionless surface. After colliding the 2 kg balls returns back with velocity 2m/s. Then find :



(a) velocity of 4 kg ball after collision
(b) coefficient of restitution e.
(c) Impulse of deformation J_D.
(d) Maximum potential energy of deformation.
(e) Impulse of reformation J_R.

Solution

(a) By momentum conservation,

$$2(4) - 4(2) = 2(-2) + 4(v_2) \quad \Rightarrow \quad v_2 = 1 \text{ m/s}$$

(b) $$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = \frac{1-(-2)}{4-(-2)} = \frac{3}{6} = 0.5$$

(c) At maximum deformed state, by conservation of momentum, common velocity is $v = 0$.

$$J_D = m_1(v - u_1) = m_2(v - u_2) = 2(0 - 4) = -8\ N\text{-}s = 4(0 - 2) = -8\ N\text{-}s$$

or $$= 4(0 - 2) = -8\ N\text{-}s$$

(d) Potential energy at maximum deformed state U = loss in kinetic energy during deformation.

or $$U = \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \left(\frac{1}{2}2(4)^2 + \frac{1}{2}4(2)^2\right) - \frac{1}{2}(2 + 4)(0)^2$$

or $$U = 24 \text{ Joule}$$

(e) $$J_R = m_1(v_1 - v) = m_2(v - v_2) = 2(-2 - 0) = -4\ N\text{-}s$$

or $= 4(0 - 1) = -4\ N\text{-}s$ or $e = \dfrac{J_R}{J_D}$

$\Rightarrow$ $J_R = eJ_D = (0.5)(-8) = -4\ N\text{-}s$

Example 110. A block of mass 1.2 kg moving at a speed of 20 cm/sec collides head on with a similar block kept at rest. The coefficient of restitution is 0.6. Find the loss of kinetic energy during collision.

Solution Suppose the fist block moves at a speed v_1 and the second at v_2 after collision. Since the collision is head on, the two blocks move along the original direction of motion of first block. Using the principle of conservation of momentum,

$$(1.2 \times 0.2) = 1.2v_1 + 1.2v_2$$

$$v_1 + v_2 = 0.2 \qquad \text{...(i)}$$

By Newton's law of restitution,

$$v_2 - v_1 = -e(u_2 - u_1)$$

$$v_2 - v_1 = -0.6(0 - 0.2)$$

$$v_2 - v_1 = 0.12 \qquad ...(ii)$$

Adding equations (i) and (ii),

$$2v_2 = 0.32$$

$$v_2 = 0.16 \text{ m/s or } 16 \text{ cm/s}$$

$$v_1 = 0.2 - 0.16 = 0.04 \text{ m/s} = 4 \text{ cm/s}$$

$$\text{Loss of } K.E. = \frac{1}{2}\times 1.2\times(0.2)^2 - \frac{1}{2}\times 1.2\times(0.16)^2 - \frac{1}{2}\times 1.2\times(0.04)^2$$

$$= 0.6[0.04 - 0.0256 - 0.0016] = 0.6\times 0.0128 = \mathbf{7.7\times 10^{-3}\ J}$$

Example 111. Hail storms are observed to strike the surface of the frozen lake at 30^0 with the vertical and rebound at 60^0 with the vertical. Assume contact to be smooth, the coefficient of restitution is:

(A) $e = \frac{1}{\sqrt{3}}$ (B) $e = \frac{1}{3}$ (C) $e = \sqrt{3}$ (D) $e = 3$.

Solution

Components of velocity before and after collision parallel to the plane are equal, So

$$v \sin 60° = u \sin 30° \qquad ...(1)$$

Components of velocity normal to the plane are related to each other

$$v \cos 60° = e\, u (\cos 30°) \qquad ...(2)$$

$$\Rightarrow \quad \cot 60° = e \cot 30°$$

$$\Rightarrow \quad e = \frac{\cos 60°}{\cot 30°} \Rightarrow e = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} \Rightarrow e = \frac{1}{3}$$

Example 112. A glass ball collides with a smooth horizontal surface with a velocity $\vec{V} = a\hat{i} - b\hat{j}$. If the coefficient of restitution of collision be e, find the velocity of the ball just after the collision.

Solution Collision takes place along the normal. Therefore the magnitude of normal component (V_y) of the velocity of the glass ball is changed to $Vy' = eV_y$ just after the collision whereas the horizontal component (V_x) of its velocity remains constant due to the absence of any horizontal force.

$\Rightarrow$ The velocity of the ball just after the impact $= \vec{V} = \vec{V}'_x + \vec{V}'_y$

$\Rightarrow$ $\vec{V}' = V'_x\hat{i} + V'_y\hat{j}$ where, $V'_x = a$ and $V'_y = eb$ ($\because \vec{V} = a\hat{i} - b\hat{j}$)

$\Rightarrow$ $\vec{V}'_x = a\hat{i} + eb\hat{j}$

Therefore the magnitude of the velocity $\vec{V} = |\vec{V}'| = \sqrt{a^2 + e^2b^2}$ and the direction is given as $\theta = \tan^{-1}\left(\frac{V'_x}{V'_y}\right) = \tan^{-1}\left(\frac{a}{eb}\right)$ to the normal (vertical).

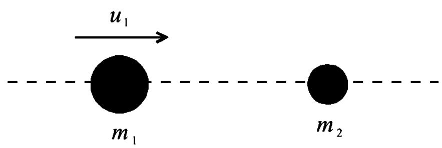
Example 113. A block of mass m moving at a speed v collides with another block of mass $2m$ at rest. The lighter block comes to rest after the collision. Find the coefficient of restitution.

Solution [Ans. $\frac{1}{2}$]

Example 114. A block of mass 1.2 kg moving at a speed of 20 cm/s collides head-on with a similar block kept at rest. The coefficient of restitution is 3/5. Find the loss of the kinetic energy during the collision.

Solution **[Ans. 7.7×10^{-3} J]**

Example 115. The sphere of mass m_1 travels with an initial velocity u_1 directed as shown and strikes the stationary sphere of mass m_2 head on. For a given coefficient of restitution e, what condition on the mass ratio $\frac{m_1}{m_2}$ ensures that the final velocity of m_2 is greater than u_1?



Solution [Ans. $\frac{m_1}{m_2} \geq \frac{1}{e}$]

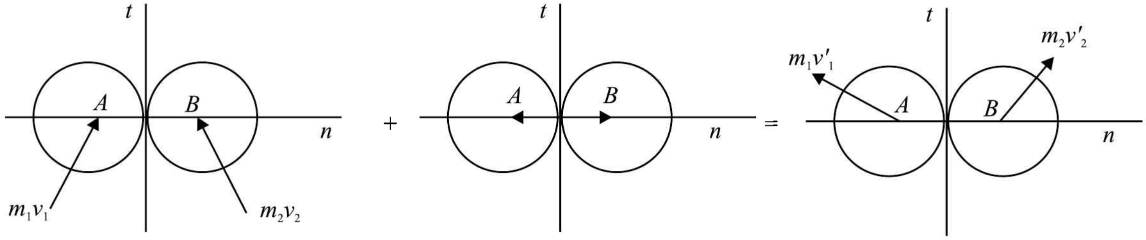
Collision in Two Dimension (Oblique)

1. A pair of equal and opposite impulses act along common normal direction. Hence, linear momentum of individual particles do change along common normal direction. If mass of the colliding particles remain constant during collision, then we can say that linear velocity of the individual particles change during collision in this direction.
2. No component of impulse act along common tangent direction. Hence, linear momentum or linear velocity of individual particles (if mass is constant) remain unchanged along this direction.
3. Net impulse on both the particles is zero during collision. Hence, net momentum of both the particles remain conserved before and after collision in any direction.
4. Definition of coefficient of restitution can be applied along common normal direction, i.e., along common normal direction we can apply

   Relative speed of separation = e (relative speed of approach)

Oblique Collision

Let us now consider the case when the velocities of the two colliding spheres are not directed along the line of impact as shown in figure. As already discussed the impact is said to be oblique. Since velocities v'_1 and v'_2 of the particles after impact are unknown in direction and magnitude, their determinaion will require the use of four independent equations. We choose as coordinate axes the n-axis along the line of impact, i.e. along the common normal to the surfaces in contact, and the t-axis along their common tangent. Assuming that the sphere are perfectly smooth and frictionless, we observe that the only impulses exerted on the sphere during the impact are due to internal forces directed along the line of impact i.e., along the n axis. It follows that



(i) The component along the t axis of the momentum of each particle, considered separately, is conserved; hence the t component of the velocity of each particle remains unchanged. We can write. $(v_1)_t = (v'_1)_t$; $(v_2)_t = (v'_2)_t$

(ii) The component along the n axis of the total momentum of the two particles is conserved. We write $m_1 (v_1)_n + m_2(v_2)_n = m_1(v'_1)_n + m_2(v'_2)_n$

(iii) The component along the n axis of the relative velocity of the two particles after impact is obtained by multiplying the n component of their relative velocity before impact by the coefficient of restitution. $(v'_2)_n - (v'_1)_n = e[(v_1)_n - (v_2)_n]$

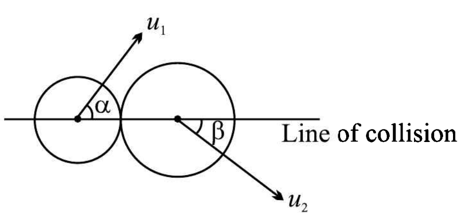
We have thus obtained four independent equations, which can be solved for the components of the velocities of A and B after impact.

Notes

Definition of coefficient of restitution can be applied along common normal direction in the case of oblique collisions .

(A) When line of collision is known (or can be known)

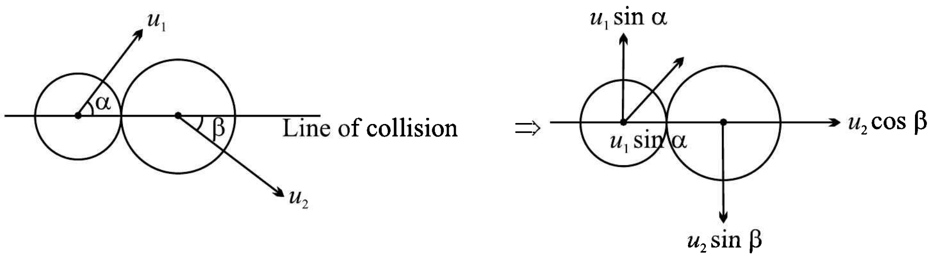
[**Use :** Impulse will be always along line of collision. So momentum will charge only along line of collision]



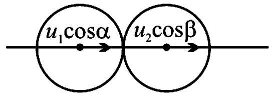
In last (after discussion)

Case (i): If one of them was at rest then after collision the body which was at rest will move in direction of line of collision.

Case (ii): If two moving bodies collide obliquely but one of them continues to move in same direction as was before colliding then that direction is direction of line of collision.



so $u_1 \sin \alpha$ & $u_2 \sin \beta$ will remain uncharged



(can treat as heat on collision)

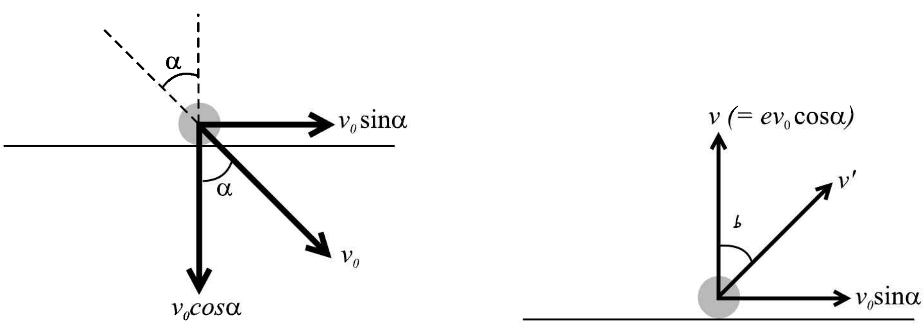
Thus $\quad v_1 = \sqrt{(v_{1x})^2 + (u_1 \sin\alpha)^2}\ /\ \tan\theta = \dfrac{u_1 \sin\alpha}{v_{1x}}$

$$v_2 = \sqrt{(v_{2x})^2 + (u_1 \sin\beta)^2}\ /\ \tan\phi = \frac{u_2 \sin\beta}{v_{2x}}$$

$\theta + \phi \rightarrow$ angle of divergence

Example 116. A ball of mass m hits a floor with a speed v_0 making an angle of incidence α with the normal. The coefficient of restitution is e. Find the speed of the reflected ball and the angle of reflection of the ball.

Solution The component of velocity v_0 along common tangent direction $v_0 \sin \alpha$ will remain unchanged. Let v be the component along common normal direction after collision. Applying



Relative speed of separation = e (relative speed of approach) along common normal direction, we get

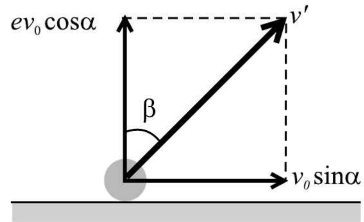
$$v = ev_0 \cos \alpha$$

Thus, after collision components of velocity v^1 are $v_0 \sin \alpha$ and $ev_0 \cos \alpha$

$$\therefore \quad v' = \sqrt{(v_0 \sin \alpha)^2 + (ev_0 \cos \alpha)^2}$$

$$\text{and} \quad \tan \beta = \frac{v_0 \sin \alpha}{ev_0 \cos \alpha}$$

$$\text{or} \quad \tan \beta = \frac{\tan \alpha}{e}$$



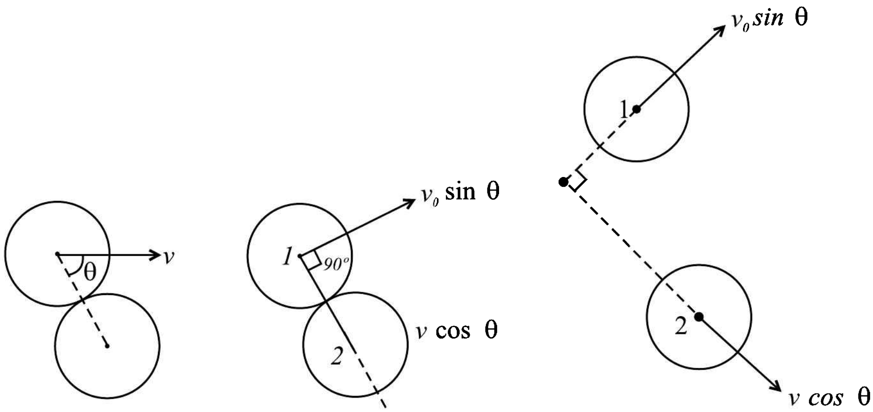
Concept

For elastic collision, $e = 1$

$\therefore v' = v_0$ and $\beta = \alpha$

Example 117. A ball of mass m makes an elastic collision with another identical ball at rest. Show that if the collision is oblique, the bodies go at right angles to each other after collision.

Solution In head on elastic collision between two particles, they exchange their velocities. In this case, the component of ball 1 along common normal direction, $v \cos \theta$



Before collision *After collision*

becomes zero after collision, while that of 2 becomes $v \cos \theta$. While the components along common tangent direction of both the particles remain unchanged. Thus, the components along common tangent and common normal direction of both the balls in tabular form are given below.

Ball	Component along common tangent direction		Component along common normal direction	
	Before collision	*After collision*	*Before collision*	*After collision*
1	$v \sin\theta$	$v \sin \theta$	$v \cos\theta$	0
2	0	0	0	$v \cos\theta$

From the above table and figure, we see that both the balls move at right angle after collision with velocities $v \sin \theta$ and $v \cos \theta$.

Concept

When two identical bodies have an oblique elastic collision, with one particle at rest before collision, then the two particles will go in $\perp$ directions.

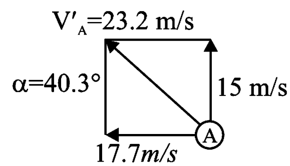
Example 118. The magnitude and direction of the velocities of two identical frictionless balls before they strike each other are as shown. Assuming $e = 0.90$, determine the magnitude and direction of the velocity of each ball after the impact.

Solution The impulsive force that the balls exert on each other during the impact are directed along a line joining the centres of the balls called the line of impact. Resolving the velocities into components directed, respectively, along the line of impact and along the common tangent to the surfaces in contact, we write

$$(V_A)_n = V_A \cos30° = +26 \text{ m/s} \qquad (V_A)_t = V_A \sin30° = +15 \text{ m/s}$$
$$(V_B)_n = -V_B \cos60° = -20 \text{ m/s} \qquad (V_B)_t = V_B \sin60° = +34.6 \text{ m/s}$$

Since the impulsive forces are directed along the line of impact, the t component of the momentum, and hence the t component of the velocity of each ball, is unchanged. We have

$$(V'_A)_t = 15 \text{ m/s}\uparrow,\ (V'_B)_t = 34.6 \text{ m/s}\uparrow$$



In the n direction, we consider the two balls as a single system and not that by Newton's third law, the internal impulses are, respectively, $F\Delta t$ and $-F\Delta t$ and cancel. We thus write that the total momentum of the balls is conserved.

$$m_A(V_A)_n + m_B(V_B)_n = m_A(V'_A)_n + m_B(V'_B)_n$$
$$m(26) + m(-20) = m(V'_A)_n + m(V'_B)_n$$
$$(V'_A)_n + (V'_B)_n = 6.0 \qquad \text{...(i)}$$

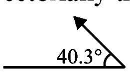
Using law of restitution,

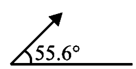
$$(V'_B)_n - (V'_A)_n = e[(V_A)_n - (V_B)_n]$$
$$(V'_B)_n - (V'_A)_n = (0.90)[26 - (-20)]$$
$$(V_A)_n - (V'_A)_n = 41.4 \qquad \text{...(ii)}$$

Solving equations (i) and (ii) simultaneously, we obtain

$$(V'_A)_n = -17.7 \text{ m/s} \qquad (V'_B)_n = +23.7 \text{ m/s}$$
$$(V'_A)_n = 17.7 \text{ m/s} \leftarrow \qquad (V'_B)_n = +23.7 \text{ m/s} \rightarrow$$

Resultant motion : Adding vectorially the velocity components of each ball, we obtain

$V'_A = 23.2$ m/s 

, $V'_B = 41.9$ m/s 

Example 119. A ball of mass m hits a floor with a speed v making an angle of incidence θ with normal. The coefficient of restitution is e. Find the speed of reflected ball and the angle of reflection.

Solution Suppose the angle of reflection is θ' and the speed after collision is v'. It is an oblique impact. Resolving the velocity v along the normal and tangent, the components are $v \cos\theta$ and $v \sin\theta$. Similarly, resolving the velocity after reflection along the normal and along the tangent the components are $-v'\cos\theta'$ and $v' \sin\theta'$.

Since there is no tangential action,

$$v \sin \theta = v' \sin \theta' \quad ...(i)$$

Applying Newton's law for collision,

$$(-v' \cos\theta' - 0) = -e(v \cos\theta - 0)$$

$$\therefore \quad v'\cos\theta' = ev \cos\theta \quad ...(ii)$$

From equations (i) and (ii),

$$v'^2 = v^2 \sin^2\theta + e^2 v^2\cos^2\theta$$

$$v' = \sqrt{v^2 \sin^2\theta + e^2 v^2 \cos^2\theta}$$

$$v' = (v\sqrt{\sin^2\theta + e^2\cos^2\theta}) \text{ and } \tan\theta' = \frac{\tan\theta}{e}$$

$$\theta' = \tan^{-1}\left(\frac{\tan\theta}{e}\right).$$

Example 120. A disc A of radius r moving on perfectly smooth surface at a speed v undergoes an elastic collision with an identical stationary disc B. Find the velocity of the disk B after collision if the impact parameter is d as shown in figure.

Solution One of the discs is at rest before impact. After the impact its velocity will be in the direction of the centre line at the moment of contact because this is the direction in which the force acted on it.

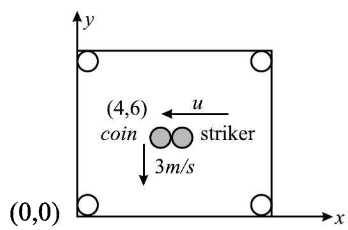
Thus, $\sin\alpha_2 = \dfrac{d}{2r}$ $\quad \alpha_1 + \alpha_2 = \dfrac{\pi}{2}$

Since the masses of both disks are equal, the triangle of momenta turns into triangle of velocities.

we have $V_1 = v\cos\alpha_1 = v\sin\alpha_2 = \dfrac{vd}{2r}$

$$v_2 = v\cos\alpha_2 = v\sqrt{1 - \frac{d^2}{4r^2}}$$

Example 121. On a smooth carom board, a coin moving in negative y-direction with a speed of 3 m/s is being hit at the point (4, 6) by a striker moving along negative x-axis. The line joining centres of the coin and the striker just before the collision is parallel to x-axis. After collision the coin goes into the hole located at the origin. Masses of the striker and the coin are equal. Considering the collision to be elastic, the initial and final speeds of the striker in m/s will be:



(A) (1.2, 0) (B) (2, 0)
(C) (3, 0) (D) none of these

Solution The line of impact for duration of collision is parallel to x-axis.

The situation of striker and coin just before the collision is given as

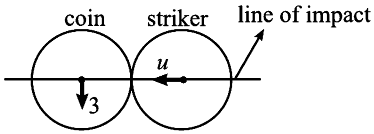


Figure (A)
before collision

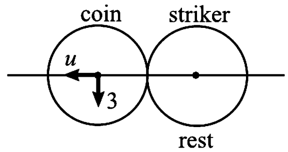
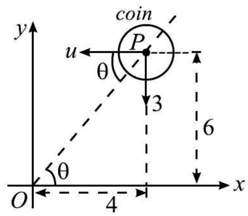


Figure (B)
after collision

Because masses of coin and striker are same, their components of velocities along line of impact shall exchange. Hence the striker comes to rest and the *x*-*y* component of velocities of coin are *u* and 3 m/s as shown in figure.

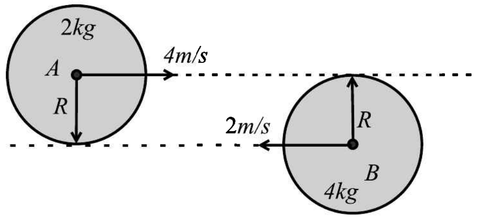


For coin to enter hole,

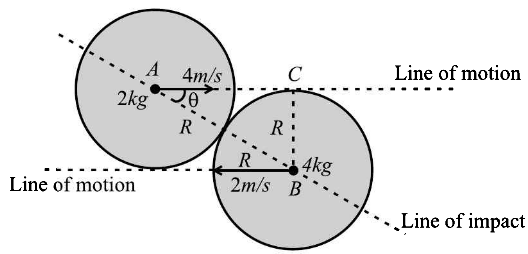
its velocity must be along *PO* $\therefore \tan\theta = \frac{6}{4} = \frac{3}{u}$ or $u = 2$ m/s

$\therefore$ (B) is the right answer.

***Example* 122.** Two spheres are moving towards each other. Both have same radius but their masses are 2 kg and 4 kg. If the velocities are 4 m/s and 2 m/s respectively and coefficient of restitution is $e = 1/3$, find.



(a) The common velocity along the line of impact.
(b) Final velocities along line of impact.
(c) Impulse of deformation.
(d) impulse of reformation.
(e) Maximum potential energy of deformation.
(f) Loss in kinetic energy due to collision.



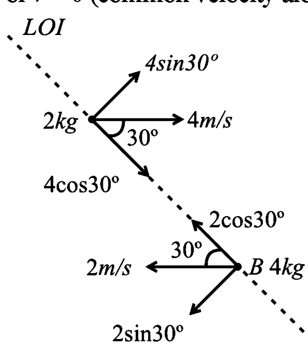
Solution In ΔABC $\sin\theta = \frac{BC}{AB} = \frac{R}{2R} = \frac{1}{2}$

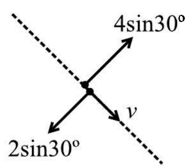
or $\theta = 30^\circ$

(a) By conservation of momentum along line of impact.

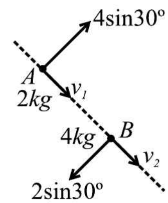
$2(4\cos 30^\circ) - 4(2\cos 30^\circ) = (2+4)v$

or $v = 0$ (common velocity along LOI)





Just Before Collision Along LOI Maximum Deformed State



Just After Collision Along LOI

(b) Let v_1 and v_2 be the final velocity of A and B respectively then, by conservation of momentum along line of impact,

$$2(4\cos 30^\circ) - 4(2\cos 30^\circ) = 2(v_1) + 4(v_2)$$

or $\quad 0 = v_1 + 2v_2 \quad \ldots (1)$

By coefficient of restitution,

$$e = \frac{\text{velocity of separation along LOI}}{\text{veloctiy of approach along LO I}}$$

or $\quad \dfrac{1}{3} = \dfrac{v_2 - v_1}{4\cos 30^\circ + 2\cos 30^\circ}$

or $\quad v_2 - v_1 = \sqrt{3} \quad \ldots (2)$

from the above two equations,

$$v_1 = \frac{2}{\sqrt{3}} m/s \text{ and } v_2 = \frac{1}{\sqrt{3}} m/s$$

(c) $J_D = m_1(v - u_1) = 2(0 - 4\cos 30^\circ) = -4\sqrt{3}\ N\text{-}s$

(d) $J_R = eJ_D = \dfrac{1}{3}(-4\sqrt{3}) = -\dfrac{4}{\sqrt{3}}\ N\text{-}s$

(e) Maximum potential energy of deformation is equal to loss in kinetic energy during deformation upto maximum deformed state,

$$U = \frac{1}{2} m_1(u_1 \cos\theta)^2 + \frac{1}{2} m_2(u_2 \cos\theta)^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2} 2(4\cos 30^\circ)^2 + \frac{1}{2} 4(-2\cos 30^\circ)^2 - \frac{1}{2}(2 + 4)(0)^2$$

or $\quad U = 18$ Joule.

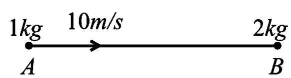
(f) Loss in kinetic energy,

$$\Delta KE = \frac{1}{2} m_1(u_1 \cos\theta)^2 + \frac{1}{2} m_2(u_2 \cos\theta)^2 - \left(\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2\right)$$

$$= \frac{1}{2}\ 2(4\cos 30^\circ)^2 + \frac{1}{2}\ 4(-2\cos 30^\circ)^2 - \left(\frac{1}{2} 2\left(\frac{2}{\sqrt{3}}\right)^2 + \frac{1}{2} 4\left(\frac{1}{\sqrt{3}}\right)^2\right)$$

$\Delta KE = 16$ Joule

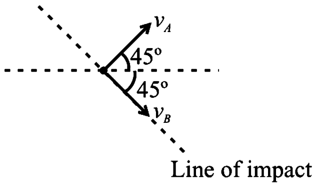
Example 123. Two point particles A and B are placed in line on a friction less horizontal plane. If particle A (mass 1 kg) is moved with velocity 10 m/s towards stationary particle B (mass 2 kg) and after collision the two move at an angle of 45° with the initial direction of motion, then find:

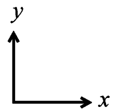


(a) Find velocities of A and B just after collision.

(b) Coefficient of restitution.

Solution The very first step to solve such problems is to find the line of impact which is along the direction of force applied by *A* on *B*, resulting the stationary *B* to move. Thus, by watching the direction of motion of *B*, line of impact can be determined. In this case line of impact is along the direction of motion of *B*. i.e. 45° with the initial direction of motion of *A*.





(a) By conservation of momentum, along *x* direction :

$m_A u_A = m_A v_A \cos 45^\circ + m_B v_B \cos 45^\circ$

or $1(10) = 1(v_A \cos 45^\circ) + 2(v_B (\cos 45^\circ)$

or $v_A + 2v_B = 10\sqrt{2}$ ... (1)

along y direction

$0 = m_A v_A \sin 45^\circ + m_B v_B \sin 45^\circ$

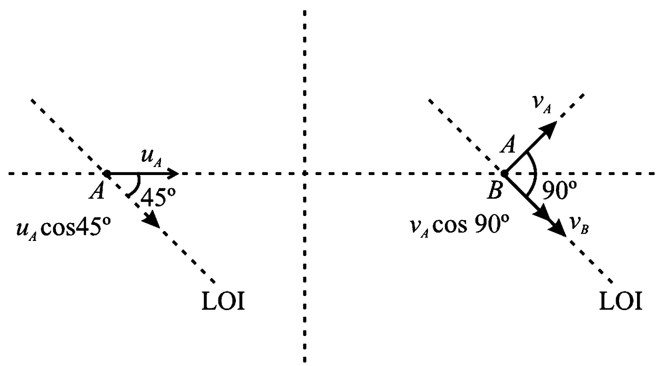
or $0 = 1(v_A \sin 45^\circ) - 2(v_B \sin 45^\circ)$

or $v_A = 2v_B$ ... (2)

solving the two equations,

$v_A = \dfrac{10}{\sqrt{2}}$ m/s and $v_B = \dfrac{5}{\sqrt{2}}$ m/s.

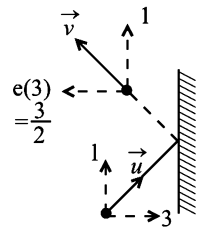
(b) $e = \dfrac{\text{velocity of separation along LOI}}{\text{veloctiy of approach along LO I}}$



Before collision *After collision*

or $e = \dfrac{v_B - v_A \cos 90^\circ}{u_A \cos 45^\circ} = \dfrac{\dfrac{5}{\sqrt{2}} - 0}{\dfrac{10}{\sqrt{2}}} = \dfrac{1}{2}$

Example 124. A smooth sphere of mass m is moving on a horizontal plane with a velocity $3\hat{i} + \hat{j}$ when it collides with a vertical wall which is parallel to the vector $\hat{j}$. If the coefficient of restitution between the sphere and the wall is $\frac{1}{2}$, find



(a) the velocity of the sphere after impact,

(b) the loss in kinetic energy caused by the impact.

(c) the impulse $\vec{J}$ that acts on the sphere.

Solution Let $\vec{v}$ be the velocity of the sphere after impact.

To find $\vec{v}$ we must separate the velocity components parallel and perpendicular to the wall.

Using the law of restitution the component of velocity parallel to the wall remains unchanged while component perpendicular to the wall becomes e times in opposite direction.

Thus, $\vec{v} = -\frac{3}{2}\hat{i} + \hat{j}$

(a) Therefore, the velocity of the sphere after impact is = $-\frac{3}{2}\hat{i} + \hat{j}$

(b) The loss in $K.E.$ = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(3^2 + 1^2) - \frac{1}{2}m\left(\left\{\frac{3}{2}\right\}^2 + 1^2\right) = \frac{27}{8}m$

(c) $\vec{J} = \Delta\vec{P} = \vec{P}_f - \vec{P}_i = m(\vec{v}) - m(\vec{u}) = m\left(-\frac{3}{2}\hat{i} + \hat{j}\right) - m\left(3\hat{i} + \hat{j}\right) = -\frac{9}{2}m\hat{i}$

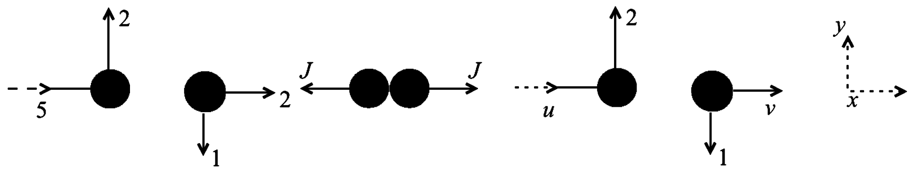
Example 125. A sphere of mass m is moving with a velocity $4\hat{i} - \hat{j}$ when it hits a wall and rebounds with velocity $\hat{i} + 3\hat{j}$. Find the impulse it receives. Find also the coefficient of restitution between the sphere and the wall.

