

CAMBRIDGE

ENDORSED BY



CAMBRIDGE
International Examinations

Karen Morrison and Lucille Dunne

Cambridge IGCSE®

Mathematics

Extended Practice Book

Completely **Cambridge**
Cambridge resources
for
Cambridge qualifications

Karen Morrison and Lucille Dunne

Cambridge IGCSE®

Mathematics

Extended Practice Book

CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

www.cambridge.org

Information on this title: www.cambridge.org/9781107672727

© Cambridge University Press 2013

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 2013

Printed and bound in the United Kingdom by the MPG Books Group

A catalogue record for this publication is available from the British Library

ISBN-13 978-1-107-67272-7 Paperback

Cover image: Seamus Ditmeyer/Alamy

Cambridge University Press has no responsibility for the persistence or
accuracy of URLs for external or third-party internet websites referred to in
this publication, and does not guarantee that any content on such websites is,
or will remain, accurate or appropriate. Information regarding prices, travel
timetables and other factual information given in this work are correct at
the time of first printing but Cambridge University Press does not guarantee
the accuracy of such information thereafter.

IGCSE® is the registered trademark of Cambridge International Examinations.

Contents

Introduction

v

Unit 1

Chapter 1: Reviewing number concepts

- 1.1 Different types of numbers
- 1.2 Multiples and factors
- 1.3 Prime numbers
- 1.4 Powers and roots
- 1.5 Working with directed numbers
- 1.6 Order of operations
- 1.7 Rounding numbers

Chapter 2: Making sense of algebra

- 2.1 Using letters to represent unknown values
- 2.2 Substitution
- 2.3 Simplifying expressions
- 2.4 Working with brackets
- 2.5 Indices

Chapter 3: Lines, angles and shapes

- 3.1 Lines and angles
- 3.2 Triangles
- 3.3 Quadrilaterals
- 3.4 Polygons
- 3.5 Circles
- 3.6 Construction

Chapter 4: Collecting, organising and displaying data

- 4.1 Collecting and classifying data
- 4.2 Organising data
- 4.3 Using charts to display data

Unit 2

Chapter 5: Fractions

- 5.1 Equivalent fractions
- 5.2 Operations on fractions
- 5.3 Percentages
- 5.4 Standard form
- 5.5 Estimation

Chapter 6: Equations and transforming formulae

- 6.1 Further expansions of brackets
- 6.2 Solving linear equations
- 6.3 Factorising algebraic expressions
- 6.4 Transformation of a formula

Chapter 7: Perimeter, area and volume

- 7.1 Perimeter and area in two dimensions
- 7.2 Three-dimensional objects
- 7.3 Surface areas and volumes of solids

Chapter 8: Introduction to probability

- 8.1 Basic probability
- 8.2 Theoretical probability
- 8.3 The probability that an event does not happen
- 8.4 Possibility diagrams
- 8.5 Combining independent and mutually exclusive events

Unit 3

Chapter 9: Sequences and sets

- 9.1 Sequences
- 9.2 Rational and irrational numbers
- 9.3 Sets

Chapter 10: Straight lines and quadratic equations

- 10.1 Straight lines
- 10.2 Quadratic expressions

Chapter 11: Pythagoras' theorem and similar shapes

- 11.1 Pythagoras' theorem
- 11.2 Understanding similar triangles
- 11.3 Understanding similar shapes
- 11.4 Understanding congruence

Chapter 12: Averages and measures of spread

- 12.1 Different types of average
- 12.2 Making comparisons using averages and ranges
- 12.3 Calculating averages and ranges for frequency data
- 12.4 Calculating averages and ranges for grouped continuous data
- 12.5 Percentiles and quartiles