Solution $\vec{J} = m(-3\hat{i} + 4\hat{j})$ and $e = \frac{9}{16}$

Example 126. Two smooth spheres, A and B, having equal radii, lie on a horizontal table. A is of mass m and B is of mass $3m$. The spheres are projected towards each other with velocity vector $5\hat{i} + 2\hat{j}$ and $2\hat{i} - \hat{j}$, respectively and when they collide the line joining their centres is parallel to the vector $\hat{i}$.

If the coefficient of restitution between A and B is $\frac{1}{3}$, find the velocities after impact and the loss in kinetic energy caused by the collision. Find also the magnitude of the impulses that act at the instant of impact.

Solution The line of centres at impact, is parallel to the vector $\hat{i}$, the velocity components of A and B perpendicular to $\hat{i}$ are unchanged by the impact.



Applying conservation of linear momentum and the law of restitution, we have

in x direction $5m + (3m)(2) = mu + 3\,mv$ .... (i)

and $\frac{1}{3}(5-2) = v - u$ ...(ii)

Solving these equations, we have $u = 2$ and $v = 3$

The velocities of A and B after impact are therefore,

$2\hat{i} + 2\hat{j}$ and $3\hat{i} - \hat{j}$ respectively **Ans.**

Before impact the kinetic energy of A is

$$\frac{1}{2}m(5^2 + 2^2) = \frac{29}{2}\,m$$

and of B is $\frac{1}{2}(3m)(2^2 + 1^2) = \frac{15}{2}\,m$

After impact the kinetic energy of A is

$$\frac{1}{2}m(2^2 + 2^2) = 4m$$

and of B is $\frac{1}{2}(3\text{m})(3^2 + 1^2) = 15\,m$

Therefore, the loss in $K.E.$ at impact is

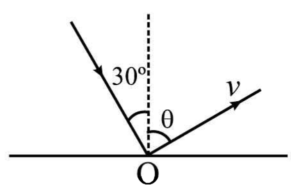
$$\frac{29}{2}m + \frac{15}{2}m - 4m - 15m = 3m$$

To find value of J, we consider the change in momentum along $\hat{i}$ for one sphere only.

For sphere B$J = 3m\,(3-2)$

or $J = 3m$

Example 127. A ball with a velocity of 4 ms^{-1} impinges at an angle of 30° with the vertical on a smooth horizontal fixed plane. If the coefficient of restitution is 0.5, find its velocity and direction of motion after the first impact.



Solution Let the ball be reflected at angle θ with the vertical with speed v. Since the plane is smooth, there is no interaction along the horizontal and so that momentum of the ball along this direction remains unchanged.

$\therefore$ $4\cos 60^\circ = v\cos(90^\circ - \theta)$ or $2 = v\sin\theta$ ...(i)

The velocity of the ball relative to the plane before collision = $4\cos 30^\circ - 0 = 4\cos 30^\circ$

The velocity of the ball relative to the plane after collision = $(-v\cos\theta) - 0 = -v\cos\theta$

By Newton's law of collision

$-v\cos\theta = -0.5\times 4\cos 30r$

or $v\cos\theta = 2\times\frac{\sqrt{3}}{2} = \sqrt{3}$ ...(ii)

Squaring and adding (i) and (ii)

$v^2 = 4 + 3 = 7$ or $v = \sqrt{7}ms^{-1}$

Dividing, $\tan\theta = 2/\sqrt{3}$. Thus the ball is reflected with velocity $\sqrt{7}ms^{-1}$ at $\tan^{-1} = 2/\sqrt{3}$ with the vertical.

Example 128. Consider a one-dimensional elastic collision between an incoming body *A* and a body *B* initially at rest. How would you choose the mass of *B*, in comparison to the mass of *A* in order that *B* should recoil with

(a) the greatest speed
(b) the greatest momentum, the
(c) the greatest kinetic energy?

Solution By the conservation of momentum

$m_1v_1 = m_1v'_1 + m_1v_2$ or $m_1(v_1 - v'_1) = m_2v_2$

By the conservation of kinetic energy

$\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2^2$ or $m_1(v_1 - v_1')(v_1 + v_1') = m_2v_2^2$

$\therefore$ $v_1 + v_1' = v_2$ $(\because v_1 \neq v_1')$

$v_1 - v_1' = \frac{m_2}{m_1}v_2$

Adding $2v_1 = \left(1 + \frac{m_2}{m_1}\right)v_2$ or $v_2 = \frac{2v_1}{1 + (m_2/m_1)}$

(a) v_2 is maximum when m_2 is negligible in comparison to m_1.

(b) $p_2 = r_2v_2 = \frac{2m_2v_1}{1+(m_2/m_1)} = \frac{2m_1m_2v_1}{m_1+m_2} = \frac{2m_1v_1}{1+(m_1/m_2)}$

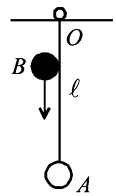
Obviously p_2 is maximum when m_2 is large in comparison to m_1.

(c) $T_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_2\frac{4v_1^2}{(1+(m_2/m_1))^2} = \frac{2m_2v_1^2m_1^2}{(m_1+m_2)^2} = \frac{2m_1^2v_1^2}{\left(\frac{m_1}{\sqrt{m_2}} + \sqrt{m_2}\right)^2}$

Now $\left(\frac{m_1}{\sqrt{m_2}} + \sqrt{m_2}\right)^2 = \left(\frac{m_1}{\sqrt{m_2}} - \sqrt{m_2}\right)^2 + 4m_1$

Obviously T_2 is maximum when $\left(\frac{m_1}{\sqrt{m_2}}+\sqrt{m_2}\right)^2$ is minimum but it is minimum when $\left(\frac{m_1}{\sqrt{m_2}}-\sqrt{m_2}\right)^2$ is minimum. This a positive quantity. Its minimum value is zero.

$\therefore$ T_2 is maximum when $\frac{m_1}{\sqrt{m_2}}-\sqrt{m_2}=0$, or $m_1 = m_2$ that is, when the masses are equal.

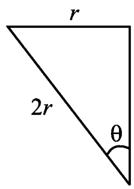


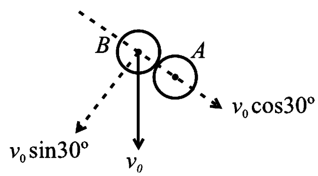
Example 129. A small steel ball A is suspended by an inextensible thread of length $\ell = 1.5$ from O. Another identical ball is thrown vertically downwards such that its surface remains just in contact with thread during downward motion and collides elastically with the suspended ball. If the suspended ball just completes vertical circle after collision, calculate the velocity of the falling ball just before collision. ($g = 10$ ms^{-2})

Solution Velocity of ball A just after collision is $\sqrt{5gl}$

Let radius of each ball be r and the joining centres of the two balls makes an angle θ with the vertical at the instant of collision, then

$\sin\theta = \frac{r}{2r} = \frac{1}{2}$ or θ = 30ŗ





Let velocity of ball B (just before collision) be v_0. This velocity can be resolved into two components, (i) v_0 cos 30ŗ, along the line joining the centre of the two balls and (ii) v_0 sin 30ŗ normal to this line. Head -on collision takes place due to v_0 cos 30ŗ and the component v_0 sin 30ŗ of velocity of ball B remains unchanged.

Since, ball A is suspended by an inextensible string, therefore, just after collision, it can move along horizontal direction only. Hence, a vertically upward impulse is exerted by thread on the ball A. This means that during collision two impulses act on ball A simultaneously. One is impulsive interaction J between the balls and the other is impulsive reaction J' of the thread.

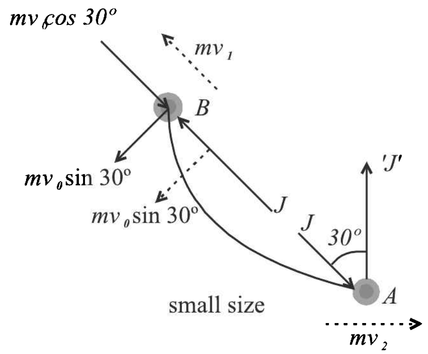
Velocity v_1 of ball B along line of collision is given by

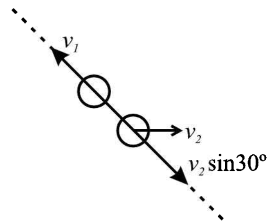
J – mv_0 cos 30ŗ = mv_1

or $v_1 = \frac{J}{m} - v_0$ cos 30ŗ ...(i)

Horizontal velocity v_2 of ball A is given by J sin 30ŗ = mv_2

or $v_2 = \frac{J}{2m}$ ...(ii)





Since, the balls collide elastically, therefore, coefficient of restitution is $e = 1$.

Hence, $e = \dfrac{v_2 \sin 30^\circ - (-v_1)}{v_0 \cos 30^\circ - 0} = 1$ ...(iii)

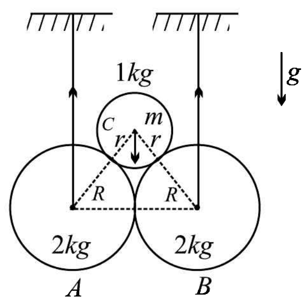
Solving Eqs. (i), (ii), and (iii), $J = 1.6\, mv_0 \cos 30^\circ$

$\therefore \quad v_1 = 0.6\, v_0 \cos 30^\circ$ and $v_2 = 0.8\, v_0 \cos 30^\circ$

Since, ball A just completes vertical circle, therefore $v_2 = \sqrt{5g\ell}$

$\therefore \quad 0.8 v_0 \cos 30^\circ = \sqrt{5g\ell}$ or $v_0 = 12.5\ \text{ms}^{-1}$

Example 130. Two identical balls A & B each of mass 2 kg & radius R are suspended vertically from inextensible strings as shown. Third ball C of mass 1 kg & radius $r = (\sqrt{2} - 1)R$ falls & hits A & B symmetrically with 10 m/s. Speed of both A & B just after the collision is 3 m/s.



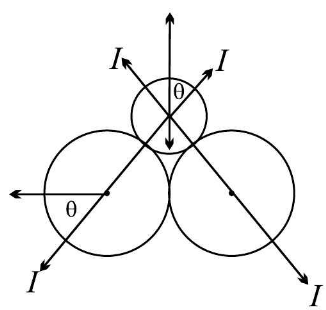
Example 131. Speed of C just after collision is

(A) 2 m/s (B) $2\sqrt{2}$ m/s

(C) 5 m/s (D) $(\sqrt{2} - 1)$ m/s

Solution As the balls A & B are constrained to move horizontally, if 'I' be the impulse imparted by ball 'C' to each of A & B, the impulse recieved by ball C from then would be $2I\cos\theta$.

Now, each of ball B & C received impulse 'I' as shown, but moves horizontally as its vertical as its vertical comp. gets balanced by impulse imparted to ball B & C by the respective strings & hence, $I\cos\theta = M_A V_A = M_B V_B$



$\Rightarrow \quad I = \dfrac{M_A V_A}{\cos\theta}$ (I = magnitude of Impulse)

Now, for ball C, if its final velocity is V_c' downwards,

we have $\quad M_c V_c' = M_c V_c - 2I\cos\theta$

$\Rightarrow \quad V_c' = V_c - 2\dfrac{M_A}{M_C} V_A$

$= -2$ m/s (–ve sign indicates that $\vec{V}_c'$ is directed upwards)]

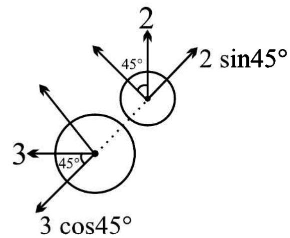
∴ (A) is the correct option.

Example 132. Impulse provided by each string during collision is

(A) $6\sqrt{2}\,N$ sec. (B) $12\,N$ sec. (C) $3\sqrt{2}\,N$ sec. (D) $6\,N$ sec.

Solution Impulse provided by each string $I\cos\theta = 6\,N$ sec.

Example 133. The value of coefficient of restitution is



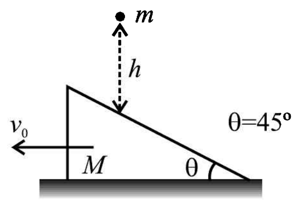
(A) $\frac{1}{4}$ (B) $\frac{1}{\sqrt{2}}$

(C) $(\sqrt{2}-1)$ (D) $\frac{1}{2}$

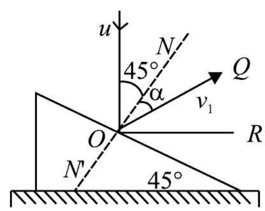
Solution $$e = \frac{2\sin 45° + 3\cos 45°}{10\sin 45°} = \frac{1}{2}$$

∴ (D) is correct.

Example 134. A triangular wedge of mass M is moving with uniform velocity v_0 along a smooth horizontal surface in the leftward direction. A particle of mass m falls from rest from h on to the inclined face, colliding elastically with it. Find velocity of the ball and wedge after the impact taking $M = 2$m.



Solution Impact line is a straight line perpendicular to the incline. Normal reaction force between the body and the wedge acts along the impact line. This normal force becomes an internal force when we consider (wedge + body) as a total system. But, normal reaction and considerable in magnitude because the impact force during collision has contribution towards making of the normal force.



→ Momentum of the system is conserved along a line perpendicular to this normal force and momentum of the system is conserved along horizontal. ...(i)

→ Momentum of the body/particle is conserved along the common tangent at the point of impact. ...(ii)

→ As this is an elastic collision, relative velocity of separation along the impact line = relative velocity of approach. ...(iii)

→ If the wedge were at rest then the body /particle would deflect in the horizontal direction after collision because it is an elastic collision ...(iv)

→ As, in this case, wedge is moving, the object would not be deflected horizontally, but at an angle ' α ' to the impact ...(v)

Velocity of body/particle before impact = $u = \sqrt{2gh}$ along $\overrightarrow{PQ}$

Velocity of body/particle after collision = v_1 along $\overrightarrow{OQ}$

Velocity of wedge before collision = v_0 along $\overrightarrow{RO}$

Velocity of wedge after collision = v_2 along $\overrightarrow{RO}$

$v_0 \cos 45^0 - u \cos 45^0 = -v_1 \cos\alpha - v_2 \cos 45^0$

$\Rightarrow v_1 \cos\alpha + \frac{v_2}{\sqrt{2}} = \frac{u}{\sqrt{2}} - \frac{v_0}{\sqrt{2}}$

$\sqrt{2}\, v_1 \cos\alpha + v_2 = u - v_0$ ...(A)

According to logic (i)

$Mv_0 = Mv_2 - mv_1 \cos(45^0 - \alpha)$

$\Rightarrow 2v_0 = 2v_2 - v_1 \cos(45^0 - \alpha)$

$\Rightarrow 2v_2 - v_1 \cos(45^0 - \alpha) = 2v_0$ ...(B)

According to logic (ii)

$mu \sin 45^0 = mv_1 \sin\alpha$

$\Rightarrow v_1 \sin\alpha = u$

solving $v_1 = \sqrt{u^2\left[\frac{u(4-\sqrt{2}) - 6v_0}{20}\right]}$ and $v_2 = \frac{v_0 + (\sqrt{2}+1)a}{5}$.

VARIABLE MASS SYSTEM

If a mass is added or ejected from a system, at rate μ kg/s and relative velocity $\vec{v}_{rel}$ (w.r.t. the system), then the force exerted by this mass on the system has magnitude $\mu|\vec{v}_{rel}|$.

Thrust Force $\vec{F}_t = \vec{v}_{ref}\left(\frac{dm}{dt}\right)$

Suppose at some moment $t = t$ mass of a body is m and its velocity is $\vec{v}$. After some time at $t = t +$ dt its mass becomes $(m - dm)$ and velocity becomes $\vec{v} + d\vec{v}$. The mass dm is ejected with relative velocity $\vec{v}_r$. Absolute velocity of mass 'dm' is therefore $(\vec{v} + \vec{v}_r)$. If no external forces are acting on the system, the linear momentum of the system will remain conserved, or

$\vec{P}_i = \vec{P}_f$

or $m = (m - dm)(\vec{v} + d\vec{v}) + dm(\vec{v} + v_r)$

or $m\vec{v} = m\vec{v} + md\vec{v} - (dm)\vec{v} - (dm)(d\vec{v}) + (dm)\vec{v} + v_r\, dm$

The term $(dm)(d\vec{v})$ is too small and can be neglected.

$\therefore md\vec{v} = -\vec{v}_r\, dm$

or $m\left(\frac{d\vec{v}}{dt}\right) = \vec{v}_r\left(-\frac{dm}{dt}\right)$

Here, $m\left(-\frac{d\vec{v}}{dt}\right)$ = thrust force $(\vec{F}_t)$ and $-\frac{dm}{dt}$ = rate at which mass is ejecting or $\vec{F}_t = \vec{v}_r \left(\frac{dm}{dt}\right)$

Problems Related to Variable Mass can be Solved in Following Four Steps

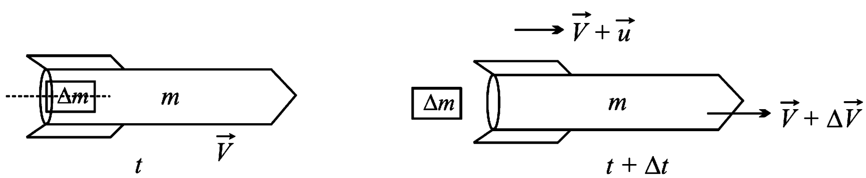
1. Make a list of all the forces acting on the main mass and apply them on it.
2. Apply an additional thrust force $\vec{F}_t$ on the mass, the magnitude of which is $\left|\vec{v}_r\left(\pm\frac{dm}{dt}\right)\right|$ and direction is given by the direction of $\vec{v}_r$ in case the mass is increasing and otherwise the direction of $-\vec{v}_r$ if it is decreasing.
3. Find net force on the mass and apply
$$\vec{F}_{net} = m\frac{d\vec{v}}{dt} \quad (m = \text{mass at the particular instant})$$
4. Integrate it with proper limits to find velocity at any time t.

Note

Problems of one-dimensional motion (which are mostly asked in JEE) can be solved in easier manner just by assigning positive and negative signs to all vector quantities. Here are few example in support of the above theory.

System of Variable Mass

Lets consider a rocket experiencing external force $\vec{F}$. From time t to $t + \Delta t$ a mass of fuel Δm is turned and expelled as gas with velocity $\vec{u}$ rel. to the rocket. (generally u is constant & is independent of velocity of rocket).



$$\vec{P}_{(t)} = (M + \Delta m)\,\vec{V}$$

$$\vec{P}_{(t+\Delta t)} = M(\vec{V} + \Delta\vec{V}) + \Delta m\,(\vec{V} + \Delta\vec{V} + \vec{u})$$

$$\Delta\vec{P} = \vec{P}\,(t + \Delta t) - \vec{P}(t) = \Delta\vec{P} = M\Delta\vec{V} + \Delta m\vec{u}$$

$$\frac{d\vec{P}}{dt} = \lim_{\Delta t \to 0}\left[M\frac{\Delta\vec{V}}{\Delta t} + \vec{u}\left(\frac{\Delta m}{\Delta t}\right)\right] = m\frac{d\vec{V}}{dt} + \vec{u}\frac{dm}{dt}$$

Notes

1. $\vec{u}$ is defined +ve in direction of $\vec{V}$, but generally it is opposite.
2. $\frac{dm}{dt}$ represents rate of releasing mass (r).

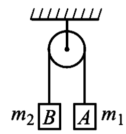
$$\frac{dm}{dt} = -\frac{dm}{dt} \text{ or } \frac{dm}{dt} = -\text{r}$$

where $\frac{dm}{dt}$ represents rate of change of mass of rocket.

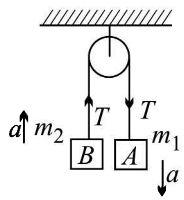
$$\frac{d\vec{P}}{dt} = \vec{F}_{ext}$$

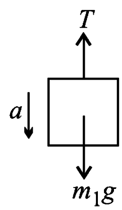
$$\vec{F}_{ext} = \frac{md\vec{V}}{dt} - \vec{u}\frac{dm}{dt}$$

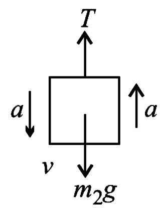
Example 135. In the figure block A has mass m_1 (constant) and B has initial mass m_0. B is filled with sand which is thrown out by some internal mechanism at constant rate μ kg/sec at velocity v relative to B in downward direction. Assuming $m_1 \geq m_0$, light string and pulley, no friction in pulley and motion in vertical plane find acceleration of A.



Solution







here $m_2 = m_0 - \mu t$

$$m_1 g - T = m_1 a \quad \text{...(1)} \quad \longrightarrow \text{(for } A\text{)}$$

$$m\frac{dv}{dt} = F_{ext} + u\frac{dm}{dt} \quad \longrightarrow \text{(for } B\text{)}$$

$$m_2 a = (T - m_2 g) - v(-\mu)$$

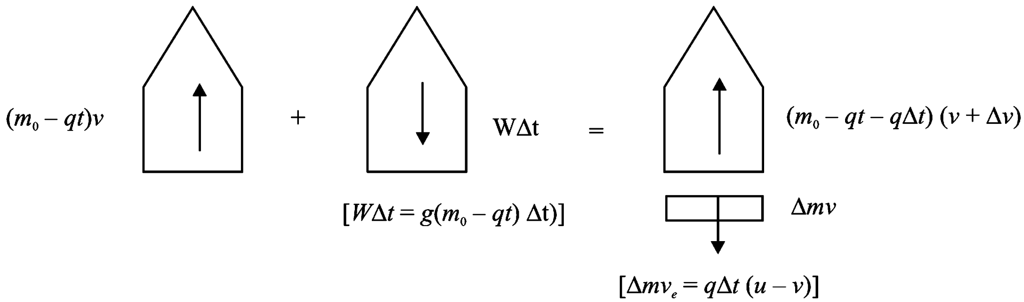
$$m_2 a = T - m_2 g + \mu v \quad \text{... (2)}$$

solving (1) & (2) $\Rightarrow$ $a = \frac{(m_1 - m_2)\, g + \mu v}{m_1 + m_2}$

[Ans. $\frac{(m_1 - m_2)\, g + \mu v}{m_1 + m_2}$; $m_2 = m_0 - \mu\, t$]

Example 136. A rocket of initial mass m_0 (including shell and fuel) is fired vertically at time $t = 0$. The fuel is consumed at a constant rate $q = dm/dt$ and is expelled at a constant speed u relative to the rocket. Derive an expression for the magnitude of the velocity of the rocket at time t, neglecting the resistance of the air and variation of acceleration due to gravity.

Solution At time t, the mass of the rocket shell and remaining fuel is $m = m_0 - qt$, and the velocity is v. During the time interval Δt, *a* mass of fuel $\Delta m = q\,\Delta t$ is expelled with a speed u relative to the rocket. Denoting by v_e the absolute velocity of expelled fuel, we apply the principle of impulse and we write



$$(m_0 - qt)v - g(m_0 - qt)\,\Delta\, t = (m_0 - qt - q\Delta t)(v + \Delta v) - q\Delta t(u - v)$$

Dividing throughout by Δt and letting Δt approach zero, we obtain

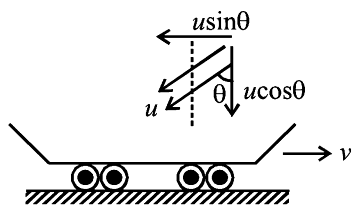
$$-g(m_0 - qt) = (m_0 - qt)\frac{dv}{dt} - qu$$

Separating variables and integrating from $t = 0$, $v = 0$ to $t = t$, $v = v$

$$dv = \left(\frac{du}{m_0 - qt} - g\right)dt \qquad \Rightarrow \int_0^v dv = \int_0^v dv\left(\frac{au}{m_0 - qt} - g\right)dt$$

$$\Rightarrow \qquad v = [u\ln(m_0 - qt) - \mathrm{g}t]_0^t \qquad \therefore\ v = u\ln\left(\frac{m_0}{m_0 - qt}\right) - gt$$

Example 137. A freight car is moving on smooth horizontal track without any external force. Rain is falling with a velocity u m/s at an angle θ with the vertical. Rain drops are collected in the car at the rate of m kg/s. If initial mass of the car is m_0 and velocity v_0 then find its velocity after time t.



Solution After time t mass of the car with water is $m_t = (m_0 + mt)$kg. Let at that momentum speed of the car be v.

$$\therefore \qquad m_t\frac{dv}{dt} = f_{ext} + v_{rel}\frac{dm}{dt}$$

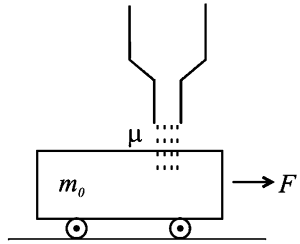
$$(m_0 + mt)\left(\frac{-dv}{dt}\right) = 0 + (u\sin\,\theta\, + v)m$$

$$\Rightarrow \qquad \int_{v_0}^{v}\frac{dv}{(u\sin\theta + v)} = -\int_0^t\frac{dt}{\left(\frac{m_0}{\mu} + t\right)} \Rightarrow \ln\left(\frac{u\sin\theta + v}{u\sin\theta + v_0}\right) = -\ln\left[\left(\frac{\frac{m_0}{\mu} + t}{m_0/\mu}\right)\right]$$

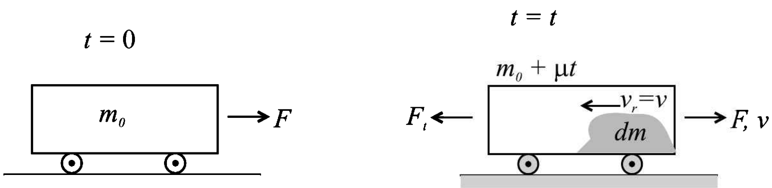
$$\Rightarrow \quad \frac{u\sin\theta + v}{u\sin\theta + v_0} = \frac{m_0}{m_0 + \mu t} \Rightarrow v = u\sin\theta\left(\frac{m_0}{m_0 + \mu t} - 1\right) + \frac{m_0 v_0}{(m_0 + \mu t)}$$

$$\Rightarrow \quad v = u\sin\theta\left(\frac{\mu t}{m_0 + \mu t}\right) + \frac{\mu_0 v_0}{(m_0 + \mu t)}.$$

Example 138. A flat car of mass m_0 starts moving to the right due to a constant horizontal force F. Sand spills on the flat car from a stationary hopper. The rate of loading is constant and equal to μ kg/s. Find the time dependence of the velocity and the acceleration of the flat car in the process of loading. The friction is negligibly small.



Solution Initial velocity of the flat car is zero. Let v be its velocity at time t and m its mass at that instant. Then



At $t = 0$, $v = 0$ and $m = m_0$ at $t = t$, $v = v$ and $m = m_0 + \mu t$

Here, $v_r = v$ (backwards)

$$\frac{dm}{dt} = \mu \qquad \therefore \qquad F_t = v_r\frac{dm}{dt} = \mu v \text{ (backwards)}$$

Net force on the flat car at time t is $F_{net} = F - F_t$

$$\text{or} \quad m\frac{dv}{dt} = F - \mu v \qquad \text{....(i)}$$

$$\text{or} \quad (m_0 + \mu t)\frac{dv}{dt} = F - \mu v \qquad \text{or} \quad \int_0^v \frac{dv}{F - \mu v} = \int_0^t \frac{dt}{m_0 + \mu t}$$

$$\therefore \quad -\frac{1}{\mu}[\ell n\,(F - \mu v)]_0^v = \frac{1}{\mu}[\ell n\,(m_0 + \mu t)]_0^t$$

$$\Rightarrow \quad \ell n\left(\frac{F}{F - \mu v}\right) = \ell n\left(\frac{m_0 + \mu t}{m_0}\right) \therefore \frac{F}{F - \mu v} = \frac{m_0 + \mu t}{m_0}$$

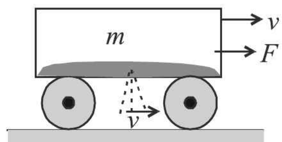
$$\text{or} \quad v = \frac{Ft}{m_0 + \mu t} \quad \textbf{Ans.}$$

From Eq. (i), $\frac{dv}{dt}$ = acceleration of flat car at time t

$$\text{or} \quad \frac{F - \mu v}{m}$$

$$a = \left(\frac{F - \dfrac{F\mu t}{m_0 + \mu t}}{m_0 + \mu t} \right) \quad \text{or} \quad a = \frac{Fm_0}{(m_0 + \mu t)^2}$$

Example 139. A cart loaded with sand moves along a horizontal floor due to a constant force F coinciding in direction with the cart's velocity vector. In the process sand spills through a hole in the bottom with a constant rate μkg/s. Find the acceleration and velocity of the cart at the moment t, if at the initial moment $t = 0$ the cart with loaded sand had the mass m_0 and its velocity was equal to zero. Friction is to be neglected.



Example In this problem the sand through a hole in the bottom of the cart. Hence, the relative velocity of the sand v_r will be zero because it will acquire the same velocity as that of the cart at the moment.

$$v_r = 0$$

Thus, $$F_t = 0 \left(as\ F_t = v_r \frac{dm}{dt} \right)$$

and the net force will be F only.

$\therefore$ $$F_{net} = F$$

or $$m\left(\frac{dv}{dt}\right) = F \qquad \text{....(i)}$$

But here $m = m_0 - \mu t$

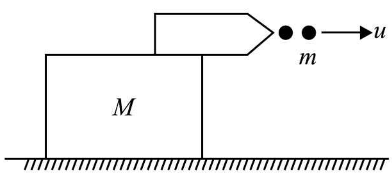
$\therefore$ $$(m_0 - \mu t)\frac{dv}{dt} = F \quad \text{or} \quad \int_0^v dv = \int_0^t \frac{F\,dt}{m_0 - \mu t}$$

$\therefore$ $$v = \frac{F}{-\mu}[\ln(m_0 - \mu t)]_0^t \quad \text{or } v = \frac{F}{\mu}\ln\left(\frac{m_0}{m_0 - \mu t}\right) \quad \textbf{Ans.}$$

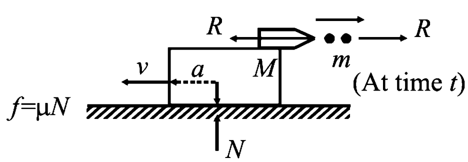
From Eq. (i), acceleration of the cart

$$a = \frac{dv}{dt} = \frac{F}{m} \quad \text{or } a = \frac{F}{m_0 - \mu t} \quad \textbf{Ans.}$$

Example 140. A cannon with shots of total mass M_0 is kept on a rough horizontal surface. The coefficient of friction between the cannon and the horizontal surface is μ. If the cannon fires the shots with a velocity u relative to it, find the velocity of the cannon when it possesses a total mass M with the remaining shots, after time t from starting. Assume that the cannon fires shots at the same fequency.



Solution Because each shot of mass m (say) leaves the cannon with a relative velocity u with a frequency n, the rate of loss of mass of the system is given as



$$r = \frac{dm}{dt} = mn$$

⇒ the impact force $R = u\frac{dm}{dt} = mnu$

⇒ The net force acting on the system after a time t

⇒ $F = R - f \Rightarrow Ma = mnu - \mu N$ where f is the force of friction

where $N = Mg \Rightarrow Ma = mnu - \mu Mg$ ...(i)

M = mass of the cannon with the shots remaining inside it after a time t given as

$M = M_0 - rt$

⇒ $M = M_0 - mnt$ ...(ii)

using (i) and (ii) $a = \dfrac{mnu}{M_0 - mnt} - \mu g$

Integrating both sides $\int_0^v dv = mnu\int_0^t \frac{dt}{M_0 - mnt} - \mu g\int_0^t dt$

$$v = mnu\left[-\frac{1}{mn}\ln|M_0 - mnt|\right]_0^t - \mu gt$$

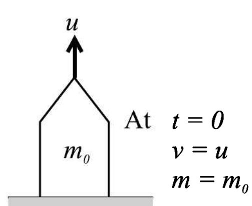
⇒ $$v = -u\ln\left(\frac{M_0 - mnt}{M_0}\right) - \mu gt = u\ln\left(\frac{M_0}{M_0 - mnt}\right) - \mu gt$$

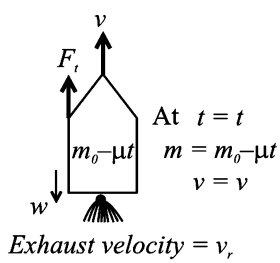
Since after a time t, the cannon + remaining shots has mass M

⇒ $M_0 - mnt = M$ ⇒ $v = u\ln(M_0/M) - \mu gt$.