Unit 4

Chapter 13: Understanding measurement	81	Chapter 15: Scale drawings, bearings and trigonometry	99
13.1 Understanding units	81	15.1 Scale drawings	99
13.2 Time	82	15.2 Bearings	100
13.3 Upper and lower bounds	84	15.3 Understanding the tangent, cosine and sine ratios	101
13.4 Conversion graphs	85	15.4 Solving problems using trigonometry	105
13.5 More money	86	15.5 Angles between 0° and 180°	105
Chapter 14: Further solving of equations and inequalities	89	15.6 The sine and cosine rules	105
14.1 Simultaneous linear equations	89	15.7 Area of a triangle	108
14.2 Linear inequalities	91	15.8 Trigonometry in three dimensions	108
14.3 Regions in a plane	92	Chapter 16: Scatter diagrams and correlation	111
14.4 Linear programming	93	16.1 Introduction to bivariate data	111
14.5 Completing the square	94		
14.6 Quadratic formula	95		
14.7 Factorising quadratics where the coefficient of x^2 is not 1	96		
14.8 Algebraic fractions	96		

Unit 5

Chapter 17: Managing money	114	Chapter 19: Symmetry and loci	126
17.1 Earning money	114	19.1 Symmetry in two dimensions	126
17.2 Borrowing and investing money	115	19.2 Symmetry in three dimensions	127
17.3 Buying and selling	116	19.3 Symmetry properties of circles	128
Chapter 18: Curved graphs	119	19.4 Angle relationships in circles	129
18.1 Plotting quadratic graphs (the parabola)	119	19.5 Locus	131
18.2 Plotting reciprocal graphs (the hyperbola)	121	Chapter 20: Histograms and frequency distribution diagrams	135
18.3 Using graphs to solve quadratic equations	122	20.1 Histograms	135
18.4 Using graphs to solve simultaneous linear and non-linear equations	122	20.2 Cumulative frequency	137
18.5 Other non-linear graphs	123		
18.6 Finding the gradient of a curve	124		

Unit 6

Chapter 21: Ratio, rate and proportion	139	Chapter 23: Transformations and matrices	153
21.1 Working with ratio	139	23.1 Simple plane transformations	153
21.2 Ratio and scale	140	23.2 Vectors	158
21.3 Rates	141	23.3 Further transformations	161
21.4 Kinematic graphs	141	23.4 Matrices and matrix transformation	163
21.5 Proportion	144	23.5 Matrices and transformations	164
21.6 Direct and inverse proportion in algebraic terms	145	Chapter 24: Probability using tree diagrams	169
21.7 Increasing and decreasing amounts by a given ratio	146	24.1 Using tree diagrams to show outcomes	169
Chapter 22: More equations, formulae and functions	149	24.2 Calculating probability from tree diagrams	169
22.1 Setting up equations to solve problems	149		
22.2 Using and transforming formulae	150		
22.3 Functions and function notation	151		

Answers	171
----------------	------------

Example practice papers can be found online, visit education.cambridge.org/extendedpracticebook

Introduction

This highly illustrated practice book has been written by experienced teachers to help students revise the *Cambridge IGCSE Mathematics* (0580) Extended syllabus. Packed full of exercises, the only narrative consists of helpful bulleted lists of key reminders and useful hints in the margins for students needing more support.

There is plenty of practice offered via ‘drill’ exercises throughout each chapter. These consist of progressive and repetitive questions that allow the student to practise methods applicable to each subtopic. At the end of each chapter there are ‘Mixed exercises’ that bring together all the subtopics of a chapter in such a way that students have to decide for themselves what methods to use. The answers to *all* of these questions are supplied at the back of the book. This encourages students to assess their progress as they go along, choosing to do more or less practice as required.

The book has been written with a clear progression from start to finish, with some later chapters requiring knowledge learned in earlier chapters. There are useful signposts throughout that link the content of the chapters, allowing the individual to follow their own course through the book: where the content in one chapter might require knowledge from a previous chapter, a comment is included in a ‘Rewind’ box; and where content will be practised in more detail later on, a comment is included in a ‘Fast forward’ box. Examples of both are included below:

REWIND
You learned how to plot lines from equations in chapter 10. ◀

FAST FORWARD
You will learn much more about sets in chapter 9. For now, just think of a set as a list of numbers or other items that are often placed inside curly brackets. ▶

Remember ‘coefficient’ is the *number* in the term.