Rocket Propulsion

Let m_0 be the mass of the rocket at time $t = 0$. m its mass at any time t and v its velocity at that moment. Initially, let us suppose that the velocity of the rocket is u.





Further, let $\left(\dfrac{-dm}{dt}\right)$ be the mass of the gas ejected per unit time and v_r the exhaust velocity of the gases with respect to rocket. Usually $\left(\dfrac{-dm}{dt}\right)$ and v_r are kept constant throughout the journey of the rocket. Now, let us write few equations which can be used in the problems of rocket propulsion. At time $t = t$,

1. Thrust force on the rocket $F_t = v_r \left(\frac{-dm}{dt}\right)$ (upwards)
2. Weight of the rocket $W = mg$ (downwards)
3. Net force on the rocket $F_{net} = F_t - W$ (upwards)

   or $F_{net} = v_r \left(\frac{-dm}{dt}\right) - mg$

4. Net acceleration of the rocket $a = \frac{F}{m}$

or $\frac{dv}{dt} = \frac{v_r}{m}\left(\frac{-dm}{dt}\right) - g$ or $dv = \frac{v_r}{m}(-dm) - g\,dt$

or $\int_u^v dv = v_r \int_{m_0}^{m} \frac{-dm}{m} - g\int_0^t dt$

Thus, $v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$ ...(i)

Notes

1. $F_t = v_r\left(-\frac{dm}{dt}\right)$ is upwards, as v_r is downwards and $\frac{dm}{dt}$ is negative.
2. If gravity is ignored and initial velocity of the rocket $u = 0$, Eq. (i) reduces to $v = v_r \ln\left(\frac{m_0}{m}\right)$.

Example 141. A rocket, with an initial mass of 1000 kg, is launched vertically upwards from rest under gravity. The rocket burns fuel at the rate of 10 kg per second. The burnt matter is ejected vertically downwards with a speed of 2000 ms^{-1} relative to the rocket. If burning after one minute. Find the maximum velocity of the rocket. (Take g as at 10 ms^{-2})

Solution Using the velocity equation

$$v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$$

Here $u = 0$, $t = 60$s, $g = 10$ m/s^2, $v_r = 2000$ m/s, $m_0 = 1000$ kg

and $m = 1000 - 10 \times 60 = 400$ kg

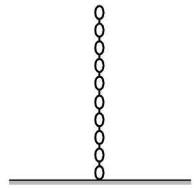
We get $v = 0 - 600 + 2000 \ln\left(\frac{1000}{400}\right)$

or $v = 2000 \ln 2.5 - 600$

The maximum velocity of the rocket is 200(10 ln 2.5 – 3) = 1232.6 ms^{-1}

Example 142. Find the mass of the rocket as a function of time, if it moves with a constant accleration a, in absence of external forces. The gas escaps with a constant velocity u relative to the rocket and its mass initially was m_0.

[Ans. $m = m_0 e^{-at/u}$**]**



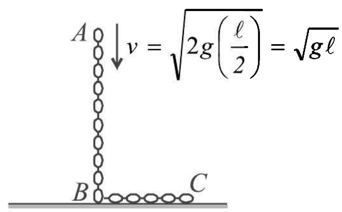
Example 143. A uniform chain of mass m and length ℓ hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the table on the chain when half of its length has fallen on the table. The fallen part does not form heap.

Solution

1. Weight of the portion BC of the chain

   lying on the table, $W = \frac{mg}{2}$ (downwards)

   Using $v = \sqrt{2gh}$



2. Thrust force $F_t = v_r\left(\frac{dm}{dt}\right)$

   $v_r = v$

   $\frac{dm}{dt} = \lambda v$

   $F_t = \lambda v^2$ (where, $\lambda = \frac{m}{\ell}$, is mass per unit length of chain)

   $v^2 = \left(\sqrt{g\ell}\right)^2 = g\ell$

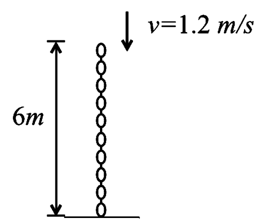
   $\therefore F_t = \left(\frac{m}{\ell}\right)(g\ell) = mg$ (downwards)

   $\therefore$ Net force exerted by the chain on the table is

   $F = W + F_t = \frac{mg}{2} + mg = \frac{3}{2}mg$

   So, from Newton's third law the force exerted by the table on the chain will be $\frac{3}{2}$ mg (vertically upwards).

Example 144. If the chain is lowered at a constant speed $v = 1.2$ m/s, determine the normal reaction exerted on the floor as a function of time. The chain has a mass of 80 kg and a total length of 6 m.



[Ans. $(19.2 + 16\,t)\,N$**]**

Example 145. A 6000 kg rocket is set for vertical firing. If the exhaust speed is 1000 ms^{-1}, the amount of gas that must be ejected per second to supply the thrust needed to overcome the weight of the rocket is ($g = 10$ ms^{-1}).

(A) 30 kg (B) 60 kg (C) 75 kg (D) 90 kg

Solution $M_0 g = \text{Thrust} = v\frac{dM}{dt}$

$\Rightarrow \frac{dM}{dt} = \frac{M_0 g}{v} = \frac{6000x10}{1000} kgs^{-1} \Rightarrow \frac{dM}{dt} = \frac{60000}{1000} = 60 kgs^{-1}$.

$\therefore$ (B) is correct answer.

Linear Momentum Conservation in Presence of External Force

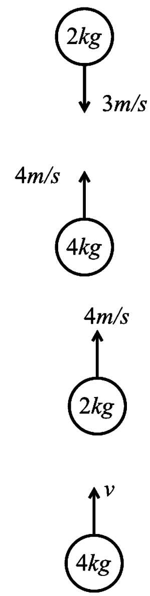
$$F_{ext} = \frac{dp}{dt} \Rightarrow \quad F_{ext}\, dt = dP$$

$\Rightarrow \quad dP = F_{ext)\ impulsive}\, dt \quad \therefore$ If $F_{ext)\ impulsive} = 0$

$\Rightarrow \quad dP = 0 \quad$ or P is constant

Note:

Momentum is conserved if the external force present is non-Impulsive. eg. Gravitation or Spring force



Example 146. Two balls are moving towards each other on a vertical line collides with each other as shown. Find their velocities just after collision.

Solution Let the final velocity of 4 kg ball just after collision be v. Since, external force is gravitational which is non-impulsive, hence, linear momentum will be conserved.

Applying linear momentum conservation:

$$2(-3) + 4(4) = 2(4) + 4(v)$$

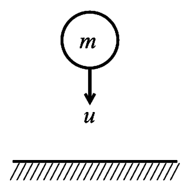
or $$v = \frac{1}{2}\text{ m/s}$$

Example 147. A ball is approaching ground with speed u. If the coefficient of restitution is e then find out:

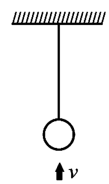
(a) the velocity just after collision.

(b) the impulse exerted by the normal due to ground on the ball.

[Ans (a) $v = eu$; (b) $J = mu(1 + e)$]



Example 148. A bullet of mass 50g is fired from below into the bob of mass 450g of a long simple pendulum as shown in figure. The bullet remains inside the bob and the bob rises through a height of 1.8 m. Find the speed of the bullet. (Take g = 10 m/s^2.)



Solution Let the speed of the bullet be v. Let the common velocity of the bullet and the bob, after the bullet is embedded into the bob, is V. By the principle of conservation of the linear momentum,

$$V = \frac{(0.05\ kg)\, v}{0.45\ kg + 0.05\ kg} = \frac{v}{10}$$

The string becomes loose and the bob will go up with a deceleration of g = 10 m/s^2. As it comes to rest at a height of 1.8 m, using the equation $v^2 = u^2 + 2ax$,

$$1.8\text{ m} = \frac{(v/10)^2}{2 \times 10\ m/s^2} \quad \text{or,} \quad v = 60\text{ m/s.}$$

Example 149. A small ball of mass m collides with a rough wall having coefficient of friction μ at an angle θ with the normal to the wall. If after collision the ball moves with angle α with the normal to the wall and the coefficient of restitution is e then find the reflected velocity v of the ball just after collision.

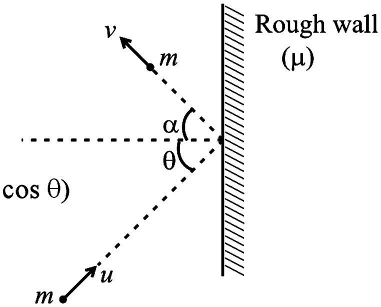
Solution $mv\cos\alpha - (m(-u\cos\theta)) = \int N\,dt$

$mv\sin\alpha - mu\sin\theta = -\mu\int N\,dt$

and $e = \dfrac{v\cos\alpha}{u\cos\theta} \Rightarrow v\cos\alpha = eu\cos\theta$

or $mv\sin\alpha - mu\sin\theta = -\mu(mv\cos\alpha + mu\cos\theta)$

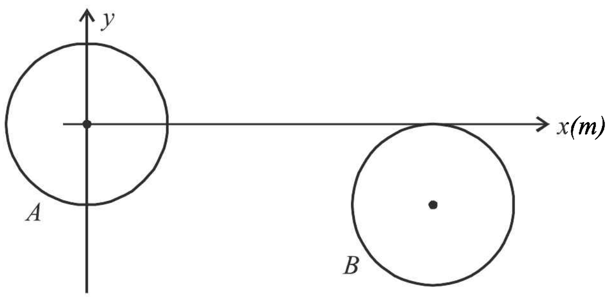
or $v = \dfrac{u}{\sin\alpha}[\sin\theta - \mu\cos\theta(\varepsilon + 1)]$



Comprehension Questions

Comprehension-1

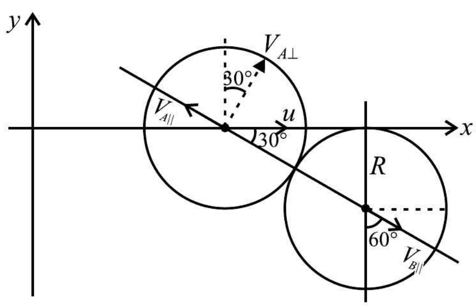
Two smooth balls A and B, each of mass m and radius R, have their centres at $(0, 0, R)$ and at $(5R, -R, R)$ respectively, in a coordinate system as shown. Ball A, moving along positive x axis, collides with ball B. Just before the collision, speed of ball A is 4 m/s and ball B is stationary. The collision between the balls is elastic.



Example 1. Velocity of the ball A just after the collision is :

(A) $(i + \sqrt{3}j)$ m/s (B) $(i - \sqrt{3}j)$ m/s (C) $(2i + \sqrt{3}j)$ m/s (D) $(2i + 2j)$ m/s

Solution (A) During collision, forces act along line of impact. As collision is elastic and both the balls have same mass, velocities are exchanged along the line of impact. Therefore ball B moves with velocity $V_{B||}$, that is equal to $u\cos 30°$. Ball A moves perpendicular to the line of impact with velocity $V_{A\perp} = u\cos 60°$. Along the line of impact, ball A does not have any velocity after the collision.



Therefore velocity of ball A in vector form after the collision

$= V_{A\perp} \cos 60° i + V_{A\perp} \cos 30° j$

$= (u \cos 60°) \cos 60° i + (u \cos 60°) \cos 30° j$

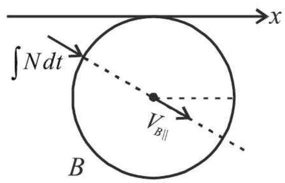
$= 4.\frac{1}{2}.\frac{1}{2}.i + 4.\frac{1}{2}.\frac{\sqrt{3}}{2}.j = (i + \sqrt{3}j)$ m/s

Example 2. Impulse of the force exerted by A on B during the collision, is equal to

(A) $(\sqrt{3}mi + 3mj)\frac{kgm}{s}$ (B) $(\frac{\sqrt{3}}{2}mi - \sqrt{3}mj)\frac{kgm}{s}$

(C) $(3mi - \sqrt{3}mj)\frac{kgm}{s}$ (D) $(2\sqrt{3}mi + 3j)\frac{kgm}{s}$

Solution (C) Using impulse-momentum equation for ball B



$\int N\,dt = \vec{p}_f - \vec{p}_i$ and as $\vec{p}_i = 0$

$\int N\,dt = \vec{p}_f$

$= (\text{mu} \cos 30°) \cos 30\, i - (\text{mu} \cos 30°) \cos 60° j$

$= m.4.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.i - m.4.\frac{\sqrt{3}}{2}.\frac{1}{2}.j$

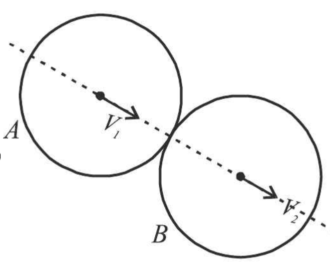
$= (3\,\text{m}\,i - \sqrt{3}\,\text{m}\,j)\ \text{kg}\frac{m}{s}$

Example 3. Coefficient of restitution during the collision is changed to $\frac{1}{2}$, keeping all other parameters unchanged. What is the velocity of the ball B after the collision ?

(A) $\frac{1}{2}(3\sqrt{3}i + 9j)$ m/s (B) $\frac{1}{4}(9i - 3\sqrt{3}j)$ m/s

(C) $(6i + 3\sqrt{3}j)$ m/s (D) $(6i - 3\sqrt{3}j)$ m/s

Solution (B) Suppose V_2 is velocity of ball B along the line of impact and V_1 is velocity of ball A along the line of impact, after the collision, as shown.



Then $\frac{1}{2}$ (Velocity of approach) = Velocity of separation

$$\frac{1}{2}\left[\frac{\sqrt{3}}{2}.u\right] = V_2 - V_1 \quad .... (1)$$

Conserving momentum along the line of impact

$$m.\frac{\sqrt{3}}{2}u = m.V_2 + mV_1 \quad .... (2)$$

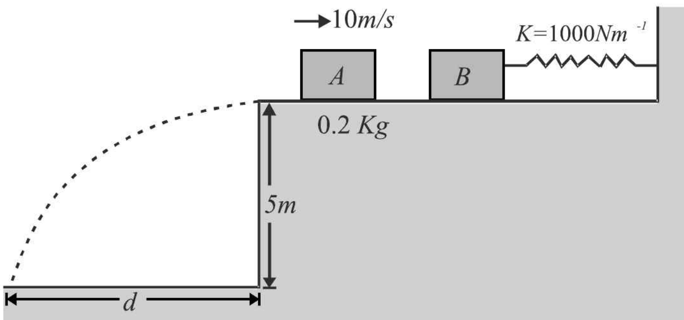
Solving and using $u = 4$ m/s

$$V_2 = \frac{3\sqrt{2}}{2}\ \text{m/s}$$

$$\vec{V}_2 = \frac{3\sqrt{3}}{2}\cos 30° i - \frac{3\sqrt{3}}{2}\cos 60° j = \left(\frac{9}{4}i - \frac{3\sqrt{3}}{4}j\right) \text{ m/s}$$

Comprehension-2

Figure shows block A of mass 0.2 kg sliding to the right over a frictionless elevated fixed surface at a speed of 10 m/s. The block undergoes a collision with stationary block B, which is connected to a nondeformed spring of spring constant 1000 Nm^{-1}. The coefficient of restitution between the blocks is 0.5. After the collision, block B oscillates in SHM with a period of 0.2 s, and block A slides off the opposite end of the elevated surface, landing a distance 'd' from the base of that surface after falling height 5m. (use $\pi^2 = 10$; $g = 10$ m/s^2) Assume that the spring does not affect the collision.



Example 1. Mass of the block B is

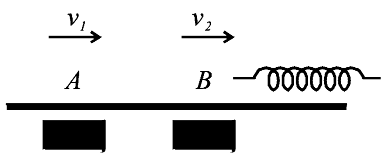
(A) 0.4 kg (B) 0.8 kg (C) 1 kg (D) 1.2 kg

Solution (C) As $t = 2\pi\sqrt{\frac{m}{k}} \Rightarrow m = \frac{T^2K}{4\pi^2} = \frac{0.2 \times 0.2 \times 1000}{4 \times 10} = 1$ kg

Example 2. Amplitude of the SHM executed by block B-spring system, is

(A) $2.5\sqrt{10}$ cm (B) 10 cm (C) $3\sqrt{10}$ cm (D) $5\sqrt{10}$ cm

Solution (A) Immediately after the collision, suppose velocities of the blocks are V_1 and V_2 as shown



$\frac{1}{2}$ vel. of approach = velocity of separation.

$\Rightarrow$ $5 = V_2 - V_1$ ...(1)

Using principal of conservation of momentum for the collision

$$2 = 0.2\, V_1 + V_2$$

or $10 = V_1 + 5V_2$ .... (2)

On solving $V_2 = 2.5$ m/s; $V_1 = -2.5$ m/s

Hence block A moves leftward after the collision with speed 2.5 m/s. And the block B moves towards right with speed 2.5 m/s.

The maximum velocity of $B = 2.5 = \omega A$

$\Rightarrow$ $A = V\sqrt{\frac{m}{K}} = 2.5\sqrt{\frac{1}{1000}}\, m = 2.5\sqrt{10}$ cm

Example 3. The distance '*d*' will be equal to

(A) 2m (B) 2.5 m (C) 4m (D) 6.25 m

Solution Time of flight = $\sqrt{\frac{2h}{g}}$ = 1 sec.

$d = (2.5 \text{ m/s}) \times 1 \text{ s} = 2.5 \text{ m}$

Comprehension - 3

Consider a system of three particles, each of mass 0.1 kg, which remains always in the same *xy*-plane. The particles interact among themselves, always in a manner consistant with Newton's third law. The particles *A*, *B* and *C* have positions at various times having equal intervals as given in the table.

Time (in sec.)	A	B	C
0	(1, 1)	(2, 2)	(3, 3)
1	(1, 0)	(0, 1)	(3, 3)
2	(0, 0)	(1, 2)	(1, 0)

Example 1. Most probable path of the centre of mass of the system is :

(A) a parabola (B) a circle (C) an ellipse (D) a straight line

Solution Coordinates of COM of the system at three instants are (2,2), $\left(\frac{4}{3}, \frac{4}{3}\right)$ and $\left(\frac{2}{3}, \frac{2}{3}\right)$.

Therefore COM is moving along a straight line.

[**Ans**. (D)]

Example 2. Magnitude of net external force acting on the system :

(A) may be zero
(B) must be zero
(C) its average value must be zero for time interval $t = 0$ to $t = 2$ sec
(D) its average value may be zero for time interval $t = 0$ to $t = 2$ sec

[**Ans**. (A), (D)]

Comprehension - 4

In the following passage we will study how the problems of collision can be solved with the help of simple geometrical construction.

Let us consider a case of elastic collision in which second mass is at rest before collision.

For collision process we can write two equations, one for momentum conservation and other for energy conservation ;

$$\vec{P} = \vec{P}_1 + \vec{P}_2 \quad \text{...(1)}$$

$$\frac{P^2}{2m_1} = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} \quad \text{...(2)}$$

where, $\vec{P}$ = Initial momentum of particle 1,

$\vec{P}_1$ = Final momentum of particle 1,

$\vec{P}_2$ = momentum of second particle after collision.

Let 'θ' be the angle between $\vec{P}$ and $\vec{P}_2$.

Then, $\vec{P}_1 = \vec{P} - \vec{P}_2$

$$P_1^2 = P^2 + P_2^2 - 2PP_2\cos\theta \qquad ...(3)$$

From equation (2) & (3) we get ;

$$P_2 = \left(\frac{2m_2}{m_1 + m_2}\right) \text{P} \cos\theta \qquad ...(4)$$

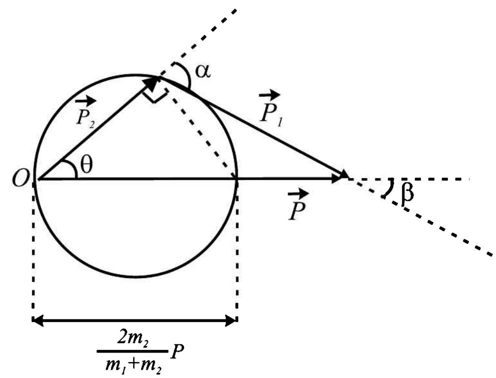
Now we can proceed for a geometrical construction :

Step 1 : Draw vector $\vec{P}$ from a certain point '*O*' to represent the momentum of the incident particle.

Step 2 : Draw a circle of diameter $\left(\frac{2m_2}{m_1 + m_2}\right) P$ with its centre lying on straight line coinciding with $\vec{P}$ in such a way that circle passes through point '*O*'.

Step 3 : Draw vector $\vec{P}_2$ from point '*O*' at an angle 'θ' with its head touching the circumference of the circle.

Step 4 : Draw $\vec{P}_1$ so as to satisfy the equation $\vec{P} = \vec{P}_1 + \vec{P}_2$.

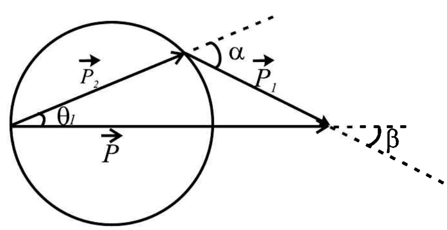


Here the angle between the momentum of the first and the second particle after collision is denoted by α, while β is the angle by which the incident particle deflects from initial direction after the collision.

Since, the angle inscribed by a diameter on the circumference is a right angle therefore all vectors drawn from point '*O*' to the circumference of the circle will satisfy the equation :

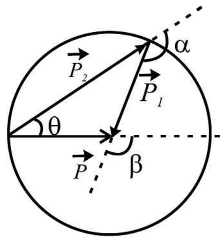
$$P_2 = \left(\frac{2m_2}{m_1 + m_2}\right) P \cos\theta$$

Here 'θ' represents the angle made by the line of impact with the initial line of motion of first particle. By changing 'θ' we can draw all possible cases of the collision.



All above diagrams are for the case when : $\dfrac{2m_2}{m_1+m_2}<1$, which implies that the mass of incident particle is larger than the mass at rest. i.e. $m_1 \geq m_2$.

But when the case changes to $\dfrac{2m_2}{m_1+m_2}>1 \quad \Rightarrow m_2 \geq m_1$; our diagram will become ;

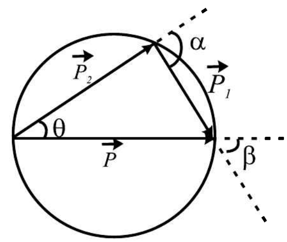


Where ; α = angle of divergence between the particles,

β = angle of deviation or deflection of the incident particle,

and θ = angle of incidence.

Similarly for $m_1 = m_2$ our diagram will be :

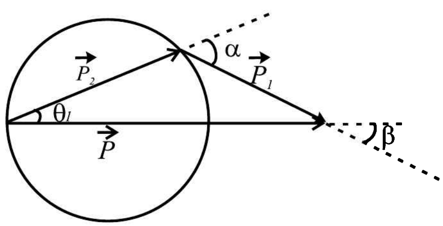


Now answer the following questions taking 'm_2' to be initially at rest :

Example 1. In which of the following cases the incident particle can not change its direction of motion by all possible angles from 0 to π ?

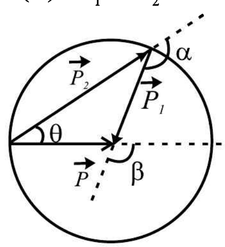
(A) $m_1 \geq m_2$ (B) $m_2 \geq m_1$ (C) $m_1 = m_2$ (D) $m_1 \geq m_2$

Solution



$m_1 > m_2$

(A)



$m_1 < m_2$

(B)

∵ $\vec{P} = \vec{P}_1 + \vec{P}_2$

and $\vec{P}_2$ is a chord of the circle and from diagram (A) it is clear that P_1 cannot reverse its direction as it can do in diagram "*B*".

In case of $m_1 = m_2$

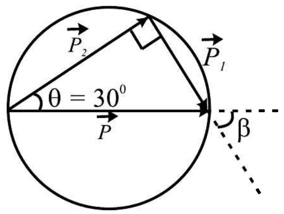
It can be judged by changing 'θ' that P_1 cannot change its direction from 0 to π.

[Ans. (D)]

Example 2. Consider the case when $m_1 = m_2$ & m_2 is initially at rest and elastic collision takes place. When θ = 30°, the angle of deflection of the incident particle from initial direction of motion will be :

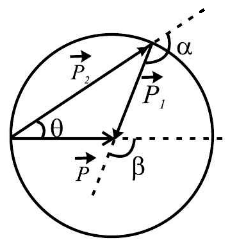
(A) 30^0 (B) 60^0 (C) 120^0 (D) 150^0

Solution



From figure $\beta = \pi - \pi/2 - 30^0 = 60^0$.

Example 3. The two colliding particles will diverge at an angle greater than 90^0 for the case :



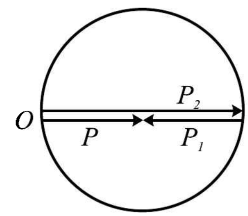
(A) $m_1 \geq m_2$ (B) $m_2 \geq m_1$
(C) $m_1 = m_2$ (D) $m_1 \geq m_2$

Solution Clearly from the passage :

$\alpha \geq 90^0$ for $m_2 \geq m_1$ (here angle of divergence is 'α')

Hence (*B*) is correct answer.

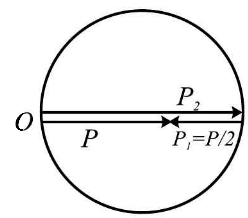
Example 4. When the incident particle reverses its direction of motion, we can draw the diagram as :



If $|\vec{P}_1| = \frac{|\vec{P}|}{2}$, Then $m_2 : m_1$ will be :

(A) 1 (B) 2
(C) 3 (D) 4

Solution Diameter = $\frac{2m_2}{m_1 + m_2} P\cos\theta = \frac{2m_2}{m_1 + m_2} P$ (since θ = 0)

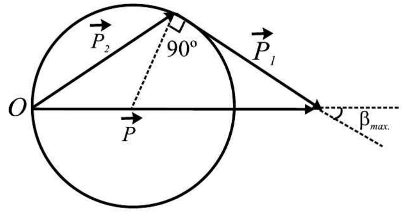


Here diameter = $|\vec{P}_2| = |\vec{P}_1| + |\vec{P}|$

$|\vec{P}_1| + |\vec{P}| = P + \frac{P}{2} = \frac{3P}{2}$

$\Rightarrow \quad \frac{2m_2}{m_1 + m_2} P = \frac{3P}{2} \quad \Rightarrow \frac{m_2}{m_1} = 3$ **Ans.**

Example 5. For $m_1 \geq m_2$, there exists an angle β_{max} beyond which the direction of motion of the incident particle cannot be changed. This angle corresponds to the case when $\vec{P}_1$ is tangential to the circle. If $m_1 = 2m_2$, for maximum deflection we must have :

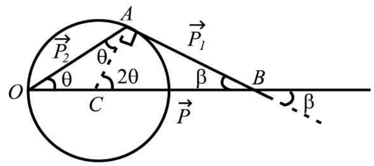


(A) $|\vec{P}_1| = \frac{|\vec{P}|}{\sqrt{3}}$ (B) $|\vec{P}_1| = \frac{2\sqrt{2}|\vec{P}|}{3}$ (C) $|\vec{P}_1| = \sqrt{5}\frac{|\vec{P}|}{3}$ (D) $|\vec{P}_1| = \frac{|\vec{P}|}{3}$

Solution For $m_1 = 2m_2$;

Diameter = $\dfrac{2m_2}{m_1 + m_2}P = \dfrac{2}{3}P \quad \Rightarrow OC = CA = \dfrac{P}{3}$

$\therefore$ In the diagram, $BC = P - \dfrac{P}{3} = \dfrac{2P}{3}$



By pythagorous theorem :

$AC^2 + AB^2 = BC^2$

$\dfrac{P^2}{9} + P_1^2 = \dfrac{4}{9}P^2 \Rightarrow P_1 = \pm\dfrac{P}{\sqrt{3}} \Rightarrow |\vec{P}_1| = \dfrac{|\vec{P}|}{\sqrt{3}}$

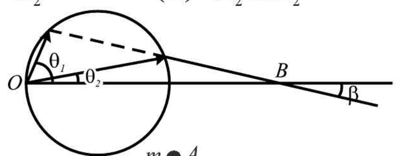
Example 6. In which of the following cases, is it possible to find two different angles of incidence corresponding to the same angle of deviation :

(A) $m_1 \geq m_2$ (B) $m_2 \geq m_1$ (C) $m_1 = m_2$ (D) $m_2 \geq m_2$

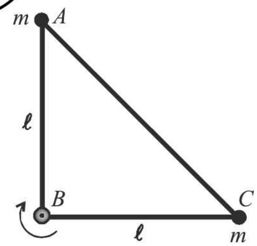
Solution For $m_1 \geq m_2$;

there can be two values of 'θ' as shown ;

Hence (A) is correct answer.



Comprehension - 6



Three massless rods are fixed to form a right angled triangular frame such that $AB = BC = \ell$. Two identical small objects of mass m are fixed at A and C. The frame is hinged about B such that the frame can rotate in vertical plane about an horizontal axis without friction. Initially AB is vertical and BC is horizontal and the system is released from rest.

Example 1. The maximum shift of centre of mass of two mass system from its initial position is

(A) $\sqrt{2}\,\ell$ (B) $1.5\,\ell$ (C) $\ell/\sqrt{2}$ (D) 2ℓ

[**Ans**. (A)]

Example 2. The magnitude of acceleration of mass A when the rod AC becomes horizontal is :

(A) $g\,(1+\sqrt{2})$ (B) $g\,(2+\sqrt{2})$ (C) $2g(1+\sqrt{2})$ (D) None of these

[**Ans**. (A)]

Example 3. Tension in the rod AC when it (rod AC) becomes horizontal.

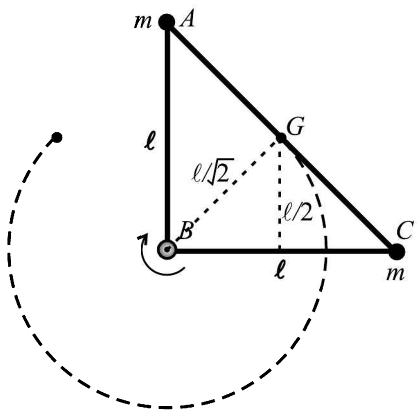
(A) mg (B) $\sqrt{2}\,mg$ (C) $\dfrac{mg}{\sqrt{2}}$ (D) None of these

[**Ans**. A]

Solution **1 to 3**

From the figure, the centre of mass of two body system (G) moves along a partial circular path with radius $\dfrac{\ell}{\sqrt{2}}$. Hence the maximum shift of centre of mass from initial position is $2\times\dfrac{\ell}{\sqrt{2}} = \sqrt{2}\ell$

The centre of mass is at lowest position when AC is horizontal. Since gravitational potential energy of system is least at this position, its kinetic energy shall be maximum.



Thus the speed of both objects shall be maximum at this position and as a result the tangential acceleration of both object is zero.