Tip

It is essential that you remember to work out *both* unknowns. Every pair of simultaneous linear equations will have a pair of solutions.

Other helpful guides in the margin of the book are as follows:

Hints: these are general comments to remind students of important or key information that is useful when tackling an exercise, or simply useful to know. They often provide extra information or support in potentially tricky topics.

Tip: these are tips that relate to good practice in examinations, and also just generally in mathematics! They cover common pitfalls based on the **authors’** experiences of their students, and give students things to be wary of or to remember in order to score marks in the exam.

The *Extended Practice Book* mirrors the chapters and subtopics of the *Cambridge IGCSE Mathematics Core and Extended Coursebook* written by Karen Morrison and Nick Hamshaw (9781107606272). **However**, this book has been written such that it can be used **without the coursebook**; it can be used as a revision tool by any student regardless of what coursebook they are using. Various aspects of the Core syllabus are also revised for complete coverage.

Also in the *Cambridge IGCSE Mathematics* series:

Cambridge IGCSE Mathematics Core and Extended Coursebook (9781107606272)

Cambridge IGCSE Mathematics Core Practice Book (9781107609884)

Cambridge IGCSE Mathematics Teacher’s Resource CD-ROM (9781107627529)

1

Reviewing number concepts

1.1 Different types of numbers

- Real numbers can be divided into rational and irrational numbers. You will deal with rational numbers in this chapter. Irrational numbers are covered in chapter 9.
- Rational numbers can be written as fractions in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$.
(Integers are negative and positive whole numbers, and zero.)
- Integers, fractions and terminating decimals are all rational numbers.

Tip

Make sure you know what the following sets of numbers are: natural numbers, integers, odd and even numbers and prime numbers.

Exercise 1.1

- 1 Tick the correct columns in the table to classify each number.

Number	Natural	Integer	Prime	Fraction
-0.2				
-57				
3.142				
0				
$0.\dot{3}$				
1				
51				
10 270				
$-\frac{1}{4}$				
$\frac{2}{7}$				
11				
$\sqrt[3]{512}$				

- 2 List:

- four square numbers greater than 100.
- four rational numbers smaller than $\frac{1}{3}$.
- two prime numbers that are > 80 .
- the prime numbers < 10 .

1.2 Multiples and factors

- A multiple of a number is the product obtained when multiplying that number and an integer. The lowest common multiple (LCM) of two or more numbers is the lowest number that is a multiple of both (or all) of the numbers.
- A factor of a number is any number that will divide into the number exactly.
- The highest common factor (HCF) of two or more numbers is the highest number that is a factor of all the given numbers.

To find the LCM of a set of numbers, you can list the multiples of each number until you find the first multiple that is in the lists for all of the numbers in the set.

FAST FORWARD

You will use LCM again when you work with fractions to find the lowest common denominator of two or more fractions. See chapter 5. ►

You need to work out whether to use LCM or HCF to find the answers. Problems involving LCM usually include repeating events. Problems involving HCF usually involve splitting things into smaller pieces or arranging things in equal groups or rows.