Further as the system moves from initial position to position of maximum kinetic energy, the vertical fall in centre of mass of system is $\frac{\ell}{\sqrt{2}}(1+\cos 45^\circ) = \frac{\ell}{\sqrt{2}} + \frac{\ell}{2}$

From conservation of energy ⇒ Gain in *K.E.* = Loss in *P.E.*

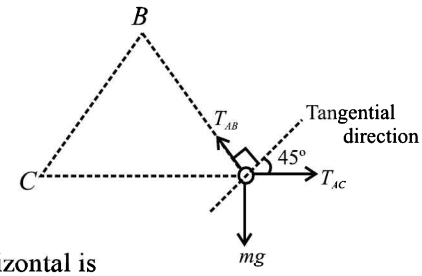
therefore total acceleration of mass A

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = (2m)g\ell\left(\frac{1}{2}+\frac{1}{\sqrt{2}}\right)$$

or $v = \sqrt{g\ell(1+\sqrt{2})}$

Therefore total acceleration of mass *A*

= normal acceleration of mass $A = \frac{v^2}{\ell} = g(1+\sqrt{2})$



The free body diagram of mass *A* when rod is horizontal is

because tangential acceleration is zero

$T_{AC}\cos 45^\circ = mg \sin 45^\circ$

or $T_{AC} = mg$

Comprehension - 6

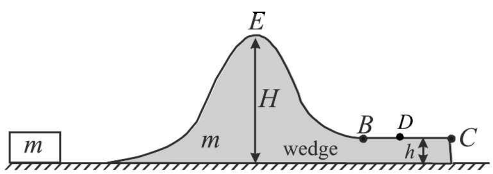


Figure shows an irregular wedge of mass m placed on a smooth horizontal surface. Part *BC* is rough.The other part of the wedge is smooth.

Example 1. What minimum velocity should be imparted to a small block of same mass *m* so that it may reach point *B* :

(A) $2\sqrt{gH}$ (B) $\sqrt{2gH}$ (C) $2\sqrt{g(H-h)}$ (D) $\sqrt{gh}$

Solution Let '*u*' be the required minimum velocity. By momentum conservation :

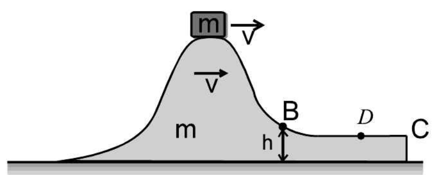
$mu = (m+m)v \quad \Rightarrow v = u/2.$

Energy equation :

$$\frac{1}{2}mu^2 = \frac{1}{2}(2m)v^2 + mgH.$$

Substituting v = u/2 :

$u = 2\sqrt{gH}$



(At highest point)

Example 2. The velocity of wedge when the block comes to rest (w.r.t. wedge) on part *BC* is :

(A) $\sqrt{gH}$ (B) $\sqrt{g(H-h}$ (C) $2\sqrt{gH}$ (D) none of these

Solution When the block comes to rest, the wedge continues to move at $V = \frac{u}{2} = \sqrt{gH}$ on the smooth surface. (since, momentum of wedge-block system remains conserved).

Example 3. If the coefficient of friction between the block and wedge is μ, and the block comes to rest with respect to wedge at a point D on the rough surface then BD will be :

(A) $\frac{H}{\mu}$ (B) $\frac{H-h}{\mu}$ (C) $\frac{h}{\mu}$ (D) none of these

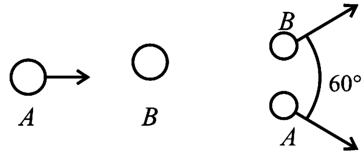
Solution $mg(H-h) - \mu mg\,(BD) = \frac{1}{2}mv^2 - \frac{1}{2}mv^2$

$\Rightarrow \quad H - h = \mu(BD)$

$\Rightarrow \quad BD = \frac{H-h}{\mu}$

Comprehension - 7

A smooth ball 'A' moving with velocity 'V' collides with another smooth identical ball at rest. After collision both the balls move with same speed with angle between their velocities 60°. No external force acts on the system of balls.



Example 1. The speed of each ball after the collision is

(A) $\frac{V}{2}$ (B) $\frac{V}{3}$ (C) $\frac{V}{\sqrt{3}}$ (D) $\frac{2V}{\sqrt{3}}$

Solution From conservation of momentum, $mv = mv'\cos 30° + mv'\cos 30°$

$\therefore v' = \frac{v}{2\cos 30°} = \frac{v}{\sqrt{3}}$

Example 2. If the kinetic energy lost is fully converted to heat then heat produced is

(A) $\frac{1}{3}mV^2$ (B) $\frac{2}{3}mV^2$ (C) 0 (D) $\frac{1}{6}mV^2$

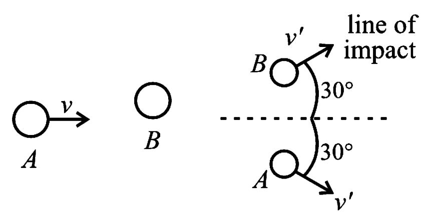
Solution Loss in kinetic energy$= \frac{1}{2}mv^2 - 2 \times \frac{1}{2}m\left(\frac{v}{\sqrt{3}}\right)^2 = \frac{1}{6}mv^2$

Example 3. The value of coefficient of restitution is

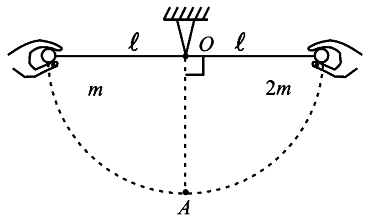
(A) 1 (B) $\frac{1}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) 0

Solution Initially B was at rest, therefore line of impact is along final velocity of B.

$\therefore e = \frac{v' - v'\cos 60°}{v\cos 30} = \frac{\frac{1}{2}\frac{v}{\sqrt{3}}}{v \times \frac{\sqrt{3}}{2}} = \frac{1}{3}$



Comprehension - 8



Two balls having masses m and 2m are fastened to two light strings of same length ℓ (figure). The other ends of the strings are fixed at O. The strings are kept in the same horizontal line and the system is released from rest. The collision between the balls is elastic. Point A is the lowest point at which either ball can reach.

Example 1. The position at which the balls will collide for the first time

(A) at point A (B) left of point A

(C) right of point A (D) None of these

Solution Initial speed of both balls is zero. Further, tangential acceleration of both balls at any time are same. Hence before the first collision, they cover same distance in same time intervals.

Hence both of them shall collide at A.

[**Ans.** (A)]

Example 2. The speed of ball of mass 2m just after collision (for the first time) is over

(A) $2\dfrac{\sqrt{2g\ell}}{3}$ (B) $2\dfrac{\sqrt{50g\ell}}{3}$ (C) $\dfrac{\sqrt{2g\ell}}{3}$ (D) $\dfrac{\sqrt{50g\ell}}{3}$

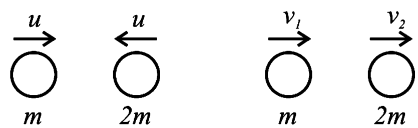
Solution The speed of both balls just before collision is $u = \sqrt{2g\ell}$.

From conservation of momentum

$$mu - 2mu = mv_1 + 2mv_2 \quad ...(1)$$

From equation of coefficient of restitution

$$e = 1 = \frac{v_2 - v_1}{2u} \quad ...(2)$$



Both balls just before collision

Both balls just after collision

Solving equation (1) and (2)

$$v_1 = \frac{u}{3} = \frac{\sqrt{2g\ell}}{3}$$

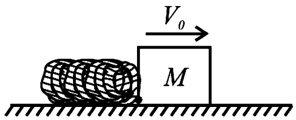
Example 3. In the duration between first and second collision, the maximum (vertical) height to which the ball of mass 2m rises above point A is

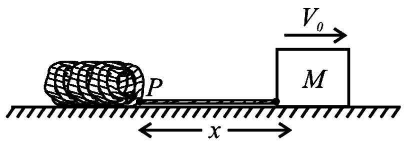
(A) $\dfrac{\ell}{9}$ (B) $\dfrac{2\ell}{9}$ (C) ℓ (D) 2ℓ

Solution The maximum height to which the wall of mass 2m rises is $h = \dfrac{v_2^2}{2g} = \dfrac{\ell}{9}$

Comprehension - 9

A smooth rope of mass m and length L lies in a heap on a smooth horizontal floor, with one end attached to a block of mass M. The block is given a sudden kick and instantaneously acquires a horizontal velocity of magnitude V_0 as shown in figure 1. As the block moves to right pulling the rope from heap, the rope being smooth, the heap remains at rest. At the instant block is at a distance x from point P as shown in figure-2 (P is a point on the rope which has just started to move at the given instant) , choose correct options for next three question.





Example 1. The speed of block of mass M is

(A) $\dfrac{mV_0}{(M+\frac{m}{L}x)}$ (B) $\dfrac{MV_0}{(M+\frac{m}{L}x)}$ (C) $\dfrac{m^2V_0}{M(M+\frac{m}{L}x)}$ (D) $\dfrac{M^2V_0}{m(M+\frac{m}{L}x)}$

Solution The mass of moving material is $M+\frac{m}{L}x$.

From conservation of momentum $MV_0 = (M+\frac{m}{L}x)\,V$

$\therefore$ velocity of moving block and moving rope is $V = \dfrac{MV_0}{(M+\frac{m}{L}x)}$

Example 2. The magnitude of acceleration of block of mass M is

(A) $\dfrac{m^3}{L}\dfrac{V_0^2}{(M+\frac{m}{L}x)^3}$ (B) $\dfrac{mM^2}{L}\dfrac{V_0^2}{(M+\frac{m}{L}x)^3}$

(C) $\dfrac{m^4}{ML}\dfrac{V_0^2}{(M+\frac{m}{L}x)^3}$ (D) $\dfrac{M^2}{L}\dfrac{V_0^2}{(M+\frac{m}{L}x)^3}$

Solution The acceleration of moving block is

$$a = -v\frac{dv}{dx} = -\frac{MV_0}{(M+\frac{m}{L}x)^2}\times\frac{m}{L}\frac{dx}{dt} = -\frac{m}{L}\frac{M^2V_0^2}{(M+\frac{m}{L}x)^3}$$

Example 3. The tension in rope at point P is

(A) $\dfrac{mM^2}{L}\dfrac{V_0^2}{(M+\frac{m}{L}x)^2}$ (B) $\dfrac{m^2M}{L}\dfrac{V_0^2}{(M+\frac{m}{L}x)^2}$

(C) $\dfrac{m^3}{L}\dfrac{V_0^2}{(M+\frac{m}{L}x)^2}$ (D) $\dfrac{M^3}{L}\dfrac{V_0^2}{(M+\frac{m}{L}x)^2}$

Solution The tension at point P is what gives momentum to next tiny piece (to left of P) that starts moving. The speed of this piece increases from 0 to V in time dt.

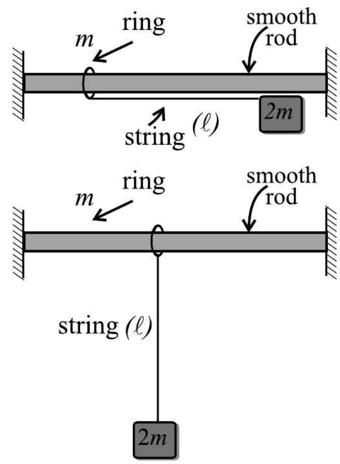
$\Rightarrow$ $dp = dmV$

or $$F = \frac{dP}{dt} = \frac{dm}{dt}V = \frac{\frac{m}{L}dx}{dt}V = \frac{m}{L}V^2$$

$\therefore \quad F_p = \frac{m}{L}\frac{M^2V_0^2}{(M+\frac{m}{L}x)^2}$

Comprehension - 10

In given figure, the small block of mass 2m is released from rest condition when the string is in horizontal position.



Example 1. Maximum possible velocity of ring of mass '*m*' is (Assuming zero friction):

(A) $\sqrt{2g\ell}$ (B) $\sqrt{\frac{4g\ell}{3}}$

(C) $\sqrt{\frac{8g\ell}{3}}$ (D) none of these

Solution By momentum conservation : (In horizontal direction)

$mv_1 = 2mv_2$

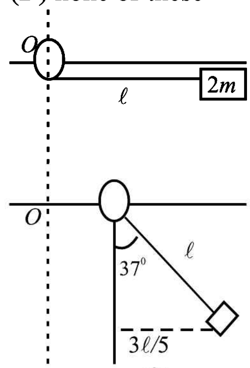
and by energy conservation:

$$2mg\ell = \frac{1}{2}mv_1^2 + \frac{1}{2}2mv_2^2$$

$$\Rightarrow \quad 2g\ell = \frac{1}{2}v_1^2 + \left(\frac{v_1}{2}\right)^2 \Rightarrow 2gl = \frac{3}{4}v_1^2 \Rightarrow \quad v_1 = \sqrt{\frac{8g\ell}{3}}$$

Example 2. Displacement of the ring when string makes an angle θ = 37° with the vertical will be:

(A) $\frac{4\ell}{15}$ (B) $\frac{\ell}{15}$ (C) $\frac{2\ell}{15}$ (D) none of these



Solution Taking '*O*' as the origin ;

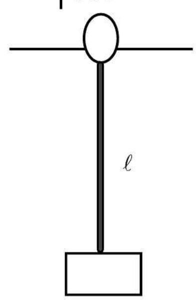
$$X_{CM(i)} = \frac{m(0)+2m(\ell)}{3m} = \frac{2\ell}{3}$$

$$\text{and } X_{CM(f)} = \frac{m(x)+2m(x+\frac{3\ell}{5})}{m+2m}$$

As ; $\Sigma F_x = 0$ $\Rightarrow X_{CM}(i) = X_{CM}(f)$

$$\Rightarrow X = \frac{4\ell}{15}$$

Example 3. Displacement of centre of mass of ring and block when string becomes vertical is:



(a) zero (B) $\frac{\ell}{3}$

(C) $\frac{2\ell}{3}$ (D) none of these

Solution At the position shown :

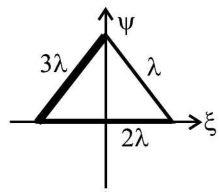
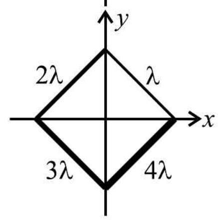
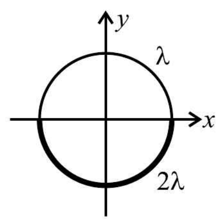
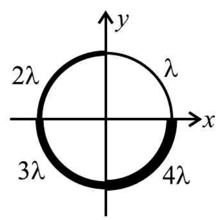
$$\Delta y_{CM} = \frac{2m(\ell)+m(0)}{2m+m} \qquad \Delta y_{CM} = \frac{2\ell}{3}$$

and $\Delta x_{CM} = 0$ As $\Sigma F_x = 0$

$\Rightarrow$ Displacement of centre of mass $= \dfrac{2\ell}{3}$

Match the following

***Example* 1.** In each situation of column-I a mass distribution is given and information regarding x and y-coordinate of centre of mass is given in column-II. Match the figures in column-I with corresponding information of centre of mass in column-II.

Column-I	Column-II	
(A) An equilateral triangular wire frame is made using three thin uniform rods of mass per unit lengths λ, 2λ and 3λ as shown	(p) $x_{cm} \geq 0$	
(B) A square frame is made using four thin uniform rods of mass per unit length lengths λ, 2λ, 3λ and 4λ as shown	(q) $y_{cm} \geq 0$	
(C) A circular wire frame is made of two uniform semicircular wires of same radius and of mass per unit length λ and 2λ as shown	(r) $x_{cm} < 0$	
(D) A circular wire frame is made of four uniform quarter circular wires of same radius and mass per unit length λ, 2λ, 3λ and 4λ as shown.	(s) $y_{cm} < 0$	

Ans. (A) *q,r* (B) *p,s* (C) *p,s* (D) *p,s*

Solution (A) Centre of mass lies in second quadrant.

(B), (C) and (D) Centre of mass lies on y-axis and below x-axis.

***Example* 2.** Two identical uniform solid spheres of mass m each approach each other with constant velocities such that net momentum of system of both spheres is zero. The speed of each sphere before collision is u. Both the spheres then collide. The condition of collision is given for each situation of column-I. In each situation of column-II information regarding speed of sphere(s) is given after the collision is over. Match the condition of collision in column-I with statements in column-II.

Column-I	Column-II
(A) Collision is perfectly elastic and head on	(p) speed of both spheres after collision is u
(B) Collision is perfectly elastic and oblique	(q) velocity of both spheres after collision is different
(C) Coefficient of restitution is $e = \frac{1}{2}$ and collision is head on	(r) speed of both spheres after collision is same but less than u.
(D) Coefficient of restitution is $e = \frac{1}{2}$ and collision is oblique	(s) speed of one sphere may be more than u.

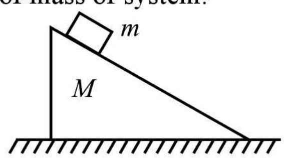
[**Ans.** (A) p,q (B) p,q (C) q,r (D) q,r]

Solution In all cases speed of balls after collision will be same. In case of elastic collision speed of both balls after collision will be u, otherwise it will be less than u.

***Example* 3.** In each situation of column-I, a system involving two bodies is given. All strings and pulleys are light and friction is absent everywhere. Initially each body of every system is at rest. Consider the system in all situation of column I from rest till any collision occurs. Then match the statements in column-I with the corresponding results in column-II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

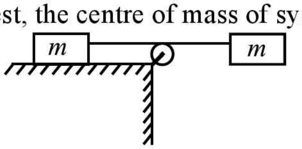
Column-I | **Column-II**

(A) The block plus wedge system is placed over smooth horizontal surface. After the system is released from rest, the centre of mass of system.



(p) Shifts towards right

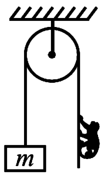
(B) The string connecting both the blocks of mass m is horizontal. Left block is placed over smooth horizontal table as shown. After the two block system is released from rest, the centre of mass of system



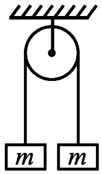
(q) Shifts downwards

(C) The block and monkey have same mass. The monkey starts climbing up the rope. After the monkey starts climbing up, the centre of mass of monkey + block system.

(r) Shifts upwards



(D) Both block of mass m are initially at rest. The left block is given initial velocity u downwards. Then, the centre of mass of two block system afterwards.



(s) Does not shift

[**Ans.** A (q) , (B) p,q (C) r (D) s]

Solution

(A) Initial velocity of centre of mass of given system is zero and net external force is in vertical direction. Since there is shift of mass downward, the centre of mass has only downward shift.

(B) Obviously there is shift of centre of mass of given system downwards. Also the pulley exerts a force on string which has a horizontal component towards right. Hence centre of mass of system has a rightward shift.

(C) Both block and monkey moves up, hence centre of mass of given system shifts vertically upwards.

(D) Net external force on given system is zero. Hence centre of mass of given system remains at rest.

Assertion Reason Questions

***Example* 1.** **STATEMENT-1 :** A sphere of mass m moving with speed u undergoes a perfectly elastic head on collision with another sphere of heavier mass M at rest ($M \geq m$), then direction of velocity of sphere of mass m is reversed due to collision. [No external force acts on system of two spheres.]

STATEMENT-2 : During a collision of spheres of unequal masses, the heavier mass exerts more force on lighter mass in comparison to the force which lighter mass exerts on heavier mass.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

Solution Statement-2 contradicts Newton's third law and hence is false.

[**Ans.** (C)]

Example **2. STATEMENT-1 :** A rocket launched vertically upward explodes at the highest point it reaches. The explosion produces three fragments with non-zero initial velocity. Then the initial velocity vectors of all the three fragments are in one plane.

STATEMENT-2 : For sum of momentum of three particles to be zero all the three momentum vectors must be coplanar.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

Solution For sum of three non null vectors to be zero, there must be coplanar. Hence Statement-2 is a correct explanation for Statement-1.

[Ans. (A)]

Example **3. STATEMENT-1 :** In a perfectly inelastic collision between two spheres, velocity of both spheres just after the collision are not always equal.

STATEMENT-2 : For two spheres undergoing collision, component of velocities of both spheres along line of impact just after the collision will be equal if the collision is perfectly inelastic. The component of velocity of each sphere perpendicular to line of impact remains unchanged due to the impact.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True.

Solution From statement-2, if the component of relative velocity normal to line of impact is non-zero, they shall not have same velocity after collision. Hence statement-2 is correct explanation of statement-1.

[Ans. (A)]

Example **4. STATEMENT-1 :** Two spheres undergo a perfectly elastic collision. The kinetic energy of system of both spheres is always constant. [There is no external force on system of both spheres.]

STATEMENT-2 : If net external force on a system is zero, the velocity of centre of mass remains constant.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

Solution During colision KE of system is not constant, hence statement-1 is false.

[Ans. (D)]

Example **5. STATEMENT-1 :** No external force acts on system of two spheres which undergo a perfectly elastic head on collision. The minimum kinetic energy of this system is zero if the net momentum of this system is zero.

STATEMENT-2 : In any two body system undergoing perfectly elastic head on collision, at the instant of maximum deformation, the complete kinetic energy of the system is converted to deformation potential energy of the system.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

Solution For a system of two isolated sphere having non zero initial kinetic energy, the complete kinetic energy can be converted to other forms of energy if the momentum of system is zero. This is due to the fact that for an isolated system, the net momentum remains conserved. If an isolated system has nonzero momentum, for the momentum to remain constant complete kinetic energy of the system cannot become zero. Hence statement 1 is true while statement 2 is false.

[**Ans.** (C)]

Example **6.** **STATEMENT-1 :** Non zero work has to be done on a moving particle to change its momentum.

STATEMENT-2 : To change momentum of a particle a non zero net force should act on it.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

Solution No work done by net force it changes only direction of momentum of particle. Hence statement-1 is false.

[**Ans.** (D)]

Example **7.** **STATEMENT-1 :** In a perfectly inelastic collision between two spheres, velocity of both spheres just after the collision are not always equal.

STATEMENT-2 : For two spheres undergoing collision, component of velocities of both spheres along line of impact just after the collision will be equal if the collision is perfectly inelastic. The component of velocity of each sphere perpendicular to line of impact remains unchanged due to the impact.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

Solution From statement-2, if the component of relative velocity normal to line of impact is non-zero, they shall not have same velocity after collision. Hence statement-2 is correct explanation of statement-1.

[**Ans.** (A)]

Example **8.** **STATEMENT-1 :** Two particles undergo rectilinear motion along different straight lines. Then the centre of mass of system of given two particles also always moves along a straight line.

STATEMENT-2 : If direction of net momentum of a system of particles (having nonzero net momentum) is fixed, the centre of mass of given system moves along a straight line.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

Solution In statement-1, nothing is said about acceleration of both particles. Hence angle between velocity and acceleration of centre of mass may not be zero. Consequently centre of mass may not move along a straight line. Hence statement-1 is false.

[Ans. (D)]

***Example* 9. STATEMENT-1 :** Gas ejected from rocket will never exert thrust on the rocket if the ejected gas and the rocket move in the same direction.

STATEMENT-2 : To exert thrust on rocket in its direction of motion, the ejected gas (w.r.t. rocket) must move opposite to velocity of rocket (w.r.t. ground).

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

Solution Even if the ejected gas follows the rocket, ejected gases shall exert thrust on rocket if the velocity of ejected gas w.r.t. rocket is non zero and opposite in direction to motion of rocket w.r.t. ground. Hence statement-1 is false and statement-2 is true.

[Ans. (D)]

***Example* 10. STATEMENT-1 :** Momentum of an isolated system is conserved.

STATEMENT-2 : Momentum of one particle within an isolated system is not necessarily conserved because other particles in the system may be interacting with it.

(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.

(C) Statement-1 is True, Statement-2 is False.

(D) Statement-1 is False, Statement-2 is True.

[Ans. (B)]

***Example* 11. STATEMENT-1 :** When a body collides elastically and head on with another identical stationary body on a frictionless surface, it losses all of its kinetic energy (No external forces on the system of the two bodies and no rotation of the bodies).

STATEMENT-2 : In elastic collisions, only momentum is conserved.

(a) If both assertion and reason are true and reason is the correct explanation of assertion.

(b) If both assertion and reason are true but reason is not the correct explanation of assertion.

(c) If assertion is true but reason is false.

(d) If assertion is false but reason is true.

[Ans. (C)]

True/False

Example 1. S_1: The magnitude of momentum of a heavy object is greater than that of a light object moving at the same speed.

S_2: In a perfectly inelastic collision, all the initial kinetic energy of the colliding bodies is converted to heat.

S_3: The momentum of a system of colliding bodies may be conserved even though the total mechanical energy may not.

S_4 :The velocity of the center of mass of a system is the system's total momentum divided by its total mass.

(A) T F T T (B) F F T T
(C) T F F T (D) F F F T

[Ans. (A)]

Solution S_1: Since speeds are same, the heavier body shall have more magnitude of momentum. Hence this statement is true.

S_2: In a perfectly inelastic collision, final momentum of system is equal to initial momentum of system. If final momentum of system is nonzero, there will be some final kinetic energy of system. Hence all initial kinetic energy of system cannot always be dissipated. Thus the statement is false.

S_3: In an inelastic collision, momentum of system is conserved but mechanical energy is not conserved. Hence the statement is true.

S_4 : Statement is true by definition.

Example 2. S_1: The locations of centre of mass and centre of gravity may be different for an object.

S_2: Internal forces can change, the momentum of a non–rigid body.

S_3: If the resultant force on a system of particles is non-zero, then the distance of the centre of mass of system may remain constant from a fixed point.

S_4: If net external force on a two body system is always zero, then direction of velocity of the centre of mass of given system may change.

(A) *FTTF* (B) *TFTT*
(C) *TFTF* (D) *FTFT*

[Ans. (C)]

Solution S_1: If the object is large so that gravitational acceleration is not same at every point, both will have different locations.

S_2: Internal forces cannot change momentum of any kind of system.

S_3: If resultant force on a system of particle is non zero, the centre of mass shall accelerate and in some condition it may move along a circular path. Thus the distance of centre of mass from centre of circle shall be constant. Hence the statement is true.

S_4: Since net external force in system is zero, velocity of its centre of mass cannot change. Hence the statement is false.

Example 3. The locations of centre of mass and centre of gravity can be different for an object.

Solution If the object is large so that gravitational acceleration is not same at every point, both will have different locations.

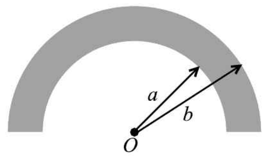
Example 4. Internal forces can change, the momentum of a non–rigid body.

Solution **(False)** Internal forces cannot change momentum of any kind of system.

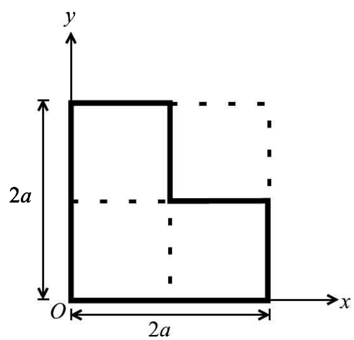
EXERCISE

Exercise–1: Subjective Problems

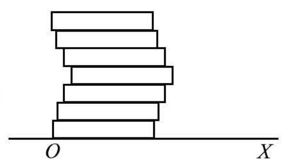
1. Can centre of mass of a body coincide with the geometrical centre of the body?
2. If one of the particles is heavier than the other, to which side will their centre of mass shift?
3. Does centre of mass of a system of two particles lie on the line joining the particles?
4. Can centre of mass of a body lie at a point where there is absolutely no mass?
5. Where does centre of mass of a uniform triangular lamina lie?
6. Three particles of mass 1 kg, 2 kg and 3 kg are placed at the corners A, B and C respectively of an equilateral triangle ABC of edge 1 m. Find the distance of their centre of mass from A.
7. Find the distance of centre of mass of a uniform plate having semicircular inner and outer boundaries of radii a and b from the centre O.



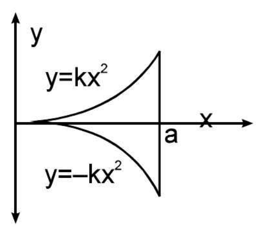
8. Find the position of centre of mass of the uniform planner section shown in figure with respect to the origin (O)



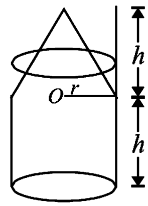
9. Seven homogeneous bricks, each of length L, are arranged as shown in figure. Each brick is displaced with respect to the one in contact by $L/10$. Find the x-coordinate of the centre of mass relative to the origin O shown.



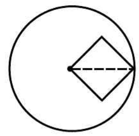
10. A uniform disc of radius R is put over another uniform disc of radius $2R$ of the same thickness and density. The peripheries of the two discs touch each other. Locate the centre of mass of the system from the centre of large disc.
11. A disc of radius R is cut out from a larger disc of radius $2R$ in such a way that the edge of the hole touches the edge of the disc. Locate the centre of mass of the residual disk.
12. A thin sheet of metal of uniform thickness is cut into the shape bounded by the line $x = a$ and $y = \pm kx^2$, as shown. Find the coordinates of the centre of mass.



13. Four particles of mass 5, 3, 2, 4 kg are at the points (1, 6), (-1, 5), (2, -3), (-1, -4). Find the coordinates of their centre of mass.
14. The linear mass density of a ladder of length l increases uniformly from one end A to the other end B,
    (a) Form an expression for linear mass density as a function of distance x from end A where linear mass density λ_0. The density at one end being twice that of the other end.
    (b) Find the position of the centre of mass from end A.



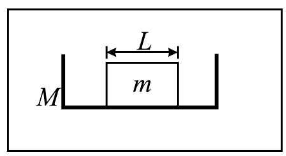
15. Find the distance of centre of mass from O of a composite solid cone and solid cylinder made of same material.



16. From a uniform circular disc of radius R, a square is cut out with radius R as its diagonal. Find the centre of mass of remainder is at a distance.(from the centre)

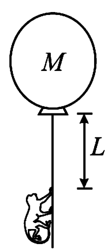
Motion of Com

17. (a) Two blocks of masses 10 kg and 20 kg are placed on the x-axis. The first mass is moved on the axis by a distance of 2 cm. By what distance should the second mass be moved to keep the position of the centre of mass unchanged?
    (b) Two blocks of masses 10 kg and 30 kg are placed along a vertical line. The first block is raised through a height of 7 cm. By what distance should the second mass be moved to raise the centre of mass by 1 cm ?
18. Consider a gravity-free hall in which a tray of mass M, carrying a cubical block of ice of mass m and edge L, is at rest in the middle (figure show). If the ice melts, by what distance does the centre of mass of " the tray plus the ice" system descend?



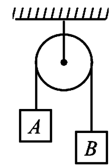
Gravity free hall

19. Mr. Verma (50 kg) and Mr. Mathur (60 kg) are sitting at the two extremes of a 4 m long boat (40 kg) standing still in water. To discuss a mechanics problem, they come to the middle of the boat. Neglecting friction with water, how far does the boat move in the water during the process ?
20. The balloon, the light rope and the monkey shown in figure are at rest in the air. If the monkey reaches the top of the rope, by what distance does the balloon descend? Mass of the balloon = M, mass of the monkey = m and the length of the rope ascended by the monkey = L.

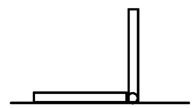


21. The figure shows the positions and velocities of two particles. If the particles move under the mutual attraction of each other, then find the position of centre of mass at $t = 1$ s.

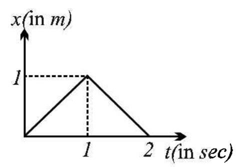
22. Two cars initially at rest are free to move in the x-direction. Car A has mass 4 kg and car B has mass 2 kg. They are tied together, compressing a spring in between them. When the spring holding them together is burned, car A moves off with a speed of 2 m/s
    (a) with what speed does car B leave.
    (b) how much energy was stored in the spring before it was burned.
23. A 45.0-kg girl is standing on a plank that has a mass of 150 kg. The plank, originally at rest, is free to slide on a frozen lake, which is a flat, frictionless supporting surface. The girl begins to walk along the plank at a constant speed of 1.50 m/s relative to the plank. (a) What is her speed relative to the ice surface? (b) What is the speed of the plank relative to the ice surface?
24. Two balls of equal masses are projected upward simultaneously, one from the ground with speed 50 m/s and other from a 40 m high tower with initial speed 30 m/s. Find the maximum height attained by their centre of mass.
25. In the arrangement shown in the figure, $m_A = 2$ kg and $m_B = 1$ kg. String is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.



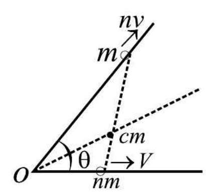
26. Two planks each of mass m and length L are connected by a frictionless, massless hinge as shown in the figure. Initially the system is at rest on a level frictionless surface. The vertical plank falls anticlockwise and finaly comes to rest on the top of the horizontal plank. Find the displacement of the hinge till the two planks come in contact.



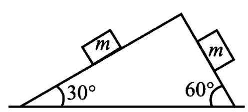
27. 2 bodies m_1 and m_2 of mass 1 and 2 kg respectively are moving along x-axis under the influence of mutual force only. The velocity of their centre of mass at a given instant is 2 m/s. The x coordinate of m_1 is plotted against time. Then plot the x coordinate of m_2 against time. (Both are initially located at origin)



28. Two masses, nm and m, start simultaneously from the intersection of two straight lines with velocities v and nv respectively. It is observed that the path of their centre of mass is a straight line bisecting the angle between the given straight lines. Find the magnitude of the velocity of centre of inertia. (here θ = angle between the lines)



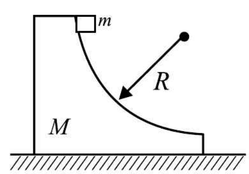
29. Two blocks of equal masses m are released from the top of a smooth fixed wedge as shown in the figure. Find the magnitude of the acceleration of the centre of mass of the two blocks.



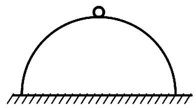
30. Two bodies of same mass tied with an inelastic string of length l lie together. One of them is projected vertically upwards with velocity $\sqrt{6gl}$. Find the maximum height up to which the centre of mass of system of the two masses rises.
31. A platform of mass m and a counter weight of mass $(m + M)$ are connected by a light cord which passes over a smooth pulley. A man of mass M is standing on the platform which is at rest. If the man leaps vertically upwards with velocity u, find the distance through which the platform will descend. Show that when the man meets the platform again both are in their original positions.

Conservation of Momentum

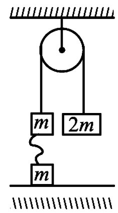
32. A projectile is fired from a gun at an angle of 45° with the horizontal and with a speed of 20 m/s relative to ground. At the highest point in its flight the projectile explodes into two fragments of equal mass. One fragment, whose initial speed is zero falls vertically. How far from the gun does the other fragment land, assuming a horizontal ground ? Take $g = 10$ m/s^2?
33. A particle of mass 2 m is projected at an angle of 45° with horizontal with a velocity of $20\sqrt{2}$ m/s. After 1 s explosion takes place and the particle is broken into two equal pieces. As a result of explosion one part comes to rest. Find the maximum height attained by the other part. Take $g = 10$ m/s^2.
34. A rail road car of mass M is at rest on frictionless rails when a man of mass m starts moving on the car towards the engine. If the car recoils with a speed v backward on the rails, with what velocity is the man approaching the engine?
35. A (trolley + child) of total mass 200 kg is moving with a uniform speed of 36 km/h on a frictionless track. The child of mass 20 kg starts running on the trolley from one end to the other (10 m away) with a speed of 10 m s^{-1} relative to the trolley in the direction of the trolley's motion and jumps out of the trolley with the same relative velocity. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?
36. A boy of mass 60 kg is standing over a platform of mass 40 kg placed over a smooth horizontal surface. He throws a stone of mass 1 kg with velocity $v = 10$ m/s at an angle of 45° with respect to the ground. Find the displacement of the platform (with boy) on the horizontal surface when the stone lands on the ground. ($g = 10$ m/s^2)
37. A uranium-238 nucleus, initially at rest, emits an alpha particle with a speed of 1.17×10^7 m/s. Calculate the recoil speed of the residual nucleus thorium-234. Assume that the mass of a nucleus is proportional to the mass number.
38. A neutron initially at rest, decays into a proton, an electron and an antineutrino. The ejected electron has a momentum of 1.4×10^{-26} kg-m/s and the antineutrino 6.5×10^{-27} kg-m/s. Find the recoil speed of the proton (a) if the electron and the antineutrino are ejected along the same direction and (b) if they are ejected along perpendicular directions. Mass of the proton = 1.67×10^{-27} kg.
39. A small cube of mass 'm' slides down a circular path of radius 'R' cut into a large block of mass 'M'. 'M' rests on a table and both blocks move without friction. The blocks initially are at rest and 'm' starts from the top of the path. Find the velocity 'v' of the cube as it leaves the block. Initially the line joining m and the centre is horizontal.



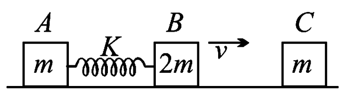
40. A hemisphere of radius R and of mass 4m is free to slide with its base on a smooth horizontal table. A particle of mass m is placed on the top of the hemisphere. Find the angular velocity of the particle relative to hemisphere at an angular displacement θ when velocity of hemisphere has become v.



41. A man whose mass is m kg jumps vertically into air from a sitting position in which his centre of mass is at a height h_1 from the ground. When his feet are just about to leave the ground his centre of mass is h_2 from the ground and finally rises to h_3 when he is at the top of the jump.
    (a) What is the upward force exerted by the ground on him treating it as a constant?
    (b) Find work done by normal reaction from ground.
42. Two trolleys A and B are free to move on a level frictionless track, and are initially stationary. A man on trolley A throws a bag of mass 10 kg with a horizontal velocity of 4 m/s with respect to himself on to trolley B of mass 100 kg. The combined mass of trolley A (excluding bag) and the man is 140 kg. Find the ratio of velocities of trolleys A and B, just after the bag lands on trolley B.
43. A bob of mass m attached with a string of length l tied to a point on ceiling is released from a position when its string is horizontal. At the bottom most point of its motion, an identical mass m gently stuck to it. Find the angle from the vertical to which it rises.
44. Bullets of mass 10 g each are fired from a machine gun at rate of 60 bullets/minute. The muzzle velocity of bullets is 100 m/s. The thrust force due to firing bullets experienced by the person holding the gun stationary is ________.
45. A spaceship is moving with constant speed v_0 in gravity free space along + Y-axis suddenly shoots out one third of its part with speed $2v_0$ along + X-axis. Find the speed of the remaining part.



46. Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring. Initially the spring is unstretched and the velocity of 2 m/s is imparted to 3 kg block as shown. Find the maximum velocity of 6 kg block during subsequent motion.



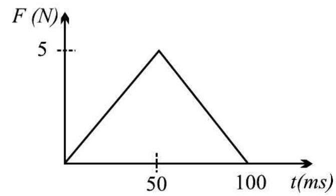
47. Two blocks A and B of masses m and $2m$ respectively are connected by a spring of force constant k. The masses are moving to the right with uniform velocity v each, the heavier mass leading the lighter one. The spring in between them is of natural length during the motion. Block B collides with a third block C of mass m, at rest. The collision being completely inelastic. Calculate the maximum compression of the spring.

Impulse

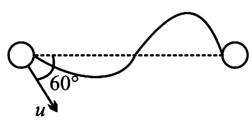
48. Velocity of a particle of mass 2 kg varies with time t according to the equation $\vec{v} = (2t\hat{i} + 4\hat{j})\,m/s$. Here t is in seconds. Find the impulse imparted to the particle in the time interval from $t = 0$ to $t = 2$ s.
49. During a heavy rain, hailstones of average size 1.0 cm in diameter fall with an average speed of 20 m/s. Suppose 2000 hailstones strike every square meter of a 10 m × 10 m roof

perpendicularly in one second and assume that the hailstones do not rebound. Calculate the average force exerted by the falling hailstones on the roof. Density of a hailstones is 900 kg/m³, take $\pi = 3.14$.

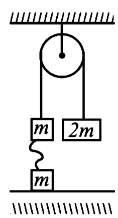
50. A steel ball of mass 0.5 kg is dropped from a height of 4 m on to a horizontal heavy steel slab. The collision is elastic and the ball rebounds to its original height.
    (a) Calculate the impulse delivered to the ball during impact.
    (b) If the ball is in contact with the slab for 0.002s, find the average reaction force on the ball during impact.
51. A particle *A* of mass 2kg lies on the edge of a table of height 1m. It is connected by a light inelastic string of length 0.7m to a second particle *B* of mass 3kg which is lying on the table 0.25m from the edge (line joining *A* & *B* is perpendicular to the edge). If *A* is pushed gently so that it start falling from table then, find the speed of *B* when it starts to move. Also find the impulsive tension in the string at that moment.
52. A smooth sphere *A* of mass 0.1 kg is moving with speed 5m/s when it collides head on with another smooth stationary sphere of same radius. If *A* is brought to rest by the impact and $e = \frac{1}{2}$, find the mass of *B*, its speed just after impact and magnitude of impulse during collision.
53. A force *F* acts on an object (mass = 1 kg) which is initially at rest as shown in the figure. Draw the graph showing the momentum of the object varying during the time for which the force acts.



54. A bullet of mass *m* strikes an obstruction and deviates off at 60° to its original direction. If its speed is also changed from *u* to *v*, find the magnitude of the impulse acting on the bullet.
55. In the figure shown, each tiny ball has mass *m*, and the string has length *L*. One of the ball is imparted a velocity *u*, in the position shown, in which the initial distance between the balls is $L/\sqrt{3}$. The motion of ball occurs on smooth horizontal plane. Find the impulse of the tension in the string when it becomes taut.



56. After scaling a wall of 3 m height a man of weight *W* drops himself to the ground. If his body comes to a complete stop 0.15 sec. After his feet touch the ground, calculate the average impulsive force in the vertical direction exerted by ground on his feet. ($g = 9.8$ m/s²)
57. The Atwood machine in fig has a third mass attached to it by a limp string. After being released, the 2m mass falls a distance *x* before the limp string becomes taut. Thereafter both the mass on the left rise at the same speed. What is the final speed? Assume that pulley is ideal.

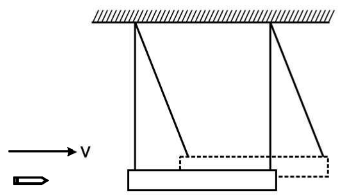


58. Two particles, each of mass *m*, are connected *y* a light inextensible string of length 2l. Initially they lie on a smooth horizontal table at points *A* and *B* distant l apart. The particle at *A* is projected across the table with velocity *u*. Find the speed with which the second particle

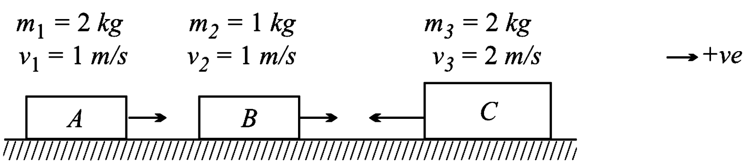
begins to move if the direction of u is, (a) along BA, (b) at n angle of 120ŗ with AB, (c) perpendicular to AB. In each case calculate (in terms of m and u) the impulsive tension in the string.

Collision

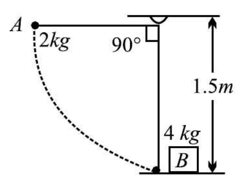
59. After an elastic collision between two balls of equal masses, one is observed to have a speed of 3 m/s along the positive x-axis and the other has a speed of 2 m/s along the negative x-axis. What were the original velocities of the balls?
60. A particle moving with kinetic energy K makes a head on elastic collision with an identical particle at rest. Find the maximum elastic potential energy of the system during collision.
61. A ball of mass m moving at a speed v makes a head on collision with an identical ball at rest. The kinetic energy of the balls after the collision is 3/4 of the original $K.E.$ Calculate the coefficient of restitution.
62. A particle of mass m moving with a speed v hits elastically another stationary particle of mass 2m on a smooth horizontal circular tube or radius r. Find the time when the next collision will take place?
63. A ball falls on the ground from a height of 2.0 m and rebounds upto a height of 1.5 m. Find the coefficient of restitution.
64. A bullet of mass 25 g is fired horizontally into a ballistic pendulum of mass 5.0 kg and gets embedded in it (figure). If the centre of the pendulum rises by a distance of 10 cm, find the speed of the bullet.



65. Three carts move on a frictionless track with inertias and velocities as shown. The carts collide and stick together after successive collisions.
    (a) Find loss of mechanical energy when B & C stick together.
    (b) Find magnitude of impulse experienced by A when it sticks to combined mass (B & C).



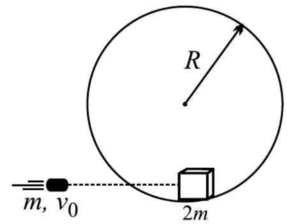
66. A body is thrown vertically upwards from ground with a speed of 10 m/s. If coefficient of restitution of ground, e = 1/2. Find
    (a) the total distance travelled by the time it almost stops.
    (b) time elapsed (after the ball has been thrown) when it is at its subsequent maximum height for the third time.
67. A sphere A is released from rest in the position shown and strikes the block B which is at rest. If e = 0.75 between A and B and μ_k = 0.5 between B and the support, determine



(a) the velocity of A just after the impact

(b) the maximum displacement of B after the impact.

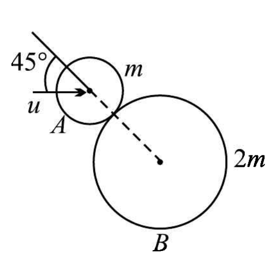
68. A small block of mass $2m$ initially rests at the bottom of a fixed circular, vertical track, which has a radius of R. The contact surface between the mass and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_0 and remain embedded in the block as the block and the bullet circle the loop. Determine each of the following in terms of m, v_0, R and g.



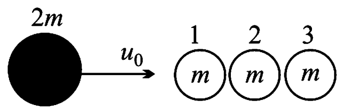
(a) The speed of the masses immediately after the impact.

(b) The minimum initial speed of the bullet if the block and the bullet are to successfully execute a complete ride on the loop

69. Disc A of mass m collides with stationary disk B of mass $2m$ as shown in figure. Find the value of coefficient of restitution for which the two disks move in perpendicular direction after collision.



70. A heav ball of mass 2m moving with a velocity u_0 collides elastically head-on with a cradle of three identical balls each of mass m as shown in figure. Determine the velocity of each ball after collision.

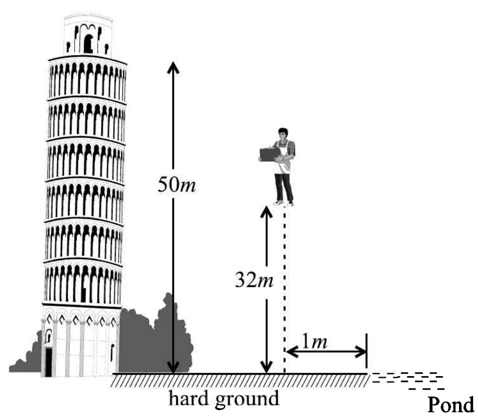


71. A sphere of mass m_1 in motion hits directly another sphere of mass m_2 at rest and sticks to it, the total kinetic energy after collision is 2/3 of their total $K.E.$ before collision. Find the ratio of $m_1 : m_2$.

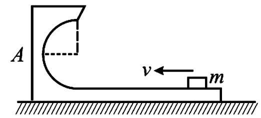
72. A ball is dropped from a height of 1 m. The coefficient of restitution between the ground and the ball is 1/3. The height of which the ball will rebound after two collisions with ground is ______.

Exercise–2

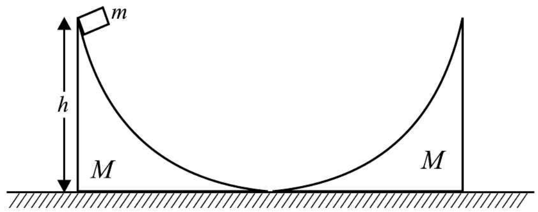
1. A man of mass 56 kg having a bag of mass 2 kg slips from the roof of a tall building of height 50 m and starts falling vertically (figure). When at a height 32 m from the ground, he notices that the ground below him is pretty hard, but there is a pond at a horizontal distance 1 m from the line of fall. In order to save himself he throws the bag horizontally (with respect to himself) in the direction opposite to the pond. Calculate the minimum horizontal velocity imparted to the bag so that the man lands in the water. If the man just succeeds to avoid the hard ground, where will the bag land?



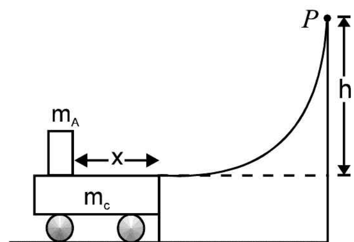
2. Figure shows a small block of mass m which is started with a speed v on the horizontal part of the bigger block of mass M placed on a horizontal floor. The curved part of the surface shown is semicircular. All the surfaces are frictionless. Find the speed of the bigger block when the smaller block reaches the point A of the surface.



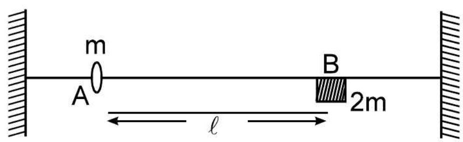
3. The inclined surfaces of two moveable wedges of the same mass M are smoothly conjugated with the horizontal plane as shown in the figure. A small block of mass 'm' slides down the left wedge from a height 'h'. To what maximum height will the block rise along the right wedge? Neglect the friction.



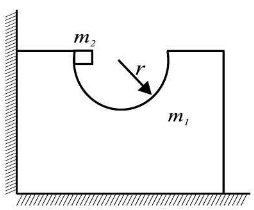
4. A block A having a mass 'm_A' is released from rest at the position P shown and slides freely down the smooth inclined ramp. When it reaches the bottom of the ramp it slides horizontally onto the surface of a cart of mass m_c for which the coefficient of friction between the cart and the box is 'μ'. If 'h' be the initial height of A, determine the final velocity of the cart once the block comes to rest in it. Also determine the position 'x' of the box on the cart after it comes to rest relative to cart. (The cart moves on smooth horizontal surface.)



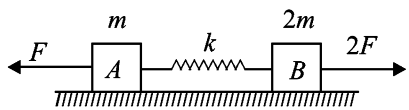
5. A small ring A of mass 'm' is attached at an end of a light string the other end of which is tied to a block B of mass 2m. The ring is free to move on a fixed smooth horizontal rod. Find the velocity of ring A and tension in the string when it becomes vertical.



6. A symmetric block of mass m_1 with a groove of hemispherical shape of radius 'r' rests on a smooth horizontal surface near the wall as shown in the figure. A small block of mass m_2 slides without friction from the initial position. Find the maximum velocity of the block m_1.

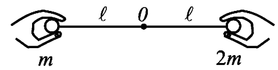


7. Two blocks A & B of mass 'm' & $2m$ respectively are joined to the ends of an under formed massless spring of spring constant 'k'. They can move on a horizontal smooth surface. Initially A & B have velocities 'u' towards left and '$2u$' towards right respectively. Constant forces of magnitudes F and $2F$ are always acting on A and B respectively in the directions shown. Find the maximum extension in the spring during the motion.



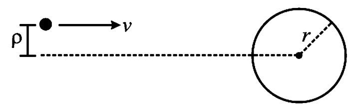
8. Two ball having masses m and 2m are fastened to two light strings of same length ℓ (figure). The other ends of the strings are fixed at O. The strings are kept in the same horizontal line

and the system is released from rest. The collision between the balls is elastic.



(a) Find the velocities of the balls just after their collision. (b) How high will the balls rise after the collision?

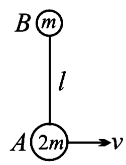
9. A small particle travelling with a velocity v collides elastically with a spherical body of equal mass and of radius r initially kept at rest. The centre of this spherical body is located at a distance ρ ($< r$) away from the direction of motion of the particle (figure). Find the final velocities of the two particles.



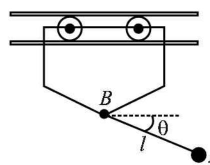
[***Hint :*** The force acts along the normal to the sphere through the contact. Treat the collision as one dimensional for this direction. In the tangential direction no force acts and the velocities do not change.]

10. A 24–kg projectile is fired at an angle of 53° above the horizontal with an initial speed of 50 m/s. At the highest point in its trajectory, the projectile explodes into two fragments of equal mass, the first of which falls vertically with zero initial speed.
    (a) How far from the point of firing does the second fragment strike the ground? (Assume the ground is level.)
    (b) How much energy was released during the explosion?

11. Two masses A and B connected with an inextensible string of length l lie on a smooth horizontal plane. A is given a velocity of v m/s along the ground perpendicular to line AB as shown in figure. Find the tension in string during their subsequent motion.

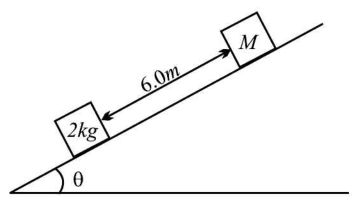


12. The simple pendulum A of mass m_A and length l is suspended from the trolley B of mass m_B. If the system is released from rest at $\theta = 0$, determine the velocity v_B of the trolley and tension in the string when $\theta = 90°$. Friction is negligible.



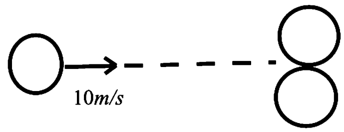
13. A massive vertical wall is approaching a man at a speed u. When it is at a distance of 10 m, the man throws a ball with speed 10 m/s at an angle of 37° which after completely elastic rebound reaches back directly into his hands. Find the velocity u of the wall .

14. Two blocks of mass 2kg and M are at rest on an indiclined plane and are separated by a distance of 6.0m as shown. The coefficient of friction between each of the blocks and the inclined plane is 0.25. The 2kg block is given a velocity of 10.0m/s up the inclined plane. It collides with M, comes back and has a velocity of 1.0m/s when it reaches its initial position. The other block M after the collision moves 0.5 m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M. [Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10\text{m/s}^2$]

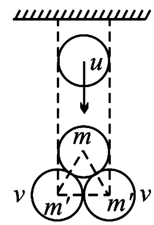


15. A stream of particles each of mass m moving with velocity v with number of particles per unit volume n strike against a wall at an angle θ to the normal elastically. Find the pressure exerted by the stream on the wall.

16. A ball with initial speed of 10m/s collides elastically with two other identical ball whose centres are on a line perpendicular to the initial velocity and which are initially in contact with each other. All the three ball are lying on a smooth horizontal table. The first ball is aimed directly at the contact point of the other two balls All the balls are smooth. Find the velocities of the three balls after the collision.

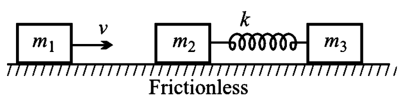


17. Two equal spheres of mass *m*' are suspended by vertical strings so that they are in contact with their centers at same level. A third equal spheres of mass *m* falls vertically and strikes elastically the other two simultaneously so that their centres at the instant of impact form an equilateral triangle in a vertical plane. If *u* is the velocity of m just before impact, find the velocities just after impact and the impulse of tension of the strings.



18. Two particles, each of mass *m*, are connected by a light inextensible string of length 2l. Initially they lie on a smooth horizontal table at points *A* and *B* distant l apart. The particle at *A* is projected across the table with velocity *u*. Find the speed with which the second particle begins to move if the direction of *u* is, (a) along *BA*, (b) at an angle of 120^0 with *AB*, (c) perpendicular to *AB*. In each case calculate (in terms of *m* and *u*) the impulse of tension in the string.

19. Mass m_1 hits & sticks with m_2 while sliding horizontally with velocity *v* along the common line of centres of the three equal masses ($m_1 = m_2 = m_3 = m$). Initially masses m_2 and m_3 are stationary and the spring is unstretched. Find
    (a) the velocities of m_1, m_2 and m_3 immediately after impact.
    (b) the maximum kinetic energy of m_3.
    (c) the minimum kinetic energy of m_2.
    (d) the maximum compression of the spring.



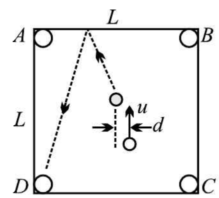
20. A sphere of mass m is moving with a velocity $4\hat{i} - \hat{j}$ when it hits a smooth wall and rebounds with velocity $\hat{i} + 3\hat{j}$. Find the impulse it receives. Find also the coefficient of restitution between the sphere and the wall.

21. Two bodies of same mass tied with an inelastic string of length *l* lie together. One of them is projected vertically upwards with velocity $\sqrt{6gl}$. Find the maximum height up to which the centre of mass of system of the two masses rises.

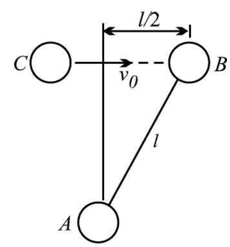
22. A flexible chain has a length *l* and mass *m*. It is lowered on the table top with constant velocity *v*. Find the force that the chain exerts on the table as a function of time.

23. A billiard table is 15 cm by 20 cm. A smooth ball of coefficient of restitution $e = 4/9$ is projected from a point on the shorter side so as to describe a rectangle and return to the point of projection after rebounding at each of the other three cushions. Find the position of the point and the direction of projection.

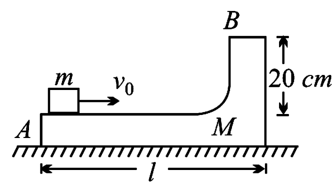
24. In a game of Carom Board, the Queen (a wooden disc of radius 2 cm and mass 50 gm) is placed at the exact center of the horizontal board. The striker is a smooth plastic disc of radius 3 cm and mass 100 gm. The board is frictionless. The striker is given an initial velocity 'u' parallel to the sides BC or AD so that it hits the Queen inelastically with coefficient of restitution = 2/3. The impact parameter for the collision is 'd' (shown in the figure). The Queen rebounds from the edge AB of the board inelastically with same coefficient of restitution = 2/3 and enters the hole D following the dotted path shown. The side of the board is L. Find the value of impact parameter 'd' and the time which the Queen takes to enter hole D after collision with the striker.



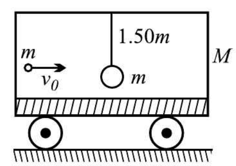
25. Three spheres, each of mass m, can slide freely on a frictionless, horizontal surface. Spheres A and B are attached to an inextensible inelastic cord of length l and are at rest in the position shown when sphere B is struck directly by sphere C which is moving to the right with a velocity v_0. Knowing that the cord is taut when sphere B is struck by sphere C and assuming perfectly elastic impact between B and C, determine the velocity of each sphere immediately after impact.



26. A wedge of mass $M = 2$m rests on a smooth horizontal plane. A small block of mass m rests over it at left end A as shown in figure. A sharp impulse is applied on the block, due to which it starts moving to the right with velocity $v_0 = 6$ ms^{-1}. At highest point of its trajectory, the block collides with a particle of same mass m moving vertically downwards with velocity $v = 2$ ms^{-1} and gets stuck with it. If the combined body lands at the end point A of body of mass M, calculate length l. Neglect friction ($g = 10$ ms^{-2})



27. A ball of mass = 1Kg is hung vertically by a thread of length $l = 1.50$ m. Upper end of the thread is attached to the ceiling of a trolley of mass M = 4 kg. Initially, trolley is stationary and it is free to move along horizontal rails without friction. A shell of mass m = 1 kg moving horizontally with velocity v_0 = 6ms^{-1} collides with the ball and gets stuck with it. As a result, thread starts to deflect towards right. Calculate its maximum deflection with the vertical. (g = 10m s^{-2})

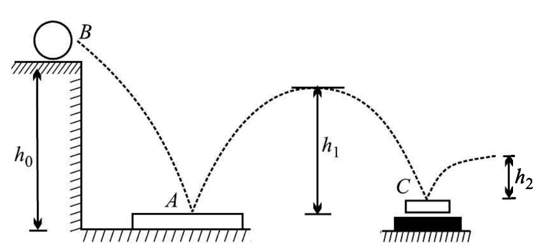


28. A 70g ball B droped from a height h_0 = 9 m reaches a height h_2 = 0.25m after bouncing twice from identical 210g plates.

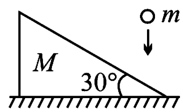
Plate A rests directly on hard ground, while plate C rests on a foam-rubber mat. Determine

(a) the coefficient of resitution between the ball and the plates,

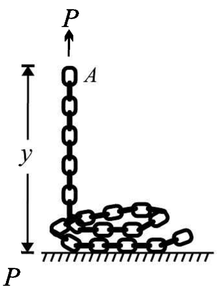
(b) the height h_1 of the ball's first bounce.



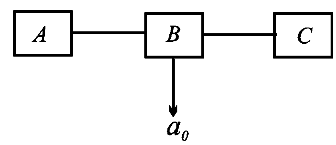
29. A ball of mass $m = 1$ kg falling vertically with a velocity $v_0 = 2$ m/s strikes a wedge of mass $M = 2$kg kept on a smooth, horizontal surface as shown in figure. The coefficient of restitution between the ball and the wedge is $e = 1/2$. Find the velocity of the wedge and the ball immediately after collision.



30. A chain of length l and m lies in a pile on the floor. It its end A is raised vertically at a constant speed v_0, express in terms of the length y of chain which is off the floor at any given instant.
   (a) the magnitude of the force P applied to end A.
   (b) the reaction of the floor.
   (c) energy lost during the lifting of the chain.



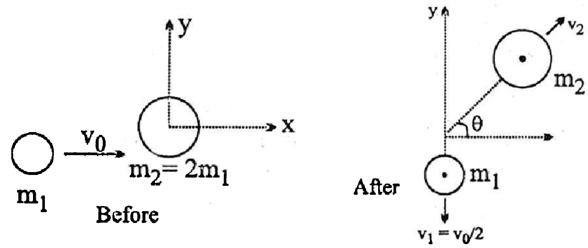
31. 3 blocks of mass 1kg each kept on horizontal smooth ground are connected by 2 taut strings of length l as shown. B is pulled with constant acceleration a_0 in direction shown. Find the relative velocity of A & C just before striking.



32. A particle is projected from point O on level ground towards a smooth vertical wall 50 cm from O and hits the wall. The initial velocity of the particle is 30 m/s at 45ŗ to the horizontal and the coefficient of restitution between the particle and the wall is e. Find the distance from of the point at which the particle hits the ground again if (a) $e = 0$, (b) $e = 1$, $e = 1/2$.

33. A massive vertical wall is approaching a man at a speed u. When it is at a distance of 10m, the man throws a ball with speed 10 m/s at an angle of 37ŗ which after completely elastic rebound reaches back directly into his hands. Find the velocity u of the wall.

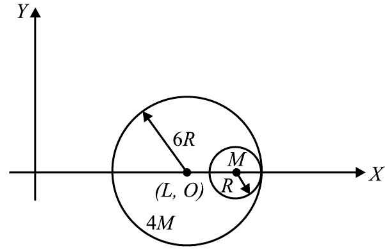
34. A mass m_1 with initial speed v_0 in the positive x-direction collides with a mass $m_2 = 2m_1$ which is initially at rest at the origin, as shown in figure. After the collision m_1 moves off with speed $v_1 = v_0/2$ in the negative y-direction, and m_2 moves off with speed v_2 at angle θ.
   (a) Find the velocity (magnitude and direction) of the centre of mass before the collision, as well as its velocity after the collision.
   (b) Write down the x and y-components of the equation of conservation of momentum for the collision.
   (c) Determine $\tan \theta$, and find v_2 in terms of v_0.
   (d) Determine how much (if any) energy was gained or lost in the collision, and state whether the collision was elastic or inelastic.