Exercise 1.2 A

1 Find the LCM of the given numbers.

- | | | | |
|--------------|-----------------|-----------------|-------------------|
| (a) 9 and 18 | (b) 12 and 18 | (c) 15 and 18 | (d) 24 and 12 |
| (e) 36 and 9 | (f) 4, 12 and 8 | (g) 3, 9 and 24 | (h) 12, 16 and 32 |

2 Find the HCF of the given numbers.

- | | | | |
|---------------|---------------|---------------|-----------------|
| (a) 12 and 18 | (b) 18 and 36 | (c) 27 and 90 | (d) 12 and 15 |
| (e) 20 and 30 | (f) 19 and 45 | (g) 60 and 72 | (h) 250 and 900 |

Exercise 1.2 B

- Amira has two rolls of cotton fabric. One roll has 72 metres on it and the other has 90 metres on it. She wants to cut the fabric to make as many equal length pieces as possible of the longest possible length. How long should each piece be?
- In a shopping mall promotion every 30th shopper gets a \$10 voucher and every 120th shopper gets a free meal. How many shoppers must enter the mall before one receives a voucher and a free meal?
- Amanda has 40 pieces of fruit and 100 sweets to share amongst the students in her class. She is able to give each student an equal number of pieces of fruit and an equal number of sweets. What is the largest possible number of students in her class?
- Francesca, Ayuba and Claire are Olympic and Paralympic contenders. They share a training slot on a running track. Francesca cycles and completes a lap in 20 seconds, Ayuba runs the lap in 84 seconds and Claire, in her wheelchair, takes 105 seconds. They start training together. After how long will all three be at the same point again and how many laps will each have completed?
- Mr Smit wants to tile a rectangular veranda with dimensions $3.2 \text{ m} \times 6.4 \text{ m}$ with a whole number of identical square tiles. Mrs Smit wants the tiles to be as large as possible.
 - Find the area of the largest possible tiles in cm^2 .
 - How many $3.2 \text{ m} \times 3.2 \text{ m}$ tiles will Mr Smit need to tile the veranda?

1.3 Prime numbers

- Prime numbers only have two factors: 1 and the number itself.
- Prime factors are factors of a number that are also prime numbers.
- You can write any number as a product of prime factors. But remember the number 1 itself is *not* a prime number so you cannot use it to write a number as the product of its prime factors.
- You can use the product of prime factors to find the HCF or LCM of two or more numbers.

You can use a tree diagram or division to find the prime factors of a composite whole number.

Exercise 1.3

1 Identify the prime numbers in each set.

- (a) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
 (b) 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60
 (c) 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105

2 Express the following numbers as a product of their prime factors.

- (a) 36 (b) 65 (c) 64 (d) 84
 (e) 80 (f) 1000 (g) 1270 (h) 1963

3 Find the LCM and the HCF of the following numbers by means of prime factors.

- (a) 27 and 14 (b) 85 and 15 (c) 96 and 27 (d) 53 and 16
 (e) 674 and 72 (f) 234 and 66 (g) 550 and 128 (h) 315 and 275

1.4 Powers and roots

- A number is squared (n^2) when it is multiplied by itself ($n \times n$).
- The square root ($\sqrt{n}$) of a number is the number that is multiplied by itself to get the number.
- A number is cubed (n^3) when it is multiplied by itself and then multiplied by itself again ($n \times n \times n$).
- The cube root ($\sqrt[3]{n}$) of a number is the number that is multiplied by itself twice to get the number.

FAST FORWARD

Powers greater than 3 are dealt with in chapter 2. See topic 2.5 indices. ►

Exercise 1.4

1 Find all the square and cube numbers between 100 and 300.

2 Simplify.

- (a) $\sqrt{9} + \sqrt{16}$ (b) $\sqrt{9+16}$ (c) $\sqrt{64} + \sqrt{36}$ (d) $\sqrt{64+36}$
 (e) $\sqrt{\frac{36}{4}}$ (f) $(\sqrt{25})^2$ (g) $\frac{\sqrt{9}}{\sqrt{16}}$ (h) $\sqrt{169-144}$
 (i) $\sqrt[3]{27} - \sqrt[3]{1}$ (j) $\sqrt{100 \div 4}$ (k) $\sqrt{1} + \sqrt{\frac{9}{16}}$ (l) $\sqrt{16} \times \sqrt[3]{27}$
 (m) $\sqrt{(-5)^2} \times \sqrt{-1}$ (n) $\sqrt{\frac{1}{4}} + \sqrt{\left(\frac{1}{3}\right)^2}$ (o) $\sqrt[3]{1} - \sqrt[3]{-125}$

3 A cube has a volume of 12 167 cm³. Calculate:

- (a) the height of the cube.
 (b) the area of one face of the cube.