Exercise-3

JEE Questions of Previous Years

1. A small sphere of radius R is held against the inner surface of a larger sphere of radius $6R$. The masses of large and small spheres are $4M$ and M, respectively. This arrangement is placed on a horizontal table. There is no friction between any surfaces of contact. The small sphere is now released. The coordinates of the centre of the large sphere when the smaller sphere reaches the other extreme position is **[JEE - 96]**



(A) $(L - 2R, 0)$ (B) $(L + 2R, 0)$ (C) $(2R, 0)$ (D) $(2R - L, 0)$

2 A body of mass 5kg moves along the x-axis with a velocity 2m/s. A second body of mass 10kg moves along the y-axis with a velocity $\sqrt{3}$ m/s.

(a) If they collide at the origin and stick together, then the final velocity of the combined mass after collision is

(A) $\frac{3}{4}$ m/s (B) $\frac{4}{3}$ m/s (C) $\frac{2}{3}$ m/s (D) $\frac{3}{2}$ m/s

(b) In the above question, the amount of heat liberated in the collision is

(A) $\frac{35}{3}J$ (B) $\frac{30}{7}J$ (C) $\frac{36}{7}J$ (D) None of these

3 A ball of mass 'm', moving with uniform speed, collides elastically with another stationary ball. The incident ball will lose maximum kinetic energy when the mass of the stationary ball is

(A) m (B) $2m$ (C) $4m$ (D) infinity

4 An isolated particle of mass m is moving in a horizontal plane $(x - y)$ along the x-axis, a certain height above the ground. It suddenly explodes into two fragments of masses $\frac{m}{4}$ and $\frac{3m}{4}$. An instant later, the smaller fragment is at $y = +15$ cm. The larger fragment at this instant is at **[JEE - 97]**

(A) $y = -5$ cm (B) $y = +20$ cm (C) $y = +5$ cm (D) $y = -20$ cm

5 A particle of mass 'm' and velocity '$\vec{v}$' collides oblique elastically with a stationary particle of mass 'm'. The angle between the velocity vectors of the two particles after the collision is

(A) 45° (B) 30° (C) 90° (D) None of these

6 A shell explodes in a region of negligible gravitational field, giving out n fragments of equal mass m. Then its total **[REE - 97]**

(A) Kinetic energy is smaller than that before the explosion
(B) Kinetic energy is greater than that before the explosion
(C) Momentum and kinetic energy depend on n
(D) Momentum is equal to that before the explosion.

7 Two particles approach each other with different velocities. After collision, one of the particles has a momentum $\vec{p}$ in their center of mass frame. In the same frame, the momentum of the other particle is

(A) 0 (B) $-\vec{p}$ (C) $-\vec{p}/2$ (D) $-2\vec{p}$

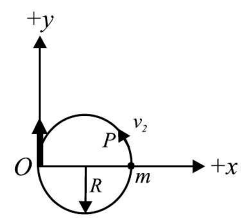
8 Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{u}_1$ and $\vec{u}_2$ respectively at time $t = 0$. They collide at time t_0. Their velocities become $\vec{v}_1$ and $\vec{v}_2$ at time $2t_0$ while still moving in air. The value of $[(m_1\vec{v}_1 + m_2\vec{v}_2) - (m_1\vec{u}_1 + m_2\vec{u}_2)]$ is **[JEE - 2001]**

(A) Zero (B) $(m_1 + m_2)gt_0$ (C) $2(m_1 + m_2)gt_0$ (D) $\frac{1}{2}(m_1 + m_2)gt_0$

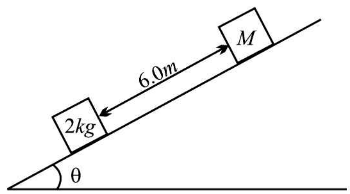
9 Two blocks of masses 10kg and 4kg are connected by a spring of negligible mass and are placed on a frictionless horizontal surface. An impulse gives a speed of 14 ms^{-1} to the heavier block in the direction of the lighter block. Then, the velocity of the centre of mass is **[JEE - 2002]**

(A) 30 ms^{-1} (B) 20 ms^{-1} (C) 10 ms^{-1} (D) 5 ms^{-1}

10 A person at the origin O starts moving with a constant speed v_1 along $+y$ axis. At the same instant, a particle of mass m starts from point P with a uniform speed v_2 along a circular path of radius R, as shown in figure. Find the momentum of the particle with respect to the person as a function of time t.



11 Two blocks of mass 2kg and M are at rest on an indiclined plane and are separated by a distance of 6.0m as shown. The coefficient of friction between each of the blocks and the inclined plane is 0.25. The 2kg block is given a velocity of 10.0 m/s up the inclined plane. It collides with M, comes back and has a velocity of 1.0m/s when it reaches its initial position. The other block M after the collision moves 0.5m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M. [Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10 m/s^2$] **[IIT 99]**



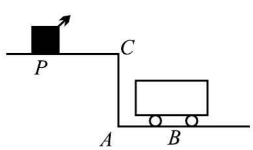
12 Two trolleys A and B of equal masses M are moving in oppsite directions with velocities $\vec{v}$ and $-\vec{v}$ respectively on separate horizontal frictionless parallel tracks. When they start crossing each other, a ball of mass m is thrown from B to A and another of same mass is thrown from A to B with velocities normal to $\vec{v}$. The balls may be thrown in following two ways: **[REE 2000]**

(i) balls from A to B and B to A are thrown simultaneously.

(ii) ball is thrown from A to B after the ball thrown from B reaches A.

Which procedure would lead to a larger change in the velocities of the trolleys?

13 A car P is moving with a uniform speed of $5(3^{1/2})$ m/s towards a carriage of mass 9 Kg at rest kept on the rails at a point B as shown in fig. The height AC is 120 m. Cannon balls of 1 Kg are fired from the car with an initial velocity 100 m/s at an angle 30^0 with the horizontal. The first canon ball hits the stationary carriage after a time t_0 and sticks to it. Determine t_0. At t_0, the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second ball also hits and sticks to the carriage. What will be the horizontal velocity of the carriage just after the second impact? **[IIT 2001]**



14 Two block of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is : **[IIT 2002]**

(A) 30 m/s (B) 20 m/s

(C) 10 m/s (D) 5 m/s

15 Two balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}'_1$ and $\vec{p}'_2$ be their final momenta. The following option(s) is(are) NOT ALLOWED for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.

[JEE 2008]

(A) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}'_2 = a_2\hat{i} + b_2\hat{j}$

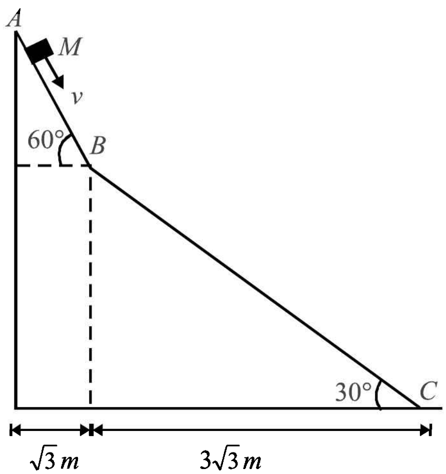
(B) $\vec{p}'_1 = c_1\hat{k}$, $\vec{p}'_2 = c_2\hat{k}$

(C) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}'_2 = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(D) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j}$, $\vec{p}'_2 = a_2\hat{i} + b_1\hat{j}$

Comprehension 16 to 18 (3 Questions)

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic (g = 10 m/s²).



16. The speed of the block at point B immediately after it strikes the second incline is **[JEE 2008]**

(A) $\sqrt{60}$ m/s (B) $\sqrt{45}$ m/s

(C) $\sqrt{30}$ m/s (D) $\sqrt{15}$ m/s

17. The speed of the block at point *C*, immediately before it leaves the second incline is **[JEE 2008]**

(A) $\sqrt{120}$ m/s (B) $\sqrt{105}$ m/s

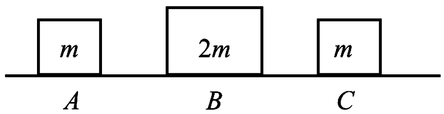
(C) $\sqrt{90}$ m/s (D) $\sqrt{75}$ m/s

18. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point *B*, immediately after it strikes the second incline is **[JEE 2008]**

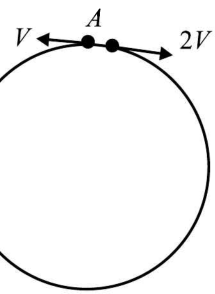
(A) $\sqrt{30}$ m/s (B) $\sqrt{15}$ m/s

(C) 0 (D) $-\sqrt{15}$ m/s

19. Three objects *A*, *B* and *C* are kept in a straight line on a frictionless horizontal surface. These have masses *m*, 2*m* and *m*, respectively. The object *A* moves towards *B* with a speed 9 m/s and makes an elastic collision with it. Thereafter, *B* makes completely inelastic collision with *C*. All motions occur on the same straight line. Find the final speed (in m/s) of the object *C*. **[JEE-2009]**

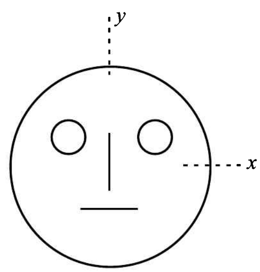


20. Two small particles of equal masses start moving in opposite directions from a point *A* in a horizontal circular orbit. Their tangential velocities are *v* and 2*v*, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at *A*, these two particles will again reach the point *A*? **[JEE-2009]**



(A) 4 (B) 3
(C) 2 (D) 1

21. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is *m*. The mass of the ink used to draw the outer circle is 6m. The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle (–*a*, *a*), right inner circle (*a*, *a*), vertical line (0, 0) and horizontal line (0, –*a*). The y-coordinate of the centre of mass of the ink in this drawing is :
[JEE-2009]



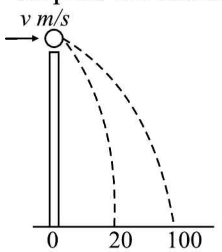
(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$

22. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[JEE 2009]**

(A) linear momentum of the system does not change in time
(B) kinetic energy of the system does not change in time
(C) angular momentum of the system does not change in time
(D) potential energy of the system does not change in time

23. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is/are correct for the system of these two masses? **[JEE 2010]**
(A) Total momentum of the system is 3 kg-ms^{-1}
(B) Momentum of 5 kg mass after collision is 4 kg-ms^{-1}
(C) Kinetic energy of the centre of mass is 0.75 *J*
(D) Total kinetic energy of the system is 4 *J*

24. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity *v* m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity *v* of the bullet is **[JEE 2011]**



(A) 250 m/s (B) $250\sqrt{2}$ m/s (C) 400 m/s (D) 500 m/s

25. A binary star consists of two stars A (mass 2.2 M_S) and *B* (mass 11 M_S), where M_S is the mass of the sun. They are separated by distance *d* and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star *B* about the centre of mass is... **[JEE 2010]**
[Ans (A)]

26. A particle of mass *m* is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is **[JEE ADVANCE 2013]**
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{4}+\alpha$ (C) $\frac{\pi}{4}-\alpha$ (D) $\frac{\pi}{2}$

27. A bob of mass m, suspended by a string of length ℓ_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length ℓ_2, which is initially at rest. Both the strings are massless and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{\ell_1}{\ell_2}$ is... **[JEE ADVANCE 2013]**

Previous years' AIEEE questions

28. Two identical particles move towards each other with velocity $2v$ and v respectively. This velocity of centre of mass is – **[AIEEE 2002]**

(A) v (B) $v/3$ (C) $v/2$ (D) zero

29. Consider the following two statements : **[AIEEE 2003]**

A. Linear momentum of a system of particles is zero

B. Kinetic energy of a system of particles is zero,

Then,

(A) A does not imply B and B does not imply A

(B) A implies B but B does not imply A

(C) A does not imply B but B implies A

(D) A implies B and B implies A

30. Two particles A and B of equal masses suspended from two massless springs of spring constant k_1 and k_2, respectively. If the maximum velocities, during oscillations are equal, the ratio of amplitudes of A and B is : **[AIEEE 2003]**

(A) $\sqrt{k_1 / k_2}$ (B) k_1/k_2 (C) $\sqrt{k_2 / k_1}$ (D) k_1/k_2

31. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an intial acceleration of 10 m/s^2. Then the initial thrust of the blast is : **[AIEEE 2003]**

(A) $3.5 \times 10^5\,N$ (B) $7.0 \times 10^5\,N$ (C) $14.0 \times 10^5\,N$ (D) $1.75 \times 10^5\,N$

32. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and, a body C of mass $\frac{2}{3}M$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards: **[AIEEE 2005]**

(A) depends on height of breaking (B) does not shift

(C) shift towards body C (D) shift towards body B

33. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is : **[AIEEE 2005]**

(A) $\sqrt{Mk}\ L$ (B) $\frac{kL^2}{2M}$ (C) zero (D) $\frac{ML^2}{k}$

34. A mass 'm' moves with a velocity 'v' and collides in elastically with another identical mass. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision : **[AIEEE 2005]**

(A) v (B) $\sqrt{3}v$ (C) $\frac{2}{\sqrt{3}}$ (D) $\frac{v}{\sqrt{3}}$

35. A bomb of mass 16 kg at rest explodes into two pieces of masses of 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1} . The kinetic energy of the other mass is : **[AIEEE 2006]**

(A) 96 J (B) 144 J (C) 288 J (D) 192 J

36. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance *d*, by what distance should the second particle be moved, so as to keep the centre of mass at the same position ?

AIEEE 2006]

(A) d (B) $\frac{m_2}{m_1}d$ (C) $\frac{m_1}{m_1+m_2}d$ (D) $\frac{m_1}{m_2}d$

37. A circular disc of radius *R* is removed from a bigger circular disc of radius 2*R* such that the circumferences of the discs coincide. The centre of mass of the new disc is $\frac{\alpha}{R}$ from the centre of the bigger disc. The value of α is : **[AIEEE 2007]**

(A) 1/3 (B) 1/2 (C) 1/6 (D) 1/4

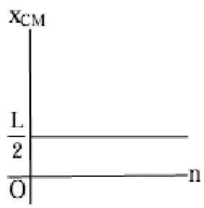
38. A body of mass $m = 3.513$ kg is moving along the *x*- axis with a speed of 5.00 ms^{-1} . The magnitude of its momentum is recorded as : **[AIEEE 2008]**

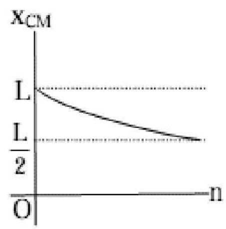
(A) 17.565 kg ms^{-1} (B) 17.56 kg ms^{-1} (C) 17.57 kg ms^{-1} (D) 17.6 kg ms^{-1}

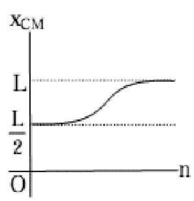
39. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is : **[AIEEE 2008]**

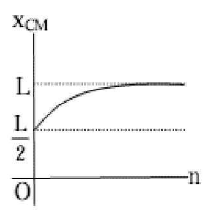
(A) 1.00 *J* (B) 0.67 *J* (C) 0.34 *J* (D) 0.16 *J*

40. A thin rod of length '*L*' is lying along the *x*-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with *x* as $k\left(\frac{x}{L}\right)^n$, where *n* can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against '*n*', which of the following graphs best approximates the dependence of x_{CM} on *n* ? **[AIEEE 2008]**

(A)


(B)


(C)


(D)


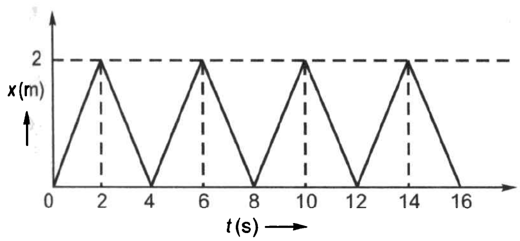
41. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

[AIEEE 2010]

(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

42. The figure shows the position-time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is **[AIEEE 2010]**



(A) 0.4 Ns (B) 0.8 Ns
(C) 1.6 Ns (D) 0.2 Ns

43. This question has statement 1 and statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[JEE Main 2013]**
Statement 1 : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$, then $f = \left(\frac{m}{M+m}\right)$

.

Statement II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.
(A) Statement I is true, Statement II is true, and Statement II is the correct explanation of Statement I.
(B) Statement I is true, Statement II is true, and Statement II is not the correct explanation of Statement I.
(C) Statement I is true, Statement II is false.
(D) Statement I is false, Statement II is true.

44. A particle of mass m moving in the x-direction with speed $2u$ is hit by another particle of mass $2m$ moving in the y-direction with speed u. If the collission is perfectly inelastic, the percentage loss in the energy during the collision is close to: **[JEE - Main 2015]**
(A) 44% (B) 50% (C) 56% (D) 62%

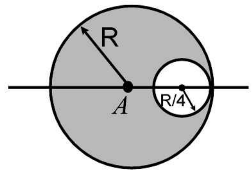
45. Distance of the centre of mass of a solid uniform cone from the its vertex is Z_o. If the radius of its base is R and its height is h, then Z_o is equal to: **[JEE - Main 2015]**
(A) $\frac{h^2}{4R}$ (B) $\frac{3h}{4}$ (C) $\frac{5h}{8}$ (D) $\frac{3h^2}{8R}$

Exercise–4: Objective Problems

Only One Option Correct

1. The centre of mass of a body:
(A) Lies always at the geometrical centre (B) Lies always inside the body
(C) Lies always outside the body (D) Lies within or outside the body

2. The centre of mass of the shaded portion of the disc is :
(The mass is uniformly distributed in the shaded portion) :



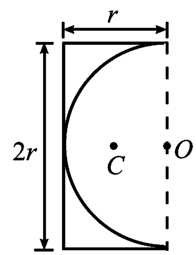
(A) $\frac{R}{20}$ to the left of A (B) $\frac{R}{12}$ to the left of A

(C) $\frac{R}{20}$ to the right of A (D) $\frac{R}{12}$ to the right of A

3. A thin uniform wire is bent to form the two equal sides AB and AC of triangle ABC, where $AB = AC = 5$ cm. The third side BC, of length 6cm, is made from uniform wire of twice the density of the first. The distance of centre of mass from A is :

(A) $\frac{34}{11}$ cm (B) $\frac{11}{34}$ cm (C) $\frac{34}{9}$ cm (D) $\frac{11}{45}$ cm

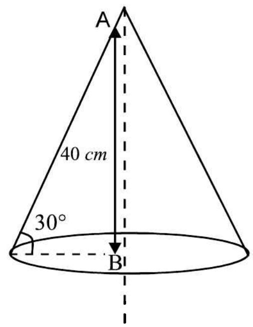
4. A semicircular portion of radius 'r' is cut from a uniform rectangular plate as shown in figure. The distance of centre of mass 'C' of remaining plate, from point 'O' is :



(A) $\frac{2r}{(3-\pi)}$ (B) $\frac{3r}{2(4-\pi)}$

(C) $\frac{2r}{(4+\pi)}$ (D) $\frac{2r}{3(4-\pi)}$

5. A uniform solid cone of height 40 cm is shown in figure. The distance of centre of mass of the cone from point B (centre of the base) is :



(A) 20 cm (B) 10/3 cm
(C) 20/3 cm (D) 10 cm

6. The centre of mass of a system of particles is at the origin. It follows that
(A) The number of particles to the right of the origin is equal to the number of particles to the left
(B) The total mass of the particles to the right of the origin is same as the total mass to the left of the origin
(C) The number of particles on X-axis should be equal to the number of particles on Y-axis.
(D) If there is a particle on the positive X-axis, there must be at least one particle on the negative X-axis.

7. A body has its centre of mass at the origin. The x-coordinates of the particles
(A) May be all positive
(B) May be all negative
(C) May be all non-negative
(D) May be positive for some cases and negative in other cases

8. In which of the following cases the centre of mass of a rod is certainly not at its centre ?
   (A) The density continuously increases from left to right
   (B) The density continuously decreases from left to right
   (C) The density decreases from left to right upto the centre and then increases
   (D) The density increases from left to right upto the centre and then decreases

Motion of Com

9. Two particles bearing mass ratio $n : 1$ are interconnected by a light inextensible string that passes over a smooth pulley. If the system is released, then the acceleration of the centre of mass of the system is
   (A) $(n-1)^2 g$ (B) $\left(\frac{n+1}{n-1}\right)^2 g$ (C) $\left(\frac{n-1}{n+1}\right)^2 g$ (D) $\left(\frac{n+1}{n-1}\right) g$
10. A bomb travelling in a parabolic path under the effect of gravity, explodes in mid air. The centre of mass of fragments will
    (A) Move vertically upwards and then downwards
    (B) Move vertically downwards
    (C) Move in irregular path
    (D) Move in the parabolic path which the unexploded bomb would have travelled.
11. If a ball is thrown upwards from the surface of earth
    (A) The earth remains stationary while the ball moves upwards
    (B) The ball remains stationary while the earth moves downwards
    (C) The ball and earth both moves towards each other
    (D) The ball and earth both move away from each other
12. Consider a system of two identical particles. One of the particles is at rest and the other has an acceleration $\vec{a}$. The centre of mass has an acceleration.
    (A) Zero (B) $\frac{1}{2}\vec{a}$ (C) $\vec{a}$ (D) $2\vec{a}$
13. Internal forces can change
    (A) The linear momentum but not the kinetic energy
    (B) The kinetic energy but not the linear momentum
    (C) Linear momentum as well as kinetic energy
    (D) Neither the linear momentum nor the kinetic energy
14. A body at rest breaks into two pieces of equal masses. The parts will move
    (A) In same direction
    (B) Along different lines
    (C) In opposite directions with equal speeds
    (D) In opposite directions with unequal speeds
15. If the external forces acting on a system have zero resultant, the centre of mass
    (A) Must not move (B) Must not accelerate
    (C) May move (D) May accelerate

16. Two particles A and B initially at rest move towards each other under a mutual force of attraction. The speed of centre of mass at the instant when the speed of A is v and the speed of B is $2v$ is :

(A) v (B) Zero (C) $2v$ (D) $3v/2$

17. Two masses of 1g and 4g are moving with equal $K.E.$ The ratio of the magnitude of their linear momentum is -

(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 1 : 4

18. A stationary body explodes into two fragments of masses m_1 and m_2. If momentum of one fragment is p, the energy of explosion is :

(A) $\dfrac{p^2}{2(m_1+m_2)}$ (B) $\dfrac{p^2}{2\sqrt{m_1 m_2}}$ (C) $\dfrac{p^2(m_1+m_2)}{2m_1 m_2}$ (D) $\dfrac{p^2}{2(m_1-m_2)}$

19. A railway flat car has an artillery gun installed on it. The combined system has a mass M and moves with a velocity v_0. The barrel of the gun makes an angle α with the horizontal. A shell of mass m leaves the barrel at a speed 'u' relative to barrel in the forward direction. The speed of the flat car so that it may stop after the firing is

(A) $\dfrac{mu}{M+m}$ (B) $\left(\dfrac{Mu}{M+m}\right)\cos\alpha$ (C) $\left(\dfrac{mu}{M}\right)\cos\alpha$ (D) $(M+m)u\cos\alpha$

20. A block moving in air breaks in two parts and the parts separate

(A) The total momentum must be conserved
(B) The total kinetic energy must be conserved
(C) The total momentum must change
(D) The total kinetic energy must change

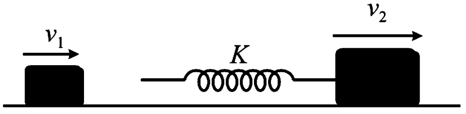
21. A shell is fired from a canon with a velocity V at an angle θ with the horizontal direction. At the highest point in its path, it explodes into two pieces of equal masses. One of the pieces retraces its path to the cannon. The speed of the other piece immediately after the explosion is

(A) $3V\cos\theta$ (B) $2V\cos\theta$ (C) $\dfrac{3}{2}V\cos\theta$ (D) $V\cos\theta$

22. A skater of mass m standing on ice throws a stone of mass M with a velocity of v m/s in a horizontal direction. The distance over which the skater will move back (the coefficient of friction between the skater and the ice is μ) :

(A) $\dfrac{M^2v^2}{2m\mu g}$ (B) $\dfrac{Mv^2}{2m^2\mu g}$ (C) $\dfrac{M^2v^2}{2m^2\mu g}$ (D) $\dfrac{M^2v^2}{2m^2\mu^2 g}$

23. Two blocks of masses m and M are moving with speeds v_1 and v_2 ($v_1 > v_2$) in the same direction on the frictionless surface respectively, M being ahead of m. An ideal spring of force constant k is attached to the backside of M (as shown). The maximum compression of the spring when the block collides is :



(A) $v_1\sqrt{\dfrac{m}{k}}$ (B) $v_2\sqrt{\dfrac{M}{k}}$ (C) $(v_1-v_2)\sqrt{\dfrac{mM}{(M+m)K}}$

(D) None of above is correct.

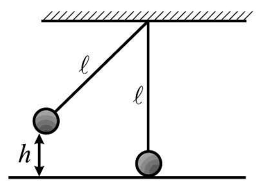
24. A bullet in motion hits and gets embedded in a solid block resting on a frictionless table. Which of the following is conserved ?
(A) Momentum and *KE* (B) Kinetic energy alone
(B) Neither *KE* nor momentum (D) Momentum alone

25. A body moving towards a finite body at rest collides with it. It is possible that
(A) Both the bodies come to rest
(B) Both the bodies move after collision
(C) The moving body comes to rest and the stationary body starts moving
(D) The stationary body remains stationary, the moving body changes its velocity.

26. In head on elastic collision of two bodies of equal masses
(A) The velocities are interchanged
(B) The speeds are interchanged
(C) The momenta are interchanged
(D) The faster body slows down and the slower body speeds up.

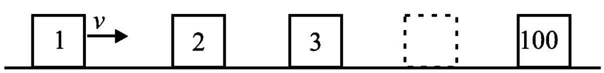
27. A particle of mass 1 g moving with a velocity $\vec{u}_1 = (3\hat{i} - 2\hat{j})\text{ms}^{-1}$ experiences a perfectly inelastic collision with another particle of mass 2 g and velocity $\vec{u}_2 = (4\hat{j} - 6\hat{k})\text{ms}^{-1}$. The velocity of the combined particle is :
(A) $\hat{i}+2\hat{j}-4\hat{k}$ (B) $\hat{i}-2\hat{j}+4\hat{k}$ (C) $\hat{i}-2\hat{j}-4\hat{k}$ (D) $\hat{i}+3.33\hat{j}+4\hat{k}$

28. In the arrangement shown, the pendulum on the left is pulled aside. It is then released and allowed to collide with other pendulum which is at rest. A perfectly inelastic collision occurs and the system rises to a height 1/4 h. The ratio of the masses of the pendulum is



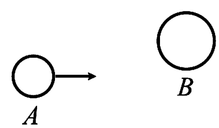
(A) 1 (B) 2
(C) 3 (D) 4

29. There are hundred identical sliders equally spaced on a frictionless track as shown in the figure. Initially all the sliders are at rest. Slider 1 is pushed with velocity *v* towards slider 2. In a collision the sliders stick together. The final velocity of the set of hundred stucked sliders will be



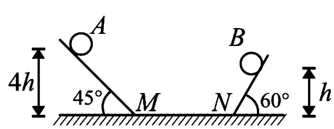
(A) $\frac{v}{99}$ (B) $\frac{v}{100}$ (C) zero (D) *v*

30. A solid iron ball *A* collides with another stationary solid iron ball *B*. If the ratio of radii of the balls is $n = 2$, then the ratio of their speeds just after the collision ($e = 0.5$) is



(A) 3 (B) 4
(C) 2 (D) 1

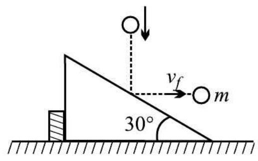
31. Two identical balls *A* and *B* are released from the positions shown in figure. They collide elastically on horizontal portion *MN*. All surfaces are smooth. The ratio of heights attained by *A* and *B* after collision will be (Neglect energy loss at *M* & *N*)



(A) 1 : 4 (B) 2 : 1 (C) 4 : 13 (D) 2 : 5

32. A ball strikes a smooth horizontal ground at an angle of 45° with the vertical. What cannot be the possible angle of its velocity with the vertical after the collision. (Assume $e \leq 1$).
(A) 45° (B) 30°
(C) 53° (D) 60°

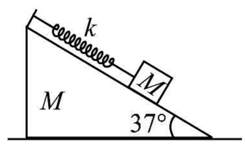
33. As shown in the figure a body of mass m moving vertically with speed 3 m/s hits a smooth fixed inclined plane and rebounds with a velocity v_f *in* the horizontal direction. If angle of inclined is 30°, the velocity v_f will be



(A) 3 m/s (B) √3 m/s
(C) 1/√3 m/s (D) this is not possible

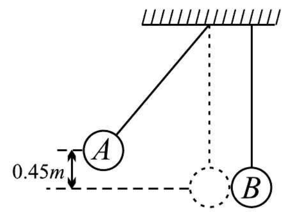
34. An ice block is melting at a constant rate $\left|\frac{dm}{dt}\right| = \mu$. Its initial mass is m_0 and it is moving with velocity on a frictionless horizontal surface. The distance travelled by it till it melts completely is :
(A) $\frac{2m_0v}{\mu}$ (B) $\frac{m_0v}{\mu}$
(C) $\frac{m_0v}{2\mu}$ (D) can't be said

35. The system of the wedge and the block connected by a massless spring as shown in the figure is released with the spring in its natural length. Friction is absent. Maximum elongation in the spring will be



(A) $\frac{3Mg}{5k}$ (B) $\frac{6Mg}{5k}$
(C) $\frac{4Mg}{5k}$ (D) $\frac{8Mg}{5k}$

36. Two massless string of length 5 m hang from the ceiling very near to each other as shown in the figure. Two balls A and B of masses 0.25 kg and 0.5 kg are attached to the string. The ball A is released from rest at a height 0.45 m as shown in the figure. The collision between two balls is completely elastic. Immediately after the collision, the kinetic energy of ball B is 1 J. The velocity of ball A just after the collision is



(A) 5 ms^{-1} to the right (B) 5 ms^{-1} to the left
(C) 1 ms^{-1} to the right (D) 1 ms^{-1} to the left

37. A smooth sphere is moving on a horizontal surface with a velocity vector $(2\hat{i} + 2\hat{j})$ m/s immediately before it hit a vertical wall. The wall is parallel to vector $\hat{j}$ and coefficient of restitution between the sphere and the wall is $e = 1/2$. The velocity of the sphere after it hits the wall is
(A) $\hat{i} - \hat{j}$ (B) $-\hat{i} + 2\hat{j}$ (C) $-\hat{i} - \hat{j}$ (D) $2\hat{i} - \hat{j}$

38. A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to other end of the plank. If the mass of the plank is $\frac{M}{3}$, then the distance that the man moves relative to ground is
(A) $\frac{3L}{4}$ (B) $\frac{L}{4}$ (C) $\frac{4L}{5}$ (D) $\frac{L}{3}$

39. Two balls A and B having masses 1 kg and 2 kg, moving with speeds 21 m/s and 4 m/s respectively in opposite direction, collide head on. After collision A moves with a speed of 1 m/s in the same direction, then the coefficient of restitution is
(A) 0.1 (B) 0.2 (C) 0.4 (D) None

40. Two particles of equal mass have velocities $2\hat{i}$ ms^{-1} and $2\hat{j}$ ms^{-1}. First particle has an acceleration $(\hat{i}+\hat{j})\,ms^{-2}$ while the acceleration of the second particle is zero. The centre of mass of the two particles moves in
(A) Circle (B) Parabola (C) Ellipse (D) Straight line

41. A particle of mass $3m$ is projected from the ground at some angle with horizontal. The horizontal range is R. At the highest point of its path it breaks into two pieces m and $2m$. The smaller mass comes to rest and larger mass finally falls at a distance x from the point of projection where x is equal to
(A) $\frac{3R}{4}$ (B) $\frac{3R}{2}$ (C) $\frac{5R}{4}$ (D) 3R

42. A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8 m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time ?
(A) 11.2 m (B) 13.8 m (C) 14.3 m (D) 15.4 m

43. From a circle of radius a, an isosceles right angled triangle with the hypotenuse as the diameter of the circle is removed. The distance of the centre of gravity of the remaining position from the centre of the circle is
(A) $3(\pi-1)a$ (B) $\frac{(\pi-1)a}{6}$ (C) $\frac{a}{3(\pi-1)}$ (D) $\frac{a}{3(\pi+1)}$

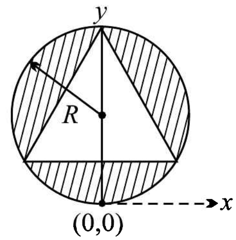
44. A sphere strikes a wall and rebounds with coefficient of restitution 1/3. If it rebounds with a velocity of 0.1 m/sec at an angle of 60° to the normal to the wall, the loss of kinetic energy is
(A) 50% (B) $33\frac{1}{3}$% (C) 40% (D) $66\frac{2}{3}$%

45. A truck moving on horizontal road towards east with velocity 20 ms^{-1} collides elastically with a light ball moving with velocity 25 ms^{-1} along west. The velocity of the ball just after collision
(A) 65 ms^{-1} towards east (B) 25 ms^{-1} towards west
(C) 65 ms^{-1} towards west (D) 20 ms^{-1} towards east

46. A spaceship of speed v_0 travelling along + y axis suddenly shots out one fourth of its part with speed $2v_0$ along + x-axis. xy axes are fixed with respect to ground. The velocity of the remaining part is

(A) $\frac{2}{3}v_0$ (B) $\frac{\sqrt{20}}{3}v_0$ (C) $\frac{\sqrt{5}}{3}v_0$ (D) $\frac{\sqrt{13}}{3}v_0$

47. From a uniform disc of radius R, an equilateral triangle of side $\sqrt{3}R$ is cut as shown. The new position of centre of mass is :



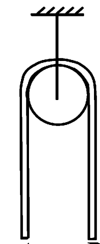
(A) (0, 0) (B) (0, R)

(C) (0, $\frac{\sqrt{3}R}{2}$) (D) None of these

48. If the linear density of a rod of length 3 m varies as $\lambda = 2 + x$, then the position of centre of gravity of the rod is :
(A) 7/3 m (B) 12/7 m (C) 10/7 m (D) 9/7 m

Question No. 49 to 50 (2 questions)

A uniform chain of length $2L$ is hanging in equilibrium position, if end B is given a slightly downward displacement the imbalance causes an acceleration. Here pulley is small and smooth and string is inextensible

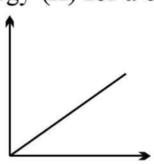


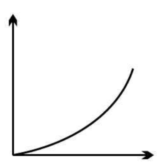
49. The acceleration of end B when it has been displaced by distance x, is
(A) $\frac{x}{L}g$ (B) $\frac{2x}{L}g$
(C) $\frac{x}{2}g$ (D) g

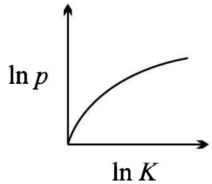
50. The velocity v of the string when it slips out of the pulley (height of pulley from floor > $2L$)
(A) $\sqrt{\frac{gL}{2}}$ (B) $\sqrt{2gL}$ (C) $\sqrt{gL}$ (D) none of these

51. A small sphere is moving at a constant speed in a vertical circle. Below is a list of quantities that could be used to describe some aspect of the motion of the sphere.
I – kinetic energy
II – gravitational potential energy
III – momentum
Which of these quantities will change as this sphere moves around the circle?
(A) I and II only (B) I and III only
(C) III only (D) II and III only

52. Which of the following graphs represents the graphical relation between momentum (p) and kinetic energy (K) for a body in motion?