1.5 Working with directed numbers

- Integers are directed whole numbers.
- Negative integers are written with a minus (–) sign. Positive integers may be written with a plus (+) sign, but usually they are not.
- In real life, negative numbers are used to represent temperatures below zero; movements downwards or left; depths; distances below sea level; bank withdrawals and overdrawn amounts, and many more things.

Exercise 1.5

- 1 If the temperature is 4°C in the evening and it drops 7°C overnight, what will the temperature be in the morning?
- 2 Which is colder in each pair of temperatures?
(a) 0°C or -2°C (b) 9°C or -9°C (c) -4°C or -12°C
- 3 An office block has three basement levels (level –1, –2 and –3), a ground floor and 15 floors above the ground floor (1 to 15). Where will the lift be in the following situations?
(a) Starts on ground and goes down one floor then up five?
(b) Starts on level –3 and goes up 10 floors?
(c) Starts on floor 12 and goes down 13 floors?
(d) Starts on floor 15 and goes down 17 floors?
(e) Starts on level –2, goes up seven floors and then down eight?

Draw a number line to help you.

1.6 Order of operations

- When there is more than one operation to be done in a calculation you must work out the parts in brackets first. Then do any division or multiplication (from left to right) before adding and subtracting (from left to right).
- The word ‘of’ means $\times$ and a fraction line means divide.
- Long fraction lines and square or cube root signs act like brackets, indicating parts of the calculation that have to be done first.

Remember the order of operations using BODMAS:

Brackets
Of
Divide
Multiply
Add
Subtract

Exercise 1.6

Tip

Most modern scientific calculators apply the rules for order of operations automatically. If there are brackets, fractions or roots in your calculation you need to enter these correctly on the calculator. When there is more than one term in the denominator, the calculator will divide by the first term only unless you enter brackets.

FAST FORWARD

The next section will remind you of the rules for rounding numbers. ►

- 1 Calculate and give your answer correct to two decimal places.

- | | | |
|-------------------------------|-----------------------------|-----------------------------|
| (a) $8 + 3 \times 6$ | (b) $(8 + 3) \times 6$ | (c) $8 \times 3 - 4 \div 5$ |
| (d) $12.64 + 2.32 \times 1.3$ | (e) $6.5 \times 1.3 - 5.06$ | (f) $(6.7 \div 8) + 1.6$ |

(g) $1.453 + \frac{7.6}{3.2}$

(h) $\frac{5.34 + 3.315}{4.03}$

(i) $\frac{6.54}{2.3} - 1.08$

(j) $\frac{5.27}{1.4 \times 1.35}$

(k) $\frac{11.5}{2.9 - 1.43}$

(l) $\frac{0.23 \times 4.26}{1.32 + 3.43}$

(m) $8.9 - \frac{8.9}{10.4}$

(n) $\frac{12.6}{8.3} - \frac{1.98}{4.62}$

(o) $12.9 - 2.03^2$

(p) $(9.4 - 2.67)^3$

(q) $12.02^2 - 7.05^2$

(r) $\left(\frac{16.8}{9.3} - 1.01\right)^2$

(s) $\frac{4.07^2}{8.2 - 4.09}$

(t) $6.8 + \frac{1.4}{6.9} - \frac{1.2}{9.3}$

(u) $4.3 + \left(1.2 + \frac{1.6}{5}\right)^2$

(v) $\frac{6.1}{2.8} + \left(\frac{2.1}{1.6}\right)^2$

(w) $6.4 - (1.2^2 + 1.9^2)^2$

(x) $\left(4.8 - \frac{1}{9.6}\right) \times 4.3$

1.7 Rounding numbers

- You may be asked to round numbers to a given number of decimal places or to a given number of significant figures.
- To round to a decimal place:
 - look at the value of the digit to the right of the place you are rounding to
 - if this value is ≥ 5 then you round up (add 1 to the digit you are rounding to)
 - if this value is ≤ 4 then leave the digit you are rounding to as it is.
- To round to a significant figure:
 - the first non-zero digit (before or after the decimal place in a number) is the first significant figure
 - find the correct digit and then round off from that digit using the rules above.

Exercise 1.7

FAST FORWARD

Rounding is very useful when you have to estimate an answer. You will deal with this in more detail in chapter 5. ►

1 Round these numbers to:

- (i) two decimal places
- (ii) one decimal place
- (iii) the nearest whole number.

- | | | |
|------------|------------|-------------|
| (a) 5.6543 | (b) 9.8774 | (c) 12.8706 |
| (d) 0.0098 | (e) 10.099 | (f) 45.439 |
| (g) 13.999 | (h) 26.001 | |

2 Round each of these numbers to three significant figures.

- | | | | |
|------------|-------------|------------|--------------|
| (a) 53 217 | (b) 712 984 | (c) 17.364 | (d) 0.007279 |
|------------|-------------|------------|--------------|

3 Round the following numbers to two significant figures.

- | | | | |
|-------------|------------|-------------|-------------|
| (a) 35.8 | (b) 5.234 | (c) 12 345 | (d) 0.00875 |
| (e) 432 128 | (f) 120.09 | (g) 0.00456 | (h) 10.002 |

Mixed exercise

- 1 State whether each number is natural, rational, an integer and/or a prime number.
 $-\frac{3}{4}$ 24 0.65 -12 $3\frac{1}{2}$ 0 0.66 17
- 2 List the factors of 36.
 - (a) How many of these factors are prime numbers?
 - (b) Express 36 as the product of its prime factors.
 - (c) List two numbers that are factors of both 36 and 72.
 - (e) What is the highest number that is a factor of both 36 and 72?
- 3 Write each number as a product of its prime factors.
 - (a) 196 (b) 1845 (c) 8820
- 4 Amira starts an exercise programme on the 3rd of March. She decides she will swim every 3 days and cycle every 4 days. On which dates in March will she swim and cycle on the same day?
- 5 State whether each equation is true or false.
 - (a) $18 \div 6 + (5 + 3 \times 4) = 20$
 - (b) $6 \times (5 - 4) + 3 = 9$
 - (c) $\frac{30+10}{30} - 10 = 1$
 - (d) $(6 + 3)^2 = 45$
- 6 Simplify:
 - (a) $\sqrt{100} \div \sqrt{4}$ (b) $\sqrt{100 \div 4}$ (c) $(\sqrt[3]{64})^3$ (d) $4^3 + 9^2$
- 7 Calculate. Give your answer correct to two decimal places.
 - (a) $\frac{5.4 \times 12.2}{4.1}$ (b) $\frac{12.2^2}{3.9^2}$ (c) $\frac{12.65}{2.04} + 1.7 \times 4.3$
 - (d) $\frac{3.8 \times 12.6}{4.35}$ (e) $\frac{2.8 \times 4.2^2}{3.3^2 \times 6.2^2}$ (f) $2.5 - \left(3.1 + \frac{0.5}{5}\right)^2$
- 8 Round each number to three significant figures.
 - (a) 1235.6 (b) 0.76513 (c) 0.0237548 (d) 31.4596
- 9 A building supply store is selling tiles with an area of 790 cm^2 .
 - (a) Is it possible to have square tiles whose area is not a square number? Explain.
 - (b) Find the length of each side of the tile correct to 3 significant figures.
 - (c) What is the minimum number of tiles you would need to tile a rectangular floor 3.6 m long and 2.4 m wide?