(A) 

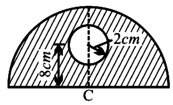
(B) 

(C) 

(D) none

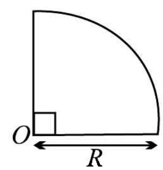
53. When the momentum of a body increases by 100%, its *KE* increases by
(A) 400% (B) 100% (C) 300% (D) none

54. A small bucket of mass *M* kg is attached to a long inextensible cord of length *L m* . The bucket is released from rest when the cord is in a horizontal position. At its lowest position, the bucket scoops up *m* kg of water and swings up to a height *h*. The height *h* in meters is
(A) $\left(\frac{M}{M+m}\right)^2 L$ (B) $\left(\frac{M}{M+m}\right) L$
(C) $\left(\frac{M+m}{M}\right)^2 L$ (D) $\left(\frac{M+m}{M}\right) L$

55. In the figure shown a hole of radius 2 cm is made in a semicircular disc of radius 6π at a distance 8 cm from the centre *C* of the disc. The distance of the centre of mass of this system from point *C* is


(A) 4 cm (B) 8 cm (C) 6 cm (D) 12 cm

56. A buggy of mass 100 kg is free to move on a frictionless horizontal track. Two men, each of mass 50 kg, are standing on the buggy, which is initially stationary. The men jump off the buggy with velocity = 10m/s relative to the buggy. In one situation, the men jump one after the other. In another situation, the men jump simultaneously. What is the ratio of the recoil velocities of the buggy in two cases?
(A) 5 : 4 (B) 5 : 3 (C) 7 : 6 (D) 7 : 5

57. In the figure one fourth part of a uniform disc of radius *R* is shown. The distance of the centre of mass of this object from centre '*O*' is


(A) $\frac{4R}{3\pi}$ (B) $\frac{2R}{3\pi}$
(C) $\sqrt{2}\frac{4R}{3\pi}$ (D) $\sqrt{2}\frac{2R}{3\pi}$

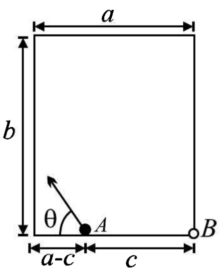
58. Two men, of masses 60 kg and 80 kg are sitting at the ends of a boat of mass 60 kg and length 4 m. The boat is stationary. If the men now exchange their positions, then
(A) The centre of mass of the two men shifts by 2 m
(B) The boat moves by 0.4 m
(C) The centre of mass of the two men shifts by 4/7 m.
(D) The boat moves by 0.6 m.

59. On a horizontal smooth surface a disc is placed at rest. Another disc of same mass is coming with impact parameter equal to its own radius. First disc is of radius *r*. What should be the radius of coming disc so that after collision first disc moves at an angle 45° to the direction of motion of incoming disc:

(A) 2r (B) $r\left(\sqrt{2}-1\right)$

(C) $\dfrac{r}{\left(\sqrt{2}-1\right)}$ (D) $r\sqrt{2}$

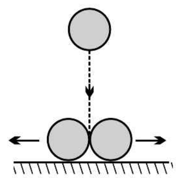
60. A billiard table whose length and width are as shown in the figure. A ball is placed at point A. At what angle 'θ' the ball be projected so that after colliding with two walls, the ball will fall in the pocket *B*. Assume that all collisions are perfectly elastic (neglect friction)



(A) $\theta = \cot^{-1}\dfrac{2a-c}{2b}$ (B) $\theta = \tan^{-1}\dfrac{2a-c}{2b}$

(C) $\theta = \cot^{-1}\dfrac{c-a}{2b}$ (D) $\theta = \cot^{-1}\dfrac{c-a}{b}$

61. In the figure shown, the two identical balls of mass *M* and radius *R* each, are placed in contact with each other on the frictionless horizontal surface. The third ball of mass *M* and radius *R*/2, is coming down vertically and has a velocity = v_0 when it simultaneously hits the two balls and itself comes to rest. Then, each of the two bigger balls will move after collision with a speed equal to



(A) $4v_0/\sqrt{5}$ (B) $2v_0/\sqrt{5}$ (C) $v_0/\sqrt{5}$ (D) None

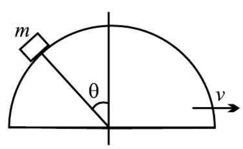
62. In the above, suppose that the smaller ball does not stop after collision, but continues to move downwards with a speed= $v_0/2$, after the collision. Then, the speed of each bigger ball after collision is

(A) $4v_0/\sqrt{5}$ (B) $2v_0/\sqrt{5}$ (C) $v_0/2\sqrt{5}$ (D) None

63. A body of mass '*m*' is dropped from a height of '*h*'. Simultaneously another body of mass 2*m* is thrown up vertically with such a velocity *v* that they collide at the height *h*/2. If the collision is perfectly inelastic, the velocity at the time of collision with the ground will be

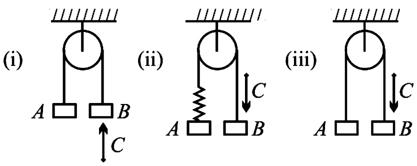
(A) $\sqrt{\dfrac{5gh}{4}}$ (B) $\sqrt{gh}$ (C) $\sqrt{\dfrac{gh}{4}}$ (D) $\dfrac{\sqrt{10gh}}{3}$

64. A hemisphere of mass 3*m* and radius *R* is free to slide with its base on a smooth horizontal table. A particle of mass *m* is placed on the top of the hemisphere. If particle is displaced with a negligible velocity, then find the angular velocity of the particle relative to the centre of the hemisphere at an angular displacement θ, when velocity of hemisphere is *v*.



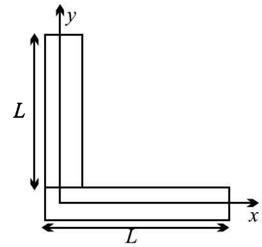
(A) $\dfrac{4v}{R\cos\theta}$ (B) $\dfrac{3v}{R\cos\theta}$ (C) $\dfrac{5v}{R\cos\theta}$ (D) $\dfrac{2v}{R\cos\theta}$

65. In the figure (i), (ii) & (iii) shown the objects A, B & C are of same mass. String, spring & pulley are massless. C strikes B with velocity 'u' in each case and sticks to it. The ratio of velocity of B in case (i) to (ii) to (iii) is



(A) 1 : 1: 1 (B) 3 : 3 : 2
(C) 3 : 2 : 2 (D) none of these

66 Centre of mass of two thin uniform rods of same length but made up of different materials & kept as shown, can be, if the meeting point is the origin of co-ordinates

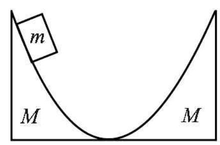


(A) ($L/2$, $L/2$)
(B) ($2L/3$, $L/2$)
(C) ($L/3$, $L/3$)
(D) ($L/3$, $L/6$)

67. A force exerts an impulse I on a particle changing its speed from u to $2u$. The applied force and the initial velocity are oppositely directed along the same line. The work done by the force is

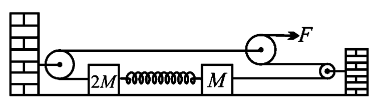
(A) $\frac{3}{2}\text{Iu}$ (B) $\frac{1}{2}\text{Iu}$ (C) $I\,u$ (D) $2\,I\,u$

68. The inclined surfaces of two movable wedges of same mass M are smoothly conjugated with the horizontal plane as shown in figure. A washer of mass m slides down the left wedge from a height h. To what maximum height will the washer rise along the right wedge? Neglect friction.



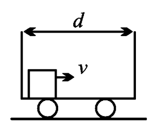
(A) $\dfrac{h}{(M+m)^2}$ (B) $\dfrac{hM}{(M+m)^2}$ (C) $h\left(\dfrac{M}{M+m}\right)^2$ (D) $h\left(\dfrac{M}{M+m}\right)$

69. In the diagram shown, no friction at any contact surface. Initially, the spring has no deformation. What will be the maximum deformation in the spring? Consider all the strings to be sufficiency large. Consider the spring constant to be K.



(A) $4F/3K$ (B) $8F/3K$ (C) $F/3K$ (D) none

70. In a smooth stationary cart of length d, a small block is projected along it's length with velocity v towards front. Coefficient of restitution for each collision is e. The cart rests on a smooth ground and can move freely. The time taken by block to come to rest w.r.t. cart is



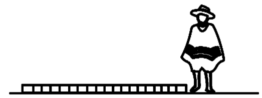
(A) $\dfrac{ed}{(1-e)v}$ (B) $\dfrac{ed}{(1+e)v}$ (C) $\dfrac{d}{e}$ (D) infinite

71. A flexible chain of length 2m and mass 1kg initially held in vertical position such that its lower end just touches a horizontal surface, is released from rest at time $t = 0$. Assuming that

any part of chain which strikes the plane immediately comes to rest and that the portion of chain lying on horizontal surface does not from any heap, the height of its centre of mass above surface at any instant $t = 1/\sqrt{5}$ (before it completely comes to rest) is

(A) 1 m (B) 0.5 m (C) 1.5 m (D) 0.25 m

72. On a smooth horizontal plane, a uniform string of mass M and length is lying in the state of rest. A man of the same mass M is standing next to one end of the string. Now, the man starts collecting the string. Finally the man collects all the string and puts it in his pocket. What is the displacement of the man with respect to earth in the process of collection?



(A) $L/2$ (B) $L/4$ (C) $L/8$ (D) none

73. An open water tight railway wagon of mass 5×10^3 kg coasts at an initial velocity 1.2 m/s without friction on a railway track. Rain drops fall vertically downwards into the wagon. The velocity of the wagon after it has collected 10^3 kg of water will be

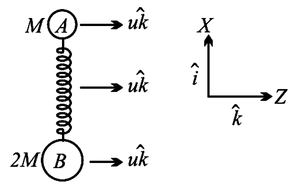
(A) 0.5 m/s (B) 2 m/s (C) 1 m/s (D) 1.5 m/s

74. A parallel beam of particles of mass m moving with velocity v impinges on a wall at an angle θ to its normal . The number of particles per unit volume in the beam is n . If the collision of particles with the wall is elastic, then the pressure exerted by this beam on the wall is

(A) $2\,mn\,v^2 \cos\theta$ (B) $2\,mn\,v^2 \cos^2\theta$

(C) $2\,mn\,v \cos\theta$ (D) $2\,mn\,v \cos^2\theta$

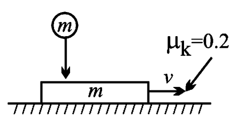
75. Two masses A and B of mass M and $2M$ respectively are connected by a compressed ideal spring. The system is placed on a horizontal frictionless table and given a velocity $u\hat{k}$ in the z-direction as shown in the figure. The spring is then released. In the subsequent motion the line from B to A always points along the $\hat{i}$ unit vector. At some instant of time mass B has a x-component of velocity as $V_x\hat{i}$. The velocity $\vec{V}_A$ of as A at that instant is



(A) $V_x\hat{i} + u\hat{k}$ (B) $-V_x\hat{i} + u\hat{k}$

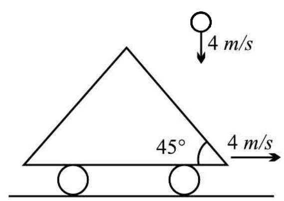
(C) $-2V_x\hat{i} + u\hat{k}$ (D) $2V_x\hat{i} + u\hat{k}$

76. A ball of mass m falls vertically from a height h and collides with a block of equal mass m moving horizontally with a velocity v on a surface. The coefficient of kinetic friction between the block and the surface is 0.2, while the coefficient of restitution e between the ball and the block is 0.5. There is no friction acting between the ball and the block. The velocity of the block decreases by



(A) 0 (B) $0.1\sqrt{2gh}$ (C) $0.3\sqrt{2gh}$ (D) Can't be said

77. A small ball falling vertically downward with constant velocity 4m/s strikes elastically a massive inclined cart moving with velocity 4m/s horizontally as shown. The velocity of the rebound of the ball is

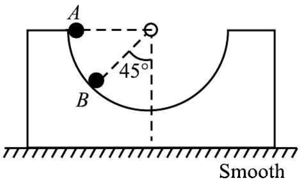


(A) $4\sqrt{2}$ m/s (B) $4\sqrt{3}$ m/s

(C) 4m/s (D) $4\sqrt{5}$m/s

78. A rocket of mass 4000 kg is set for vertical firing. How much gas must be ejected per second so that the rocket may have initial upwards acceleration of magnitude 19.6 m/s^2. [Exhaust speed of fuel = 980 m/s.]
(A) 240 kg s^{-1} (B) 60 kg s^{-1} (C) 120 kg s^{-1} (D) None

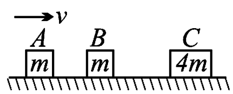
79. A ball of mass m is released from A inside a smooth wedge of mass m as shown in the figure. What is the speed of the wedge when the ball reaches point B?



(A) $\left(\frac{gR}{3\sqrt{2}}\right)^{1/2}$ (B) $\sqrt{2gR}$

(C) $\left(\frac{5gR}{2\sqrt{3}}\right)^{1/2}$ (D) $\sqrt{\frac{3}{2}gR}$

80. Three blocks are initially placed as shown in the figure. Block A has mass m and initial velocity v to the right. Block B with mass m and block C with mass 4m are both initially at rest. Neglect friction. All collisions are elastic. The final velocity of block A is

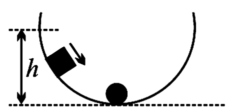


(A) 0.6v to the left (B) 1.4v to the left
(C) v to the left (D) 0.4v to the right

81. Two identical spheres move in opposite directions with speeds v_1 and v_2 and pass behind an opaque screen, where they may either cross without touching (Event 1) or make an elastic head–on collision (Event 2)
(A) We can never make out which event has occurred
(B) We cannot make out which event has occurred only if $v_1 = v_2$
(C) We can always make out which event has occurred
(D) We can make out which event has occurred only if $v_1 = v_2$

82. There are some passengers inside a stationary railway compartment. The track is frictionless. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the 'compartment plus passengers' system is C_2. If the passengers move about inside the compartment along the track.
(A) both C_1 and C_2 will move with respect to the ground
(B) neither C_1 nor C_2 will move with respect to the ground
(C) C_1 will move but C_2 will be stationary with respect to the ground
(D) C_2 will move but C_1 will be stationary with respect to the ground

83. A block of mass m starts from rest and slides down a frictionless semi–circular track from a height h as shown. When it reaches the lowest point of the track, it collides with a stationary piece of putty also having mass m. If the block and the putty stick together and continue to slide, the maximum height that the block-putty system could reach is :



(A) $h/4$ (B) $h/2$
(C) h (D) independent of h

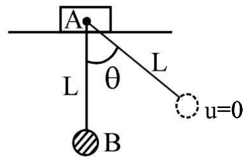
84. A boy hits a baseball with a bat and imparts an impulse J to the ball. The boy hits the ball again with the same force, except that the ball and the bat are in contact for twice the amount of time as in the first hit. The new impulse equals:

(A) Half the original impulse (B) The original impulse
(C) Twice the original impulse (D) Four times the original impulse

85. Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of $v/3$ in the same direction. What type of collision has occured?
(A) Inelastic
(B) Elastic
(C) Completely inelastic
(D) Cannot be determined from the information given

Question No. 86 to 89 (4 questions)

A small ball B of mass m is suspended with light inelastic string of length L from a block A of same mass m which can move on smooth horizontal surface as shown in the figure. The ball is displaced by angle θ from equilibrium position & then released.



86. The displacement of block when ball reaches the equilibrium position is
(A) $\frac{L\sin\theta}{2}$ (B) $L\sin\theta$
(C) L (D) None of these

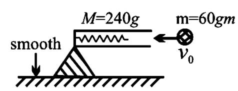
87. Tension in string when it is vertical, is
(A) mg (B) $mg(2-\cos\theta)$
(C) $mg\,(3-2\cos\theta)$ (D) None of these

88. Maximum velocity of block during subsequent motion of the system after release of ball is
(A) $[gl\,(1-\cos\theta)]^{1/2}$ (B) $[2gl(1-\cos\theta)]^{1/2}$
(C) $[gl\cos\theta]^{1/2}$ (D) informations are insufficient to decide

89. The displacement of centre of mass of $A + B$ system till the string becomes vertical is
(A) zero (B) $\frac{L}{2}(1-\cos\theta)$
(C) $\frac{L}{2}(1-\sin\theta)$ (D) None of these

Question No. 90 & 91 (2 questions)

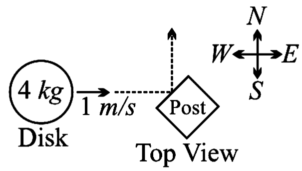
A ball of mass m = 60gm is shot with speed v_0 = 22m/s into the barrel of spring gun of mass M = 240g initially at rest on a frictionless surface. The ball sticks in the barrel at the point of maximum compression of the spring.



90. The speed of the spring gun after the ball stops relative to the barrel, is
(A) 2.2 m/s (B) 4.4 m/s (C) 4.4 cm/s (D) None

91. What fraction of initial kinetic energy of the ball is now stored in the spring?
(A) 0.2 (B) 0.8 (C) 0.4 (D) 0.6

92. In an elastic collision of two billiard balls which of the following quantities is not conserved during the short time of collision
 (A) Momentum (B) Total mechanical energy
 (C) Kinetic energy (D) None
93. A 4-kilogram disk slides over level ice toward the east at a velocity of 1meter per second, as shown. The disk strikes a post and rebounds toward the north at the same speed. The change in the magnitude of the eastward component of the momentum of the disk is



 (A) –4 kg·m/s (B) –1 kg·m/s
 (C) 0 kg·m/s (D) 4 kg·m/s
94. A system of N particles is free from any external forces.
(a) Which of the following is true for the magnitude of the total momentum of the system?
 (A) It must be zero
 (B) It could be non–zero, but it must be constant
 (C) It could be non–zero, and it might not be constant
 (D) The answer depends on the nature of the internal forces in the system
(b) Which of the following must be true for the sum of the magnitudes of the momenta of the individual particles in the system?
 (A) It must be zero
 (B) It could be non–zero, but it must be constant
 (C) It could be non–zero, and it might not be constant
 (D) It could be zero, even if the magnitude of the total momentum is not zero
95. An isolated rail car of mass M is moving along a straight, frictionless track at an initial speed v_0. The car is passing under a bridge when a crate filled with N bowling balls, each of mass m, is dropped from the bridge into the bed of the rail car. The crate splits open and the bowling balls bounce around inside the rail car, but none of them fall out.
(a) Is the momentum of the rail car + bowling balls system conserved in this collision?
 (A) Yes, the momentum is completely conserved.
 (B) Only the momentum component in the vertical direction is conserved.
 (C) Only the momentum component parallel to the track is conserved.
 (D) No components are conserved.
(b) What is the average speed of the rail car + bowling balls system some time after the collision?
 (A) $(M + Nm)v_0/M$
 (B) $Mv_0/(Nm + M)$
 (C) Nmv_0/M
 (D) The speed cannot be determined because there is not enough information
96. Consider a one-dimensional collision that involves a body of mass m_1 originally moving in the positive x direction with speed v_0 colliding with a second body of mass m_2 originally at rest. The collision could be completely inelastic, with the two bodies sticking together, completely elastic, or somewhere in between. After the collision, m_1 moves with velocity v_1 while m_2 moves with velocity v_2.

(a) If $m_1 > m_2$, then

(A) $-v_0 < v_1 < 0$ (B) $0 < v_1 < v_0$ (C) $0 < v_1 < 2v_0$ (D) $v_0 < v_1 < 2v_0$

(b) and

(A) $-v_0 < v_2 < 0$ (B) $0 < v_2 < v_0$ (C) $v_0/2 < v_2 < 2v_0$ (D) $v_0 < v_2 < 2v_0$

(c) If $m_1 < m_2$ then

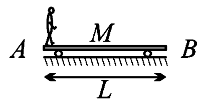
(A) $-v_0 < v_1 < 0$ (B) $-v_0 < v_1 < v_0/2$ (C) $0 < v_1 < v_0/2$ (D) $0 < v_1 < v_0$

(d) and

(A) $-v_0 < v_2 < 0$ (B) $-v_0 < v_2 < v_0/2$ (C) $0 < v_2 < v_0/2$ (D) $0 < v_2 < v_0$

Question No. 97 to 103 (7 questions)

The figure shows a man of mass m standing at the end A of a trolley of mass M placed at rest on a smooth horizontal surface. The man starts moving towards the end B with a velocity u_{rel} with respect to the trolley. The length of the trolley is L.



97. When the man starts moving, then the velocity of the trolley v_2 with respect to ground will be

(A) $\frac{Mu_{rel}}{m+M}$ (B) $\frac{mu_{rel}}{m+M}$ (C) $\frac{m}{M}u_{rel}$ (D) $\frac{M}{m}u_{rel}$

98. The velocity of the man with respect to ground v_1 will be

(A) $\frac{Mu_{rel}}{m+M}$ (B) $\frac{mu_{rel}}{m+M}$ (C) $\frac{m}{M}u_{rel}$ (D) $\frac{M}{m}u_{rel}$

99. The time taken by the man to reach the other end is

(A) $\left(\frac{m+M}{M}\right)\frac{L}{u_{rel}}$ (B) $\left(\frac{m+M}{m}\right)\frac{L}{u_{rel}}$ (C) $\frac{L}{u_{rel}}$ (D) none of these

100. As the man walks on the trolley, the centre of mass of the system (man + trolley)

(A) Accelerates towards left (B) Accelerates towards right

(C) Moves with u_{rel} (D) Remains stationary

101. When the man reaches the end B, the distance moved by the trolley with respect to ground is

(A) $\frac{mL}{m+M}$ (B) $\frac{ML}{m+M}$ (C) $\frac{m}{M}L$ (D) $\frac{M}{m}L$

102. The distance moved by the man with respect to ground is

(A) $\frac{mL}{m+M}$ (B) $\frac{ML}{m+M}$ (C) $\frac{m}{M}L$ (D) $\frac{M}{m}L$

103. Choose the correct statement

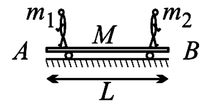
(A) As the man starts moving the trolley must move backward

(B) The distance moved by the trolley is independent of the speed of the man

(C) The distance moved by the trolley can never exceed L

(D) All the above

Question No. 104 to 108 (5 questions)



Two persons of mass m_1 and m_2 are standing at the two ends A and B respectively, of a trolley of mass M as shown.

104. When the person standing at A jumps from the trolley towards left with u_{rel} with respect to the trolley, then
(A) The trolley moves towards right
(B) The trolley rebounds with velocity $\dfrac{m_1 u_{rel}}{m_1 + m_2 + M}$
(C) The centre of mass of the system remains stationary
(D) All the above

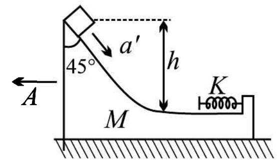
105. When only the person standing at B jumps from the trolley towards right while the person at A keeps standing, then
(A) The trolley moves towards left
(B) The trolley moves with velocity $\dfrac{m_2 u_{rel}}{m_1 + m_2 + M}$
(C) The centre of mass of the system remains stationary
(D) All the above

106. When both the persons jump simultaneously with same speed then
(A) The centre of mass of the system remains stationary
(B) The trolley remains stationary
(C) The trolley moves toward the end where the person with heavier mass is standing
(D) None of these

107. When both the persons jump simultaneously with u_{rel} with respect to the trolley, then the velocity of the trolley is
(A) $\dfrac{|m_1 - m_2| u_{rel}}{m_1 + m_2 + M}$ (B) $\dfrac{|m_1 - m_2| u_{rel}}{M}$
(C) $\left|\dfrac{m_1 u_{rel}}{m_2 + M} - \dfrac{m_2 u_{rel}}{m_1 + M}\right|$ (D) none of these

108. Choose the incorrect statement, if $m_1 = m_2 = m$ and both the persons jump one by one, then
(A) The centre of mass of the system remains stationary
(B) The final velocity of the trolley is in the direction of the person who jumps first
(C) The final velocity of the trolley is $\left(\dfrac{mu_{rel}}{M + m} - \dfrac{mu_{rel}}{M + 2m}\right)$
(D) None of these

Question No. 109 to 111 (3 questions)



109. A small block of mass m is placed on a wedge of mass M as shown, which is initially at rest. All the surfaces are frictionless. The spring attached to the other end of wedge has force constant k. If a' is the acceleration of m relative to the wedge as it starts coming down and A is the acceleration acquired by the wedge as the block starts coming down, then

(A) $\frac{a'}{\sqrt{2}} < A < a'$ (B) $A < \frac{a'}{\sqrt{2}}$ (C) $A > a'$ (D) None

110. Maximum velocity of M is:

(A) $\sqrt{2gh}$ (B) $\sqrt{\frac{2ghm}{m+M}}$ (C) $\sqrt{\frac{2m^2gh}{mM+M^2}}$ (D) None

111. Maximum retardation of M is:

(A) $\sqrt{\frac{2mghk}{M^2}}$ (B) $\sqrt{\frac{2kgh}{M}}$ (C) $\sqrt{\frac{2kgh}{m}}$ (D) None

112. In a one-dimensional collision, a particle of mass $2m$ collides with a particle of mass m at rest. If the particles stick together after the collision, what fraction of the initial kinetic energy is lost in the collision?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) none

113. A ball is dropped from a height h. As it bounces off the floor, its speed is 80 percent of what it was just before it hit the floor. The ball will then rise to a height of most nearly

(A) 0.80 h (B) 0.75 h (C) 0.64 h (D) 0.50 h

Question No.114 to 115 (2 questions)

A projectile of mass "m" is projected from ground with a speed of 50 m/s at an angle of 53° with the horizontal. It breaks up into two equal parts at the highest point of the trajectory. One particle coming to rest immediately after the explosion.

114. The ratio of the radii of curvatures of the moving particle just before and just after the explosion are:

(A) 1 : 4 (B) 1 : 3 (C) 2 : 3 (D) 4 : 9

115. The distance between the pieces of the projectile when they reach the ground are:

(A) 240 (B) 360 (C) 120 (D) none

116. A ball is thrown vertically downwards with velocity $\sqrt{2gh}$ from a height h. After colliding with the ground it just reaches the starting point. Coefficient of restitution is

(A) $1/\sqrt{2}$ (B) 1/2 (C) 1 (D) $\sqrt{2}$

117. A ball is dropped from height 5m. The time after which ball stops rebounding if coefficient of restitution between ball and ground $e = 1/2$, is

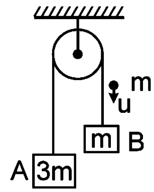
(A) 1 sec (B) 2 sec (C) 3 sec (D) infinite

118. A ball is projected from ground with a velocity V at an angle θ to the vertical. On its path it makes an elastic collison with a vertical wall and returns to ground. The total time of flight of the ball is

(A) $\frac{2v\sin\theta}{g}$ (B) $\frac{2v\cos\theta}{g}$ (C) $\frac{v\sin 2\theta}{g}$ (D) $\frac{v\cos\theta}{g}$

One or More than One Option may be correct

119. A system of two blocks A and B are connected by an inextensible massless strings as shown. The pulley is massless and frictionless. Initially the system is at rest when, a bullet of mass 'm' moving with a velocity 'u' as shown hits the block 'B' and gets embedded into it. The impulse imparted by tension force to the block of mass 3m is



(A) $\frac{5mu}{4}$ (B) $\frac{4mu}{5}$ (C) $\frac{2mu}{5}$ (D) $\frac{3mu}{5}$

120. Consider the following two statements :
(a) linear momentum of the system remains constant
(b) centre of mass of the system remains at rest.
(A) *a* implies *b* and *b* implies *a* (B) *a* does not imply *b* and *b* does not imply *a*
(C) *a* implies *b* but *b* does not imply *a* (D) *b* implies a but *a* does not imply *b*

121. Consider the following two statements:
(a) Linear momentum of the system of particles is zero
(b) Kinetic energy of a system of particles is zero
(A) *a* implies *b* and *b* implies *a* (B) *a* does not imply *b* and *b* does not imply *a*
(C) *a* implies *b* but *b* does not imply *a* (D) *b* implies *a* but *a* does not imply *b*

122. Consider the following two statements :
(a) the linear momentum of a particle is independent of the frame of reference
(b) the kinetic energy of a particle is independent of the frame of reference
(A) both *a* and *b* are true (B) *a* is true but *b* is false
(C) *a* is false but *b* is true (D) both *a* and *b* are false

123. A ball kept in a closed box moves in the box making collisions with the walls. The box is kept on a smooth surface. The velocity of the centre of mass
(A) of the box remains constant
(B) of the box plus the ball system remains constant
(C) of the ball remains constant
(D) of the ball relative to the box remains constant

124. A heavy ring of mass m is clamped on the periphery of a light circular disc. A small particle having equal mass is clamped at the centre of the disc. The system is rotated in such a way that the centre moves in a circle of radius r with a uniform speed v. We conclude that an external force

(A) $\frac{mv^2}{r}$ must be acting on the central particle

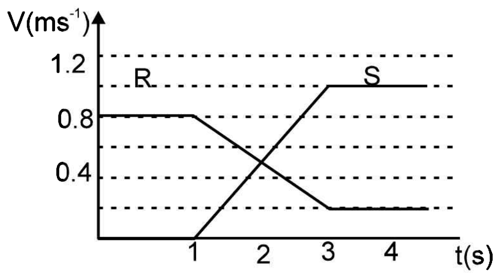
(B) $\frac{2mv^2}{r}$ must be acting on the central particle

(C) $\dfrac{2mv^2}{r}$ must be acting on the system

(D) $\dfrac{2mv^2}{r}$ must be acting on the ring.

125. A ball hits a floor and rebounds after an inelastic collision. In this case
(A) the momentum of the ball just after the collision is same as that just before the collision
(B) the mechanical energy of the ball remains the same during the collision
(C) the total momentum of the ball and the earth is conserved
(D) the total energy of the ball and the earth remains the same

126. The diagram shows the velocity-time graph for two masses R and S that collided elastically. Which of the following statements is true?

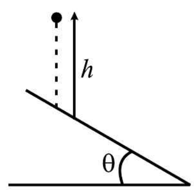


I. R and S moved in the same direction after the collision.
II. The velocities of R and S were equal at the mid time of the collision.
III. The mass of R was greater than mass of S.

(A) I only (B) II only
(C) I and II only (D) I, II and III

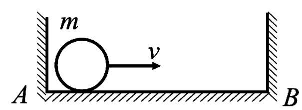
127. A ball collides with an inclined plane of inclination θ after falling through a distance h. If it moves horizontally just after the impact, the coefficient of restitution is :
(A) $\tan^2\theta$ (B) $\cot^2\theta$
(C) $\tan\theta$ (D) $\cot\theta$

128. A ball of mass m strikes the fixed inclined plane after falling through a height h. If it rebounds elastically, the impulse on the ball is



(A) $2\,m\cos\theta\,\sqrt{2gh}$ (B) $2\,m\cos\theta\,\sqrt{gh}$
(C) $\dfrac{2m\sqrt{2gh}}{\cos\theta}$ (D) $2m\sqrt{2gh}$

129. A small ball moves towards right with a velocity V. It collides with the wall and returns back and continues to and fro motion. If the average speed for first to and fro motion of the ball is $\left(\dfrac{2}{3}\right)V$, then the coefficient of restitution of impact is :



(A) 0.5 (B) 0.8 (C) 0.25 (D) 0.75

130. A wagon filled with sand has a hole so that sand leaks through the bottom at a constant rate λ. An external force $\vec{F}$ acts on the wagon in the direction of motion. Assuming instantaneous velocity of the wagon to be $\vec{v}$ and initial mass of system to be m_0, the force equation governing the motion of the wagon is :

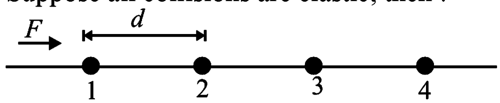
(A) $\vec{F} = m_0 \frac{d\vec{v}}{dt} + \lambda\vec{v}$ (B) $\vec{F} = m_0 \frac{d\vec{v}}{dt} - \lambda\vec{v}$

(C) $\vec{F} = (m_0 - \lambda t)\frac{d\vec{v}}{dt}$ (D) $\vec{F} = (m_0 - \lambda t)\frac{d\vec{v}}{dt} + \lambda\vec{v}$

131. A particle strikes a horizontal smooth floor with a velocity u making an angle θ with the floor and rebounds with velocity v making an angle ϕ with the floor. If the coefficient of restitution between the particle and the floor is e, then :
(A) the impulse delivered by the floor to the body is $mu(1 + e)\sin\theta$.
(B) $\tan\phi = e\tan\theta$.
(C) $v = u\sqrt{1-(1-e^2)\sin^2\theta}$.
(D) The ratio of the final kinetic energy to the initial kinetic energy is $(\cos^2\theta + e^2\sin^2\theta)$

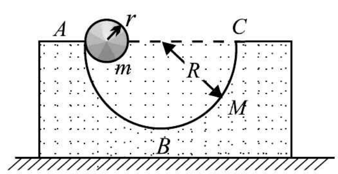
132. A ball moving with a velocity v hits a massive wall moving towards the ball with a velocity u. An elastic impact lasts for a time Δt.
(A) The average elastic force acting on the ball is $\frac{m(u+v)}{\Delta t}$
(B) The average elastic force acting on the ball is $\frac{2m(u+v)}{\Delta t}$
(C) The kinetic energy of the ball increases by $2mu(u + v)$
(D) The kinetic energy of the ball remains the same after the collision.

133. The fig. shows a string of equally spaced beads of mass m, separated by distance d. The beads are free to slide without friction on a thin wire. A constant force F acts on the first bead initially at rest till it makes collision with the second bead. The second bead then collides with the third and so on. Suppose all collisions are elastic, then :



(A) Speed of the first bead immediately before and immediately after its collision with the second bead is $\sqrt{\frac{2Fd}{m}}$ and zero respectively.
(B) Speed of the first bead immediately before and immediately after its collision with the second bead is $\sqrt{\frac{2Fd}{m}}$ and $\frac{1}{2}\sqrt{\frac{2Fd}{m}}$ respectively.
(C) Speed of the second bead immediately after its collision with third bead is zero.
(D) The average speed of the first bead is $\frac{1}{2}\sqrt{\frac{2Fd}{m}}$.

134. A block of mass M with a semicircular track of radius R rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest from the top point A. The cylinder slips on the semicircular frictionless track. The distance travelled by the block when the cylinder reaches the point B is:



(A) $\frac{M(R-r)}{M+m}$ (B) $\frac{m(R-r)}{M+m}$

(C) $\frac{(M+m)R}{M}$ (D) none

135. Two blocks A and B each of mass 'm' are connected by a massless spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal. Block C also of mass m moves on the floor with a speed 'v' along the line joining A and B and collides elastically with A then which of the following is/are correct :
(A) KE of the AB system at maximum compression of the spring is zero
(B) The KE of AB system at maximum compression is (1/4) mv^2
(C) The maximum compression of spring is $v\sqrt{m/k}$
(D) The maximum compression of spring is $v\sqrt{m/2k}$

136. A uniform thin rod of mass M and Length L is standing vertically along the y-axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance at $t = 0$ causes the lower end to slip on the smooth surface along the positive x-axis, and the rod starts falling. The acceleration vector of centre of mass of the rod during its fall is : [$\vec{R}$ is reaction from surface] **[JEE - 93]**
(A) $\vec{a}_{CM} = \frac{M\vec{g}+\vec{R}}{M}$ (B) $\vec{a}_{CM} = \frac{M\vec{g}-\vec{R}}{M}$
(C) $\vec{a}_{CM} = M\vec{g}-\vec{R}$ (D) None of these

137. A set of n-identical cubical blocks lie at rest parallel to each other along a line on a smooth horizontal surface. The separation between the near surfaces of any two adjacent blocks is L. The block at one end is given a speed V towards the next one at time $t = 0$. All collisions are completely inelastic, then
(A) The last block starts moving at $t = n(n-1)\frac{L}{2V}$
(B) The last block starts moving at $t = (n-1)\frac{L}{V}$
(C) The centre of mass of the system will have a final speed v/n
(D) The centre of mass of the system will have a final speed v

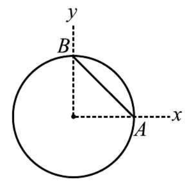
138. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle which has twice its mass during the impact.
(A) The minimum kinetic energy of the system is 1 joule.
(B) The maximum elastic potential energy of the system is 2 joule.
(C) Momentum and total kinetic energy of the system are conserved at every instant.
(D) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

139. A ball of mass m collides elastically with an identical ball at rest with some impact parameter.
(A) 100 % energy transfer can never take place
(B) 100 % energy transfer may take place

(C) angle of divergence between the two balls must be 90°
(D) angle of divergence between the two balls depend on impact parameter

140. Two balls A and B having masses 1 kg and 2 kg, moving with speeds 21 m/s and 4 m/s respectively in opposite direction, collide head on. After collision A moves with a speed of 1 m/s in the same direction, then correct statements is
(A) The velocity of B after collision is 6 m/s opposite to its direction of motion before collision.
(B) The coefficient of restitution is 0.2.
(C) The loss of kinetic energy due to collision is 200 J.
(D) The impulse of the force between the two balls is 40 Ns.

141. An object comprises of a uniform ring of radius R and its uniform chord AB (not necessarily made of the same material) as shown. Which of the following can not be the centre of mass of the object

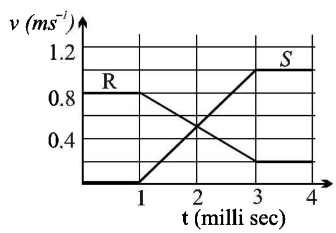


(A) $(R/3, R/3)$ (B) $(R/3, R/2)$
(C) $(R/4, R/4)$ (D) $(R/\sqrt{2}, R/\sqrt{2})$

142. A ball A collides elastically with another identical ball B initially at rest A is moving with velocity of 10m/s at an angle of 60° from the line joining their centres. Select correct alternative
(A) velocity of ball A after collision is 5 m/s
(B) velocity of ball B after collision is $5\sqrt{3}$ m/s
(C) velocity of ball A after collision is 7.5 m/s
(D) velocity of ball B after collision is 5 m/s.

143. Consider following statements
[1] CM of a uniform semicircular disc of radius $R = 2R/\pi$ from the centre
[2] CM of a uniform semicircular ring of radius $R = 4R/3\pi$ from the centre
[3] CM of a solid hemisphere of radius $R = 4R/3\pi$ from the centre
[4] CM of a hemisphere shell of radius $R = R/2$ from the centre
Which statements are correct?
(A) 1, 2, 4 (B) 1, 3, 4 (C) 4 only (D) 1, 2 only

144. The diagram to the right shows the velocity-time graph for two masses R and S that collided elastically. Which of the following statements is true?



(I) R and S moved in the same direction after the collision.
(II) Kinetic energy of the system (R & S) is minimum at t = 2 milli sec.
(III) The mass of R was greater than mass of S.
(A) I only (B) II only
(C) I and II only (D) I, II and III

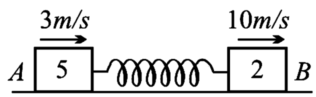
145. In an inelastic collision,
(A) the velocity of both the particles may be same after the collision
(B) kinetic energy is not conserved
(C) linear momentum of the system is conserved.
(D) velocity of separation will be less than velocity of approach.

146. A man of mass 40 kg is standing on a trolley A of mass 140 kg . He pushes another trolley B of same material of mass 60 kg, so that they are set in motion . Then
   (A) Speed of trolley A is 3 times that of trolley B immediately after the interaction.
   (B) Speed of trolley B is 3 times that of trolley A immediately after the interaction.
   (C) Distance travelled by trolley B is 3 times that of trolley A before they stop.
   (D) Distance travelled by trolley B is 9 times that of trolley A before they stop.

147. Two identical balls are interconnected with a massless and inextensible thread. The system is in gravity free space with the thread just taut. Each ball is imparted a velocity v, one towards the other ball and the other perpendicular to the first, at $t = 0$. Then,
   (A) The thread will become taut at $t = (L/v)$
   (B) The thread will become taut at some time $t < (L/v)$.
   (C) The thread will always remain taut for $t > (L/v)$.
   (D) The kinetic energy of the system will always remain mv^2.

148. In a one dimensional collision between two identical particles A and B, B is stationary and A has momentum p before impact. During impact, B gives impulse J to A.
   (A) The total momentum of the 'A plus B' system is p before and after the impact, and (p-J) during the mpact.
   (B) During the impact A gives impulse J to B
   (C) The coefficient of restitution is $\frac{2J}{p} - 1$
   (D) The coefficient of restitution is $\frac{J}{p} + 1$

149. Two blocks A (5kg) and B (2kg) attached to the ends of a spring constant 1120N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10 m/s along the line of the spring in the same direction are imparted to A and B then



   (A) When the extension of the spring is maximum the velocities of A and B are zero.
   (B) The maximum extension of the spring is 25cm.
   (C) Maximum extension and maximum compression occur alternately.
   (D) The maximum compression occur for the first time after $\frac{\pi}{56}$ sec.
   (E) The minimum speed of B is 0.

150. In a one–dimensional collision between two particles, their relative velocity is $\vec{v}_1$ before the collision and $\vec{v}_2$ after the collision
   (A) $\vec{v}_1 = \vec{v}_2$ if the collision is elastic
   (B) $\vec{v}_1 = -\vec{v}_2$ if the collision is elastic
   (C) $|\vec{v}_2| = |\vec{v}_1|$ in all cases
   (D) $\vec{v}_1 = -k\vec{v}_2$ in all cases, where $k \geq 1$

151. In an elastic collision between disks A and B of equal mass but unequal radii, A moves along the x–axis and B is stationary before impact. Which of the following is possible after impact?
   (A) A comes to rest
   (B) The velocity of B relative to A remains the same in magnitude but reverses in direction

(C) A and B move with equal speeds, making an angle of 45° each with the x–axis

(D) A and B move with unequal speeds, making angles of 30° and 60° with the x–axis respectively

152. An isolated rail car originally moving with speed v_0 on a straight, frictionles, level track contains a large amount of sand. A release valve on the bottom of the car malfunctions, and sand begins to pour out straight down relative to the rail car.

(a) Is momentum conserved in this process?

(A) The momentum of the rail car alone is conserved

(B) The momentum of the rail car + sand remaining within the car is conserved

(C) The momentum of the rail car + all of the sand, both inside and outside the rail car, is conserved

(D) None of the three previous systems have momentum conservation

(b) What happens to the speed of the rail car as the sand pours out?

(A) The car begins to roll faster

(B) The car maintains the same speed

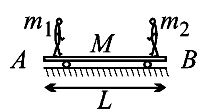
(C) The car begins to slow down

(D) The problem cannot be solved since momentum is not conserved

Question No. 153 to 156 (4 questions)

Two men of mass m_1 and m_2 are standing at the ends A and B of the trolley, respectively. The mass of the trolley is M and its length is L

The two men can exchange their positions in three different ways:



Case I : m_1 moves towards B with u_{rel} and m_2 remains stationary until m_1 reaches its position; and then m_2 starts moving and reaches the end A.

Case II: m_2 moves towards A with u_{rel} and m_1 remains stationary until m_2 reaches its position, and then m_1 starts moving and reaches the end B.

Case III : Both moves with u_{rel} with respect to trolley towards each other and reach then opposite ends.

153. Choose the correct statement(s) related to **Case I**

(A) As the man m_1 moves, the trolley moves toward left and its velocity becomes maximum when it reaches the end B.

(B) When m_1 reaches the end B, the distance moved by the trolley is $\dfrac{m_1 L}{m_1 + m_2 + M}$

(C) When m_1 and m_2 has exchanged their positions, the displacement of the centre of mass of the system is zero.

(D) When the men have exchanged their positions, the final velocity of the trolley is zero

154. Choose the correct statement(s) related to **Case II**

(A) When the man m_2 reaches the position of m_1, the distance moved by the trolley is $\dfrac{m_2 L}{m_1 + m_2 + M}$

(B) When the man m_1 reaches the position of m_2, the distance moved by the trolley is $\dfrac{m_1 L}{m_1 + m_2 + M}$

(C) When the men have exchanged their positions, the distance moved by the center of mass is $\left(\dfrac{m_1 + m_2}{m_1 + m_2 + M}\right)L$

(D) When the men have exchanged their position, the displacement of the centre of mass is $\dfrac{(m_1 - m_2)L}{m_1 + m_2 + M}$

155. Choose the correct statement(s) related to **Case III**

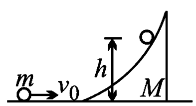
(A) As both the men move simultaneously, the velocity of the trolley at any instant is zero
(B) Both men reach their opposite ends simultaneously
(C) The distance travelled by both the men with respect to ground is same
(D) All the above

156. Choose the correct statement(s) related to all the three cases

(A) The centre mass remains stationary at all instants
(B) The displacement of the trolley cannot exceed L
(C) The displacement of the trolley is independent of the velocity of each man
(D) The displacement of the trolley in all the three cases is same

Question No. 157 to 163 (7 questions)

A particle of mass m moving horizontally with v_0 strikes a smooth wedge of mass M, as shown in figure. After collision, the ball starts moving up the inclined face of the wedge and rises to a height h.



157. The final velocity of the wedge v_2 is

(A) $\dfrac{mv_0}{M}$ (B) $\dfrac{mv_0}{M + m}$ (C) v_0 (D) insufficient data

158. When the particle has risen to a height h on the wedge, then choose the correct alternative(s)

(A) The particle is stationary with respect to ground
(B) Both are stationary with respect to the centre of mass
(C) The kinetic energy of the centre of mass remians constant
(D) The kinetic energy with respect to centre of mass is converted into potential energy

159. The maximum height h attained by the particle is

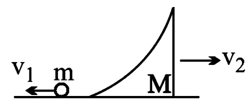
(A) $\left(\dfrac{m}{m + M}\right)\dfrac{v_0^2}{2g}$ (B) $\left(\dfrac{m}{M}\right)\dfrac{v_0^2}{2g}$

(C) $\left(\dfrac{M}{m + M}\right)\dfrac{v_0^2}{2g}$ (D) None of these

160. Identify the correct statement(s) related to the situation when the particle starts moving downward.

(A) The centre of mass of the system remains stationary
(B) Both the particle and the wedge remain stationary with respect to centre of mass
(C) When the particle reaches the horizontal surface it velocity relative to the wedge is v_0
(D) None of these

161. Suppose the particle when reaches the horizontal surfaces, its velocity with respect to ground is v_1 and that of wedge is v_2. Choose the correct statement (s)



(A) $mv_1 = Mv_2$ (B) $Mv_2 - mv_1 = mv_0$
(C) $v_1 + v_2 = v_0$ (D) $v_1 + v_2 < v_0$

162. Choose the correct statement(s) related to particle m

(A) Its kinetic energy is $K_f = \left(\frac{mM}{m+M}\right)gh$

(B) $v_1 = v_0\left(\frac{M-m}{M+m}\right)$

(C) The ratio of its final kinetic energy to its initial kinetic energy is $\frac{K_f}{K_i} = \left(\frac{M}{m+M}\right)^2$

(D) It moves opposite to its initial direction of motion

163. Choose the correct statement related to the wedge M

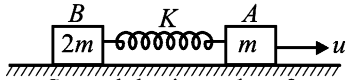
(A) Its kinetic energy is $K_f = \left(\frac{4m^2}{m+M}\right)gh$

(B) $v_2 = \left(\frac{2m}{m+M}\right)v_0$

(C) Its gain in kinetic energy is $\Delta K = \left(\frac{4mM}{(m+M)^2}\right)\left(\frac{1}{2}mv_0^2\right)$

(D) Its velocity is more than the velocity of centre of mass

164. Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At $t = 0$ the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant.



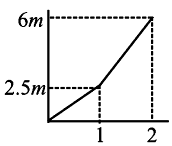
Column-I	Column-II
(A) The velocity of block A	(P) can never be zero
(B) The velocity of block B	(Q) may be zero at certain instants of time
(C) The kinetic energy of system of spring	(R) is minimum at maximum compression of two block
(D) The potential energy of spring	(S) is maximum at maximum extension of spring

Answer Key

Exercise-1 : Calculation of Com

1. Yes, when a body has a uniform mass density, its centre of mass of shall coincide with its geometrical centre.
2. The centre of mass will shift closer to the heavier particle.
3. Yes, always.
4. Yes, it can. For example, centre of mass of a uniform circular ring lies at the centre of ring, where there is no mass.
5. It lies at the centroid of the triangular lamina i.e. where the three medians of the triangle intersect.
6. $\frac{\sqrt{19}}{6}$
7. $\frac{4}{3\pi}\frac{(b^3 - a^3)}{(b^2 - a^2)}$
8. $(5a/6, 5a/6)$
9. $22L/35$
10. At $R/5$ from the centre of the bigger disc towards the centre of the smaller disk.
11. At $R/3$ from the centre of the original disc away from the centre of the hole.
12. $\frac{3}{4}a$
13. (1/7, 23/14)
14. (a) $l(x) = 1 + \frac{\lambda x}{L}$, (b) $\frac{5}{9}L$
15. $\frac{5h}{16}$
16. $\frac{R}{4\pi - 2}$

Motion of Com

17. (a) 1 cm (b) 1 cm downward.
18. zero
19. 40/3 cm
20. $mL/(m+M)$
21. $x = 6$m
22. 4 m/s, 24 J
23. (a) 1.5 m/s (b) –0.50 m/s
24. 100 m
25. $g/9$ downwards
26. L/4
27. 

28. $\frac{2nv\cos(\theta/2)}{n+1}$

29. $g/2$

30. l

31. $\dfrac{Mu^2}{2g(M+2m)}$

Conservation of Momentum

32. 60 m.

33. 35 m.

34. $\left(1+\dfrac{M}{m}\right)v$

35. 9m/s, 9m

36. 10 cm.

37. 2.0×10^5 m/s

38. (a) 12.3 m/s (b) 9.4 m/s

39. $v = \sqrt{\dfrac{2gR}{1+\frac{m}{M}}}$

40. $\dfrac{5v}{R\cos\theta}$

41. (a) $\dfrac{mg(h_3-h_2)}{(h_2-h_1)}$; (b) 0

42. 11/14

43. $\cos^{-1}(3/4)$

44. 1 N

45. $\dfrac{\sqrt{13}}{2}v_0$

46. 4/3 m/s

47. $\sqrt{\dfrac{mv^2}{12k}}$

Impulse

48. $8\hat{i}\ m/s$

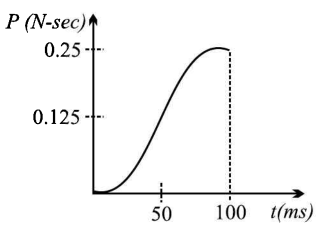
49. 1884 N

50. (a) $4\sqrt{5}$ Ns (b) $2000\sqrt{5}$ N

51. 1.2 m/s, 3.6 Ns

52. 0.2kg, 2.5m/s, 0.5Ns

53.



54. $m \times \sqrt{u^2 - uv + v^2}$

55. $\dfrac{[mu\sqrt{3}]}{4}$

56. 6.21 W

57. $\sqrt{\frac{3gx}{8}}$

58. (a) $u/2$, $mu/2$ (b) $u\sqrt{13}/8$, $mu\sqrt{13}/8$ (c) $u\sqrt{3}/4$, $mu\sqrt{3}/4$

Collision

59. 2 m/s negative axis and 3m/s positive axis respectively

60. $K/2$.

61. $e = \frac{1}{\sqrt{2}}$

62. $t = \frac{2\pi r}{v}$

63. $\sqrt{3}/2$

64. 280 m/s

65. (a) 3 J, (b) $\frac{12}{5}$ $N{\cdot}s$

66. (a) $\frac{40}{3}$ m, (b) $t_e = 3.25$ s

67. $v_A = \sqrt{g/12}$ m/s, $S_{max} = 49/48$ m

68. (a) $v_0/3$, (b) $3\sqrt{5gR}$.

69. $\frac{1}{2}$

70. $v_{\text{heavy ball}} = \frac{u_0}{27}$, $v_{\text{first ball}} = \frac{4u_0}{27}$, $v_{\text{second ball}} = \frac{4u_0}{9}$, $v_{\text{third ball}} = \frac{4u_0}{3}$

71. $\sqrt{2g}$ rad/s

72. (1/81) m

Exercise-2

1. 22 m/s, 28 m left to the line of fall.

2. $\frac{mv}{M+m}$

3. $\frac{M^2 h}{(M+m)^2}$

4. $\frac{m_A}{m_A + m_C}\sqrt{2gh}$, $\frac{h}{\mu\left(1+\frac{m_A}{m_C}\right)}$

5. $v = \sqrt{\frac{8g\ell}{3}}$, $T = 14$ mg

6. $\frac{2m_2\sqrt{2gr}}{m_1 + m_2}$

7. $x_{max} = \frac{4F + \sqrt{16F^2 + 54mu^2k}}{3k}$

8. (a) Light ball $\frac{\sqrt{50g\ell}}{3}$ towards left, heavy ball $\frac{\sqrt{2g\ell}}{3}$ towards right

(b) Light ball 2ℓ and heavy ball $\frac{\ell}{9}$

9. The small particle goes along the tangent with a speed of $v\rho/r$ and the spherical body goes perpendicular to the smaller particle with a speed of $\frac{v}{r}\sqrt{r^2 - \rho^2}$.

10. (a) 360 m, (b) 10800 J

11. $2mv^2/3l$

12. $v_B = \frac{m_A}{m_B}\sqrt{\frac{2gl}{1+m_A/m_B}}$; $T = 3m_A g + \frac{2m_A^2 g}{m_B}$

13. 13/3m/s

14. $e = (5 + \sqrt{3})/8$, $M = 26/\sqrt{3}$ kg

15. $2\,mnv^2 \cos^2\theta$

16. –2m/s, 6.93m/s ∠30°

17. $v = \frac{2\sqrt{3}u}{7}$, $u' = \frac{5u}{7}$, $T = \frac{6}{7}mv$

18. (a) $u/2$, $mu/2$; (b) $u\sqrt{13}/8$, $mu\sqrt{13}/8$; (c) $u\sqrt{3}/4$, $mu\sqrt{3}/4$

19. (a) $v/2$, $v/2$, 0; (b) $2mv^2/9$; (c) $mv^2/72$; (d) $x = \sqrt{m/6k}\,v$

20. $m\left(-3\hat{i} + 4\hat{j}\right)$, $e = \frac{9}{16}$

21. l

21. $\frac{m}{\ell}v(v + gt)$

23. $x = 3$ units, $\tan\theta = 2/3$

24. $5/\sqrt{17}$ cm, $153L/80u$

25. $v_c = -\frac{v_0}{15}$, $v_B = \frac{\sqrt{208}\,v_0}{15}$, $v_A = \frac{4v_0}{15}$

26. 40 cm

27. 37^0

28. (a) 0.66, (b) 4 m

29. $v_1 = \frac{1}{\sqrt{3}}$ m/s, $v_2 = \frac{2}{\sqrt{3}}$ m/s

30. (a) $\frac{m}{l}(gy + v_0^2)$, (b) $mg\left(1-\frac{y}{l}\right)$, (c) $\frac{mv_0^{\,2}y}{2l}$

31. $2\sqrt{2a_0 1}$

32. (a) 50 m, (b) 10 m, (c) 30 m

33. 13/3m/s

31. (a) $v_0/3$, (b) $mv_0 = 2mv_2\cos\theta$, $0 = 2\,mv_2\sin\theta - \frac{mv_0}{2}$ (c) $\frac{1}{2}, \frac{\sqrt{5}}{4}v_0$ (d) $\frac{mv_0^2}{16}$

Exercise-3

1. *B*
2. (a) *B* (b) *A*
3. *A*
4. *A*
5. *C*
6. *BD*
7. *B*
8. *C*
9. C

10. $m\left(-v_2\sin\left(\frac{v_2}{R}t\right)\hat{i} + v_2\cos\left(\frac{v_2}{R}t\right)\hat{j} - v_1\hat{j}\right)$

11. $(5+\sqrt{3})/8)$, $M = 26/\sqrt{3}$ kg

12. 2 in case I
13. $t_0 = 12$ sec, $v = 100\sqrt{3}/11$
14. *C*
15. *A, D*
16. *B*
17. B
18. *C*
19. 4m/s
20. *C*
21. *A*
22. *A*
23. *A, C*
24. *D*
25.
26. *A*
27.
28. *C*
29. *A*
30. *C*
31. *A*
32. *B*

33. *A*	**34.** *C*	**35.** *C*
36. *D*	**37.** *A*	**38.** *D*
39. *B*	**40.** *D*	**41.** *A*
42. *B*	**43.** *D*	**44.** *C*
45. *B*		

Exercise-4

1. *D*	**2.** *A*	**3.** *A*
4. *D*	**5.** *D*	**6.** **None**
7. *CD*	**8.** *AB*	**9.** *C*
10. *D*	**11.** *D*	**12.** *B*
13. *B*	**14.** *C*	**15.** *C*
16. *B*	**17.** *B*	**18.** *C*
19. *C*	**20.** *A D*	**21.** *A*
22. *C*	**23.** *C*	**24.** *D*
25. *BC*	**26.** C D	**27.** A C
28. A	**29.** B	**30.** C
31. C	**32.** B	**33.** B
34. B	**35.** B	**36.** D
37. B	**38.** B	**39.** B
40. D	**41.** C	**42.** C
43. C	**44.** D	**45.** A
46. B	**47.** B	**48.** B
49. A	**50.** C	**51.** D
52. D	**53.** C	**54.** A
55. B	**56.** C	**57.** C
58. B	**59.** C	**60.** A
61. C	**62.** C	**63.** D
64. A	**65.** B	**66.** D
67. B	**68.** C	**69.** B
70. D	**71.** D	**72.** B
73. C	**74.** B	**75.** C
76. D	**77.** D	**78.** C
79. A	**80.** A	**81.** A
82. C	**83.** A	**84.** C
85. B	**86.** A	**87.** D
88. A	**89.** B	**90.** B

91. B	**92.** C	**93.** A
94. (a) B, (b) C	**95.** (a) C, (b) B	**96.** (a) B, (b) C, (c) B, (d) D
97. B	**98.** A	**99.** C
100. D	**101.** A	**102.** B
103. D	**104.** D	**105.** D
106. A	**107.** A	**108.** D
109. B	**110.** C	**111.** A
112. B	**113.** C	**114.** A
115. A	**116.** A	**117.** C
118. B		

One or More than One Option Correct

119. D	**120.** D	**121.** D
122. D	**123.** B	**124.** C
125. CD	**126.** D	**127.** A
128. A	**129.** A	**130.** C
131. BCD	**132.** BC	**133.** ACD
134. B	**135.** BD	**136.** B
137. AC	**138.** A,B,D	**139.** A,C
140. A,B,C	**141.** B,D	**142.** D
143. C	**144.** D	**145.** A,B,C,D
146. B,D	**147.** A,C	**148.** B,C
149. B,C,E	**150.** B,D	**151.** A,B,C,D
152. (a) A,C; (b) B	**153.** B,C,D	**154.** A
155. B	**156.** A,B,C,D	**157.** B
158. B,D	**159.** C	**160.** C
161. B,C	**162.** B	**163.** A,B,C,D

164. (A)-Q, (B)-Q, (C)-P,R , (D)-Q,S